

Computer algebra independent integration tests

7-Inverse-hyperbolic-functions/7.5-Inverse-hyperbolic-secant/7.5.1-u-a+b-arcsech-c-x-^n

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3.148	$\int \frac{(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{x^6} dx$	626
3.149	$\int \frac{(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{x^8} dx$	631

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3.160	$\int \frac{x^3(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^{3/2}} dx$	673
3.161	$\int \frac{x(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^{3/2}} dx$	678
3.162	$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x(d+ex^2)^{3/2}} dx$	682
3.163	$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^3(d+ex^2)^{3/2}} dx$	684
3.164	$\int \frac{x^4(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^{3/2}} dx$	686
3.165	$\int \frac{x^2(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^{3/2}} dx$	688
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3.167	$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^2(d+ex^2)^{3/2}} dx$	693
3.168	$\int \frac{x^5(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^{5/2}} dx$	697
3.169	$\int \frac{x^3(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^{5/2}} dx$	702
3.170	$\int \frac{x(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^{5/2}} dx$	707
3.171	$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x(d+ex^2)^{5/2}} dx$	711
3.172	$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^3(d+ex^2)^{5/2}} dx$	713
3.173	$\int \frac{x^6(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^{5/2}} dx$	715
3.174	$\int \frac{x^4(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^{5/2}} dx$	717
3.175	$\int \frac{x^2(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^{5/2}} dx$	719
3.176	$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{(d+ex^2)^{5/2}} dx$	723

3.177	$\int (fx)^m (d + ex^2)^3 (a + b\operatorname{sech}^{-1}(cx)) dx$	727
3.178	$\int (fx)^m (d + ex^2)^2 (a + b\operatorname{sech}^{-1}(cx)) dx$	731
3.179	$\int (fx)^m (d + ex^2) (a + b\operatorname{sech}^{-1}(cx)) dx$	735
3.180	$\int \frac{(fx)^m (a + b\operatorname{sech}^{-1}(cx))}{d + ex^2} dx$	738
3.181	$\int \frac{(fx)^m (a + b\operatorname{sech}^{-1}(cx))}{(d + ex^2)^2} dx$	740
3.182	$\int (fx)^m (d + ex^2)^{3/2} (a + b\operatorname{sech}^{-1}(cx)) dx$	742
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3.186	$\int \frac{x^{11} (a + b\operatorname{sech}^{-1}(cx))}{\sqrt{1 - c^4 x^4}} dx$	750
3.187	$\int \frac{x^7 (a + b\operatorname{sech}^{-1}(cx))}{\sqrt{1 - c^4 x^4}} dx$	755
3.188	$\int \frac{x^3 (a + b\operatorname{sech}^{-1}(cx))}{\sqrt{1 - c^4 x^4}} dx$	760
3.189	$\int \frac{a + b\operatorname{sech}^{-1}(cx)}{x\sqrt{1 - c^4 x^4}} dx$	764
3.190	$\int \frac{a + b\operatorname{sech}^{-1}(cx)}{x^5\sqrt{1 - c^4 x^4}} dx$	766

4 Listing of Grading functions

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Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [190]. This is test number [200].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (190)	% 0. (0)
Mathematica	% 97.37 (185)	% 2.63 (5)
Maple	% 81.05 (154)	% 18.95 (36)
Maxima	% 31.58 (60)	% 68.42 (130)
Fricas	% 63.16 (120)	% 36.84 (70)
Sympy	% 20. (38)	% 80. (152)
Giac	% 23.68 (45)	% 76.32 (145)

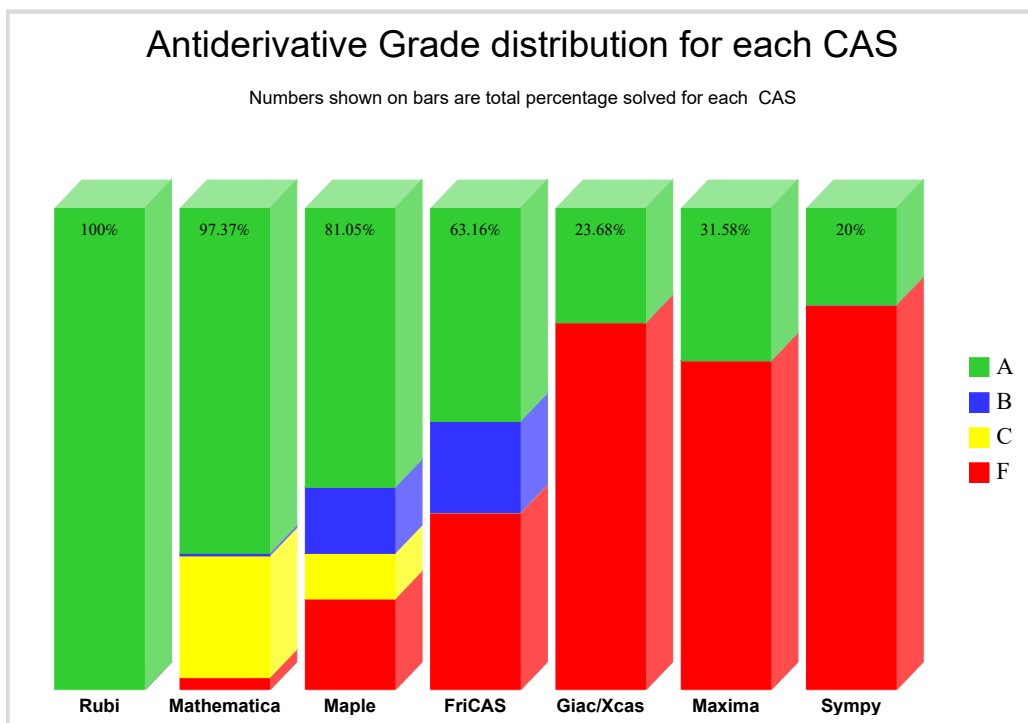
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

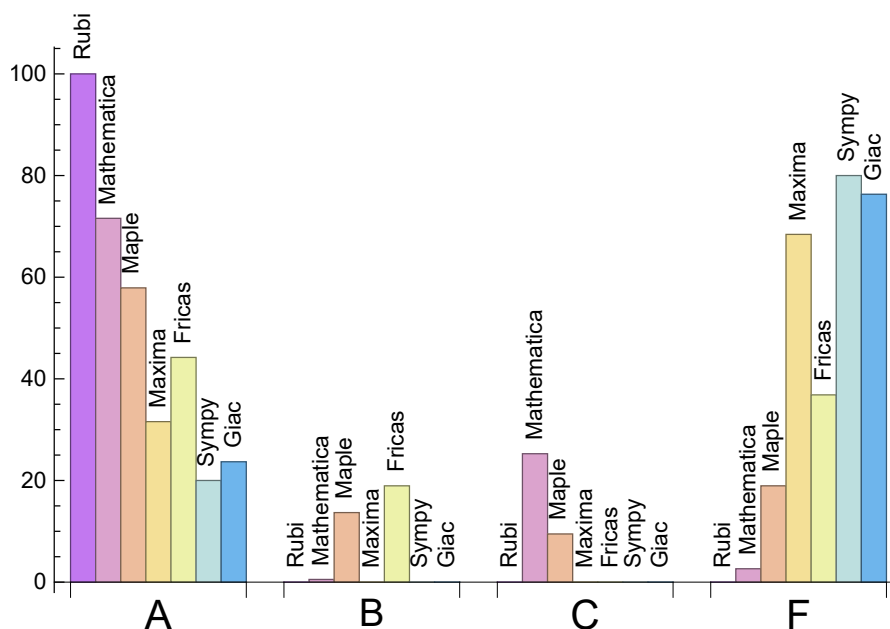
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	71.58	0.53	25.26	2.63
Maple	57.89	13.68	9.47	18.95
Maxima	31.58	0.	0.	68.42
Fricas	44.21	18.95	0.	36.84
Sympy	20.	0.	0.	80.
Giac	23.68	0.	0.	76.32

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.39	195.62	0.76	138.5	1.
Mathematica	3.23	359.14	1.3	135.	0.95
Maple	0.84	412.95	1.47	148.	1.06
Maxima	0.78	114.67	0.8	97.	0.99
Fricas	1.81	638.77	3.28	262.	1.93
Sympy	5.09	38.53	0.27	0.	0.
Giac	0.	0.	0.	0.	0.

1.4 list of integrals that has no closed form antiderivative

{51, 52, 53, 57, 58, 59, 63, 64, 65, 69, 70, 72, 73, 87, 133, 134, 135, 136, 137, 142, 143, 144, 145, 146, 147, 153, 154, 155, 156, 162, 163, 164, 165, 171, 172, 173, 174, 180, 181, 182, 183, 184, 185, 189, 190}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {26, 111, 115, 116, 123}

Mathematica {1, 3, 5, 6, 10, 11, 12, 13, 14, 15, 26, 34, 36, 37, 42, 43, 44, 45, 46, 68, 78, 81, 82, 83, 84, 85, 86, 98, 99, 108, 109, 110, 111, 112, 113, 114, 115, 116, 119, 120, 121, 122, 123, 127, 128, 129}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error Exception raised: NotImplementedError

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function LeafSize is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

#see <https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in->

```
def tree(expr):
    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()+map(tree, expr.operands())
```

```
try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

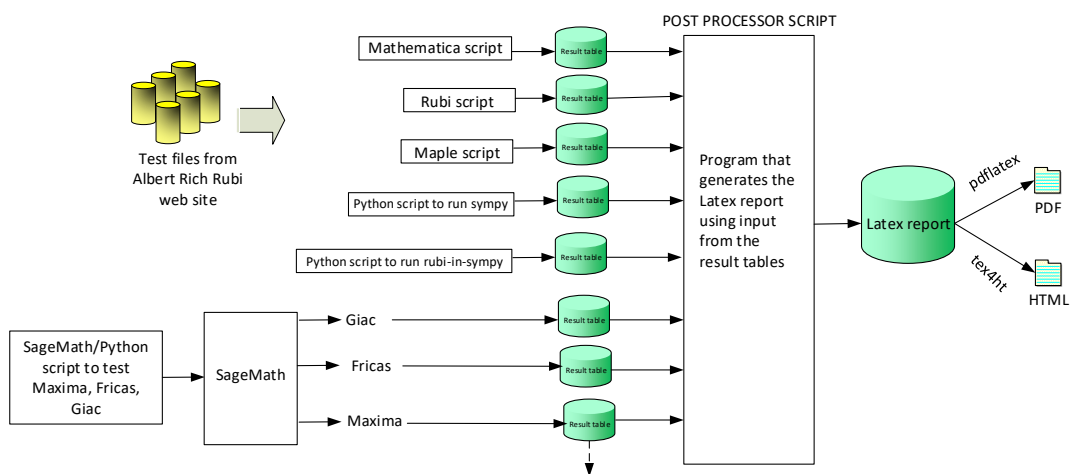
```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

When these cas systems have a builtin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

Naser M. Abbasi
June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190 }
}

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 76, 77, 79, 87, 91, 92, 93, 94, 95, 96, 97, 98, 99, 104, 105, 106, 107, 108, 109, 130, 131, 132, 133, 134, 135, 136, 137, 140, 141, 142, 143, 144, 145, 146, 147, 150, 151, 152, 153, 154, 155, 156, 159, 160, 161, 162, 163, 164, 165, 168, 169, 170, 171, 172, 173, 174, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190 }
}

B grade: { 45 }

C grade: { 19, 21, 23, 74, 75, 78, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 100, 101, 102, 103, 110, 111, 112, 113, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 138, 139, 148, 149, 157, 158, 166, 167, 175, 176 }
}

F grade: { 118, 126, 177, 178, 179 }

2.1.3 Maple

A grade: { 1, 2, 3, 5, 6, 7, 8, 9, 11, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 36, 37, 39, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 63, 64, 65, 69, 70, 72, 73, 74, 75, 76, 77, 79, 82, 83, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 133, 134, 135, 136, 137, 142, 143, 144, 145, 146, 147, 153, 154, 155, 156, 162, 163, 164, 165, 171, 172, 173, 174, 180, 181, 182, 183, 184, 185, 189, 190 }
}

B grade: { 4, 33, 35, 38, 40, 41, 42, 44, 46, 47, 48, 49, 50, 61, 62, 66, 67, 68, 80, 81, 84, 85, 86, 117, 124, 125 }

C grade: { 78, 110, 111, 112, 113, 114, 115, 116, 118, 119, 120, 121, 122, 123, 126, 127, 128, 129 }

F grade: { 10, 12, 14, 43, 45, 71, 130, 131, 132, 138, 139, 140, 141, 148, 149, 150, 151, 152, 157, 158, 159, 160, 161, 166, 167, 168, 169, 170, 175, 176, 177, 178, 179, 186, 187, 188 }

2.1.4 Maxima

A grade: { 4, 7, 16, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 35, 38, 47, 51, 52, 53, 57, 58, 59, 63, 64, 65, 72, 73, 74, 75, 76, 77, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 100, 101, 102, 103, 104, 105, 106, 107, 180, 181, 182, 183, 184, 185, 189, 190 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 26, 33, 34, 36, 37, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 54, 55, 56, 60, 61, 62, 66, 67, 68, 69, 70, 71, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 98, 99, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 186, 187, 188 }

2.1.5 FriCAS

A grade: { 2, 8, 9, 16, 17, 18, 19, 20, 22, 27, 28, 29, 30, 31, 32, 39, 40, 41, 48, 49, 50, 51, 52, 53, 57, 58, 59, 63, 64, 65, 69, 70, 72, 73, 87, 88, 92, 93, 94, 95, 96, 97, 100, 104, 105, 106, 107, 130, 131, 133, 134, 135, 136, 137, 140, 142, 143, 144, 145, 146, 147, 150, 153, 154, 155, 156, 162, 163, 164, 165, 171, 172, 173, 174, 180, 181, 182, 183, 184, 185, 186, 187, 189, 190 }

B grade: { 4, 7, 21, 23, 24, 25, 33, 35, 38, 47, 74, 75, 76, 77, 79, 80, 89, 90, 91, 101, 102, 103, 117, 124, 125, 132, 141, 151, 152, 159, 160, 161, 168, 169, 170, 188 }

C grade: { }

F grade: { 1, 3, 5, 6, 10, 11, 12, 13, 14, 15, 26, 34, 36, 37, 42, 43, 44, 45, 46, 54, 55, 56, 60, 61, 62, 66, 67, 68, 71, 78, 81, 82, 83, 84, 85, 86, 98, 99, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 119, 120, 121, 122, 123, 126, 127, 128, 129, 138, 139, 148, 149, 157, 158, 166, 167, 175, 176, 177, 178, 179 }

2.1.6 Sympy

A grade: { 4, 20, 22, 24, 35, 51, 52, 53, 57, 58, 59, 63, 64, 65, 69, 70, 72, 73, 87, 95, 96, 97, 106, 107, 133, 134, 135, 136, 137, 153, 154, 155, 156, 165, 180, 183, 184, 189 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 23, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 54, 55, 56, 60, 61, 62, 66, 67, 68, 71, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 93, 94, 98, 99, 100, 101, 102, 103, 104, 105, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 157, 158, 159, 160, 161, 162, 163, 164, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 181, 182, 185, 186, 187, 188, 190 }

2.1.7 Giac

A grade: { 51, 52, 53, 57, 58, 59, 63, 64, 65, 69, 70, 72, 73, 87, 133, 134, 135, 136, 137, 142, 143, 144, 145, 146, 147, 153, 154, 155, 156, 162, 163, 164, 165, 171, 172, 173, 174, 180, 181, 182, 183, 184, 185, 189, 190 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 54, 55, 56, 60, 61, 62, 66, 67, 68, 71, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 138, 139, 140, 141, 148, 149, 150, 151, 152, 157, 158, 159, 160, 161, 166, 167, 168, 169, 170, 175, 176, 177, 178, 179, 186, 187, 188 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	164	164	182	289	0	0	0	0
normalized size	1	1.	1.11	1.76	0.	0.	0.	0.
time (sec)	N/A	0.124	0.362	0.404	0.	0.	0.	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	77	151	0	273	0	0
normalized size	1	1.	0.74	1.45	0.	2.62	0.	0.
time (sec)	N/A	0.088	0.101	0.328	0.	1.699	0.	0.

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	117	117	138	240	0	0	0	0
normalized size	1	1.	1.18	2.05	0.	0.	0.	0.
time (sec)	N/A	0.097	0.231	0.325	0.	0.	0.	0.

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	53	101	54	236	42	0
normalized size	1	1.	1.	1.91	1.02	4.45	0.79	0.
time (sec)	N/A	0.057	0.06	0.258	1.048	1.636	3.11	0.

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	63	63	90	190	0	0	0	0
normalized size	1	1.	1.43	3.02	0.	0.	0.	0.
time (sec)	N/A	0.055	0.185	0.272	0.	0.	0.	0.

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	63	136	0	0	0	0
normalized size	1	1.	0.98	2.12	0.	0.	0.	0.
time (sec)	N/A	0.092	0.031	0.241	0.	0.	0.	0.

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	42	61	47	208	0	0
normalized size	1	1.	0.86	1.24	0.96	4.24	0.	0.
time (sec)	N/A	0.048	0.087	0.213	1.016	1.709	0.	0.

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	54	77	0	235	0	0
normalized size	1	1.	0.6	0.86	0.	2.61	0.	0.
time (sec)	N/A	0.059	0.041	0.213	0.	1.631	0.	0.

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	73	146	0	258	0	0
normalized size	1	1.	0.72	1.43	0.	2.53	0.	0.
time (sec)	N/A	0.084	0.095	0.237	0.	1.696	0.	0.

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	297	297	281	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.203	0.554	0.651	0.	0.	0.	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	184	184	188	246	0	0	0	0
normalized size	1	1.	1.02	1.34	0.	0.	0.	0.
time (sec)	N/A	0.177	0.576	0.349	0.	0.	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	198	198	199	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.141	0.46	0.434	0.	0.	0.	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	102	102	101	152	0	0	0	0
normalized size	1	1.	0.99	1.49	0.	0.	0.	0.
time (sec)	N/A	0.122	0.36	0.267	0.	0.	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	111	111	128	0	0	0	0	0
normalized size	1	1.	1.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	0.116	0.304	0.	0.	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	88	88	84	181	0	0	0	0
normalized size	1	1.	0.95	2.06	0.	0.	0.	0.
time (sec)	N/A	0.108	0.049	0.244	0.	0.	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	75	98	74	333	0	0
normalized size	1	1.	0.9	1.18	0.89	4.01	0.	0.
time (sec)	N/A	0.071	0.072	0.211	1.034	1.937	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	147	126	0	382	0	0
normalized size	1	1.	1.07	0.92	0.	2.79	0.	0.
time (sec)	N/A	0.086	0.133	0.217	0.	2.035	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	120	226	0	406	0	0
normalized size	1	1.	0.67	1.26	0.	2.27	0.	0.
time (sec)	N/A	0.125	0.113	0.229	0.	1.975	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	143	138	182	406	0	0
normalized size	1	1.	1.01	0.97	1.28	2.86	0.	0.
time (sec)	N/A	0.061	0.188	0.217	1.492	2.331	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	97	81	105	217	94	0
normalized size	1	1.	0.89	0.74	0.96	1.99	0.86	0.
time (sec)	N/A	0.047	0.084	0.193	0.99	1.984	16.626	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	123	118	143	378	0	0
normalized size	1	1.	1.12	1.07	1.3	3.44	0.	0.
time (sec)	N/A	0.04	0.118	0.225	1.515	2.011	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	77	72	77	193	68	0
normalized size	1	1.	1.	0.94	1.	2.51	0.88	0.
time (sec)	N/A	0.031	0.077	0.187	1.008	1.987	5.803	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	103	96	99	352	0	0
normalized size	1	1.	1.32	1.23	1.27	4.51	0.	0.
time (sec)	N/A	0.027	0.087	0.192	1.53	2.12	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	57	63	49	157	46	0
normalized size	1	1.	1.27	1.4	1.09	3.49	1.02	0.
time (sec)	N/A	0.014	0.052	0.178	0.979	1.91	1.319	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	60	42	42	262	0	0
normalized size	1	1.	1.5	1.05	1.05	6.55	0.	0.
time (sec)	N/A	0.015	0.099	0.159	0.973	2.	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	56	56	47	100	0	0	0	0
normalized size	1	1.	0.84	1.79	0.	0.	0.	0.
time (sec)	N/A	0.088	0.044	0.256	0.	0.	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	42	58	43	138	0	0
normalized size	1	1.	1.05	1.45	1.08	3.45	0.	0.
time (sec)	N/A	0.02	0.054	0.178	0.982	1.965	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	117	112	142	170	0	0
normalized size	1	1.	1.24	1.19	1.51	1.81	0.	0.
time (sec)	N/A	0.04	0.068	0.183	0.989	1.858	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	74	77	76	174	0	0
normalized size	1	1.	0.96	1.	0.99	2.26	0.	0.
time (sec)	N/A	0.033	0.063	0.181	0.979	1.89	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	137	135	198	198	0	0
normalized size	1	1.	1.09	1.07	1.57	1.57	0.	0.
time (sec)	N/A	0.055	0.095	0.182	0.988	1.901	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	94	85	99	200	0	0
normalized size	1	1.	0.86	0.78	0.91	1.83	0.	0.
time (sec)	N/A	0.05	0.086	0.18	0.996	1.873	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	157	155	250	227	0	0
normalized size	1	1.	0.99	0.98	1.58	1.44	0.	0.
time (sec)	N/A	0.073	0.142	0.189	1.015	1.833	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	212	264	0	525	0	0
normalized size	1	1.	1.71	2.13	0.	4.23	0.	0.
time (sec)	N/A	0.119	0.33	0.29	0.	2.161	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	140	140	224	372	0	0	0	0
normalized size	1	1.	1.6	2.66	0.	0.	0.	0.
time (sec)	N/A	0.123	1.154	0.278	0.	0.	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	112	168	113	446	99	0
normalized size	1	1.	1.72	2.58	1.74	6.86	1.52	0.
time (sec)	N/A	0.075	0.209	0.253	1.02	2.066	3.947	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	78	78	126	250	0	0	0	0
normalized size	1	1.	1.62	3.21	0.	0.	0.	0.
time (sec)	N/A	0.07	0.224	0.221	0.	0.	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	83	83	116	250	0	0	0	0
normalized size	1	1.	1.4	3.01	0.	0.	0.	0.
time (sec)	N/A	0.125	0.157	0.237	0.	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	87	124	105	301	0	0
normalized size	1	1.	1.43	2.03	1.72	4.93	0.	0.
time (sec)	N/A	0.07	0.229	0.217	1.029	1.66	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	183	192	0	355	0	0
normalized size	1	1.	1.55	1.63	0.	3.01	0.	0.
time (sec)	N/A	0.085	0.163	0.236	0.	1.618	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	134	226	0	392	0	0
normalized size	1	1.	1.1	1.85	0.	3.21	0.	0.
time (sec)	N/A	0.102	0.25	0.225	0.	1.654	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	268	298	0	439	0	0
normalized size	1	1.	1.77	1.97	0.	2.91	0.	0.
time (sec)	N/A	0.12	0.259	0.233	0.	1.67	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	223	223	315	546	0	0	0	0
normalized size	1	1.	1.41	2.45	0.	0.	0.	0.
time (sec)	N/A	0.239	1.558	0.385	0.	0.	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	242	242	440	0	0	0	0	0
normalized size	1	1.	1.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.195	1.018	0.49	0.	0.	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	126	126	219	343	0	0	0	0
normalized size	1	1.	1.74	2.72	0.	0.	0.	0.
time (sec)	N/A	0.157	0.868	0.302	0.	0.	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	140	140	282	0	0	0	0	0
normalized size	1	1.	2.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.111	0.409	0.329	0.	0.	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	114	114	182	454	0	0	0	0
normalized size	1	1.	1.6	3.98	0.	0.	0.	0.
time (sec)	N/A	0.145	0.229	0.272	0.	0.	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	165	227	194	485	0	0
normalized size	1	1.	1.62	2.23	1.9	4.75	0.	0.
time (sec)	N/A	0.103	0.322	0.252	1.035	1.689	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	245	321	0	583	0	0
normalized size	1	1.	1.5	1.97	0.	3.58	0.	0.
time (sec)	N/A	0.116	0.461	0.274	0.	1.642	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	256	455	0	649	0	0
normalized size	1	1.	1.2	2.14	0.	3.05	0.	0.
time (sec)	N/A	0.166	0.383	0.274	0.	1.785	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	332	553	0	756	0	0
normalized size	1	1.	1.37	2.29	0.	3.12	0.	0.
time (sec)	N/A	0.196	0.714	0.277	0.	1.639	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.017	2.21	0.736	0.	0.	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.007	0.033	0.367	0.	0.	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.294	0.298	0.	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	43	54	0	0	0	0
normalized size	1	1.	0.93	1.17	0.	0.	0.	0.
time (sec)	N/A	0.11	0.075	0.211	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	56	60	0	0	0	0
normalized size	1	1.	0.89	0.95	0.	0.	0.	0.
time (sec)	N/A	0.142	0.076	0.207	0.	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	91	110	0	0	0	0
normalized size	1	1.	0.78	0.94	0.	0.	0.	0.
time (sec)	N/A	0.236	0.156	0.234	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	16.38	0.677	0.	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.007	8.593	0.3	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	4.993	0.286	0.	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	82	164	0	0	0	0
normalized size	1	1.	0.95	1.91	0.	0.	0.	0.
time (sec)	N/A	0.145	0.334	0.258	0.	0.	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	92	186	0	0	0	0
normalized size	1	1.	1.08	2.19	0.	0.	0.	0.
time (sec)	N/A	0.165	0.335	0.237	0.	0.	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	170	420	0	0	0	0
normalized size	1	1.	0.89	2.21	0.	0.	0.	0.
time (sec)	N/A	0.295	0.623	0.273	0.	0.	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.016	5.791	0.67	0.	0.	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.006	3.587	0.589	0.	0.	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.025	2.499	0.276	0.	0.	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	103	244	0	0	0	0
normalized size	1	1.	0.9	2.14	0.	0.	0.	0.
time (sec)	N/A	0.176	0.288	0.231	0.	0.	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	122	277	0	0	0	0
normalized size	1	1.	1.09	2.47	0.	0.	0.	0.
time (sec)	N/A	0.206	0.405	0.293	0.	0.	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	240	240	204	628	0	0	0	0
normalized size	1	1.	0.85	2.62	0.	0.	0.	0.
time (sec)	N/A	0.371	0.555	0.329	0.	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.024	5.077	1.97	0.	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.026	3.336	1.681	0.	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	97	0	0	0	0	0
normalized size	1	1.	1.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.147	1.601	0.	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.028	0.394	1.291	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.027	0.806	1.267	0.	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	190	283	298	764	0	0
normalized size	1	1.	0.72	1.07	1.13	2.89	0.	0.
time (sec)	N/A	0.362	0.397	0.196	1.516	2.417	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	147	215	205	601	0	0
normalized size	1	1.	0.73	1.07	1.02	2.99	0.	0.
time (sec)	N/A	0.217	0.214	0.213	1.507	2.13	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	122	125	95	401	0	0
normalized size	1	1.	0.86	0.88	0.67	2.82	0.	0.
time (sec)	N/A	0.118	0.344	0.193	0.99	1.852	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	60	42	42	262	0	0
normalized size	1	1.	1.5	1.05	1.05	6.55	0.	0.
time (sec)	N/A	0.015	0.087	0.165	0.971	1.694	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	229	229	393	514	0	0	0	0
normalized size	1	1.	1.72	2.24	0.	0.	0.	0.
time (sec)	N/A	0.933	0.523	0.271	0.	0.	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	222	231	0	1162	0	0
normalized size	1	1.	1.51	1.57	0.	7.9	0.	0.
time (sec)	N/A	0.119	0.226	0.29	0.	1.824	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	306	342	1090	0	2423	0	0
normalized size	1	1.	1.12	3.56	0.	7.92	0.	0.
time (sec)	N/A	0.193	0.629	0.274	0.	2.689	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	343	343	2653	830	0	0	0	0
normalized size	1	1.	7.73	2.42	0.	0.	0.	0.
time (sec)	N/A	0.621	9.876	0.395	0.	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	279	279	2938	415	0	0	0	0
normalized size	1	1.	10.53	1.49	0.	0.	0.	0.
time (sec)	N/A	0.372	12.983	0.324	0.	0.	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	187	187	1707	288	0	0	0	0
normalized size	1	1.	9.13	1.54	0.	0.	0.	0.
time (sec)	N/A	0.247	10.784	0.275	0.	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	105	1675	253	0	0	0	0
normalized size	1	1.	15.95	2.41	0.	0.	0.	0.
time (sec)	N/A	0.178	10.398	0.271	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	278	278	4527	902	0	0	0	0
normalized size	1	1.	16.28	3.24	0.	0.	0.	0.
time (sec)	N/A	0.313	12.662	0.335	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	609	609	8675	1632	0	0	0	0
normalized size	1	1.	14.24	2.68	0.	0.	0.	0.
time (sec)	N/A	0.594	12.924	0.352	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	86	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.048	1.836	1.454	0.	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	162	224	329	590	0	0
normalized size	1	1.	0.71	0.98	1.44	2.58	0.	0.
time (sec)	N/A	0.127	0.318	0.185	1.48	2.984	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	144	182	246	531	0	0
normalized size	1	1.	0.83	1.05	1.41	3.05	0.	0.
time (sec)	N/A	0.103	0.208	0.185	1.506	2.732	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	169	135	144	460	0	0
normalized size	1	1.	1.51	1.21	1.29	4.11	0.	0.
time (sec)	N/A	0.051	0.356	0.175	1.491	2.526	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	107	114	89	401	0	0
normalized size	1	1.	1.11	1.19	0.93	4.18	0.	0.
time (sec)	N/A	0.066	0.239	0.21	1.007	2.085	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	76	123	123	236	0	0
normalized size	1	1.	0.6	0.98	0.98	1.87	0.	0.
time (sec)	N/A	0.079	0.104	0.184	0.996	1.755	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	101	142	178	297	0	0
normalized size	1	1.	0.55	0.78	0.97	1.62	0.	0.
time (sec)	N/A	0.099	0.165	0.184	1.001	1.894	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	117	160	223	352	0	0
normalized size	1	1.	0.49	0.67	0.94	1.48	0.	0.
time (sec)	N/A	0.122	0.201	0.191	1.027	1.955	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	126	150	239	381	228	0
normalized size	1	1.	0.54	0.65	1.03	1.64	0.98	0.
time (sec)	N/A	0.164	0.21	0.185	1.001	2.054	69.773	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	106	132	186	325	177	0
normalized size	1	1.	0.59	0.73	1.03	1.81	0.98	0.
time (sec)	N/A	0.135	0.178	0.183	1.009	1.973	14.968	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	85	113	130	269	126	0
normalized size	1	1.	0.52	0.69	0.79	1.64	0.77	0.
time (sec)	N/A	0.193	0.126	0.178	0.998	2.044	4.156	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	296	296	98	166	0	0	0	0
normalized size	1	1.	0.33	0.56	0.	0.	0.	0.
time (sec)	N/A	0.875	0.292	0.365	0.	0.	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	309	309	149	170	0	0	0	0
normalized size	1	1.	0.48	0.55	0.	0.	0.	0.
time (sec)	N/A	0.78	0.591	0.334	0.	0.	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	207	300	443	775	0	0
normalized size	1	1.	0.75	1.09	1.61	2.82	0.	0.
time (sec)	N/A	0.235	0.457	0.212	1.515	3.602	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	174	228	302	679	0	0
normalized size	1	1.	0.85	1.12	1.48	3.33	0.	0.
time (sec)	N/A	0.127	0.279	0.178	1.499	3.004	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	158	197	205	614	0	0
normalized size	1	1.	0.89	1.11	1.16	3.47	0.	0.
time (sec)	N/A	0.132	0.242	0.187	1.524	2.452	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	149	205	181	590	0	0
normalized size	1	1.	0.85	1.16	1.03	3.35	0.	0.
time (sec)	N/A	0.139	0.269	0.185	0.979	2.235	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	134	193	236	385	0	0
normalized size	1	1.	0.63	0.91	1.11	1.81	0.	0.
time (sec)	N/A	0.167	0.266	0.185	0.987	1.794	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	281	160	225	313	483	0	0
normalized size	1	1.	0.57	0.8	1.11	1.72	0.	0.
time (sec)	N/A	0.201	0.333	0.189	1.018	1.871	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	168	212	331	509	332	0
normalized size	1	1.	0.6	0.76	1.19	1.83	1.19	0.
time (sec)	N/A	0.237	0.287	0.181	0.994	2.088	51.106	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	139	180	250	413	252	0
normalized size	1	1.	0.6	0.78	1.09	1.8	1.1	0.
time (sec)	N/A	0.252	0.257	0.175	0.992	2.014	22.678	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	370	370	176	286	0	0	0	0
normalized size	1	1.	0.48	0.77	0.	0.	0.	0.
time (sec)	N/A	1.098	0.421	0.485	0.	0.	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	373	373	212	252	0	0	0	0
normalized size	1	1.	0.57	0.68	0.	0.	0.	0.
time (sec)	N/A	1.047	0.808	0.417	0.	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	519	519	921	411	0	0	0	0
normalized size	1	1.	1.77	0.79	0.	0.	0.	0.
time (sec)	N/A	1.273	1.42	2.277	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	459	441	860	513	0	0	0	0
normalized size	1	0.96	1.87	1.12	0.	0.	0.	0.
time (sec)	N/A	1.229	0.317	0.454	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	469	469	849	302	0	0	0	0
normalized size	1	1.	1.81	0.64	0.	0.	0.	0.
time (sec)	N/A	0.966	0.37	1.344	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	417	417	386	3157	0	0	0	0
normalized size	1	1.	0.93	7.57	0.	0.	0.	0.
time (sec)	N/A	0.989	0.885	0.605	0.	0.	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	523	523	933	372	0	0	0	0
normalized size	1	1.	1.78	0.71	0.	0.	0.	0.
time (sec)	N/A	1.261	1.739	2.456	0.	0.	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	631	611	1278	870	0	0	0	0
normalized size	1	0.97	2.03	1.38	0.	0.	0.	0.
time (sec)	N/A	1.548	4.816	0.779	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	580	562	1208	661	0	0	0	0
normalized size	1	0.97	2.08	1.14	0.	0.	0.	0.
time (sec)	N/A	1.463	1.216	0.461	0.	0.	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	345	844	0	1265	0	0
normalized size	1	1.	2.35	5.74	0.	8.61	0.	0.
time (sec)	N/A	0.245	0.963	0.283	0.	1.975	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	542	542	0	3326	0	0	0	0
normalized size	1	1.	0.	6.14	0.	0.	0.	0.
time (sec)	N/A	1.363	40.277	0.757	0.	0.	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	840	840	1270	2016	0	0	0	0
normalized size	1	1.	1.51	2.4	0.	0.	0.	0.
time (sec)	N/A	3.088	1.49	17.249	0.	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	786	786	1226	1880	0	0	0	0
normalized size	1	1.	1.56	2.39	0.	0.	0.	0.
time (sec)	N/A	1.574	1.57	2.401	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	786	786	1216	1870	0	0	0	0
normalized size	1	1.	1.55	2.38	0.	0.	0.	0.
time (sec)	N/A	2.836	1.543	1.619	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	844	844	1305	1952	0	0	0	0
normalized size	1	1.	1.55	2.31	0.	0.	0.	0.
time (sec)	N/A	2.953	1.087	11.697	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD
size	778	760	2000	1779	0	0	0	0
normalized size	1	0.98	2.57	2.29	0.	0.	0.	0.
time (sec)	N/A	1.709	7.487	0.846	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	486	3343	0	2743	0	0
normalized size	1	1.	2.81	19.32	0.	15.86	0.	0.
time (sec)	N/A	0.189	1.486	0.362	0.	2.978	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	486	3301	0	2538	0	0
normalized size	1	1.	2.24	15.21	0.	11.7	0.	0.
time (sec)	N/A	0.293	1.022	0.354	0.	2.895	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	C	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	741	741	0	5713	0	0	0	0
normalized size	1	1.	0.	7.71	0.	0.	0.	0.
time (sec)	N/A	1.542	62.378	1.322	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1272	1272	2022	3455	0	0	0	0
normalized size	1	1.	1.59	2.72	0.	0.	0.	0.
time (sec)	N/A	2.263	6.201	3.527	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1276	1276	2030	2537	0	0	0	0
normalized size	1	1.	1.59	1.99	0.	0.	0.	0.
time (sec)	N/A	4.04	6.157	2.858	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1272	1272	2015	3446	0	0	0	0
normalized size	1	1.	1.58	2.71	0.	0.	0.	0.
time (sec)	N/A	4.91	6.063	6.369	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	447	447	409	0	0	4373	0	0
normalized size	1	1.	0.91	0.	0.	9.78	0.	0.
time (sec)	N/A	1.395	3.014	1.95	0.	15.839	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	365	0	0	3636	0	0
normalized size	1	1.	1.11	0.	0.	11.05	0.	0.
time (sec)	N/A	0.427	1.452	1.626	0.	7.637	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	307	0	0	3011	0	0
normalized size	1	1.	1.39	0.	0.	13.62	0.	0.
time (sec)	N/A	0.357	1.287	1.395	0.	4.037	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	5.545	1.325	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.104	6.152	1.473	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.098	12.809	1.685	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.036	2.936	1.279	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.087	1.841	1.034	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	312	312	576	0	0	0	0	0
normalized size	1	1.	1.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.38	3.969	1.318	0.	0.	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	446	446	641	0	0	0	0	0
normalized size	1	1.	1.44	0.	0.	0.	0.	0.
time (sec)	N/A	0.561	6.121	1.605	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	418	418	382	0	0	4365	0	0
normalized size	1	1.	0.91	0.	0.	10.44	0.	0.
time (sec)	N/A	0.53	2.856	1.135	0.	17.743	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	297	342	0	0	3606	0	0
normalized size	1	1.	1.15	0.	0.	12.14	0.	0.
time (sec)	N/A	0.432	1.48	0.888	0.	8.491	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.12	6.205	0.828	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.125	6.563	0.937	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.122	14.233	1.113	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	4.293	0.824	0.	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.104	6.724	0.806	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.128	16.079	1.039	0.	0.	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	409	409	620	0	0	0	0	0
normalized size	1	1.	1.52	0.	0.	0.	0.	0.
time (sec)	N/A	0.533	6.274	1.224	0.	0.	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	556	556	728	0	0	0	0	0
normalized size	1	1.	1.31	0.	0.	0.	0.	0.
time (sec)	N/A	0.758	7.763	1.421	0.	0.	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	366	0	0	3663	0	0
normalized size	1	1.	1.03	0.	0.	10.29	0.	0.
time (sec)	N/A	1.16	1.564	1.832	0.	9.047	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	406	0	0	3032	0	0
normalized size	1	1.	1.62	0.	0.	12.08	0.	0.
time (sec)	N/A	0.33	1.266	1.794	0.	5.226	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	239	0	0	2419	0	0
normalized size	1	1.	1.56	0.	0.	15.81	0.	0.
time (sec)	N/A	0.295	0.536	1.265	0.	3.582	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.095	3.1	1.22	0.	0.	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.111	22.189	1.105	0.	0.	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.098	6.828	1.578	0.	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.035	1.012	1.223	0.	0.	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	501	0	0	0	0	0
normalized size	1	1.	2.27	0.	0.	0.	0.	0.
time (sec)	N/A	0.258	4.01	1.015	0.	0.	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	612	0	0	0	0	0
normalized size	1	1.	1.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.485	4.64	1.263	0.	0.	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	278	278	436	0	0	3760	0	0
normalized size	1	1.	1.57	0.	0.	13.53	0.	0.
time (sec)	N/A	1.117	1.423	1.541	0.	4.15	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	249	0	0	2867	0	0
normalized size	1	1.	1.41	0.	0.	16.2	0.	0.
time (sec)	N/A	0.272	1.666	1.487	0.	2.889	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	135	0	0	829	0	0
normalized size	1	1.	1.55	0.	0.	9.53	0.	0.
time (sec)	N/A	0.245	0.591	0.918	0.	2.396	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.116	30.269	0.841	0.	0.	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.133	36.645	0.939	0.	0.	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.111	9.462	1.534	0.	0.	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	4.674	1.47	0.	0.	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	334	0	0	0	0	0
normalized size	1	1.	3.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.079	1.324	1.067	0.	0.	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	501	0	0	0	0	0
normalized size	1	1.	2.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.287	4.461	0.809	0.	0.	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-2)	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	272	348	0	0	5094	0	0
normalized size	1	1.	1.28	0.	0.	18.73	0.	0.
time (sec)	N/A	1.281	1.955	1.441	0.	4.44	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	218	0	0	1635	0	0
normalized size	1	1.	1.22	0.	0.	9.13	0.	0.
time (sec)	N/A	0.264	0.369	1.405	0.	3.22	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	204	0	0	1447	0	0
normalized size	1	1.	1.32	0.	0.	9.4	0.	0.
time (sec)	N/A	0.291	0.31	0.935	0.	3.175	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.123	42.658	0.829	0.	0.	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.142	52.642	0.951	0.	0.	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.122	13.681	1.511	0.	0.	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.111	13.229	1.481	0.	0.	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	246	246	488	0	0	0	0	0
normalized size	1	1.	1.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.246	2.474	1.457	0.	0.	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	517	0	0	0	0	0
normalized size	1	1.	1.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.206	5.085	1.012	0.	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	596	576	0	0	0	0	0	0
normalized size	1	0.97	0.	0.	0.	0.	0.	0.
time (sec)	N/A	2.545	0.241	2.759	0.	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	372	352	0	0	0	0	0	0
normalized size	1	0.95	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.432	0.153	2.111	0.	0.	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	206	192	0	0	0	0	0	0
normalized size	1	0.93	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.179	0.117	1.873	0.	0.	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	2.357	1.319	0.	0.	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	2.632	1.441	0.	0.	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.11	1.051	0.862	0.	0.	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.099	0.118	1.079	0.	0.	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.1	1.344	1.279	0.	0.	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.11	1.752	1.191	0.	0.	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	473	473	213	0	0	852	0	0
normalized size	1	1.	0.45	0.	0.	1.8	0.	0.
time (sec)	N/A	1.579	0.419	1.938	0.	2.176	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	316	178	0	0	705	0	0
normalized size	1	1.	0.56	0.	0.	2.23	0.	0.
time (sec)	N/A	1.383	0.36	2.082	0.	1.902	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	140	0	0	575	0	0
normalized size	1	1.	0.88	0.	0.	3.62	0.	0.
time (sec)	N/A	0.164	0.336	1.24	0.	2.112	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	0.391	1.691	0.	0.	0.	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	3.373	1.526	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [11] had the largest ratio of [1.]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	9	6	1.	10	0.6
2	A	5	5	1.	10	0.5
3	A	8	6	1.	10	0.6
4	A	4	4	1.	8	0.5
5	A	7	5	1.	6	0.833
6	A	6	6	1.	10	0.6
7	A	4	3	1.	10	0.3
8	A	4	4	1.	10	0.4
9	A	5	5	1.	10	0.5
10	A	14	9	1.	10	0.9
11	A	10	10	1.	10	1.
12	A	11	8	1.	10	0.8
13	A	7	7	1.	8	0.875
14	A	9	6	1.	6	1.
15	A	7	7	1.	10	0.7
16	A	5	3	1.	10	0.3
17	A	6	6	1.	10	0.6
18	A	8	6	1.	10	0.6
19	A	8	6	1.	12	0.5
20	A	6	4	1.	12	0.333
21	A	6	6	1.	12	0.5
22	A	4	4	1.	12	0.333
23	A	4	4	1.	12	0.333
24	A	2	2	1.	10	0.2
25	A	3	2	1.	8	0.25
26	A	6	6	1.	12	0.5
27	A	2	2	1.	12	0.167
28	A	5	5	1.	12	0.417
29	A	4	4	1.	12	0.333
30	A	7	5	1.	12	0.417
31	A	6	4	1.	12	0.333
32	A	9	5	1.	12	0.417
33	A	5	5	1.	14	0.357
34	A	8	6	1.	14	0.429
35	A	4	4	1.	12	0.333
36	A	7	5	1.	10	0.5
37	A	6	6	1.	14	0.429
38	A	4	3	1.	14	0.214
39	A	4	3	1.	14	0.214
40	A	5	5	1.	14	0.357
41	A	5	3	1.	14	0.214
42	A	10	10	1.	14	0.714
43	A	11	8	1.	14	0.571

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
44	A	7	7	1.	12	0.583
45	A	9	6	1.	10	0.6
46	A	7	7	1.	14	0.5
47	A	5	3	1.	14	0.214
48	A	6	6	1.	14	0.429
49	A	8	6	1.	14	0.429
50	A	10	6	1.	14	0.429
51	A	0	0	0.	0	0.
52	A	0	0	0.	0	0.
53	A	0	0	0.	0	0.
54	A	4	4	1.	14	0.286
55	A	6	6	1.	14	0.429
56	A	9	5	1.	14	0.357
57	A	0	0	0.	0	0.
58	A	0	0	0.	0	0.
59	A	0	0	0.	0	0.
60	A	5	5	1.	14	0.357
61	A	7	7	1.	14	0.5
62	A	11	6	1.	14	0.429
63	A	0	0	0.	0	0.
64	A	0	0	0.	0	0.
65	A	0	0	0.	0	0.
66	A	6	5	1.	14	0.357
67	A	8	7	1.	14	0.5
68	A	13	6	1.	14	0.429
69	A	0	0	0.	0	0.
70	A	0	0	0.	0	0.
71	A	3	3	1.	14	0.214
72	A	0	0	0.	0	0.
73	A	0	0	0.	0	0.
74	A	9	7	1.	16	0.438
75	A	8	7	1.	16	0.438
76	A	7	7	1.	14	0.5
77	A	3	2	1.	8	0.25
78	A	4	2	1.	16	0.125
79	A	8	7	1.	16	0.438
80	A	11	8	1.	16	0.5
81	A	21	12	1.	18	0.667
82	A	14	10	1.	18	0.556
83	A	8	8	1.	18	0.444
84	A	5	5	1.	18	0.278
85	A	11	10	1.	18	0.556
86	A	18	13	1.	18	0.722
87	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
88	A	6	6	1.	19	0.316
89	A	5	6	1.	19	0.316
90	A	4	4	1.	16	0.25
91	A	3	4	1.	19	0.21
92	A	4	5	1.	19	0.263
93	A	5	6	1.	19	0.316
94	A	6	6	1.	19	0.316
95	A	5	5	1.	19	0.263
96	A	5	5	1.	19	0.263
97	A	7	6	1.	17	0.353
98	A	12	12	1.	19	0.632
99	A	14	14	1.	19	0.737
100	A	6	7	1.	21	0.333
101	A	5	6	1.	18	0.333
102	A	5	6	1.	21	0.286
103	A	5	6	1.	21	0.286
104	A	5	6	1.	21	0.286
105	A	6	7	1.	21	0.333
106	A	5	6	1.	21	0.286
107	A	7	6	1.	19	0.316
108	A	13	14	1.	21	0.667
109	A	15	16	1.	21	0.762
110	A	24	11	1.	21	0.524
111	A	26	9	0.96	19	0.474
112	A	19	7	1.	18	0.389
113	A	19	7	1.	21	0.333
114	A	24	10	1.	21	0.476
115	A	32	15	0.97	21	0.714
116	A	30	13	0.97	21	0.619
117	A	8	6	1.	19	0.316
118	A	25	11	1.	21	0.524
119	A	50	13	1.	21	0.619
120	A	27	10	1.	21	0.476
121	A	47	11	1.	18	0.611
122	A	50	13	1.	21	0.619
123	A	35	14	0.98	21	0.667
124	A	6	7	1.	21	0.333
125	A	9	7	1.	19	0.368
126	A	30	12	1.	21	0.571
127	A	35	11	1.	21	0.524
128	A	63	12	1.	21	0.571
129	A	81	12	1.	18	0.667
130	A	12	12	1.	23	0.522
131	A	11	12	1.	23	0.522

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
132	A	10	10	1.	21	0.476
133	A	0	0	0.	0	0.
134	A	0	0	0.	0	0.
135	A	0	0	0.	0	0.
136	A	0	0	0.	0	0.
137	A	0	0	0.	0	0.
138	A	9	10	1.	23	0.435
139	A	10	11	1.	23	0.478
140	A	12	12	1.	23	0.522
141	A	11	11	1.	21	0.524
142	A	0	0	0.	0	0.
143	A	0	0	0.	0	0.
144	A	0	0	0.	0	0.
145	A	0	0	0.	0	0.
146	A	0	0	0.	0	0.
147	A	0	0	0.	0	0.
148	A	10	11	1.	23	0.478
149	A	11	11	1.	23	0.478
150	A	11	12	1.	23	0.522
151	A	10	12	1.	23	0.522
152	A	9	9	1.	21	0.429
153	A	0	0	0.	0	0.
154	A	0	0	0.	0	0.
155	A	0	0	0.	0	0.
156	A	0	0	0.	0	0.
157	A	9	10	1.	23	0.435
158	A	9	11	1.	23	0.478
159	A	10	11	1.	23	0.478
160	A	9	11	1.	23	0.478
161	A	5	5	1.	21	0.238
162	A	0	0	0.	0	0.
163	A	0	0	0.	0	0.
164	A	0	0	0.	0	0.
165	A	0	0	0.	0	0.
166	A	4	5	1.	20	0.25
167	A	8	10	1.	23	0.435
168	A	10	11	1.	23	0.478
169	A	7	8	1.	23	0.348
170	A	6	6	1.	21	0.286
171	A	0	0	0.	0	0.
172	A	0	0	0.	0	0.
173	A	0	0	0.	0	0.
174	A	0	0	0.	0	0.
175	A	8	9	1.	23	0.391

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
176	A	8	10	1.	20	0.5
177	A	5	6	0.97	23	0.261
178	A	5	6	0.95	23	0.261
179	A	4	5	0.93	21	0.238
180	A	0	0	0.	0	0.
181	A	0	0	0.	0	0.
182	A	0	0	0.	0	0.
183	A	0	0	0.	0	0.
184	A	0	0	0.	0	0.
185	A	0	0	0.	0	0.
186	A	15	10	1.	26	0.385
187	A	12	10	1.	26	0.385
188	A	7	8	1.	26	0.308
189	A	0	0	0.	0	0.
190	A	0	0	0.	0	0.

Chapter 3

Listing of integrals

3.1 $\int x^4 \operatorname{sech}^{-1}(ax)^2 dx$

Optimal. Leaf size=164

$$\frac{3i \operatorname{PolyLog}\left(2, -ie^{\operatorname{sech}^{-1}(ax)}\right)}{20a^5} - \frac{3i \operatorname{PolyLog}\left(2, ie^{\operatorname{sech}^{-1}(ax)}\right)}{20a^5} - \frac{x^3}{30a^2} - \frac{x^3 \sqrt{\frac{1-ax}{ax+1}}(ax+1) \operatorname{sech}^{-1}(ax)}{10a^2} - \frac{3x}{20a^4} - \frac{3x \sqrt{\frac{1-ax}{ax+1}}}{20a^4}$$

```
[Out] (-3*x)/(20*a^4) - x^3/(30*a^2) - (3*x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x])/(20*a^4) - (x^3*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x])/(10*a^2) + (x^5*ArcSech[a*x]^2)/5 - (3*ArcSech[a*x]*ArcTan[E^ArcSech[a*x]])/(10*a^5) + (((3*I)/20)*PolyLog[2, (-I)*E^ArcSech[a*x]])/a^5 - (((3*I)/20)*PolyLog[2, I*E^ArcSech[a*x]])/a^5
```

Rubi [A] time = 0.123648, antiderivative size = 164, normalized size of antiderivative = 1, number of steps used = 9, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$, Rules used = {6285, 5418, 4185, 4180, 2279, 2391}

$$\frac{3i \operatorname{PolyLog}\left(2, -ie^{\operatorname{sech}^{-1}(ax)}\right)}{20a^5} - \frac{3i \operatorname{PolyLog}\left(2, ie^{\operatorname{sech}^{-1}(ax)}\right)}{20a^5} - \frac{x^3}{30a^2} - \frac{x^3 \sqrt{\frac{1-ax}{ax+1}}(ax+1) \operatorname{sech}^{-1}(ax)}{10a^2} - \frac{3x}{20a^4} - \frac{3x \sqrt{\frac{1-ax}{ax+1}}}{20a^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcSech[a*x]^2,x]
```

```
[Out] (-3*x)/(20*a^4) - x^3/(30*a^2) - (3*x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x])/(20*a^4) - (x^3*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x])/(10*a^2) + (x^5*ArcSech[a*x]^2)/5 - (3*ArcSech[a*x]*ArcTan[E^ArcSech[a*x]])/(10*a^5) + (((3*I)/20)*PolyLog[2, (-I)*E^ArcSech[a*x]])/a^5 - (((3*I)/20)*PolyLog[2, I*E^ArcSech[a*x]])/a^5
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))^n_.*(x_)^m_.), x_Symbol] := -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])
```

Rule 5418

```
Int[(x_)^m_.*Sech[(a_.) + (b_.)*(x_)^n_.]^p_.*Tanh[(a_.) + (b_.)*(x_)^n_.]^q_.), x_Symbol] := -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^p)/(b*n*p)
```

```
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x] /;
FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(n_.)*((c_.) + (d_.)*(x_.)), x_Symbol] :>
-Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_.)]*(c_.) + (d_.)*(x_
))^m_., x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
- E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_.) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_.))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_.) + (e_.)*(x_.)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned} \int x^4 \operatorname{sech}^{-1}(ax)^2 dx &= \frac{\operatorname{Subst}\left(\int x^2 \operatorname{sech}^5(x) \tanh(x) dx, x, \operatorname{sech}^{-1}(ax)\right)}{a^5} \\ &= \frac{1}{5} x^5 \operatorname{sech}^{-1}(ax)^2 - \frac{2 \operatorname{Subst}\left(\int x \operatorname{sech}^5(x) dx, x, \operatorname{sech}^{-1}(ax)\right)}{5a^5} \\ &= -\frac{x^3}{30a^2} - \frac{x^3 \sqrt{\frac{1-ax}{1+ax}}(1+ax) \operatorname{sech}^{-1}(ax)}{10a^2} + \frac{1}{5} x^5 \operatorname{sech}^{-1}(ax)^2 - \frac{3 \operatorname{Subst}\left(\int x \operatorname{sech}^3(x) dx, x, \operatorname{sech}^{-1}(ax)\right)}{10a^5} \\ &= -\frac{3x}{20a^4} - \frac{x^3}{30a^2} - \frac{3x \sqrt{\frac{1-ax}{1+ax}}(1+ax) \operatorname{sech}^{-1}(ax)}{20a^4} - \frac{x^3 \sqrt{\frac{1-ax}{1+ax}}(1+ax) \operatorname{sech}^{-1}(ax)}{10a^2} + \frac{1}{5} x^5 \operatorname{sech}^{-1}(ax)^2 \\ &= -\frac{3x}{20a^4} - \frac{x^3}{30a^2} - \frac{3x \sqrt{\frac{1-ax}{1+ax}}(1+ax) \operatorname{sech}^{-1}(ax)}{20a^4} - \frac{x^3 \sqrt{\frac{1-ax}{1+ax}}(1+ax) \operatorname{sech}^{-1}(ax)}{10a^2} + \frac{1}{5} x^5 \operatorname{sech}^{-1}(ax)^2 \\ &= -\frac{3x}{20a^4} - \frac{x^3}{30a^2} - \frac{3x \sqrt{\frac{1-ax}{1+ax}}(1+ax) \operatorname{sech}^{-1}(ax)}{20a^4} - \frac{x^3 \sqrt{\frac{1-ax}{1+ax}}(1+ax) \operatorname{sech}^{-1}(ax)}{10a^2} + \frac{1}{5} x^5 \operatorname{sech}^{-1}(ax)^2 \\ &= -\frac{3x}{20a^4} - \frac{x^3}{30a^2} - \frac{3x \sqrt{\frac{1-ax}{1+ax}}(1+ax) \operatorname{sech}^{-1}(ax)}{20a^4} - \frac{x^3 \sqrt{\frac{1-ax}{1+ax}}(1+ax) \operatorname{sech}^{-1}(ax)}{10a^2} + \frac{1}{5} x^5 \operatorname{sech}^{-1}(ax)^2 \end{aligned}$$

Mathematica [A] time = 0.362456, size = 182, normalized size = 1.11

$$9i \operatorname{PolyLog}\left(2, -ie^{-\operatorname{sech}^{-1}(ax)}\right) - 9i \operatorname{PolyLog}\left(2, ie^{-\operatorname{sech}^{-1}(ax)}\right) - 2a^3 x^3 + 12a^5 x^5 \operatorname{sech}^{-1}(ax)^2 - 6a^3 x^3 \sqrt{\frac{1-ax}{ax+1}}(ax+1) \operatorname{sech}^{-1}(ax)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^4*ArcSech[a*x]^2,x]

[Out] $(-9ax - 2a^3x^3 - 9a^2x\sqrt{(1-ax)/(1+ax)})(1+ax)\operatorname{ArcSech}[ax] - 6a^3x^3\sqrt{(1-ax)/(1+ax)}(1+ax)\operatorname{ArcSech}[ax] + 12a^5x^5\operatorname{ArcSech}[ax]^2 + (9I)\operatorname{ArcSech}[ax]\operatorname{Log}[1 - I/E^{\operatorname{ArcSech}[ax]}] - (9I)\operatorname{ArcSech}[ax]\operatorname{Log}[1 + I/E^{\operatorname{ArcSech}[ax]}] + (9I)\operatorname{PolyLog}[2, (-I)/E^{\operatorname{ArcSech}[ax]}] - (9I)\operatorname{PolyLog}[2, I/E^{\operatorname{ArcSech}[ax]}])/(60a^5)$

Maple [A] time = 0.404, size = 289, normalized size = 1.8

$$\frac{x^5 (\operatorname{arcsech}(ax))^2}{5} - \frac{\operatorname{arcsech}(ax) x^4}{10a} \sqrt{\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}} - \frac{3 \operatorname{arcsech}(ax) x^2}{20a^3} \sqrt{\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}} - \frac{x^3}{30a^2} - \frac{3x}{20a^4} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*arcsech(a*x)^2,x)

[Out] $1/5x^5\operatorname{arcsech}(ax)^2 - 1/10/a\operatorname{arcsech}(ax)*(-(ax-1)/a/x)^{(1/2)}*((ax+1)/a/x)^{(1/2)}*x^4 - 3/20/a^3\operatorname{arcsech}(ax)*(-(ax-1)/a/x)^{(1/2)}*((ax+1)/a/x)^{(1/2)}*x^2 - 1/30*x^3/a^2 - 3/20*x/a^4 + 3/20*I/a^5*\operatorname{arcsech}(ax)*\ln(1+I*(1/a/x+(1/a/x-1))^{(1/2)}*(1+1/a/x)^{(1/2)}) - 3/20*I/a^5*\operatorname{arcsech}(ax)*\ln(1-I*(1/a/x+(1/a/x-1))^{(1/2)}*(1+1/a/x)^{(1/2)}) + 3/20*I/a^5*\operatorname{dilog}(1+I*(1/a/x+(1/a/x-1))^{(1/2)}*(1+1/a/x)^{(1/2)}) - 3/20*I/a^5*\operatorname{dilog}(1-I*(1/a/x+(1/a/x-1))^{(1/2)}*(1+1/a/x)^{(1/2)})$

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*arcsech(a*x)^2,x, algorithm="maxima")

[Out] Timed out

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(x^4 \operatorname{arsech}(ax)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*arcsech(a*x)^2,x, algorithm="fricas")

[Out] integral(x^4*arcsech(a*x)^2, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{asech}^2(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*asech(a*x)**2,x)
```

```
[Out] Integral(x**4*asech(a*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{arsech}(ax)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arcsech(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^4*arcsech(a*x)^2, x)
```


3.2 $\int x^3 \operatorname{sech}^{-1}(ax)^2 dx$

Optimal. Leaf size=104

$$-\frac{x^2}{12a^2} - \frac{x^2 \sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)}{6a^2} - \frac{\log(x)}{3a^4} - \frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)}{3a^4} + \frac{1}{4}x^4\operatorname{sech}^{-1}(ax)^2$$

[Out] $-x^2/(12*a^2) - (\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]*(1 + a*x)*\operatorname{ArcSech}[a*x])/(3*a^4) - (x^2*\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]*(1 + a*x)*\operatorname{ArcSech}[a*x])/(6*a^2) + (x^4*\operatorname{ArcSech}[a*x]^2)/4 - \operatorname{Log}[x]/(3*a^4)$

Rubi [A] time = 0.0884237, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6285, 5418, 4185, 4184, 3475}

$$-\frac{x^2}{12a^2} - \frac{x^2 \sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)}{6a^2} - \frac{\log(x)}{3a^4} - \frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)}{3a^4} + \frac{1}{4}x^4\operatorname{sech}^{-1}(ax)^2$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^3*\operatorname{ArcSech}[a*x]^2, x]$

[Out] $-x^2/(12*a^2) - (\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]*(1 + a*x)*\operatorname{ArcSech}[a*x])/(3*a^4) - (x^2*\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]*(1 + a*x)*\operatorname{ArcSech}[a*x])/(6*a^2) + (x^4*\operatorname{ArcSech}[a*x]^2)/4 - \operatorname{Log}[x]/(3*a^4)$

Rule 6285

$\operatorname{Int}[(a_.) + \operatorname{ArcSech}[(c_.)*(x_.)]*(b_.)]^{(n_.)}(x_.)^{(m_.)}, x_Symbol] \rightarrow -\operatorname{Dist}[(c^{(m+1)})^{(-1)}, \operatorname{Subst}[\operatorname{Int}[(a + b*x)^n*\operatorname{Sech}[x]^{(m+1)}*\operatorname{Tanh}[x], x], x, \operatorname{ArcSech}[c*x]], x] /; \operatorname{FreeQ}\{a, b, c\}, x \ \&\& \operatorname{IntegerQ}[n] \ \&\& \operatorname{IntegerQ}[m] \ \&\& (\operatorname{GtQ}[n, 0] \ \|\ \operatorname{LtQ}[m, -1])$

Rule 5418

$\operatorname{Int}[(x_.)^{(m_.)}*\operatorname{Sech}[(a_.) + (b_.)*(x_.)^{(n_.)}]^{(p_.)}*\operatorname{Tanh}[(a_.) + (b_.)*(x_.)^{(n_.)}]^{(q_.)}, x_Symbol] \rightarrow -\operatorname{Simp}[(x^{(m-n+1)}*\operatorname{Sech}[a + b*x^n]^p)/(b*n*p), x] + \operatorname{Dist}[(m-n+1)/(b*n*p), \operatorname{Int}[x^{(m-n)}*\operatorname{Sech}[a + b*x^n]^p, x], x] /; \operatorname{FreeQ}\{a, b, p\}, x \ \&\& \operatorname{RationalQ}[m] \ \&\& \operatorname{IntegerQ}[n] \ \&\& \operatorname{GeQ}[m-n, 0] \ \&\& \operatorname{EqQ}[q, 1]$

Rule 4185

$\operatorname{Int}[(\operatorname{csc}[(e_.) + (f_.)*(x_.)]*(b_.))^{(n_.)}*((c_.) + (d_.)*(x_.)), x_Symbol] \rightarrow -\operatorname{Simp}[(b^2*(c + d*x)*\operatorname{Cot}[e + f*x]*(b*\operatorname{Csc}[e + f*x])^{(n-2)})/(f*(n-1)), x] + (\operatorname{Dist}[(b^2*(n-2))/(n-1), \operatorname{Int}[(c + d*x)*(b*\operatorname{Csc}[e + f*x])^{(n-2)}, x], x] - \operatorname{Simp}[(b^2*d*(b*\operatorname{Csc}[e + f*x])^{(n-2)})/(f^2*(n-1)*(n-2)), x]) /; \operatorname{FreeQ}\{b, c, d, e, f\}, x \ \&\& \operatorname{GtQ}[n, 1] \ \&\& \operatorname{NeQ}[n, 2]$

Rule 4184

$\operatorname{Int}[\operatorname{csc}[(e_.) + (f_.)*(x_.)]^2*((c_.) + (d_.)*(x_.))^{(m_.)}, x_Symbol] \rightarrow -\operatorname{Simp}[(c + d*x)^m*\operatorname{Cot}[e + f*x]/f, x] + \operatorname{Dist}[(d*m)/f, \operatorname{Int}[(c + d*x)^{(m-1)}*\operatorname{Cot}[e + f*x], x], x] /; \operatorname{FreeQ}\{c, d, e, f\}, x \ \&\& \operatorname{GtQ}[m, 0]$

Rule 3475

`Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d *x], x]]/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\begin{aligned} \int x^3 \operatorname{sech}^{-1}(ax)^2 dx &= -\frac{\operatorname{Subst}\left(\int x^2 \operatorname{sech}^4(x) \tanh(x) dx, x, \operatorname{sech}^{-1}(ax)\right)}{a^4} \\ &= \frac{1}{4} x^4 \operatorname{sech}^{-1}(ax)^2 - \frac{\operatorname{Subst}\left(\int x \operatorname{sech}^4(x) dx, x, \operatorname{sech}^{-1}(ax)\right)}{2a^4} \\ &= -\frac{x^2}{12a^2} - \frac{x^2 \sqrt{\frac{1-ax}{1+ax}} (1+ax) \operatorname{sech}^{-1}(ax)}{6a^2} + \frac{1}{4} x^4 \operatorname{sech}^{-1}(ax)^2 - \frac{\operatorname{Subst}\left(\int x \operatorname{sech}^2(x) dx, x, \operatorname{sech}^{-1}(ax)\right)}{3a^4} \\ &= -\frac{x^2}{12a^2} - \frac{\sqrt{\frac{1-ax}{1+ax}} (1+ax) \operatorname{sech}^{-1}(ax)}{3a^4} - \frac{x^2 \sqrt{\frac{1-ax}{1+ax}} (1+ax) \operatorname{sech}^{-1}(ax)}{6a^2} + \frac{1}{4} x^4 \operatorname{sech}^{-1}(ax)^2 + \frac{\operatorname{Subst}\left(\int \frac{1}{x} dx, x, \operatorname{sech}^{-1}(ax)\right)}{3a^4} \\ &= -\frac{x^2}{12a^2} - \frac{\sqrt{\frac{1-ax}{1+ax}} (1+ax) \operatorname{sech}^{-1}(ax)}{3a^4} - \frac{x^2 \sqrt{\frac{1-ax}{1+ax}} (1+ax) \operatorname{sech}^{-1}(ax)}{6a^2} + \frac{1}{4} x^4 \operatorname{sech}^{-1}(ax)^2 - \frac{\log(x)}{3a^4} \end{aligned}$$

Mathematica [A] time = 0.100651, size = 77, normalized size = 0.74

$$\frac{a^2 x^2 - 3a^4 x^4 \operatorname{sech}^{-1}(ax)^2 + 2\sqrt{\frac{1-ax}{ax+1}} (a^3 x^3 + a^2 x^2 + 2ax + 2) \operatorname{sech}^{-1}(ax) + 4 \log(x)}{12a^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*ArcSech[a*x]^2,x]

[Out] -(a^2*x^2 + 2*sqrt[(1 - a*x)/(1 + a*x)]*(2 + 2*a*x + a^2*x^2 + a^3*x^3)*ArcSech[a*x] - 3*a^4*x^4*ArcSech[a*x]^2 + 4*Log[x])/(12*a^4)

Maple [A] time = 0.328, size = 151, normalized size = 1.5

$$-\frac{\operatorname{arcsech}(ax)}{3a^4} + \frac{x^4 (\operatorname{arcsech}(ax))^2}{4} - \frac{\operatorname{arcsech}(ax) x^3}{6a} \sqrt{\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}} - \frac{\operatorname{arcsech}(ax) x}{3a^3} \sqrt{\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}} - \frac{x^2}{12a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*arcsech(a*x)^2,x)

[Out] -1/3/a^4*arcsech(a*x)+1/4*x^4*arcsech(a*x)^2-1/6/a*arcsech(a*x)*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)*x^3-1/3/a^3*arcsech(a*x)*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)*x-1/12*x^2/a^2+1/3/a^4*ln(1+(1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))^2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{ar} \operatorname{sech}(ax)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arcsech(a*x)^2,x, algorithm="maxima")

[Out] integrate(x^3*arcsech(a*x)^2, x)

Fricas [A] time = 1.69881, size = 273, normalized size = 2.62

$$\frac{3 a^4 x^4 \log\left(\frac{ax \sqrt{-\frac{a^2 x^2 - 1}{a^2 x^2}} + 1}{ax}\right)^2 - a^2 x^2 - 2(a^3 x^3 + 2ax) \sqrt{-\frac{a^2 x^2 - 1}{a^2 x^2}} \log\left(\frac{ax \sqrt{-\frac{a^2 x^2 - 1}{a^2 x^2}} + 1}{ax}\right) - 4 \log(x)}{12 a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arcsech(a*x)^2,x, algorithm="fricas")

[Out] 1/12*(3*a^4*x^4*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x))^2 - a^2*x^2 - 2*(a^3*x^3 + 2*a*x)*sqrt(-(a^2*x^2 - 1)/(a^2*x^2))*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x)) - 4*log(x))/a^4

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{asech}^2(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*asech(a*x)**2,x)

[Out] Integral(x**3*asech(a*x)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{arsech}(ax)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arcsech(a*x)^2,x, algorithm="giac")

[Out] integrate(x^3*arcsech(a*x)^2, x)

3.3 $\int x^2 \operatorname{sech}^{-1}(ax)^2 dx$

Optimal. Leaf size=117

$$\frac{i \operatorname{PolyLog}\left(2, -ie^{\operatorname{sech}^{-1}(ax)}\right)}{3a^3} - \frac{i \operatorname{PolyLog}\left(2, ie^{\operatorname{sech}^{-1}(ax)}\right)}{3a^3} - \frac{x}{3a^2} - \frac{x \sqrt{\frac{1-ax}{ax+1}}(ax+1) \operatorname{sech}^{-1}(ax)}{3a^2} - \frac{2 \operatorname{sech}^{-1}(ax) \tan^{-1}\left(e^{\operatorname{sech}^{-1}(ax)}\right)}{3a^3}$$

```
[Out] -x/(3*a^2) - (x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x])/(3*a^2) +
(x^3*ArcSech[a*x]^2)/3 - (2*ArcSech[a*x]*ArcTan[E^ArcSech[a*x]])/(3*a^3) +
((I/3)*PolyLog[2, (-I)*E^ArcSech[a*x]])/a^3 - ((I/3)*PolyLog[2, I*E^ArcSech[a*x]])/a^3
```

Rubi [A] time = 0.0970499, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$, Rules used = {6285, 5418, 4185, 4180, 2279, 2391}

$$\frac{i \operatorname{PolyLog}\left(2, -ie^{\operatorname{sech}^{-1}(ax)}\right)}{3a^3} - \frac{i \operatorname{PolyLog}\left(2, ie^{\operatorname{sech}^{-1}(ax)}\right)}{3a^3} - \frac{x}{3a^2} - \frac{x \sqrt{\frac{1-ax}{ax+1}}(ax+1) \operatorname{sech}^{-1}(ax)}{3a^2} - \frac{2 \operatorname{sech}^{-1}(ax) \tan^{-1}\left(e^{\operatorname{sech}^{-1}(ax)}\right)}{3a^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*ArcSech[a*x]^2,x]
```

```
[Out] -x/(3*a^2) - (x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x])/(3*a^2) +
(x^3*ArcSech[a*x]^2)/3 - (2*ArcSech[a*x]*ArcTan[E^ArcSech[a*x]])/(3*a^3) +
((I/3)*PolyLog[2, (-I)*E^ArcSech[a*x]])/a^3 - ((I/3)*PolyLog[2, I*E^ArcSech[a*x]])/a^3
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))^ (n_.)*(x_)^(m_.), x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 5418

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)
^(n_.)]^(q_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^p)/(b*n*p)
, x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x] /;
FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ
[q, 1]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^ (n_.)*((c_.) + (d_.)*(x_.)), x_Symbol] :>
-Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_.)]*((c_.) + (d_.)*(x_)
)^ (m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(-
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
```

- E^{-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^{-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]}}

Rule 2279

Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))ⁿ], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] :> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rubi steps

$$\begin{aligned} \int x^2 \operatorname{sech}^{-1}(ax)^2 dx &= -\frac{\operatorname{Subst}\left(\int x^2 \operatorname{sech}^3(x) \tanh(x) dx, x, \operatorname{sech}^{-1}(ax)\right)}{a^3} \\ &= \frac{1}{3} x^3 \operatorname{sech}^{-1}(ax)^2 - \frac{2 \operatorname{Subst}\left(\int x \operatorname{sech}^3(x) dx, x, \operatorname{sech}^{-1}(ax)\right)}{3a^3} \\ &= -\frac{x}{3a^2} - \frac{x \sqrt{\frac{1-ax}{1+ax}} (1+ax) \operatorname{sech}^{-1}(ax)}{3a^2} + \frac{1}{3} x^3 \operatorname{sech}^{-1}(ax)^2 - \frac{\operatorname{Subst}\left(\int x \operatorname{sech}(x) dx, x, \operatorname{sech}^{-1}(ax)\right)}{3a^3} \\ &= -\frac{x}{3a^2} - \frac{x \sqrt{\frac{1-ax}{1+ax}} (1+ax) \operatorname{sech}^{-1}(ax)}{3a^2} + \frac{1}{3} x^3 \operatorname{sech}^{-1}(ax)^2 - \frac{2 \operatorname{sech}^{-1}(ax) \tan^{-1}\left(e^{\operatorname{sech}^{-1}(ax)}\right)}{3a^3} + \dots \\ &= -\frac{x}{3a^2} - \frac{x \sqrt{\frac{1-ax}{1+ax}} (1+ax) \operatorname{sech}^{-1}(ax)}{3a^2} + \frac{1}{3} x^3 \operatorname{sech}^{-1}(ax)^2 - \frac{2 \operatorname{sech}^{-1}(ax) \tan^{-1}\left(e^{\operatorname{sech}^{-1}(ax)}\right)}{3a^3} + \dots \\ &= -\frac{x}{3a^2} - \frac{x \sqrt{\frac{1-ax}{1+ax}} (1+ax) \operatorname{sech}^{-1}(ax)}{3a^2} + \frac{1}{3} x^3 \operatorname{sech}^{-1}(ax)^2 - \frac{2 \operatorname{sech}^{-1}(ax) \tan^{-1}\left(e^{\operatorname{sech}^{-1}(ax)}\right)}{3a^3} + \dots \end{aligned}$$

Mathematica [A] time = 0.231495, size = 138, normalized size = 1.18

$$\frac{i \operatorname{PolyLog}\left(2, -ie^{-\operatorname{sech}^{-1}(ax)}\right) - i \operatorname{PolyLog}\left(2, ie^{-\operatorname{sech}^{-1}(ax)}\right) + a^3 x^3 \operatorname{sech}^{-1}(ax)^2 - ax - ax \sqrt{\frac{1-ax}{ax+1}} (ax+1) \operatorname{sech}^{-1}(ax) + i \operatorname{sech}^{-1}(ax)}{3a^3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2*ArcSech[a*x]^2,x]

[Out] $(-(a*x) - a*x*\sqrt{((1 - a*x)/(1 + a*x))}*(1 + a*x)*\operatorname{ArcSech}[a*x] + a^3*x^3*\operatorname{ArcSech}[a*x]^2 + I*\operatorname{ArcSech}[a*x]*\operatorname{Log}[1 - I/E^{\operatorname{ArcSech}[a*x]}] - I*\operatorname{ArcSech}[a*x]*\operatorname{Log}[1 + I/E^{\operatorname{ArcSech}[a*x]}] + I*\operatorname{PolyLog}[2, (-I)/E^{\operatorname{ArcSech}[a*x]}] - I*\operatorname{PolyLog}[2, I/E^{\operatorname{ArcSech}[a*x]}])/(3*a^3)$

Maple [A] time = 0.325, size = 240, normalized size = 2.1

$$\frac{x^3 (\operatorname{arcsech}(ax))^2}{3} - \frac{\operatorname{arcsech}(ax) x^2}{3a} \sqrt{\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}} - \frac{x}{3a^2} + \frac{i}{3} \frac{\operatorname{arcsech}(ax)}{a^3} \ln\left(1 + i\left(\frac{1}{ax} + \sqrt{\frac{1}{ax} - 1} \sqrt{1 + \frac{1}{ax}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*arcsech(a*x)^2,x)`

[Out] $\frac{1}{3}x^3\operatorname{arcsech}(ax)^2 - \frac{1}{3}a\operatorname{arcsech}(ax)\left(-\frac{ax-1}{a/x}\right)^{1/2}\left(\frac{ax+1}{a/x}\right)^{1/2}x^2 - \frac{1}{3}x/a^2 + \frac{1}{3}I/a^3\operatorname{arcsech}(ax)\ln\left(1+I\left(\frac{1}{a/x}+\left(\frac{1}{a/x}-1\right)^{1/2}\right)\left(1+\frac{1}{a/x}\right)^{1/2}\right)\right) - \frac{1}{3}I/a^3\operatorname{arcsech}(ax)\ln\left(1-I\left(\frac{1}{a/x}+\left(\frac{1}{a/x}-1\right)^{1/2}\right)\left(1+\frac{1}{a/x}\right)^{1/2}\right)\right) + \frac{1}{3}I/a^3\operatorname{dilog}\left(1+I\left(\frac{1}{a/x}+\left(\frac{1}{a/x}-1\right)^{1/2}\right)\left(1+\frac{1}{a/x}\right)^{1/2}\right)\right) - \frac{1}{3}I/a^3\operatorname{dilog}\left(1-I\left(\frac{1}{a/x}+\left(\frac{1}{a/x}-1\right)^{1/2}\right)\left(1+\frac{1}{a/x}\right)^{1/2}\right)\right)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arsech}(ax)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arcsech(a*x)^2,x, algorithm="maxima")`

[Out] `integrate(x^2*arcsech(a*x)^2, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(x^2 \operatorname{arsech}(ax)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arcsech(a*x)^2,x, algorithm="fricas")`

[Out] `integral(x^2*arcsech(a*x)^2, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{asech}^2(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*asech(a*x)**2,x)`

[Out] `Integral(x**2*asech(a*x)**2, x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arsech}(ax)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*arcsech(a*x)^2,x, algorithm="giac")`

[Out] `integrate(x^2*arcsech(a*x)^2, x)`

3.4 $\int x \operatorname{sech}^{-1}(ax)^2 dx$

Optimal. Leaf size=53

$$-\frac{\log(x)}{a^2} - \frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)}{a^2} + \frac{1}{2}x^2\operatorname{sech}^{-1}(ax)^2$$

[Out] $-\left(\frac{\sqrt{(1-ax)/(1+ax)}(1+ax)\operatorname{ArcSech}[ax]}{a^2}\right) + (x^2\operatorname{ArcSech}[ax]^2)/2 - \operatorname{Log}[x]/a^2$

Rubi [A] time = 0.0567211, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6285, 5418, 4184, 3475}

$$-\frac{\log(x)}{a^2} - \frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)}{a^2} + \frac{1}{2}x^2\operatorname{sech}^{-1}(ax)^2$$

Antiderivative was successfully verified.

[In] Int[x*ArcSech[a*x]^2,x]

[Out] $-\left(\frac{\sqrt{(1-ax)/(1+ax)}(1+ax)\operatorname{ArcSech}[ax]}{a^2}\right) + (x^2\operatorname{ArcSech}[ax]^2)/2 - \operatorname{Log}[x]/a^2$

Rule 6285

Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))^n_*(x_)^(m_.), x_Symbol] := -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

Rule 5418

Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)^(n_.)]^(q_.), x_Symbol] := -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^p)/(b*n*p), x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x] /; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ[q, 1]

Rule 4184

Int[csc[(e_.) + (f_.)*(x_.)]^2*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol] := -Simp[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3475

Int[tan[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int x \operatorname{sech}^{-1}(ax)^2 dx &= -\frac{\operatorname{Subst}\left(\int x^2 \operatorname{sech}^2(x) \tanh(x) dx, x, \operatorname{sech}^{-1}(ax)\right)}{a^2} \\
&= \frac{1}{2} x^2 \operatorname{sech}^{-1}(ax)^2 - \frac{\operatorname{Subst}\left(\int x \operatorname{sech}^2(x) dx, x, \operatorname{sech}^{-1}(ax)\right)}{a^2} \\
&= -\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax) \operatorname{sech}^{-1}(ax)}{a^2} + \frac{1}{2} x^2 \operatorname{sech}^{-1}(ax)^2 + \frac{\operatorname{Subst}\left(\int \tanh(x) dx, x, \operatorname{sech}^{-1}(ax)\right)}{a^2} \\
&= -\frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax) \operatorname{sech}^{-1}(ax)}{a^2} + \frac{1}{2} x^2 \operatorname{sech}^{-1}(ax)^2 - \frac{\log(x)}{a^2}
\end{aligned}$$

Mathematica [A] time = 0.0595508, size = 53, normalized size = 1.

$$-\frac{\log(x)}{a^2} - \frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1) \operatorname{sech}^{-1}(ax)}{a^2} + \frac{1}{2} x^2 \operatorname{sech}^{-1}(ax)^2$$

Antiderivative was successfully verified.

[In] Integrate[x*ArcSech[a*x]^2,x]

[Out] -((Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x])/a^2) + (x^2*ArcSech[a*x]^2)/2 - Log[x]/a^2

Maple [B] time = 0.258, size = 101, normalized size = 1.9

$$-\frac{\operatorname{arcsech}(ax)}{a^2} + \frac{x^2 (\operatorname{arcsech}(ax))^2}{2} - \frac{\operatorname{arcsech}(ax) x}{a} \sqrt{\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}} + \frac{1}{a^2} \ln\left(1 + \left(\frac{1}{ax} + \sqrt{\frac{1}{ax} - 1} \sqrt{1 + \frac{1}{ax}}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*arcsech(a*x)^2,x)

[Out] -1/a^2*arcsech(a*x)+1/2*x^2*arcsech(a*x)^2-1/a*arcsech(a*x)*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)*x+1/a^2*ln(1+(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2))^2)

Maxima [A] time = 1.04775, size = 54, normalized size = 1.02

$$\frac{1}{2} x^2 \operatorname{ar} \operatorname{sech}(ax)^2 - \frac{x \sqrt{\frac{1}{a^2 x^2} - 1} \operatorname{ar} \operatorname{sech}(ax)}{a} - \frac{\log(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*arcsech(a*x)^2,x, algorithm="maxima")

[Out] 1/2*x^2*arcsech(a*x)^2 - x*sqrt(1/(a^2*x^2) - 1)*arcsech(a*x)/a - log(x)/a^2

Fricas [B] time = 1.63633, size = 236, normalized size = 4.45

$$\frac{a^2 x^2 \log\left(\frac{ax\sqrt{-\frac{a^2 x^2 - 1}{a^2 x^2}} + 1}{ax}\right)^2 - 2ax\sqrt{-\frac{a^2 x^2 - 1}{a^2 x^2}} \log\left(\frac{ax\sqrt{-\frac{a^2 x^2 - 1}{a^2 x^2}} + 1}{ax}\right) - 2 \log(x)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*arcsech(a*x)^2,x, algorithm="fricas")

[Out] 1/2*(a^2*x^2*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x))^2 - 2*a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2))*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x)) - 2*log(x))/a^2

Sympy [A] time = 3.11023, size = 42, normalized size = 0.79

$$\begin{cases} \frac{x^2 \operatorname{asech}^2(ax)}{2} - \frac{\sqrt{-a^2 x^2 + 1} \operatorname{asech}(ax)}{a^2} - \frac{\log(x)}{a^2} & \text{for } a \neq 0 \\ \infty x^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*asech(a*x)**2,x)

[Out] Piecewise((x**2*asech(a*x)**2/2 - sqrt(-a**2*x**2 + 1)*asech(a*x)/a**2 - log(x)/a**2, Ne(a, 0)), (oo*x**2, True))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{arsech}(ax)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*arcsech(a*x)^2,x, algorithm="giac")

[Out] integrate(x*arcsech(a*x)^2, x)

3.5 $\int \operatorname{sech}^{-1}(ax)^2 dx$

Optimal. Leaf size=63

$$\frac{2i\operatorname{PolyLog}\left(2, -ie^{\operatorname{sech}^{-1}(ax)}\right)}{a} - \frac{2i\operatorname{PolyLog}\left(2, ie^{\operatorname{sech}^{-1}(ax)}\right)}{a} + x\operatorname{sech}^{-1}(ax)^2 - \frac{4\operatorname{sech}^{-1}(ax)\tan^{-1}\left(e^{\operatorname{sech}^{-1}(ax)}\right)}{a}$$

[Out] $x*\operatorname{ArcSech}[a*x]^2 - (4*\operatorname{ArcSech}[a*x]*\operatorname{ArcTan}[E^{\operatorname{ArcSech}[a*x]}])/a + ((2*I)*\operatorname{PolyLog}[2, (-I)*E^{\operatorname{ArcSech}[a*x]}])/a - ((2*I)*\operatorname{PolyLog}[2, I*E^{\operatorname{ArcSech}[a*x]}])/a$

Rubi [A] time = 0.0550174, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {6279, 5418, 4180, 2279, 2391}

$$\frac{2i\operatorname{PolyLog}\left(2, -ie^{\operatorname{sech}^{-1}(ax)}\right)}{a} - \frac{2i\operatorname{PolyLog}\left(2, ie^{\operatorname{sech}^{-1}(ax)}\right)}{a} + x\operatorname{sech}^{-1}(ax)^2 - \frac{4\operatorname{sech}^{-1}(ax)\tan^{-1}\left(e^{\operatorname{sech}^{-1}(ax)}\right)}{a}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{ArcSech}[a*x]^2, x]$

[Out] $x*\operatorname{ArcSech}[a*x]^2 - (4*\operatorname{ArcSech}[a*x]*\operatorname{ArcTan}[E^{\operatorname{ArcSech}[a*x]}])/a + ((2*I)*\operatorname{PolyLog}[2, (-I)*E^{\operatorname{ArcSech}[a*x]}])/a - ((2*I)*\operatorname{PolyLog}[2, I*E^{\operatorname{ArcSech}[a*x]}])/a$

Rule 6279

$\operatorname{Int}[(a_. + \operatorname{ArcSech}[c_.*(x_.)]*(b_.))^n, x_Symbol] :> -\operatorname{Dist}[c^{(-1)}, \operatorname{Subst}[\operatorname{Int}[(a + b*x)^n*\operatorname{Sech}[x]*\operatorname{Tanh}[x], x], x, \operatorname{ArcSech}[c*x]], x] /; \operatorname{FreeQ}\{a, b, c, n\}, x\} \&\& \operatorname{IGtQ}[n, 0]$

Rule 5418

$\operatorname{Int}[(x_)^{(m_.)*\operatorname{Sech}[(a_. + (b_.)*(x_)^{(n_.)]^{(p_.)*\operatorname{Tanh}[(a_. + (b_.)*(x_)^{(n_.)]^{(q_.)}], x_Symbol] :> -\operatorname{Simp}[(x^{(m-n+1)}*\operatorname{Sech}[a + b*x^n]^p)/(b*n*p), x] + \operatorname{Dist}[(m-n+1)/(b*n*p), \operatorname{Int}[x^{(m-n)}*\operatorname{Sech}[a + b*x^n]^p, x], x] /; \operatorname{FreeQ}\{a, b, p\}, x\} \&\& \operatorname{RationalQ}[m] \&\& \operatorname{IntegerQ}[n] \&\& \operatorname{GeQ}[m-n, 0] \&\& \operatorname{EqQ}[q, 1]$

Rule 4180

$\operatorname{Int}[\operatorname{csc}[(e_. + \operatorname{Pi}*(k_. + (\operatorname{Complex}[0, fz_])*(f_.)*(x_.)]*((c_. + (d_.)*(x_.))^{(m_.)}], x_Symbol] :> \operatorname{Simp}[(-2*(c + d*x)^m*\operatorname{ArcTanh}[E^{-(I*e) + f*fz*x}]/E^{(I*k*Pi)})/(f*fz*I), x] + (-\operatorname{Dist}[(d*m)/(f*fz*I), \operatorname{Int}[(c + d*x)^{(m-1)}*\operatorname{Log}[1 - E^{-(I*e) + f*fz*x}]/E^{(I*k*Pi)}], x], x] + \operatorname{Dist}[(d*m)/(f*fz*I), \operatorname{Int}[(c + d*x)^{(m-1)}*\operatorname{Log}[1 + E^{-(I*e) + f*fz*x}]/E^{(I*k*Pi)}], x], x] /; \operatorname{FreeQ}\{c, d, e, f, fz\}, x\} \&\& \operatorname{IntegerQ}[2*k] \&\& \operatorname{IGtQ}[m, 0]$

Rule 2279

$\operatorname{Int}[\operatorname{Log}[(a_. + (b_.)*((F_)^{((e_.)*((c_. + (d_.)*(x_.)))^{(n_.)}], x_Symbol] :> \operatorname{Dist}[1/(d*e*n*\operatorname{Log}[F]), \operatorname{Subst}[\operatorname{Int}[\operatorname{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})^n], x] /; \operatorname{FreeQ}\{F, a, b, c, d, e, n\}, x\} \&\& \operatorname{GtQ}[a, 0]$

Rule 2391

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rubi steps

$$\begin{aligned}
 \int \operatorname{sech}^{-1}(ax)^2 dx &= -\frac{\operatorname{Subst}\left(\int x^2 \operatorname{sech}(x) \tanh(x) dx, x, \operatorname{sech}^{-1}(ax)\right)}{a} \\
 &= x \operatorname{sech}^{-1}(ax)^2 - \frac{2 \operatorname{Subst}\left(\int x \operatorname{sech}(x) dx, x, \operatorname{sech}^{-1}(ax)\right)}{a} \\
 &= x \operatorname{sech}^{-1}(ax)^2 - \frac{4 \operatorname{sech}^{-1}(ax) \tan^{-1}\left(e^{\operatorname{sech}^{-1}(ax)}\right)}{a} + \frac{(2i) \operatorname{Subst}\left(\int \log(1 - ie^x) dx, x, \operatorname{sech}^{-1}(ax)\right)}{a} \\
 &= x \operatorname{sech}^{-1}(ax)^2 - \frac{4 \operatorname{sech}^{-1}(ax) \tan^{-1}\left(e^{\operatorname{sech}^{-1}(ax)}\right)}{a} + \frac{(2i) \operatorname{Subst}\left(\int \frac{\log(1-ix)}{x} dx, x, e^{\operatorname{sech}^{-1}(ax)}\right)}{a} - \frac{(2i) \operatorname{Subst}\left(\int \frac{\log(1+ix)}{x} dx, x, e^{\operatorname{sech}^{-1}(ax)}\right)}{a} \\
 &= x \operatorname{sech}^{-1}(ax)^2 - \frac{4 \operatorname{sech}^{-1}(ax) \tan^{-1}\left(e^{\operatorname{sech}^{-1}(ax)}\right)}{a} + \frac{2i \operatorname{Li}_2\left(-ie^{\operatorname{sech}^{-1}(ax)}\right)}{a} - \frac{2i \operatorname{Li}_2\left(ie^{\operatorname{sech}^{-1}(ax)}\right)}{a}
 \end{aligned}$$

Mathematica [A] time = 0.185302, size = 90, normalized size = 1.43

$$\frac{i \left(2 \operatorname{PolyLog}\left(2, -ie^{-\operatorname{sech}^{-1}(ax)}\right) - 2 \operatorname{PolyLog}\left(2, ie^{-\operatorname{sech}^{-1}(ax)}\right) + \operatorname{sech}^{-1}(ax) \left(-iax \operatorname{sech}^{-1}(ax) + 2 \log\left(1 - ie^{-\operatorname{sech}^{-1}(ax)}\right) \right) \right)}{a}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcSech[a*x]^2, x]

[Out] (I*(ArcSech[a*x]*((-I)*a*x*ArcSech[a*x] + 2*Log[1 - I/E^ArcSech[a*x]]) - 2*Log[1 + I/E^ArcSech[a*x]]) + 2*PolyLog[2, (-I)/E^ArcSech[a*x]] - 2*PolyLog[2, I/E^ArcSech[a*x]]))/a

Maple [A] time = 0.272, size = 190, normalized size = 3.

$$x (\operatorname{arcsech}(ax))^2 + \frac{2i \operatorname{arcsech}(ax)}{a} \ln\left(1 + i\left(\frac{1}{ax} + \sqrt{\frac{1}{ax} - 1} \sqrt{1 + \frac{1}{ax}}\right)\right) - \frac{2i \operatorname{arcsech}(ax)}{a} \ln\left(1 - i\left(\frac{1}{ax} + \sqrt{\frac{1}{ax} - 1} \sqrt{1 + \frac{1}{ax}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsech(a*x)^2, x)

[Out] x*arcsech(a*x)^2+2*I/a*arcsech(a*x)*ln(1+I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))-2*I/a*arcsech(a*x)*ln(1-I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))-2*I/a*dilog(1-I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))+2*I/a*dilog(1+I*(1/a/x+(1/a/x-1)^(1/2)*(1+1/a/x)^(1/2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$x \log\left(\sqrt{ax+1}\sqrt{-ax+1}+1\right)^2 - \int -\frac{a^2x^2 \log(a)^2 + (a^2x^2 - 1) \log(x)^2 + (a^2x^2 \log(a)^2 + (a^2x^2 - 1) \log(x)^2 - \log(a))}{a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x)^2,x, algorithm="maxima")
```

```
[Out] x*log(sqrt(a*x + 1)*sqrt(-a*x + 1) + 1)^2 - integrate(-(a^2*x^2*log(a)^2 +
(a^2*x^2 - 1)*log(x)^2 + (a^2*x^2*log(a)^2 + (a^2*x^2 - 1)*log(x)^2 - log(a)
)^2 + 2*(a^2*x^2*log(a) - log(a))*log(x))*sqrt(a*x + 1)*sqrt(-a*x + 1) - 2*
(a^2*x^2*log(a) + (a^2*x^2*(log(a) + 1) + (a^2*x^2 - 1)*log(x) - log(a))*sq
rt(a*x + 1)*sqrt(-a*x + 1) + (a^2*x^2 - 1)*log(x) - log(a))*log(sqrt(a*x +
1)*sqrt(-a*x + 1) + 1) - log(a)^2 + 2*(a^2*x^2*log(a) - log(a))*log(x))/(a^
2*x^2 + (a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(-a*x + 1) - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(\text{arsech}(ax)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(arcsech(a*x)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \text{asech}^2(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(asech(a*x)**2,x)
```

```
[Out] Integral(asech(a*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \text{arsech}(ax)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(arcsech(a*x)^2, x)
```

3.6 $\int \frac{\operatorname{sech}^{-1}(ax)^2}{x} dx$

Optimal. Leaf size=64

$$-\operatorname{sech}^{-1}(ax)\operatorname{PolyLog}\left(2, -e^{2\operatorname{sech}^{-1}(ax)}\right) + \frac{1}{2}\operatorname{PolyLog}\left(3, -e^{2\operatorname{sech}^{-1}(ax)}\right) + \frac{1}{3}\operatorname{sech}^{-1}(ax)^3 - \operatorname{sech}^{-1}(ax)^2 \log\left(e^{2\operatorname{sech}^{-1}(ax)}\right)$$

```
[Out] ArcSech[a*x]^3/3 - ArcSech[a*x]^2*Log[1 + E^(2*ArcSech[a*x])] - ArcSech[a*x]
]*PolyLog[2, -E^(2*ArcSech[a*x])] + PolyLog[3, -E^(2*ArcSech[a*x])]/2
```

Rubi [A] time = 0.0915422, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$, Rules used = {6285, 3718, 2190, 2531, 2282, 6589}

$$-\operatorname{sech}^{-1}(ax)\operatorname{PolyLog}\left(2, -e^{2\operatorname{sech}^{-1}(ax)}\right) + \frac{1}{2}\operatorname{PolyLog}\left(3, -e^{2\operatorname{sech}^{-1}(ax)}\right) + \frac{1}{3}\operatorname{sech}^{-1}(ax)^3 - \operatorname{sech}^{-1}(ax)^2 \log\left(e^{2\operatorname{sech}^{-1}(ax)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a*x]^2/x, x]
```

```
[Out] ArcSech[a*x]^3/3 - ArcSech[a*x]^2*Log[1 + E^(2*ArcSech[a*x])] - ArcSech[a*x]
]*PolyLog[2, -E^(2*ArcSech[a*x])] + PolyLog[3, -E^(2*ArcSech[a*x])]/2
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_)])*(b_.))^(n_.)*(x_)^(m_.), x_Symbol] := -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_)^(m_.))*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
+ d*x)^m*E^(2*(-I*e) + f*fz*x))]/(1 + E^(2*(-I*e) + f*fz*x)), x], x] /;
FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_)^(m_.)))/
((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_)^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)
))^n]])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n]], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{sech}^{-1}(ax)^2}{x} dx &= -\operatorname{Subst}\left(\int x^2 \tanh(x) dx, x, \operatorname{sech}^{-1}(ax)\right) \\ &= \frac{1}{3} \operatorname{sech}^{-1}(ax)^3 - 2 \operatorname{Subst}\left(\int \frac{e^{2x} x^2}{1 + e^{2x}} dx, x, \operatorname{sech}^{-1}(ax)\right) \\ &= \frac{1}{3} \operatorname{sech}^{-1}(ax)^3 - \operatorname{sech}^{-1}(ax)^2 \log\left(1 + e^{2 \operatorname{sech}^{-1}(ax)}\right) + 2 \operatorname{Subst}\left(\int x \log(1 + e^{2x}) dx, x, \operatorname{sech}^{-1}(ax)\right) \\ &= \frac{1}{3} \operatorname{sech}^{-1}(ax)^3 - \operatorname{sech}^{-1}(ax)^2 \log\left(1 + e^{2 \operatorname{sech}^{-1}(ax)}\right) - \operatorname{sech}^{-1}(ax) \operatorname{Li}_2\left(-e^{2 \operatorname{sech}^{-1}(ax)}\right) + \operatorname{Subst}\left(\int \operatorname{Li}_2\left(-e^{2x}\right) dx, x, \operatorname{sech}^{-1}(ax)\right) \\ &= \frac{1}{3} \operatorname{sech}^{-1}(ax)^3 - \operatorname{sech}^{-1}(ax)^2 \log\left(1 + e^{2 \operatorname{sech}^{-1}(ax)}\right) - \operatorname{sech}^{-1}(ax) \operatorname{Li}_2\left(-e^{2 \operatorname{sech}^{-1}(ax)}\right) + \frac{1}{2} \operatorname{Subst}\left(\int \frac{\operatorname{Li}_2\left(-e^{2x}\right)}{x} dx, x, \operatorname{sech}^{-1}(ax)\right) \\ &= \frac{1}{3} \operatorname{sech}^{-1}(ax)^3 - \operatorname{sech}^{-1}(ax)^2 \log\left(1 + e^{2 \operatorname{sech}^{-1}(ax)}\right) - \operatorname{sech}^{-1}(ax) \operatorname{Li}_2\left(-e^{2 \operatorname{sech}^{-1}(ax)}\right) + \frac{1}{2} \operatorname{Li}_3\left(-e^{2 \operatorname{sech}^{-1}(ax)}\right) \end{aligned}$$

Mathematica [A] time = 0.0309726, size = 63, normalized size = 0.98

$$\operatorname{sech}^{-1}(ax) \operatorname{PolyLog}\left(2, -e^{-2 \operatorname{sech}^{-1}(ax)}\right) + \frac{1}{2} \operatorname{PolyLog}\left(3, -e^{-2 \operatorname{sech}^{-1}(ax)}\right) - \frac{1}{3} \operatorname{sech}^{-1}(ax)^3 - \operatorname{sech}^{-1}(ax)^2 \log\left(e^{-2 \operatorname{sech}^{-1}(ax)} + 1\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcSech[a*x]^2/x, x]
```

```
[Out] -ArcSech[a*x]^3/3 - ArcSech[a*x]^2*Log[1 + E^(-2*ArcSech[a*x])] + ArcSech[a
*x]*PolyLog[2, -E^(-2*ArcSech[a*x])] + PolyLog[3, -E^(-2*ArcSech[a*x])]/2
```

Maple [A] time = 0.241, size = 136, normalized size = 2.1

$$\frac{(\operatorname{arcsech}(ax))^3}{3} - (\operatorname{arcsech}(ax))^2 \ln\left(1 + \left(\frac{1}{ax} + \sqrt{\frac{1}{ax} - 1} \sqrt{1 + \frac{1}{ax}}\right)^2\right) - \operatorname{arcsech}(ax) \operatorname{polylog}\left(2, -\left(\frac{1}{ax} + \sqrt{\frac{1}{ax} - 1} \sqrt{1 + \frac{1}{ax}}\right)^2\right) + \frac{1}{2} \operatorname{polylog}\left(3, -\left(\frac{1}{ax} + \sqrt{\frac{1}{ax} - 1} \sqrt{1 + \frac{1}{ax}}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(a*x)^2/x, x)
```

```
[Out] 1/3*arcsech(a*x)^3-arcsech(a*x)^2*ln(1+(1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/
2))^2)-arcsech(a*x)*polylog(2,-(1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))^2)+1
/2*polylog(3,-(1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arosech}(ax)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(a*x)^2/x,x, algorithm="maxima")

[Out] integrate(arcsech(a*x)^2/x, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arosech}(ax)^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(a*x)^2/x,x, algorithm="fricas")

[Out] integral(arcsech(a*x)^2/x, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asech}^2(ax)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asech(a*x)**2/x,x)

[Out] Integral(asech(a*x)**2/x, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arosech}(ax)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(a*x)^2/x,x, algorithm="giac")

[Out] integrate(arcsech(a*x)^2/x, x)

3.7 $\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^2} dx$

Optimal. Leaf size=49

$$-\frac{\operatorname{sech}^{-1}(ax)^2}{x} + \frac{2\sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)}{x} - \frac{2}{x}$$

[Out] $-2/x + (2*\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]*(1 + a*x)*\operatorname{ArcSech}[a*x])/x - \operatorname{ArcSech}[a*x]^2/x$

Rubi [A] time = 0.0483221, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {6285, 3296, 2638}

$$-\frac{\operatorname{sech}^{-1}(ax)^2}{x} + \frac{2\sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)}{x} - \frac{2}{x}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{ArcSech}[a*x]^2/x^2, x]$

[Out] $-2/x + (2*\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]*(1 + a*x)*\operatorname{ArcSech}[a*x])/x - \operatorname{ArcSech}[a*x]^2/x$

Rule 6285

$\operatorname{Int}[(a_. + \operatorname{ArcSech}[(c_.)*(x_.)]*(b_.))^n*(x_.)^m, x_Symbol] := -\operatorname{Dist}[(c^{m+1})^{-1}, \operatorname{Subst}[\operatorname{Int}[(a + b*x)^n*\operatorname{Sech}[x]^{m+1}*\operatorname{Tanh}[x], x], x, \operatorname{ArcSech}[c*x]], x] /;$ $\operatorname{FreeQ}\{a, b, c, x\} \ \&\& \ \operatorname{IntegerQ}[n] \ \&\& \ \operatorname{IntegerQ}[m] \ \&\& \ (\operatorname{GtQ}[n, 0] \ || \ \operatorname{LtQ}[m, -1])$

Rule 3296

$\operatorname{Int}[(c_. + (d_.)*(x_.))^m*\sin[(e_.) + (f_.)*(x_.)], x_Symbol] := -\operatorname{Simp}[(c + d*x)^m*\operatorname{Cos}[e + f*x]/f, x] + \operatorname{Dist}[(d*m)/f, \operatorname{Int}[(c + d*x)^{m-1}*\operatorname{Cos}[e + f*x], x], x] /;$ $\operatorname{FreeQ}\{c, d, e, f, x\} \ \&\& \ \operatorname{GtQ}[m, 0]$

Rule 2638

$\operatorname{Int}[\sin[(c_.) + (d_.)*(x_.)], x_Symbol] := -\operatorname{Simp}[\operatorname{Cos}[c + d*x]/d, x] /;$ $\operatorname{FreeQ}\{c, d, x\}$

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{sech}^{-1}(ax)^2}{x^2} dx &= -\left(a \operatorname{Subst}\left(\int x^2 \sinh(x) dx, x, \operatorname{sech}^{-1}(ax)\right)\right) \\ &= -\frac{\operatorname{sech}^{-1}(ax)^2}{x} + (2a) \operatorname{Subst}\left(\int x \cosh(x) dx, x, \operatorname{sech}^{-1}(ax)\right) \\ &= \frac{2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)}{x} - \frac{\operatorname{sech}^{-1}(ax)^2}{x} - (2a) \operatorname{Subst}\left(\int \sinh(x) dx, x, \operatorname{sech}^{-1}(ax)\right) \\ &= -\frac{2}{x} + \frac{2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)}{x} - \frac{\operatorname{sech}^{-1}(ax)^2}{x} \end{aligned}$$

Mathematica [A] time = 0.0870271, size = 42, normalized size = 0.86

$$\frac{\operatorname{sech}^{-1}(ax)^2 - 2\sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax) + 2}{x}$$

Antiderivative was successfully verified.

[In] Integrate[ArcSech[a*x]^2/x^2,x]

[Out] -((2 - 2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x] + ArcSech[a*x]^2)/x)

Maple [A] time = 0.213, size = 61, normalized size = 1.2

$$a \left(-\frac{(\operatorname{arcsech}(ax))^2}{ax} + 2 \operatorname{arcsech}(ax) \sqrt{-\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}} - 2 \frac{1}{ax} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsech(a*x)^2/x^2,x)

[Out] a*(-arcsech(a*x)^2/a/x+2*arcsech(a*x)*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)-2/a/x)

Maxima [A] time = 1.01563, size = 47, normalized size = 0.96

$$2a\sqrt{\frac{1}{a^2x^2}-1}\operatorname{ar\,sech}(ax) - \frac{\operatorname{ar\,sech}(ax)^2}{x} - \frac{2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(a*x)^2/x^2,x, algorithm="maxima")

[Out] 2*a*sqrt(1/(a^2*x^2) - 1)*arcsech(a*x) - arcsech(a*x)^2/x - 2/x

Fricas [B] time = 1.70892, size = 208, normalized size = 4.24

$$\frac{2ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}} \log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right) - \log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)^2 - 2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(a*x)^2/x^2,x, algorithm="fricas")

[Out] (2*a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2))*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x)) - log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x))^2 - 2)/x

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asech}^2(ax)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asech(a*x)**2/x**2,x)

[Out] Integral(asech(a*x)**2/x**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsech}(ax)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(a*x)^2/x^2,x, algorithm="giac")

[Out] integrate(arcsech(a*x)^2/x^2, x)

3.8 $\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^3} dx$

Optimal. Leaf size=90

$$-\frac{1}{4}a^2\operatorname{sech}^{-1}(ax)^2 - \frac{(1-ax)(ax+1)}{4x^2} - \frac{(1-ax)(ax+1)\operatorname{sech}^{-1}(ax)^2}{2x^2} + \frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)}{2x^2}$$

[Out] $-\frac{((1-ax)*(1+ax))/(4*x^2) + (\operatorname{Sqrt}[(1-ax)/(1+ax)]*(1+ax)*\operatorname{ArcSech}[a*x])/(2*x^2) - (a^2*\operatorname{ArcSech}[a*x]^2)/4 - ((1-ax)*(1+ax)*\operatorname{ArcSech}[a*x]^2)/(2*x^2)}$

Rubi [A] time = 0.0593064, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {6285, 5372, 3310, 30}

$$-\frac{1}{4}a^2\operatorname{sech}^{-1}(ax)^2 - \frac{(1-ax)(ax+1)}{4x^2} - \frac{(1-ax)(ax+1)\operatorname{sech}^{-1}(ax)^2}{2x^2} + \frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)}{2x^2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{ArcSech}[a*x]^2/x^3, x]$

[Out] $-\frac{((1-ax)*(1+ax))/(4*x^2) + (\operatorname{Sqrt}[(1-ax)/(1+ax)]*(1+ax)*\operatorname{ArcSech}[a*x])/(2*x^2) - (a^2*\operatorname{ArcSech}[a*x]^2)/4 - ((1-ax)*(1+ax)*\operatorname{ArcSech}[a*x]^2)/(2*x^2)}$

Rule 6285

$\operatorname{Int}[(c + \operatorname{ArcSech}[c*(x)]*(b))^n*(x)^m, x_Symbol] \rightarrow -\operatorname{Dist}[(c^{m+1})^{-1}, \operatorname{Subst}[\operatorname{Int}[(a + b*x)^n*\operatorname{Sech}[x]^{m+1}*\operatorname{Tanh}[x], x], x, \operatorname{ArcSech}[c*x]], x] /; \operatorname{FreeQ}\{a, b, c, x\} \ \&\& \ \operatorname{IntegerQ}[n] \ \&\& \ \operatorname{IntegerQ}[m] \ \&\& \ (\operatorname{GtQ}[n, 0] \ || \ \operatorname{LtQ}[m, -1])$

Rule 5372

$\operatorname{Int}[\operatorname{Cosh}[a + (b)*(x)^n]^{p+1}*(x)^m*\operatorname{Sinh}[a + (b)*(x)^n]^{p+1}, x_Symbol] \rightarrow \operatorname{Simp}[(x^{m-n+1}*\operatorname{Sinh}[a + b*x^n]^{p+1})/(b*n*(p+1)), x] - \operatorname{Dist}[(m-n+1)/(b*n*(p+1)), \operatorname{Int}[x^{m-n}*\operatorname{Sinh}[a + b*x^n]^{p+1}, x], x] /; \operatorname{FreeQ}\{a, b, p, x\} \ \&\& \ \operatorname{LtQ}[0, n, m+1] \ \&\& \ \operatorname{NeQ}[p, -1]$

Rule 3310

$\operatorname{Int}[(c + (d)*(x))*(b*\sin[(e + (f)*(x)])]^n, x_Symbol] \rightarrow \operatorname{Simp}[(d*(b*\sin[e + f*x])^n)/(f^2*n^2), x] + (\operatorname{Dist}[(b^2*(n-1))/n, \operatorname{Int}[(c + d*x)*(b*\sin[e + f*x])^{n-2}, x], x] - \operatorname{Simp}[(b*(c + d*x)*\cos[e + f*x]*(b*\sin[e + f*x])^{n-1})/(f*n), x]) /; \operatorname{FreeQ}\{b, c, d, e, f, x\} \ \&\& \ \operatorname{GtQ}[n, 1]$

Rule 30

$\operatorname{Int}[(x)^m, x_Symbol] \rightarrow \operatorname{Simp}[x^{m+1}/(m+1), x] /; \operatorname{FreeQ}[m, x] \ \&\& \ \operatorname{NeQ}[m, -1]$

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^3} dx &= -\left(a^2 \operatorname{Subst}\left(\int x^2 \cosh(x) \sinh(x) dx, x, \operatorname{sech}^{-1}(ax)\right)\right) \\
&= -\frac{(1-ax)(1+ax)\operatorname{sech}^{-1}(ax)^2}{2x^2} + a^2 \operatorname{Subst}\left(\int x \sinh^2(x) dx, x, \operatorname{sech}^{-1}(ax)\right) \\
&= -\frac{(1-ax)(1+ax)}{4x^2} + \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)}{2x^2} - \frac{(1-ax)(1+ax)\operatorname{sech}^{-1}(ax)^2}{2x^2} - \frac{1}{2}a^2 \operatorname{Subst}\left(\int x dx, x, \operatorname{sech}^{-1}(ax)\right) \\
&= -\frac{(1-ax)(1+ax)}{4x^2} + \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)}{2x^2} - \frac{1}{4}a^2\operatorname{sech}^{-1}(ax)^2 - \frac{(1-ax)(1+ax)\operatorname{sech}^{-1}(ax)^2}{2x^2}
\end{aligned}$$

Mathematica [A] time = 0.041276, size = 54, normalized size = 0.6

$$\frac{(a^2x^2 - 2)\operatorname{sech}^{-1}(ax)^2 + 2\sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax) - 1}{4x^2}$$

Antiderivative was successfully verified.

[In] Integrate[ArcSech[a*x]^2/x^3,x]

[Out] (-1 + 2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x] + (-2 + a^2*x^2)*ArcSech[a*x]^2)/(4*x^2)

Maple [A] time = 0.213, size = 77, normalized size = 0.9

$$a^2 \left(-\frac{(\operatorname{arcsech}(ax))^2}{2a^2x^2} + \frac{\operatorname{arcsech}(ax)}{2ax} \sqrt{-\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}} + \frac{(\operatorname{arcsech}(ax))^2}{4} - \frac{1}{4a^2x^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsech(a*x)^2/x^3,x)

[Out] a^2*(-1/2*arcsech(a*x)^2/a^2/x^2+1/2*arcsech(a*x)/a/x*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)+1/4*arcsech(a*x)^2-1/4/a^2/x^2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arosech}(ax)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(a*x)^2/x^3,x, algorithm="maxima")

[Out] integrate(arcsech(a*x)^2/x^3, x)

Fricas [A] time = 1.6307, size = 235, normalized size = 2.61

$$\frac{2ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right) + (a^2x^2-2)\log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)^2 - 1}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(a*x)^2/x^3,x, algorithm="fricas")

[Out] 1/4*(2*a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2))*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x)) + (a^2*x^2 - 2)*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x))^2 - 1)/x^2

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asech}^2(ax)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asech(a*x)**2/x**3,x)

[Out] Integral(asech(a*x)**2/x**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsech}(ax)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(a*x)^2/x^3,x, algorithm="giac")

[Out] integrate(arcsech(a*x)^2/x^3, x)

3.9 $\int \frac{\operatorname{sech}^{-1}(ax)^2}{x^4} dx$

Optimal. Leaf size=102

$$-\frac{4a^2}{9x} + \frac{4a^2 \sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)}{9x} - \frac{\operatorname{sech}^{-1}(ax)^2}{3x^3} + \frac{2\sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)}{9x^3} - \frac{2}{27x^3}$$

[Out] $-2/(27*x^3) - (4*a^2)/(9*x) + (2*\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]*(1 + a*x)*\operatorname{ArcSech}[a*x])/(9*x^3) + (4*a^2*\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]*(1 + a*x)*\operatorname{ArcSech}[a*x])/(9*x) - \operatorname{ArcSech}[a*x]^2/(3*x^3)$

Rubi [A] time = 0.084085, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6285, 5373, 3310, 3296, 2638}

$$-\frac{4a^2}{9x} + \frac{4a^2 \sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)}{9x} - \frac{\operatorname{sech}^{-1}(ax)^2}{3x^3} + \frac{2\sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)}{9x^3} - \frac{2}{27x^3}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{ArcSech}[a*x]^2/x^4, x]$

[Out] $-2/(27*x^3) - (4*a^2)/(9*x) + (2*\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]*(1 + a*x)*\operatorname{ArcSech}[a*x])/(9*x^3) + (4*a^2*\operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]*(1 + a*x)*\operatorname{ArcSech}[a*x])/(9*x) - \operatorname{ArcSech}[a*x]^2/(3*x^3)$

Rule 6285

$\operatorname{Int}[(a_.) + \operatorname{ArcSech}(c_.*(x_.))*(b_.)]^{(n_.)}*(x_.)^{(m_.)}, x_Symbol] \rightarrow -\operatorname{Dist}[(c^{(m+1)})^{(-1)}, \operatorname{Subst}[\operatorname{Int}[(a + b*x)^n*\operatorname{Sech}[x]^{(m+1)}*\operatorname{Tanh}[x], x], x, \operatorname{ArcSech}[c*x]], x] /; \operatorname{FreeQ}\{a, b, c\}, x \ \&\& \operatorname{IntegerQ}[n] \ \&\& \operatorname{IntegerQ}[m] \ \&\& (\operatorname{GtQ}[n, 0] \ \|\ \operatorname{LtQ}[m, -1])$

Rule 5373

$\operatorname{Int}[\operatorname{Cosh}(a_.) + (b_.)*(x_.)^{(n_.)}]^{(p_.)}*(x_.)^{(m_.)}*\operatorname{Sinh}(a_.) + (b_.)*(x_.)^{(n_.)}, x_Symbol] \rightarrow \operatorname{Simp}[(x^{(m-n+1)}*\operatorname{Cosh}[a + b*x^n]^{(p+1)})/(b*n*(p+1)), x] - \operatorname{Dist}[(m-n+1)/(b*n*(p+1)), \operatorname{Int}[x^{(m-n)}*\operatorname{Cosh}[a + b*x^n]^{(p+1)}, x], x] /; \operatorname{FreeQ}\{a, b, p\}, x \ \&\& \operatorname{LtQ}[0, n, m+1] \ \&\& \operatorname{NeQ}[p, -1]$

Rule 3310

$\operatorname{Int}[(c_.) + (d_.)*(x_.)]*(b_.)*\sin[(e_.) + (f_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow \operatorname{Simp}[(d*(b*\sin[e + f*x])^n)/(f^2*n^2), x] + (\operatorname{Dist}[(b^2*(n-1))/n, \operatorname{Int}[(c + d*x)*(b*\sin[e + f*x])^{(n-2)}, x], x] - \operatorname{Simp}[(b*(c + d*x)*\cos[e + f*x]*(b*\sin[e + f*x])^{(n-1)})/(f*n), x]) /; \operatorname{FreeQ}\{b, c, d, e, f\}, x \ \&\& \operatorname{GtQ}[n, 1]$

Rule 3296

$\operatorname{Int}[(c_.) + (d_.)*(x_.)]^{(m_.)}*\sin[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow -\operatorname{Simp}[(c + d*x)^m*\cos[e + f*x]/f, x] + \operatorname{Dist}[(d*m)/f, \operatorname{Int}[(c + d*x)^{(m-1)}*\cos[e + f*x], x], x] /; \operatorname{FreeQ}\{c, d, e, f\}, x \ \&\& \operatorname{GtQ}[m, 0]$

Rule 2638

Int[sin[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
 \int \frac{\operatorname{sech}^{-1}(ax)^2}{x^4} dx &= -\left(a^3 \operatorname{Subst}\left(\int x^2 \cosh^2(x) \sinh(x) dx, x, \operatorname{sech}^{-1}(ax)\right)\right) \\
 &= -\frac{\operatorname{sech}^{-1}(ax)^2}{3x^3} + \frac{1}{3} (2a^3) \operatorname{Subst}\left(\int x \cosh^3(x) dx, x, \operatorname{sech}^{-1}(ax)\right) \\
 &= -\frac{2}{27x^3} + \frac{2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)}{9x^3} - \frac{\operatorname{sech}^{-1}(ax)^2}{3x^3} + \frac{1}{9} (4a^3) \operatorname{Subst}\left(\int x \cosh(x) dx, x, \operatorname{sech}^{-1}(ax)\right) \\
 &= -\frac{2}{27x^3} + \frac{2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)}{9x^3} + \frac{4a^2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)}{9x} - \frac{\operatorname{sech}^{-1}(ax)^2}{3x^3} - \frac{1}{9} (4a^3) \operatorname{Subst}\left(\int x \cosh(x) dx, x, \operatorname{sech}^{-1}(ax)\right) \\
 &= -\frac{2}{27x^3} - \frac{4a^2}{9x} + \frac{2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)}{9x^3} + \frac{4a^2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)}{9x} - \frac{\operatorname{sech}^{-1}(ax)^2}{3x^3}
 \end{aligned}$$

Mathematica [A] time = 0.0949477, size = 73, normalized size = 0.72

$$\frac{-2(6a^2x^2 + 1) + 6\sqrt{\frac{1-ax}{ax+1}}(2a^3x^3 + 2a^2x^2 + ax + 1)\operatorname{sech}^{-1}(ax) - 9\operatorname{sech}^{-1}(ax)^2}{27x^3}$$

Antiderivative was successfully verified.

[In] Integrate[ArcSech[a*x]^2/x^4, x]

[Out] (-2*(1 + 6*a^2*x^2) + 6*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x + 2*a^2*x^2 + 2*a^3*x^3)*ArcSech[a*x] - 9*ArcSech[a*x]^2)/(27*x^3)

Maple [A] time = 0.237, size = 146, normalized size = 1.4

$$a^3 \left(\frac{(\operatorname{arcsech}(ax))^2(ax-1)(ax+1)}{3x^3a^3} - \frac{(\operatorname{arcsech}(ax))^2}{3ax} + \frac{2\operatorname{arcsech}(ax)}{9a^2x^2} \sqrt{\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}} + \frac{4\operatorname{arcsech}(ax)}{9} \sqrt{\frac{ax-1}{ax}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsech(a*x)^2/x^4, x)

[Out] a^3*(1/3*arcsech(a*x)^2/a^3/x^3*(a*x-1)*(a*x+1)-1/3*arcsech(a*x)^2/a/x+2/9*arcsech(a*x)/a^2/x^2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)+4/9*arcsech(a*x)*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)+2/27*(a*x-1)/a^3/x^3*(a*x+1)-4/27/a/x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arosech}(ax)^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(a*x)^2/x^4,x, algorithm="maxima")

[Out] integrate(arcsech(a*x)^2/x^4, x)

Fricas [A] time = 1.69588, size = 258, normalized size = 2.53

$$\frac{12 a^2 x^2 - 6 (2 a^3 x^3 + a x) \sqrt{-\frac{a^2 x^2 - 1}{a^2 x^2}} \log\left(\frac{a x \sqrt{-\frac{a^2 x^2 - 1}{a^2 x^2}} + 1}{a x}\right) + 9 \log\left(\frac{a x \sqrt{-\frac{a^2 x^2 - 1}{a^2 x^2}} + 1}{a x}\right)^2 + 2}{27 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(a*x)^2/x^4,x, algorithm="fricas")

[Out] -1/27*(12*a^2*x^2 - 6*(2*a^3*x^3 + a*x)*sqrt(-(a^2*x^2 - 1)/(a^2*x^2))*log(a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x)) + 9*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x))^2 + 2)/x^3

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asech}^2(ax)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asech(a*x)**2/x**4,x)

[Out] Integral(asech(a*x)**2/x**4, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsech}(ax)^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(a*x)^2/x^4,x, algorithm="giac")

[Out] integrate(arcsech(a*x)^2/x^4, x)

3.10 $\int x^4 \operatorname{sech}^{-1}(ax)^3 dx$

Optimal. Leaf size=297

$$\frac{9i \operatorname{sech}^{-1}(ax) \operatorname{PolyLog}\left(2, -ie^{\operatorname{sech}^{-1}(ax)}\right)}{20a^5} - \frac{9i \operatorname{sech}^{-1}(ax) \operatorname{PolyLog}\left(2, ie^{\operatorname{sech}^{-1}(ax)}\right)}{20a^5} - \frac{9i \operatorname{PolyLog}\left(3, -ie^{\operatorname{sech}^{-1}(ax)}\right)}{20a^5} + \dots$$

```
[Out] (x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/(20*a^4) - (9*x*ArcSech[a*x])/(20*a^4) - (x^3*ArcSech[a*x])/(10*a^2) - (9*x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x]^2)/(40*a^4) - (3*x^3*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x]^2)/(20*a^2) + (x^5*ArcSech[a*x]^3)/5 - (9*ArcSech[a*x]^2*ArcTan[E^ArcSech[a*x]])/(20*a^5) + ArcTan[(Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/(a*x)]/(2*a^5) + (((9*I)/20)*ArcSech[a*x]*PolyLog[2, (-I)*E^ArcSech[a*x]])/a^5 - (((9*I)/20)*ArcSech[a*x]*PolyLog[2, I*E^ArcSech[a*x]])/a^5 - (((9*I)/20)*PolyLog[3, (-I)*E^ArcSech[a*x]])/a^5 + (((9*I)/20)*PolyLog[3, I*E^ArcSech[a*x]])/a^5
```

Rubi [A] time = 0.203194, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.9$, Rules used = {6285, 5418, 4186, 3768, 3770, 4180, 2531, 2282, 6589}

$$\frac{9i \operatorname{sech}^{-1}(ax) \operatorname{PolyLog}\left(2, -ie^{\operatorname{sech}^{-1}(ax)}\right)}{20a^5} - \frac{9i \operatorname{sech}^{-1}(ax) \operatorname{PolyLog}\left(2, ie^{\operatorname{sech}^{-1}(ax)}\right)}{20a^5} - \frac{9i \operatorname{PolyLog}\left(3, -ie^{\operatorname{sech}^{-1}(ax)}\right)}{20a^5} + \dots$$

Antiderivative was successfully verified.

```
[In] Int[x^4*ArcSech[a*x]^3,x]
```

```
[Out] (x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/(20*a^4) - (9*x*ArcSech[a*x])/(20*a^4) - (x^3*ArcSech[a*x])/(10*a^2) - (9*x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x]^2)/(40*a^4) - (3*x^3*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x]^2)/(20*a^2) + (x^5*ArcSech[a*x]^3)/5 - (9*ArcSech[a*x]^2*ArcTan[E^ArcSech[a*x]])/(20*a^5) + ArcTan[(Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/(a*x)]/(2*a^5) + (((9*I)/20)*ArcSech[a*x]*PolyLog[2, (-I)*E^ArcSech[a*x]])/a^5 - (((9*I)/20)*ArcSech[a*x]*PolyLog[2, I*E^ArcSech[a*x]])/a^5 - (((9*I)/20)*PolyLog[3, (-I)*E^ArcSech[a*x]])/a^5 + (((9*I)/20)*PolyLog[3, I*E^ArcSech[a*x]])/a^5
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))^(n_.)*(x_)^(m_.), x_Symbol] := -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])
```

Rule 5418

```
Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)^(n_.)]^(q_.), x_Symbol] := -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^p)/(b*n*p), x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x] /; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ[q, 1]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(n_)*((c_.) + (d_.)*(x_.))^(m_), x_Symbol]
:> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x]
+ (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(m - 2)*
(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)^(m - 1)*
(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m - 1)*(b*Csc[e + f*x])^(n - 2))/
(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x]
)*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_.)]*(c_.) + (d_.)*(x_.))^(m_.), x_Symbol]
:> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^(c_.)*((a_.) + (b_.)*(x_.)))^(n_.)]*(f_.) + (g_.)*(x_.))^(m_.), x_Symbol]
:> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_.))^(p_.)]/((d_.) + (e_.)*(x_.)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int x^4 \operatorname{sech}^{-1}(ax)^3 dx &= -\frac{\operatorname{Subst}\left(\int x^3 \operatorname{sech}^5(x) \tanh(x) dx, x, \operatorname{sech}^{-1}(ax)\right)}{a^5} \\
&= \frac{1}{5} x^5 \operatorname{sech}^{-1}(ax)^3 - \frac{3 \operatorname{Subst}\left(\int x^2 \operatorname{sech}^5(x) dx, x, \operatorname{sech}^{-1}(ax)\right)}{5a^5} \\
&= -\frac{x^3 \operatorname{sech}^{-1}(ax)}{10a^2} - \frac{3x^3 \sqrt{\frac{1-ax}{1+ax}} (1+ax) \operatorname{sech}^{-1}(ax)^2}{20a^2} + \frac{1}{5} x^5 \operatorname{sech}^{-1}(ax)^3 + \frac{\operatorname{Subst}\left(\int \operatorname{sech}^3(x) dx, x, \operatorname{sech}^{-1}(ax)\right)}{10a^5} \\
&= \frac{x \sqrt{\frac{1-ax}{1+ax}} (1+ax)}{20a^4} - \frac{9x \operatorname{sech}^{-1}(ax)}{20a^4} - \frac{x^3 \operatorname{sech}^{-1}(ax)}{10a^2} - \frac{9x \sqrt{\frac{1-ax}{1+ax}} (1+ax) \operatorname{sech}^{-1}(ax)^2}{40a^4} - \frac{3x^3 \sqrt{\frac{1-ax}{1+ax}}}{10a^5} \\
&= \frac{x \sqrt{\frac{1-ax}{1+ax}} (1+ax)}{20a^4} - \frac{9x \operatorname{sech}^{-1}(ax)}{20a^4} - \frac{x^3 \operatorname{sech}^{-1}(ax)}{10a^2} - \frac{9x \sqrt{\frac{1-ax}{1+ax}} (1+ax) \operatorname{sech}^{-1}(ax)^2}{40a^4} - \frac{3x^3 \sqrt{\frac{1-ax}{1+ax}}}{10a^5} \\
&= \frac{x \sqrt{\frac{1-ax}{1+ax}} (1+ax)}{20a^4} - \frac{9x \operatorname{sech}^{-1}(ax)}{20a^4} - \frac{x^3 \operatorname{sech}^{-1}(ax)}{10a^2} - \frac{9x \sqrt{\frac{1-ax}{1+ax}} (1+ax) \operatorname{sech}^{-1}(ax)^2}{40a^4} - \frac{3x^3 \sqrt{\frac{1-ax}{1+ax}}}{10a^5} \\
&= \frac{x \sqrt{\frac{1-ax}{1+ax}} (1+ax)}{20a^4} - \frac{9x \operatorname{sech}^{-1}(ax)}{20a^4} - \frac{x^3 \operatorname{sech}^{-1}(ax)}{10a^2} - \frac{9x \sqrt{\frac{1-ax}{1+ax}} (1+ax) \operatorname{sech}^{-1}(ax)^2}{40a^4} - \frac{3x^3 \sqrt{\frac{1-ax}{1+ax}}}{10a^5} \\
&= \frac{x \sqrt{\frac{1-ax}{1+ax}} (1+ax)}{20a^4} - \frac{9x \operatorname{sech}^{-1}(ax)}{20a^4} - \frac{x^3 \operatorname{sech}^{-1}(ax)}{10a^2} - \frac{9x \sqrt{\frac{1-ax}{1+ax}} (1+ax) \operatorname{sech}^{-1}(ax)^2}{40a^4} - \frac{3x^3 \sqrt{\frac{1-ax}{1+ax}}}{10a^5}
\end{aligned}$$

Mathematica [A] time = 0.554282, size = 281, normalized size = 0.95

$$18i \operatorname{sech}^{-1}(ax) \operatorname{PolyLog}\left(2, -ie^{-\operatorname{sech}^{-1}(ax)}\right) - 18i \operatorname{sech}^{-1}(ax) \operatorname{PolyLog}\left(2, ie^{-\operatorname{sech}^{-1}(ax)}\right) + 18i \operatorname{PolyLog}\left(3, -ie^{-\operatorname{sech}^{-1}(ax)}\right) - 18i \operatorname{PolyLog}\left(3, ie^{-\operatorname{sech}^{-1}(ax)}\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^4*ArcSech[a*x]^3,x]

[Out] (2*a*x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x) - 18*a*x*ArcSech[a*x] - 4*a^3*x^3*ArcSech[a*x] - 9*a*x*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x]^2 - 6*a^3*x^3*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x]^2 + 8*a^5*x^5*ArcSech[a*x]^3 + 40*ArcTan[Tanh[ArcSech[a*x]/2]] + (9*I)*ArcSech[a*x]^2*Log[1 - I/E^ArcSech[a*x]] - (9*I)*ArcSech[a*x]^2*Log[1 + I/E^ArcSech[a*x]] + (18*I)*ArcSech[a*x]*PolyLog[2, (-I)/E^ArcSech[a*x]] - (18*I)*ArcSech[a*x]*PolyLog[2, I/E^ArcSech[a*x]] + (18*I)*PolyLog[3, (-I)/E^ArcSech[a*x]] - (18*I)*PolyLog[3, I/E^ArcSech[a*x]])/(40*a^5)

Maple [F] time = 0.651, size = 0, normalized size = 0.

$$\int x^4 (\operatorname{arcsech}(ax))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*arcsech(a*x)^3,x)

[Out] int(x^4*arcsech(a*x)^3,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{arsech}(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*arcsech(a*x)^3,x, algorithm="maxima")

[Out] integrate(x^4*arcsech(a*x)^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(x^4 \operatorname{arsech}(ax)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*arcsech(a*x)^3,x, algorithm="fricas")

[Out] integral(x^4*arcsech(a*x)^3, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{asech}^3(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*asech(a*x)**3,x)

[Out] Integral(x**4*asech(a*x)**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 \operatorname{arsech}(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*arcsech(a*x)^3,x, algorithm="giac")

[Out] integrate(x^4*arcsech(a*x)^3, x)

3.11 $\int x^3 \operatorname{sech}^{-1}(ax)^3 dx$

Optimal. Leaf size=184

$$\frac{\operatorname{PolyLog}\left(2, -e^{2\operatorname{sech}^{-1}(ax)}\right)}{2a^4} - \frac{x^2 \sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)^2}{4a^2} - \frac{x^2 \operatorname{sech}^{-1}(ax)}{4a^2} + \frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)}{4a^4} - \frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)}{2a^4}$$

[Out] (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/(4*a^4) - (x^2*ArcSech[a*x])/(4*a^2) - ArcSech[a*x]^2/(2*a^4) - (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x]^2)/(2*a^4) - (x^2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x]^2)/(4*a^2) + (x^4*ArcSech[a*x]^3)/4 + (ArcSech[a*x]*Log[1 + E^(2*ArcSech[a*x])])/(a^4 + PolyLog[2, -E^(2*ArcSech[a*x])])/(2*a^4)

Rubi [A] time = 0.17673, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 1.$, Rules used = {6285, 5418, 4186, 3767, 8, 4184, 3718, 2190, 2279, 2391}

$$\frac{\operatorname{PolyLog}\left(2, -e^{2\operatorname{sech}^{-1}(ax)}\right)}{2a^4} - \frac{x^2 \sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)^2}{4a^2} - \frac{x^2 \operatorname{sech}^{-1}(ax)}{4a^2} + \frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)}{4a^4} - \frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)}{2a^4}$$

Antiderivative was successfully verified.

[In] Int[x^3*ArcSech[a*x]^3, x]

[Out] (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/(4*a^4) - (x^2*ArcSech[a*x])/(4*a^2) - ArcSech[a*x]^2/(2*a^4) - (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x]^2)/(2*a^4) - (x^2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x]^2)/(4*a^2) + (x^4*ArcSech[a*x]^3)/4 + (ArcSech[a*x]*Log[1 + E^(2*ArcSech[a*x])])/(a^4 + PolyLog[2, -E^(2*ArcSech[a*x])])/(2*a^4)

Rule 6285

Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))^n_*(x_)^(m_.), x_Symbol] :> -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

Rule 5418

Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)^(n_.)]^(q_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^p)/(b*n*p), x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x] /; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ[q, 1]

Rule 4186

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^n_*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m - 1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Simp[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x))/(1 + E^(2*(-I*e) + f*fz*x)), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int x^3 \operatorname{sech}^{-1}(ax)^3 dx &= -\frac{\operatorname{Subst}\left(\int x^3 \operatorname{sech}^4(x) \tanh(x) dx, x, \operatorname{sech}^{-1}(ax)\right)}{a^4} \\
&= \frac{1}{4} x^4 \operatorname{sech}^{-1}(ax)^3 - \frac{3 \operatorname{Subst}\left(\int x^2 \operatorname{sech}^4(x) dx, x, \operatorname{sech}^{-1}(ax)\right)}{4a^4} \\
&= -\frac{x^2 \operatorname{sech}^{-1}(ax)}{4a^2} - \frac{x^2 \sqrt{\frac{1-ax}{1+ax}} (1+ax) \operatorname{sech}^{-1}(ax)^2}{4a^2} + \frac{1}{4} x^4 \operatorname{sech}^{-1}(ax)^3 + \frac{\operatorname{Subst}\left(\int \operatorname{sech}^2(x) dx, x, \operatorname{sech}^{-1}(ax)\right)}{4a^4} \\
&= -\frac{x^2 \operatorname{sech}^{-1}(ax)}{4a^2} - \frac{\sqrt{\frac{1-ax}{1+ax}} (1+ax) \operatorname{sech}^{-1}(ax)^2}{2a^4} - \frac{x^2 \sqrt{\frac{1-ax}{1+ax}} (1+ax) \operatorname{sech}^{-1}(ax)^2}{4a^2} + \frac{1}{4} x^4 \operatorname{sech}^{-1}(ax)^3 \\
&= \frac{\sqrt{\frac{1-ax}{1+ax}} (1+ax)}{4a^4} - \frac{x^2 \operatorname{sech}^{-1}(ax)}{4a^2} - \frac{\operatorname{sech}^{-1}(ax)^2}{2a^4} - \frac{\sqrt{\frac{1-ax}{1+ax}} (1+ax) \operatorname{sech}^{-1}(ax)^2}{2a^4} - \frac{x^2 \sqrt{\frac{1-ax}{1+ax}} (1+ax) \operatorname{sech}^{-1}(ax)^2}{4a^2} \\
&= \frac{\sqrt{\frac{1-ax}{1+ax}} (1+ax)}{4a^4} - \frac{x^2 \operatorname{sech}^{-1}(ax)}{4a^2} - \frac{\operatorname{sech}^{-1}(ax)^2}{2a^4} - \frac{\sqrt{\frac{1-ax}{1+ax}} (1+ax) \operatorname{sech}^{-1}(ax)^2}{2a^4} - \frac{x^2 \sqrt{\frac{1-ax}{1+ax}} (1+ax) \operatorname{sech}^{-1}(ax)^2}{4a^2} \\
&= \frac{\sqrt{\frac{1-ax}{1+ax}} (1+ax)}{4a^4} - \frac{x^2 \operatorname{sech}^{-1}(ax)}{4a^2} - \frac{\operatorname{sech}^{-1}(ax)^2}{2a^4} - \frac{\sqrt{\frac{1-ax}{1+ax}} (1+ax) \operatorname{sech}^{-1}(ax)^2}{2a^4} - \frac{x^2 \sqrt{\frac{1-ax}{1+ax}} (1+ax) \operatorname{sech}^{-1}(ax)^2}{4a^2} \\
&= \frac{\sqrt{\frac{1-ax}{1+ax}} (1+ax)}{4a^4} - \frac{x^2 \operatorname{sech}^{-1}(ax)}{4a^2} - \frac{\operatorname{sech}^{-1}(ax)^2}{2a^4} - \frac{\sqrt{\frac{1-ax}{1+ax}} (1+ax) \operatorname{sech}^{-1}(ax)^2}{2a^4} - \frac{x^2 \sqrt{\frac{1-ax}{1+ax}} (1+ax) \operatorname{sech}^{-1}(ax)^2}{4a^2}
\end{aligned}$$

Mathematica [A] time = 0.576463, size = 188, normalized size = 1.02

$$\frac{-2 \operatorname{PolyLog}\left(2, -e^{-2 \operatorname{sech}^{-1}(ax)}\right) + a^4 x^4 \operatorname{sech}^{-1}(ax)^3 - \left(a^3 x^3 \sqrt{\frac{1-ax}{ax+1}} + a^2 x^2 \sqrt{\frac{1-ax}{ax+1}} + 2ax \sqrt{\frac{1-ax}{ax+1}} + 2 \sqrt{\frac{1-ax}{ax+1}} - 2\right) \operatorname{sech}^{-1}(ax)}{4a^4}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^3*ArcSech[a*x]^3,x]

[Out] (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x) - (-2 + 2*Sqrt[(1 - a*x)/(1 + a*x)]) + 2*a*x*Sqrt[(1 - a*x)/(1 + a*x)] + a^2*x^2*Sqrt[(1 - a*x)/(1 + a*x)] + a^3*x^3*Sqrt[(1 - a*x)/(1 + a*x)])*ArcSech[a*x]^2 + a^4*x^4*ArcSech[a*x]^3 + ArcSech[a*x]*(-a^2*x^2 + 4*Log[1 + E^(-2*ArcSech[a*x])]) - 2*PolyLog[2, -E^(-2*ArcSech[a*x])])/(4*a^4)

Maple [A] time = 0.349, size = 246, normalized size = 1.3

$$\frac{x^4 (\operatorname{arcsech}(ax))^3}{4} - \frac{x^3 (\operatorname{arcsech}(ax))^2}{4a} \sqrt{\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}} - \frac{x (\operatorname{arcsech}(ax))^2}{2a^3} \sqrt{\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}} - \frac{x^2 \operatorname{arcsech}(ax)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*arcsech(a*x)^3,x)

[Out] 1/4*x^4*arcsech(a*x)^3-1/4/a*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)*arcsech(a*x)^2*x^3-1/2/a^3*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)*arcsech(a*x)^2*x-1/4*x^2*arcsech(a*x)/a^2+1/4/a^3*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)*x-1/2*arcsech(a*x)^2/a^4-1/4/a^4*arcsech(a*x)*ln(1+(1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))^2)/a^4+1/2*polylog(2,-(1/a/x+(1/a/x-1)^(1/2))*(1+1/a/x)^(1/2))^2)/a^4

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{arsech}(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arcsech(a*x)^3,x, algorithm="maxima")

[Out] integrate(x^3*arcsech(a*x)^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(x^3 \operatorname{arsech}(ax)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arcsech(a*x)^3,x, algorithm="fricas")

[Out] integral(x^3*arcsech(a*x)^3, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{asech}^3(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*asech(a*x)**3,x)

[Out] Integral(x**3*asech(a*x)**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 \operatorname{arsech}(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*arcsech(a*x)^3,x, algorithm="giac")

[Out] integrate(x^3*arcsech(a*x)^3, x)

3.12 $\int x^2 \operatorname{sech}^{-1}(ax)^3 dx$

Optimal. Leaf size=198

$$\frac{\operatorname{isech}^{-1}(ax)\operatorname{PolyLog}\left(2, -ie^{\operatorname{sech}^{-1}(ax)}\right)}{a^3} - \frac{\operatorname{isech}^{-1}(ax)\operatorname{PolyLog}\left(2, ie^{\operatorname{sech}^{-1}(ax)}\right)}{a^3} - \frac{i\operatorname{PolyLog}\left(3, -ie^{\operatorname{sech}^{-1}(ax)}\right)}{a^3} + \frac{i\operatorname{PolyLog}\left(3, ie^{\operatorname{sech}^{-1}(ax)}\right)}{a^3}$$

[Out] $-\left(\frac{x\operatorname{ArcSech}[a*x]}{a^2}\right) - \left(\frac{x\sqrt{\frac{1-a*x}{1+a*x}}(1+a*x)\operatorname{ArcSech}[a*x]^2}{2*a^2}\right) + \frac{x^3\operatorname{ArcSech}[a*x]^3}{3} - \frac{\operatorname{ArcSech}[a*x]^2\operatorname{ArcTan}\left[\frac{E^{\operatorname{ArcSech}[a*x]}}{a*x}\right]}{a^3} + \frac{\operatorname{ArcTan}\left[\frac{\sqrt{\frac{1-a*x}{1+a*x}}(1+a*x)}{a*x}\right]}{a^3} + \frac{I\operatorname{ArcSech}[a*x]\operatorname{PolyLog}\left[2, (-I)*E^{\operatorname{ArcSech}[a*x]}\right]}{a^3} - \frac{I\operatorname{ArcSech}[a*x]\operatorname{PolyLog}\left[2, I*E^{\operatorname{ArcSech}[a*x]}\right]}{a^3} - \frac{I\operatorname{PolyLog}\left[3, (-I)*E^{\operatorname{ArcSech}[a*x]}\right]}{a^3} + \frac{I\operatorname{PolyLog}\left[3, I*E^{\operatorname{ArcSech}[a*x]}\right]}{a^3}$

Rubi [A] time = 0.140893, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.8$, Rules used = {6285, 5418, 4186, 3770, 4180, 2531, 2282, 6589}

$$\frac{\operatorname{isech}^{-1}(ax)\operatorname{PolyLog}\left(2, -ie^{\operatorname{sech}^{-1}(ax)}\right)}{a^3} - \frac{\operatorname{isech}^{-1}(ax)\operatorname{PolyLog}\left(2, ie^{\operatorname{sech}^{-1}(ax)}\right)}{a^3} - \frac{i\operatorname{PolyLog}\left(3, -ie^{\operatorname{sech}^{-1}(ax)}\right)}{a^3} + \frac{i\operatorname{PolyLog}\left(3, ie^{\operatorname{sech}^{-1}(ax)}\right)}{a^3}$$

Antiderivative was successfully verified.

[In] Int[x^2*ArcSech[a*x]^3, x]

[Out] $-\left(\frac{x\operatorname{ArcSech}[a*x]}{a^2}\right) - \left(\frac{x\sqrt{\frac{1-a*x}{1+a*x}}(1+a*x)\operatorname{ArcSech}[a*x]^2}{2*a^2}\right) + \frac{x^3\operatorname{ArcSech}[a*x]^3}{3} - \frac{\operatorname{ArcSech}[a*x]^2\operatorname{ArcTan}\left[\frac{E^{\operatorname{ArcSech}[a*x]}}{a*x}\right]}{a^3} + \frac{\operatorname{ArcTan}\left[\frac{\sqrt{\frac{1-a*x}{1+a*x}}(1+a*x)}{a*x}\right]}{a^3} + \frac{I\operatorname{ArcSech}[a*x]\operatorname{PolyLog}\left[2, (-I)*E^{\operatorname{ArcSech}[a*x]}\right]}{a^3} - \frac{I\operatorname{ArcSech}[a*x]\operatorname{PolyLog}\left[2, I*E^{\operatorname{ArcSech}[a*x]}\right]}{a^3} - \frac{I\operatorname{PolyLog}\left[3, (-I)*E^{\operatorname{ArcSech}[a*x]}\right]}{a^3} + \frac{I\operatorname{PolyLog}\left[3, I*E^{\operatorname{ArcSech}[a*x]}\right]}{a^3}$

Rule 6285

Int[(a_.) + ArcSech[(c_.)*(x_.)]*(b_.)]^(n_.)*(x_)^(m_.), x_Symbol] :> -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

Rule 5418

Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)^(n_.)]^(q_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^p)/(b*n*p), x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x] /; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ[q, 1]

Rule 4186

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.)]^(n_.)*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -

1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 4180

Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-I*e) + f*fz*x]/E^(I*k*Pi))]/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-I*e) + f*fz*x]/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-I*e) + f*fz*x]/E^(I*k*Pi)], x], x] /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]

Rule 2531

Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 2282

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rule 6589

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]

Rubi steps

$$\begin{aligned}
\int x^2 \operatorname{sech}^{-1}(ax)^3 dx &= -\frac{\operatorname{Subst}\left(\int x^3 \operatorname{sech}^3(x) \tanh(x) dx, x, \operatorname{sech}^{-1}(ax)\right)}{a^3} \\
&= \frac{1}{3} x^3 \operatorname{sech}^{-1}(ax)^3 - \frac{\operatorname{Subst}\left(\int x^2 \operatorname{sech}^3(x) dx, x, \operatorname{sech}^{-1}(ax)\right)}{a^3} \\
&= -\frac{x \operatorname{sech}^{-1}(ax)}{a^2} - \frac{x \sqrt{\frac{1-ax}{1+ax}} (1+ax) \operatorname{sech}^{-1}(ax)^2}{2a^2} + \frac{1}{3} x^3 \operatorname{sech}^{-1}(ax)^3 - \frac{\operatorname{Subst}\left(\int x^2 \operatorname{sech}(x) dx, x, \operatorname{sech}^{-1}(ax)\right)}{2a^3} \\
&= -\frac{x \operatorname{sech}^{-1}(ax)}{a^2} - \frac{x \sqrt{\frac{1-ax}{1+ax}} (1+ax) \operatorname{sech}^{-1}(ax)^2}{2a^2} + \frac{1}{3} x^3 \operatorname{sech}^{-1}(ax)^3 - \frac{\operatorname{sech}^{-1}(ax)^2 \tan^{-1}\left(e^{\operatorname{sech}^{-1}(ax)}\right)}{a^3} \\
&= -\frac{x \operatorname{sech}^{-1}(ax)}{a^2} - \frac{x \sqrt{\frac{1-ax}{1+ax}} (1+ax) \operatorname{sech}^{-1}(ax)^2}{2a^2} + \frac{1}{3} x^3 \operatorname{sech}^{-1}(ax)^3 - \frac{\operatorname{sech}^{-1}(ax)^2 \tan^{-1}\left(e^{\operatorname{sech}^{-1}(ax)}\right)}{a^3} \\
&= -\frac{x \operatorname{sech}^{-1}(ax)}{a^2} - \frac{x \sqrt{\frac{1-ax}{1+ax}} (1+ax) \operatorname{sech}^{-1}(ax)^2}{2a^2} + \frac{1}{3} x^3 \operatorname{sech}^{-1}(ax)^3 - \frac{\operatorname{sech}^{-1}(ax)^2 \tan^{-1}\left(e^{\operatorname{sech}^{-1}(ax)}\right)}{a^3} \\
&= -\frac{x \operatorname{sech}^{-1}(ax)}{a^2} - \frac{x \sqrt{\frac{1-ax}{1+ax}} (1+ax) \operatorname{sech}^{-1}(ax)^2}{2a^2} + \frac{1}{3} x^3 \operatorname{sech}^{-1}(ax)^3 - \frac{\operatorname{sech}^{-1}(ax)^2 \tan^{-1}\left(e^{\operatorname{sech}^{-1}(ax)}\right)}{a^3}
\end{aligned}$$

Mathematica [A] time = 0.460283, size = 199, normalized size = 1.01

$$3i \left(2 \operatorname{sech}^{-1}(ax) \operatorname{PolyLog}\left(2, -ie^{-\operatorname{sech}^{-1}(ax)}\right) - 2 \operatorname{sech}^{-1}(ax) \operatorname{PolyLog}\left(2, ie^{-\operatorname{sech}^{-1}(ax)}\right) + 2 \operatorname{PolyLog}\left(3, -ie^{-\operatorname{sech}^{-1}(ax)}\right) - 2 \operatorname{PolyLog}\left(3, ie^{-\operatorname{sech}^{-1}(ax)}\right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2*ArcSech[a*x]^3,x]

[Out] $(-6*a*x*ArcSech[a*x] - 3*a*x*sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x]^2 + 2*a^3*x^3*ArcSech[a*x]^3 + (3*I)*((-4*I)*ArcTan[Tanh[ArcSech[a*x]/2]] + ArcSech[a*x]^2*Log[1 - I/E^ArcSech[a*x]] - ArcSech[a*x]^2*Log[1 + I/E^ArcSech[a*x]] + 2*ArcSech[a*x]*PolyLog[2, (-I)/E^ArcSech[a*x]] - 2*ArcSech[a*x]*PolyLog[2, I/E^ArcSech[a*x]] + 2*PolyLog[3, (-I)/E^ArcSech[a*x]] - 2*PolyLog[3, I/E^ArcSech[a*x]])/(6*a^3)$

Maple [F] time = 0.434, size = 0, normalized size = 0.

$$\int x^2 (\operatorname{arcsech}(ax))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*arcsech(a*x)^3,x)

[Out] int(x^2*arcsech(a*x)^3,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arosech}(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*arcsech(a*x)^3,x, algorithm="maxima")

[Out] integrate(x^2*arcsech(a*x)^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(x^2 \operatorname{arsech}(ax)^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*arcsech(a*x)^3,x, algorithm="fricas")

[Out] integral(x^2*arcsech(a*x)^3, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{asech}^3(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*asech(a*x)**3,x)

[Out] Integral(x**2*asech(a*x)**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 \operatorname{arsech}(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*arcsech(a*x)^3,x, algorithm="giac")

[Out] integrate(x^2*arcsech(a*x)^3, x)

3.13 $\int x \operatorname{sech}^{-1}(ax)^3 dx$

Optimal. Leaf size=102

$$\frac{3 \operatorname{PolyLog}\left(2, -e^{2 \operatorname{sech}^{-1}(ax)}\right)}{2a^2} - \frac{3 \sqrt{\frac{1-ax}{ax+1}}(ax+1) \operatorname{sech}^{-1}(ax)^2}{2a^2} - \frac{3 \operatorname{sech}^{-1}(ax)^2}{2a^2} + \frac{3 \operatorname{sech}^{-1}(ax) \log\left(e^{2 \operatorname{sech}^{-1}(ax)} + 1\right)}{a^2} + \frac{1}{2}x$$

[Out] $(-3 \operatorname{ArcSech}[a*x]^2)/(2*a^2) - (3 \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]*(1 + a*x) \operatorname{ArcSech}[a*x]^2)/(2*a^2) + (x^2 \operatorname{ArcSech}[a*x]^3)/2 + (3 \operatorname{ArcSech}[a*x] \operatorname{Log}[1 + E^{(2 \operatorname{ArcSech}[a*x])}])/a^2 + (3 \operatorname{PolyLog}[2, -E^{(2 \operatorname{ArcSech}[a*x])}])/(2*a^2)$

Rubi [A] time = 0.121762, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {6285, 5418, 4184, 3718, 2190, 2279, 2391}

$$\frac{3 \operatorname{PolyLog}\left(2, -e^{2 \operatorname{sech}^{-1}(ax)}\right)}{2a^2} - \frac{3 \sqrt{\frac{1-ax}{ax+1}}(ax+1) \operatorname{sech}^{-1}(ax)^2}{2a^2} - \frac{3 \operatorname{sech}^{-1}(ax)^2}{2a^2} + \frac{3 \operatorname{sech}^{-1}(ax) \log\left(e^{2 \operatorname{sech}^{-1}(ax)} + 1\right)}{a^2} + \frac{1}{2}x$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x \operatorname{ArcSech}[a*x]^3, x]$

[Out] $(-3 \operatorname{ArcSech}[a*x]^2)/(2*a^2) - (3 \operatorname{Sqrt}[(1 - a*x)/(1 + a*x)]*(1 + a*x) \operatorname{ArcSech}[a*x]^2)/(2*a^2) + (x^2 \operatorname{ArcSech}[a*x]^3)/2 + (3 \operatorname{ArcSech}[a*x] \operatorname{Log}[1 + E^{(2 \operatorname{ArcSech}[a*x])}])/a^2 + (3 \operatorname{PolyLog}[2, -E^{(2 \operatorname{ArcSech}[a*x])}])/(2*a^2)$

Rule 6285

$\operatorname{Int}[(c + \operatorname{ArcSech}[(c_*)*(x_*)])*(b_*)^n*(x_*)^m, x_Symbol] \rightarrow -\operatorname{Dist}[(c^{(m+1)})^{-1}, \operatorname{Subst}[\operatorname{Int}[(a + b*x)^n \operatorname{Sech}[x]^{(m+1)} \operatorname{Tanh}[x], x], x, \operatorname{ArcSech}[c*x]], x] /; \operatorname{FreeQ}\{a, b, c\}, x] \&\& \operatorname{IntegerQ}[n] \&\& \operatorname{IntegerQ}[m] \&\& (\operatorname{GtQ}[n, 0] \mid \mid \operatorname{LtQ}[m, -1])$

Rule 5418

$\operatorname{Int}[(x_*)^m \operatorname{Sech}[(a_*) + (b_*)*(x_*)^n]^{(p_*)} \operatorname{Tanh}[(a_*) + (b_*)*(x_*)^n]^{(q_*)}, x_Symbol] \rightarrow -\operatorname{Simp}[(x^{(m-n+1)} \operatorname{Sech}[a + b*x^n]^p)/(b*n*p), x] + \operatorname{Dist}[(m-n+1)/(b*n*p), \operatorname{Int}[x^{(m-n)} \operatorname{Sech}[a + b*x^n]^p, x], x] /; \operatorname{FreeQ}\{a, b, p\}, x] \&\& \operatorname{RationalQ}[m] \&\& \operatorname{IntegerQ}[n] \&\& \operatorname{GeQ}[m-n, 0] \&\& \operatorname{EqQ}[q, 1]$

Rule 4184

$\operatorname{Int}[\operatorname{csc}[(e_*) + (f_*)*(x_*)]^{2*} ((c_*) + (d_*)*(x_*)^m), x_Symbol] \rightarrow -\operatorname{Simp}[(c + d*x)^m \operatorname{Cot}[e + f*x]/f, x] + \operatorname{Dist}[(d*m)/f, \operatorname{Int}[(c + d*x)^{(m-1)} \operatorname{Cot}[e + f*x], x], x] /; \operatorname{FreeQ}\{c, d, e, f\}, x] \&\& \operatorname{GtQ}[m, 0]$

Rule 3718

$\operatorname{Int}[(c_*) + (d_*)*(x_*)^m \operatorname{tan}[(e_*) + (\operatorname{Complex}[0, fz_])*(f_*)*(x_*)], x_Symbol] \rightarrow -\operatorname{Simp}[(I*(c + d*x)^{(m+1)})/(d*(m+1)), x] + \operatorname{Dist}[2*I, \operatorname{Int}[(c + d*x)^m E^{(2*(-I*e) + f*fz*x))}/(1 + E^{(2*(-I*e) + f*fz*x))}, x], x] /; \operatorname{FreeQ}\{c, d, e, f, fz\}, x] \&\& \operatorname{IGtQ}[m, 0]$

Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_)))^(n_))], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned} \int x \operatorname{sech}^{-1}(ax)^3 dx &= -\frac{\operatorname{Subst}\left(\int x^3 \operatorname{sech}^2(x) \tanh(x) dx, x, \operatorname{sech}^{-1}(ax)\right)}{a^2} \\ &= \frac{1}{2} x^2 \operatorname{sech}^{-1}(ax)^3 - \frac{3 \operatorname{Subst}\left(\int x^2 \operatorname{sech}^2(x) dx, x, \operatorname{sech}^{-1}(ax)\right)}{2a^2} \\ &= -\frac{3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)^2}{2a^2} + \frac{1}{2} x^2 \operatorname{sech}^{-1}(ax)^3 + \frac{3 \operatorname{Subst}\left(\int x \tanh(x) dx, x, \operatorname{sech}^{-1}(ax)\right)}{a^2} \\ &= -\frac{3\operatorname{sech}^{-1}(ax)^2}{2a^2} - \frac{3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)^2}{2a^2} + \frac{1}{2} x^2 \operatorname{sech}^{-1}(ax)^3 + \frac{6 \operatorname{Subst}\left(\int \frac{e^{2x}}{1+e^{2x}} dx, x, \operatorname{sech}^{-1}(ax)\right)}{a^2} \\ &= -\frac{3\operatorname{sech}^{-1}(ax)^2}{2a^2} - \frac{3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)^2}{2a^2} + \frac{1}{2} x^2 \operatorname{sech}^{-1}(ax)^3 + \frac{3\operatorname{sech}^{-1}(ax) \log\left(1 + e^{2\operatorname{sech}^{-1}(ax)}\right)}{a^2} \\ &= -\frac{3\operatorname{sech}^{-1}(ax)^2}{2a^2} - \frac{3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)^2}{2a^2} + \frac{1}{2} x^2 \operatorname{sech}^{-1}(ax)^3 + \frac{3\operatorname{sech}^{-1}(ax) \log\left(1 + e^{2\operatorname{sech}^{-1}(ax)}\right)}{a^2} \\ &= -\frac{3\operatorname{sech}^{-1}(ax)^2}{2a^2} - \frac{3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)^2}{2a^2} + \frac{1}{2} x^2 \operatorname{sech}^{-1}(ax)^3 + \frac{3\operatorname{sech}^{-1}(ax) \log\left(1 + e^{2\operatorname{sech}^{-1}(ax)}\right)}{a^2} \end{aligned}$$

Mathematica [A] time = 0.360192, size = 101, normalized size = 0.99

$$\frac{\operatorname{sech}^{-1}(ax) \left(a^2 x^2 \operatorname{sech}^{-1}(ax)^2 - 3 \left(ax \sqrt{\frac{1-ax}{ax+1}} + \sqrt{\frac{1-ax}{ax+1}} - 1 \right) \operatorname{sech}^{-1}(ax) + 6 \log \left(e^{-2\operatorname{sech}^{-1}(ax)} + 1 \right) \right) - 3 \operatorname{PolyLog} \left(2, -e^{-2\operatorname{sech}^{-1}(ax)} \right)}{2a^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x*ArcSech[a*x]^3, x]

[Out] (ArcSech[a*x]*(-3*(-1 + Sqrt[(1 - a*x)/(1 + a*x)] + a*x*Sqrt[(1 - a*x)/(1 + a*x]))*ArcSech[a*x] + a^2*x^2*ArcSech[a*x]^2 + 6*Log[1 + E^(-2*ArcSech[a*x]])]) - 3*PolyLog[2, -E^(-2*ArcSech[a*x])])/(2*a^2)

Maple [A] time = 0.267, size = 152, normalized size = 1.5

$$\frac{x^2 (\operatorname{arcsech}(ax))^3}{2} - \frac{3x (\operatorname{arcsech}(ax))^2}{2a} \sqrt{\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}} - \frac{3 (\operatorname{arcsech}(ax))^2}{2a^2} + 3 \frac{\operatorname{arcsech}(ax)}{a^2} \ln \left(1 + \left(\frac{1}{ax} + \sqrt{\frac{1}{ax}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*arcsech(a*x)^3,x)`

[Out] $\frac{1}{2}x^2\operatorname{arcsech}(ax)^3 - \frac{3}{2}a\operatorname{arcsech}(ax)^2 \left(-\frac{a*x-1}{a/x}\right)^{\frac{1}{2}} \left(\frac{a*x+1}{a/x}\right)^{\frac{1}{2}} x - \frac{3}{2}a^2\operatorname{arcsech}(ax)^2/a^2 + 3\operatorname{arcsech}(ax) \ln\left(1 + \left(\frac{1}{a/x} + \left(\frac{1}{a/x} - 1\right)^{\frac{1}{2}}\right)^2\right) / a^2 + 3/2 \operatorname{polylog}\left(2, -\left(\frac{1}{a/x} + \left(\frac{1}{a/x} - 1\right)^{\frac{1}{2}}\right) \left(1 + \frac{1}{a/x}\right)^{\frac{1}{2}}\right)^2 / a^2$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{ar} \operatorname{sech}(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arcsech(a*x)^3,x, algorithm="maxima")`

[Out] `integrate(x*arcsech(a*x)^3, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(x \operatorname{ar} \operatorname{sech}(ax)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arcsech(a*x)^3,x, algorithm="fricas")`

[Out] `integral(x*arcsech(a*x)^3, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{asech}^3(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*asech(a*x)**3,x)`

[Out] `Integral(x*asech(a*x)**3, x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int x \operatorname{ar} \operatorname{sech}(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*arcsech(a*x)^3,x, algorithm="giac")`

[Out] `integrate(x*arcsech(a*x)^3, x)`

3.14 $\int \operatorname{sech}^{-1}(ax)^3 dx$

Optimal. Leaf size=111

$$\frac{6i \operatorname{sech}^{-1}(ax) \operatorname{PolyLog}\left(2, -ie^{\operatorname{sech}^{-1}(ax)}\right)}{a} - \frac{6i \operatorname{sech}^{-1}(ax) \operatorname{PolyLog}\left(2, ie^{\operatorname{sech}^{-1}(ax)}\right)}{a} - \frac{6i \operatorname{PolyLog}\left(3, -ie^{\operatorname{sech}^{-1}(ax)}\right)}{a} + \frac{6i \operatorname{PolyLog}\left(3, ie^{\operatorname{sech}^{-1}(ax)}\right)}{a}$$

[Out] $x \operatorname{ArcSech}[a*x]^3 - (6 \operatorname{ArcSech}[a*x]^2 \operatorname{ArcTan}[E^{\operatorname{ArcSech}[a*x]}])/a + ((6*I) \operatorname{ArcSech}[a*x] \operatorname{PolyLog}[2, (-I)E^{\operatorname{ArcSech}[a*x]}])/a - ((6*I) \operatorname{ArcSech}[a*x] \operatorname{PolyLog}[2, I E^{\operatorname{ArcSech}[a*x]}])/a - ((6*I) \operatorname{PolyLog}[3, (-I)E^{\operatorname{ArcSech}[a*x]}])/a + ((6*I) \operatorname{PolyLog}[3, I E^{\operatorname{ArcSech}[a*x]}])/a$

Rubi [A] time = 0.0884604, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 1.$, Rules used = {6279, 5418, 4180, 2531, 2282, 6589}

$$\frac{6i \operatorname{sech}^{-1}(ax) \operatorname{PolyLog}\left(2, -ie^{\operatorname{sech}^{-1}(ax)}\right)}{a} - \frac{6i \operatorname{sech}^{-1}(ax) \operatorname{PolyLog}\left(2, ie^{\operatorname{sech}^{-1}(ax)}\right)}{a} - \frac{6i \operatorname{PolyLog}\left(3, -ie^{\operatorname{sech}^{-1}(ax)}\right)}{a} + \frac{6i \operatorname{PolyLog}\left(3, ie^{\operatorname{sech}^{-1}(ax)}\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[ArcSech[a*x]^3,x]

[Out] $x \operatorname{ArcSech}[a*x]^3 - (6 \operatorname{ArcSech}[a*x]^2 \operatorname{ArcTan}[E^{\operatorname{ArcSech}[a*x]}])/a + ((6*I) \operatorname{ArcSech}[a*x] \operatorname{PolyLog}[2, (-I)E^{\operatorname{ArcSech}[a*x]}])/a - ((6*I) \operatorname{ArcSech}[a*x] \operatorname{PolyLog}[2, I E^{\operatorname{ArcSech}[a*x]}])/a - ((6*I) \operatorname{PolyLog}[3, (-I)E^{\operatorname{ArcSech}[a*x]}])/a + ((6*I) \operatorname{PolyLog}[3, I E^{\operatorname{ArcSech}[a*x]}])/a$

Rule 6279

Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))^ (n_.), x_Symbol] :> -Dist[c^(-1), Subst[Int[(a + b*x)^n*Sech[x]*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[n, 0]

Rule 5418

Int[(x_)^(m_.)*Sech[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*Tanh[(a_.) + (b_.)*(x_)^(n_.)]^(q_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Sech[a + b*x^n]^p)/(b*n*p), x] + Dist[(m - n + 1)/(b*n*p), Int[x^(m - n)*Sech[a + b*x^n]^p, x], x] /; FreeQ[{a, b, p}, x] && RationalQ[m] && IntegerQ[n] && GeQ[m - n, 0] && EqQ[q, 1]

Rule 4180

Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_.)]*((c_.) + (d_.)*(x_)^(m_.)), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(I*k*Pi)])/ (f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]

Rule 2531

Int[Log[1 + (e_.)*((F_)^(c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)])/ (b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -

1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 2282

Int[u_, x_Symbol] :=> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

Rule 6589

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] :=> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]

Rubi steps

$$\begin{aligned} \int \operatorname{sech}^{-1}(ax)^3 dx &= -\frac{\operatorname{Subst}\left(\int x^3 \operatorname{sech}(x) \tanh(x) dx, x, \operatorname{sech}^{-1}(ax)\right)}{a} \\ &= x \operatorname{sech}^{-1}(ax)^3 - \frac{3 \operatorname{Subst}\left(\int x^2 \operatorname{sech}(x) dx, x, \operatorname{sech}^{-1}(ax)\right)}{a} \\ &= x \operatorname{sech}^{-1}(ax)^3 - \frac{6 \operatorname{sech}^{-1}(ax)^2 \tan^{-1}\left(e^{\operatorname{sech}^{-1}(ax)}\right)}{a} + \frac{(6i) \operatorname{Subst}\left(\int x \log(1 - ie^x) dx, x, \operatorname{sech}^{-1}(ax)\right)}{a} \\ &= x \operatorname{sech}^{-1}(ax)^3 - \frac{6 \operatorname{sech}^{-1}(ax)^2 \tan^{-1}\left(e^{\operatorname{sech}^{-1}(ax)}\right)}{a} + \frac{6i \operatorname{sech}^{-1}(ax) \operatorname{Li}_2\left(-ie^{\operatorname{sech}^{-1}(ax)}\right)}{a} - \frac{6i \operatorname{sech}^{-1}(ax)}{a} \\ &= x \operatorname{sech}^{-1}(ax)^3 - \frac{6 \operatorname{sech}^{-1}(ax)^2 \tan^{-1}\left(e^{\operatorname{sech}^{-1}(ax)}\right)}{a} + \frac{6i \operatorname{sech}^{-1}(ax) \operatorname{Li}_2\left(-ie^{\operatorname{sech}^{-1}(ax)}\right)}{a} - \frac{6i \operatorname{sech}^{-1}(ax)}{a} \\ &= x \operatorname{sech}^{-1}(ax)^3 - \frac{6 \operatorname{sech}^{-1}(ax)^2 \tan^{-1}\left(e^{\operatorname{sech}^{-1}(ax)}\right)}{a} + \frac{6i \operatorname{sech}^{-1}(ax) \operatorname{Li}_2\left(-ie^{\operatorname{sech}^{-1}(ax)}\right)}{a} - \frac{6i \operatorname{sech}^{-1}(ax)}{a} \end{aligned}$$

Mathematica [A] time = 0.116257, size = 128, normalized size = 1.15

$$x \operatorname{sech}^{-1}(ax)^3 - \frac{3i \left(-2 \operatorname{sech}^{-1}(ax) \left(\operatorname{PolyLog}\left(2, -ie^{-\operatorname{sech}^{-1}(ax)}\right) - \operatorname{PolyLog}\left(2, ie^{-\operatorname{sech}^{-1}(ax)}\right) \right) - 2 \left(\operatorname{PolyLog}\left(3, -ie^{-\operatorname{sech}^{-1}(ax)}\right) - \operatorname{PolyLog}\left(3, ie^{-\operatorname{sech}^{-1}(ax)}\right) \right) \right)}{a}$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcSech[a*x]^3, x]

[Out] x*ArcSech[a*x]^3 - ((3*I)*(-(ArcSech[a*x]^2*(Log[1 - I/E^ArcSech[a*x]] - Log[1 + I/E^ArcSech[a*x]])) - 2*ArcSech[a*x]*(PolyLog[2, (-I)/E^ArcSech[a*x]] - PolyLog[2, I/E^ArcSech[a*x]])) - 2*(PolyLog[3, (-I)/E^ArcSech[a*x]] - PolyLog[3, I/E^ArcSech[a*x]])))/a

Maple [F] time = 0.304, size = 0, normalized size = 0.

$$\int (\operatorname{arcsech}(ax))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsech(a*x)^3,x)

[Out] int(arcsech(a*x)^3,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$x \log\left(\sqrt{ax+1}\sqrt{-ax+1}+1\right)^3 - \int \frac{a^2x^2 \log(a)^3 + (a^2x^2 - 1) \log(x)^3 + 3(a^2x^2 \log(a) + (a^2x^2(\log(a) + 1) + (a^2x^2 - 1) \log(a)) \log(x) - \log(a)) \sqrt{ax+1} \sqrt{-ax+1} + (a^2x^2 - 1) \log(x) - \log(a)) \log(\sqrt{ax+1} \sqrt{-ax+1} + 1)^2 - \log(a)^3 + 3(a^2x^2 \log(a) - \log(a)) \log(x)^2 + (a^2x^2 \log(a)^3 + (a^2x^2 - 1) \log(x)^3 - \log(a)^3 + 3(a^2x^2 \log(a) - \log(a)) \log(x)^2 + 3(a^2x^2 \log(a)^2 - \log(a)^2) \log(x)) \sqrt{ax+1} \sqrt{-ax+1} - 3(a^2x^2 \log(a)^2 + (a^2x^2 - 1) \log(x)^2 + (a^2x^2 \log(a)^2 + (a^2x^2 - 1) \log(x)^2 - \log(a)^2 + 2(a^2x^2 \log(a) - \log(a)) \log(x)) \sqrt{ax+1} \sqrt{-ax+1} - \log(a)^2 + 2(a^2x^2 \log(a) - \log(a)) \log(x)) \log(\sqrt{ax+1} \sqrt{-ax+1} + 1) + 3(a^2x^2 \log(a)^2 - \log(a)^2) \log(x)}{(a^2x^2 + (a^2x^2 - 1) \sqrt{ax+1} \sqrt{-ax+1} - 1), x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(a*x)^3,x, algorithm="maxima")

[Out] x*log(sqrt(a*x + 1)*sqrt(-a*x + 1) + 1)^3 - integrate((a^2*x^2*log(a)^3 + (a^2*x^2 - 1)*log(x)^3 + 3*(a^2*x^2*log(a) + (a^2*x^2*(log(a) + 1) + (a^2*x^2 - 1)*log(x) - log(a))*sqrt(a*x + 1)*sqrt(-a*x + 1) + (a^2*x^2 - 1)*log(x) - log(a))*log(sqrt(a*x + 1)*sqrt(-a*x + 1) + 1)^2 - log(a)^3 + 3*(a^2*x^2*log(a) - log(a))*log(x)^2 + (a^2*x^2*log(a)^3 + (a^2*x^2 - 1)*log(x)^3 - log(a)^3 + 3*(a^2*x^2*log(a) - log(a))*log(x)^2 + 3*(a^2*x^2*log(a)^2 - log(a)^2)*log(x))*sqrt(a*x + 1)*sqrt(-a*x + 1) - 3*(a^2*x^2*log(a)^2 + (a^2*x^2 - 1)*log(x)^2 + (a^2*x^2*log(a)^2 + (a^2*x^2 - 1)*log(x)^2 - log(a)^2 + 2*(a^2*x^2*log(a) - log(a))*log(x))*sqrt(a*x + 1)*sqrt(-a*x + 1) - log(a)^2 + 2*(a^2*x^2*log(a) - log(a))*log(x))*log(sqrt(a*x + 1)*sqrt(-a*x + 1) + 1) + 3*(a^2*x^2*log(a)^2 - log(a)^2)*log(x))/(a^2*x^2 + (a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(-a*x + 1) - 1), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(\text{arsech}(ax)^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(a*x)^3,x, algorithm="fricas")

[Out] integral(arcsech(a*x)^3, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \text{asech}^3(ax) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asech(a*x)**3,x)

[Out] Integral(asech(a*x)**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \operatorname{arsech}(ax)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(arcsech(a*x)^3, x)
```

3.15 $\int \frac{\operatorname{sech}^{-1}(ax)^3}{x} dx$

Optimal. Leaf size=88

$$-\frac{3}{2}\operatorname{sech}^{-1}(ax)^2\operatorname{PolyLog}\left(2, -e^{2\operatorname{sech}^{-1}(ax)}\right) + \frac{3}{2}\operatorname{sech}^{-1}(ax)\operatorname{PolyLog}\left(3, -e^{2\operatorname{sech}^{-1}(ax)}\right) - \frac{3}{4}\operatorname{PolyLog}\left(4, -e^{2\operatorname{sech}^{-1}(ax)}\right) + \frac{1}{4}\operatorname{sech}^{-1}(ax)$$

[Out] ArcSech[a*x]^4/4 - ArcSech[a*x]^3*Log[1 + E^(2*ArcSech[a*x])] - (3*ArcSech[a*x]^2*PolyLog[2, -E^(2*ArcSech[a*x])])/2 + (3*ArcSech[a*x]*PolyLog[3, -E^(2*ArcSech[a*x])])/2 - (3*PolyLog[4, -E^(2*ArcSech[a*x])])/4

Rubi [A] time = 0.107518, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.7$, Rules used = {6285, 3718, 2190, 2531, 6609, 2282, 6589}

$$-\frac{3}{2}\operatorname{sech}^{-1}(ax)^2\operatorname{PolyLog}\left(2, -e^{2\operatorname{sech}^{-1}(ax)}\right) + \frac{3}{2}\operatorname{sech}^{-1}(ax)\operatorname{PolyLog}\left(3, -e^{2\operatorname{sech}^{-1}(ax)}\right) - \frac{3}{4}\operatorname{PolyLog}\left(4, -e^{2\operatorname{sech}^{-1}(ax)}\right) + \frac{1}{4}\operatorname{sech}^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Int[ArcSech[a*x]^3/x, x]

[Out] ArcSech[a*x]^4/4 - ArcSech[a*x]^3*Log[1 + E^(2*ArcSech[a*x])] - (3*ArcSech[a*x]^2*PolyLog[2, -E^(2*ArcSech[a*x])])/2 + (3*ArcSech[a*x]*PolyLog[3, -E^(2*ArcSech[a*x])])/2 - (3*PolyLog[4, -E^(2*ArcSech[a*x])])/4

Rule 6285

Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))^n*(x_)^m, x_Symbol] :> -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

Rule 3718

Int[((c_.) + (d_.)*(x_)^m)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

Rule 2190

Int[(((F_)^(g_)*((e_.) + (f_.)*(x_)))^n)*((c_.) + (d_.)*(x_)^m)/((a_) + (b_.)*((F_)^(g_)*((e_.) + (f_.)*(x_)))^n), x_Symbol] :> Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2531

Int[Log[1 + (e_.)*((F_)^(c_)*((a_.) + (b_.)*(x_)))^n]*((f_.) + (g_.)*(x_)^m), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]

Rule 6609

```
Int[((e_.) + (f_.)*(x_.))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_.)))^(p_.)], x_Symbol] := Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_.))^(p_.)]/((d_.) + (e_.)*(x_.)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{sech}^{-1}(ax)^3}{x} dx &= -\operatorname{Subst}\left(\int x^3 \tanh(x) dx, x, \operatorname{sech}^{-1}(ax)\right) \\ &= \frac{1}{4} \operatorname{sech}^{-1}(ax)^4 - 2 \operatorname{Subst}\left(\int \frac{e^{2x} x^3}{1 + e^{2x}} dx, x, \operatorname{sech}^{-1}(ax)\right) \\ &= \frac{1}{4} \operatorname{sech}^{-1}(ax)^4 - \operatorname{sech}^{-1}(ax)^3 \log\left(1 + e^{2\operatorname{sech}^{-1}(ax)}\right) + 3 \operatorname{Subst}\left(\int x^2 \log(1 + e^{2x}) dx, x, \operatorname{sech}^{-1}(ax)\right) \\ &= \frac{1}{4} \operatorname{sech}^{-1}(ax)^4 - \operatorname{sech}^{-1}(ax)^3 \log\left(1 + e^{2\operatorname{sech}^{-1}(ax)}\right) - \frac{3}{2} \operatorname{sech}^{-1}(ax)^2 \operatorname{Li}_2\left(-e^{2\operatorname{sech}^{-1}(ax)}\right) + 3 \operatorname{Subst}\left(\int x \log(1 + e^{2x}) dx, x, \operatorname{sech}^{-1}(ax)\right) \\ &= \frac{1}{4} \operatorname{sech}^{-1}(ax)^4 - \operatorname{sech}^{-1}(ax)^3 \log\left(1 + e^{2\operatorname{sech}^{-1}(ax)}\right) - \frac{3}{2} \operatorname{sech}^{-1}(ax)^2 \operatorname{Li}_2\left(-e^{2\operatorname{sech}^{-1}(ax)}\right) + \frac{3}{2} \operatorname{sech}^{-1}(ax) \log\left(1 + e^{2\operatorname{sech}^{-1}(ax)}\right) \\ &= \frac{1}{4} \operatorname{sech}^{-1}(ax)^4 - \operatorname{sech}^{-1}(ax)^3 \log\left(1 + e^{2\operatorname{sech}^{-1}(ax)}\right) - \frac{3}{2} \operatorname{sech}^{-1}(ax)^2 \operatorname{Li}_2\left(-e^{2\operatorname{sech}^{-1}(ax)}\right) + \frac{3}{2} \operatorname{sech}^{-1}(ax) \log\left(1 + e^{2\operatorname{sech}^{-1}(ax)}\right) \\ &= \frac{1}{4} \operatorname{sech}^{-1}(ax)^4 - \operatorname{sech}^{-1}(ax)^3 \log\left(1 + e^{2\operatorname{sech}^{-1}(ax)}\right) - \frac{3}{2} \operatorname{sech}^{-1}(ax)^2 \operatorname{Li}_2\left(-e^{2\operatorname{sech}^{-1}(ax)}\right) + \frac{3}{2} \operatorname{sech}^{-1}(ax) \log\left(1 + e^{2\operatorname{sech}^{-1}(ax)}\right) \end{aligned}$$

Mathematica [A] time = 0.0493463, size = 84, normalized size = 0.95

$$\frac{1}{4} \left(6 \operatorname{sech}^{-1}(ax)^2 \operatorname{PolyLog}\left(2, -e^{-2\operatorname{sech}^{-1}(ax)}\right) + 6 \operatorname{sech}^{-1}(ax) \operatorname{PolyLog}\left(3, -e^{-2\operatorname{sech}^{-1}(ax)}\right) + 3 \operatorname{PolyLog}\left(4, -e^{-2\operatorname{sech}^{-1}(ax)}\right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[ArcSech[a*x]^3/x, x]

[Out] $(-\operatorname{ArcSech}[a*x]^4 - 4*\operatorname{ArcSech}[a*x]^3*\operatorname{Log}[1 + E^{(-2*\operatorname{ArcSech}[a*x])}] + 6*\operatorname{ArcSech}[a*x]^2*\operatorname{PolyLog}[2, -E^{(-2*\operatorname{ArcSech}[a*x])}] + 6*\operatorname{ArcSech}[a*x]*\operatorname{PolyLog}[3, -E^{(-2*\operatorname{ArcSech}[a*x])}] + 3*\operatorname{PolyLog}[4, -E^{(-2*\operatorname{ArcSech}[a*x])}])/4$

Maple [A] time = 0.244, size = 181, normalized size = 2.1

$$\frac{(\operatorname{arcsech}(ax))^4}{4} - (\operatorname{arcsech}(ax))^3 \ln\left(1 + \left(\frac{1}{ax} + \sqrt{\frac{1}{ax} - 1}\sqrt{1 + \frac{1}{ax}}\right)^2\right) - \frac{3(\operatorname{arcsech}(ax))^2}{2} \operatorname{polylog}\left(2, -\left(\frac{1}{ax} + \sqrt{\frac{1}{ax} - 1}\sqrt{1 + \frac{1}{ax}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsech(a*x)^3/x,x)

[Out] $\frac{1}{4}\operatorname{arcsech}(ax)^4 - \operatorname{arcsech}(ax)^3 \ln(1 + (1/a/x + (1/a/x - 1)^{1/2}) * (1 + 1/a/x)^{1/2})^2 - 3/2 \operatorname{arcsech}(ax)^2 \operatorname{polylog}(2, -(1/a/x + (1/a/x - 1)^{1/2}) * (1 + 1/a/x)^{1/2})^2 + 3/2 \operatorname{arcsech}(ax) \operatorname{polylog}(3, -(1/a/x + (1/a/x - 1)^{1/2}) * (1 + 1/a/x)^{1/2})^2 - 3/4 \operatorname{polylog}(4, -(1/a/x + (1/a/x - 1)^{1/2}) * (1 + 1/a/x)^{1/2})^2$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arosech}(ax)^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(a*x)^3/x,x, algorithm="maxima")

[Out] integrate(arcsech(a*x)^3/x, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{arosech}(ax)^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(a*x)^3/x,x, algorithm="fricas")

[Out] integral(arcsech(a*x)^3/x, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asech}^3(ax)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asech(a*x)**3/x,x)

[Out] Integral(asech(a*x)**3/x, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arosech}(ax)^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x)^3/x,x, algorithm="giac")
```

```
[Out] integrate(arcsech(a*x)^3/x, x)
```

3.16 $\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^2} dx$

Optimal. Leaf size=83

$$\frac{6\sqrt{\frac{1-ax}{ax+1}}(ax+1)}{x} - \frac{\operatorname{sech}^{-1}(ax)^3}{x} + \frac{3\sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)^2}{x} - \frac{6\operatorname{sech}^{-1}(ax)}{x}$$

[Out] (6*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/x - (6*ArcSech[a*x])/x + (3*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x]^2)/x - ArcSech[a*x]^3/x

Rubi [A] time = 0.0711182, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {6285, 3296, 2637}

$$\frac{6\sqrt{\frac{1-ax}{ax+1}}(ax+1)}{x} - \frac{\operatorname{sech}^{-1}(ax)^3}{x} + \frac{3\sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)^2}{x} - \frac{6\operatorname{sech}^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

[In] Int[ArcSech[a*x]^3/x^2,x]

[Out] (6*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/x - (6*ArcSech[a*x])/x + (3*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x]^2)/x - ArcSech[a*x]^3/x

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))^n*(x_)^m, x_Symbol] :> -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_)^m)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^2} dx &= -\left(a \operatorname{Subst}\left(\int x^3 \sinh(x) dx, x, \operatorname{sech}^{-1}(ax)\right)\right) \\
&= -\frac{\operatorname{sech}^{-1}(ax)^3}{x} + (3a) \operatorname{Subst}\left(\int x^2 \cosh(x) dx, x, \operatorname{sech}^{-1}(ax)\right) \\
&= \frac{3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)^2}{x} - \frac{\operatorname{sech}^{-1}(ax)^3}{x} - (6a) \operatorname{Subst}\left(\int x \sinh(x) dx, x, \operatorname{sech}^{-1}(ax)\right) \\
&= -\frac{6\operatorname{sech}^{-1}(ax)}{x} + \frac{3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)^2}{x} - \frac{\operatorname{sech}^{-1}(ax)^3}{x} + (6a) \operatorname{Subst}\left(\int \cosh(x) dx, x, \operatorname{sech}^{-1}(ax)\right) \\
&= \frac{6\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{x} - \frac{6\operatorname{sech}^{-1}(ax)}{x} + \frac{3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)^2}{x} - \frac{\operatorname{sech}^{-1}(ax)^3}{x}
\end{aligned}$$

Mathematica [A] time = 0.0718714, size = 75, normalized size = 0.9

$$\frac{6\sqrt{\frac{1-ax}{ax+1}}(ax+1) - \operatorname{sech}^{-1}(ax)^3 + 3\sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)^2 - 6\operatorname{sech}^{-1}(ax)}{x}$$

Antiderivative was successfully verified.

[In] Integrate[ArcSech[a*x]^3/x^2,x]

[Out] (6*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x) - 6*ArcSech[a*x] + 3*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x]^2 - ArcSech[a*x]^3)/x

Maple [A] time = 0.211, size = 98, normalized size = 1.2

$$a \left(-\frac{(\operatorname{arcsech}(ax))^3}{ax} + 3(\operatorname{arcsech}(ax))^2 \sqrt{-\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}} - 6\frac{\operatorname{arcsech}(ax)}{ax} + 6\sqrt{-\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsech(a*x)^3/x^2,x)

[Out] a*(-arcsech(a*x)^3/a/x+3*arcsech(a*x)^2*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2)-6/a/x*arcsech(a*x)+6*(-(a*x-1)/a/x)^(1/2)*((a*x+1)/a/x)^(1/2))

Maxima [A] time = 1.03425, size = 74, normalized size = 0.89

$$3a\sqrt{\frac{1}{a^2x^2}-1}\operatorname{arosech}(ax)^2 - \frac{\operatorname{arosech}(ax)^3}{x} + 6a\sqrt{\frac{1}{a^2x^2}-1} - \frac{6\operatorname{arosech}(ax)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(a*x)^3/x^2,x, algorithm="maxima")

[Out] 3*a*sqrt(1/(a^2*x^2) - 1)*arcsech(a*x)^2 - arcsech(a*x)^3/x + 6*a*sqrt(1/(a^2*x^2) - 1) - 6*arcsech(a*x)/x

Fricas [A] time = 1.93658, size = 333, normalized size = 4.01

$$\frac{3ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}} \log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)^2 - \log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)^3 + 6ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}} - 6 \log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(a*x)^3/x^2,x, algorithm="fricas")

[Out] (3*a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2))*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x))^2 - log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x))^3 + 6*a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) - 6*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x)))/x

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asech}^3(ax)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asech(a*x)**3/x**2,x)

[Out] Integral(asech(a*x)**3/x**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arsech}(ax)^3}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(a*x)^3/x^2,x, algorithm="giac")

[Out] integrate(arcsech(a*x)^3/x^2, x)

3.17 $\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^3} dx$

Optimal. Leaf size=137

$$-\frac{1}{4}a^2\operatorname{sech}^{-1}(ax)^3 - \frac{3}{8}a^2\operatorname{sech}^{-1}(ax) + \frac{3\sqrt{\frac{1-ax}{ax+1}}(ax+1)}{8x^2} - \frac{(1-ax)(ax+1)\operatorname{sech}^{-1}(ax)^3}{2x^2} + \frac{3\sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)^2}{4x^2}$$

```
[Out] (3*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/(8*x^2) - (3*a^2*ArcSech[a*x])/8 -
(3*(1 - a*x)*(1 + a*x)*ArcSech[a*x])/(4*x^2) + (3*Sqrt[(1 - a*x)/(1 + a*x)]
*(1 + a*x)*ArcSech[a*x]^2)/(4*x^2) - (a^2*ArcSech[a*x]^3)/4 - ((1 - a*x)*(1
+ a*x)*ArcSech[a*x]^3)/(2*x^2)
```

Rubi [A] time = 0.0862309, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$, Rules used = {6285, 5372, 3311, 30, 2635, 8}

$$-\frac{1}{4}a^2\operatorname{sech}^{-1}(ax)^3 - \frac{3}{8}a^2\operatorname{sech}^{-1}(ax) + \frac{3\sqrt{\frac{1-ax}{ax+1}}(ax+1)}{8x^2} - \frac{(1-ax)(ax+1)\operatorname{sech}^{-1}(ax)^3}{2x^2} + \frac{3\sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)^2}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcSech[a*x]^3/x^3,x]
```

```
[Out] (3*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/(8*x^2) - (3*a^2*ArcSech[a*x])/8 -
(3*(1 - a*x)*(1 + a*x)*ArcSech[a*x])/(4*x^2) + (3*Sqrt[(1 - a*x)/(1 + a*x)]
*(1 + a*x)*ArcSech[a*x]^2)/(4*x^2) - (a^2*ArcSech[a*x]^3)/4 - ((1 - a*x)*(1
+ a*x)*ArcSech[a*x]^3)/(2*x^2)
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))^(n_.)*(x_)^(m_.), x_Symbol] := -Dist
[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, Ar
cSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 5372

```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] := Simp[(x^(m - n + 1)*Sinh[a + b*x^n]^(p + 1))/(b*n*(p
+ 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sinh[a + b*x^n]^(
p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_)^(m_.))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbo
l] := Simp[(d*m*(c + d*x)^(m - 1)*(b*Sine[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sine[e + f*x])^(n - 2), x], x] - Dist[
(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sine[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sine[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]
)*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{sech}^{-1}(ax)^3}{x^3} dx &= -\left(a^2 \operatorname{Subst}\left(\int x^3 \cosh(x) \sinh(x) dx, x, \operatorname{sech}^{-1}(ax)\right)\right) \\ &= -\frac{(1-ax)(1+ax)\operatorname{sech}^{-1}(ax)^3}{2x^2} + \frac{1}{2}(3a^2) \operatorname{Subst}\left(\int x^2 \sinh^2(x) dx, x, \operatorname{sech}^{-1}(ax)\right) \\ &= -\frac{3(1-ax)(1+ax)\operatorname{sech}^{-1}(ax)}{4x^2} + \frac{3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)^2}{4x^2} - \frac{(1-ax)(1+ax)\operatorname{sech}^{-1}(ax)^3}{2x^2} - \frac{1}{4}(3) \\ &= \frac{3\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{8x^2} - \frac{3(1-ax)(1+ax)\operatorname{sech}^{-1}(ax)}{4x^2} + \frac{3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)^2}{4x^2} - \frac{1}{4}a^2\operatorname{sech}^{-1}(ax)^3 - \\ &= \frac{3\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{8x^2} - \frac{3}{8}a^2\operatorname{sech}^{-1}(ax) - \frac{3(1-ax)(1+ax)\operatorname{sech}^{-1}(ax)}{4x^2} + \frac{3\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)^2}{4x^2} - \end{aligned}$$

Mathematica [A] time = 0.133262, size = 147, normalized size = 1.07

$$\frac{-3a^2x^2 \log(x) + 3a^2x^2 \log\left(ax\sqrt{\frac{1-ax}{ax+1}} + \sqrt{\frac{1-ax}{ax+1}} + 1\right) + 2(a^2x^2 - 2) \operatorname{sech}^{-1}(ax)^3 + 3\sqrt{\frac{1-ax}{ax+1}}(ax+1) + 6\sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSech[a*x]^3/x^3, x]
```

```
[Out] (3*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x) - 6*ArcSech[a*x] + 6*Sqrt[(1 - a*x)/(
1 + a*x)]*(1 + a*x)*ArcSech[a*x]^2 + 2*(-2 + a^2*x^2)*ArcSech[a*x]^3 - 3*a
^2*x^2*Log[x] + 3*a^2*x^2*Log[1 + Sqrt[(1 - a*x)/(1 + a*x)] + a*x*Sqrt[(1 -
a*x)/(1 + a*x)]])/(8*x^2)
```

Maple [A] time = 0.217, size = 126, normalized size = 0.9

$$a^2 \left(-\frac{(\operatorname{arcsech}(ax))^3}{2a^2x^2} + \frac{3(\operatorname{arcsech}(ax))^2}{4ax} \sqrt{-\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}} + \frac{(\operatorname{arcsech}(ax))^3}{4} - \frac{3\operatorname{arcsech}(ax)}{4a^2x^2} + \frac{3}{8ax} \sqrt{-\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arcsech(a*x)^3/x^3, x)
```

```
[Out] a^2*(-1/2*arcsech(a*x)^3/a^2/x^2+3/4*arcsech(a*x)^2/a/x*(-(a*x-1)/a/x)^(1/2)
)*(a*x+1)/a/x)^(1/2)+1/4*arcsech(a*x)^3-3/4/a^2/x^2*arcsech(a*x)+3/8/a/x*(
-(a*x-1)/a/x)^(1/2)*(a*x+1)/a/x)^(1/2)+3/8*arcsech(a*x))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arosech}(ax)^3}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(a*x)^3/x^3,x, algorithm="maxima")

[Out] integrate(arcsech(a*x)^3/x^3, x)

Fricas [A] time = 2.03459, size = 382, normalized size = 2.79

$$\frac{6ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}} \log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)^2 + 2(a^2x^2-2) \log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)^3 + 3ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}} + 3(a^2x^2-2) \log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(a*x)^3/x^3,x, algorithm="fricas")

[Out] 1/8*(6*a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2))*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x))^2 + 2*(a^2*x^2 - 2)*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x))^3 + 3*a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 3*(a^2*x^2 - 2)*log((a*x*sqrt(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x)))/x^2

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{asech}^3(ax)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asech(a*x)**3/x**3,x)

[Out] Integral(asech(a*x)**3/x**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{arosech}(ax)^3}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(a*x)^3/x^3,x, algorithm="giac")

[Out] integrate(arcsech(a*x)^3/x^3, x)

3.18 $\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^4} dx$

Optimal. Leaf size=179

$$\frac{14a^2 \sqrt{\frac{1-ax}{ax+1}}(ax+1)}{9x} + \frac{2a^2 \sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)^2}{3x} - \frac{4a^2 \operatorname{sech}^{-1}(ax)}{3x} + \frac{2\left(\frac{1-ax}{ax+1}\right)^{3/2}(ax+1)^3}{27x^3} + \frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)}{3x^3}$$

[Out] (14*a^2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/(9*x) + (2*((1 - a*x)/(1 + a*x))^(3/2)*(1 + a*x)^3)/(27*x^3) - (2*ArcSech[a*x])/(9*x^3) - (4*a^2*ArcSech[a*x])/(3*x) + (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x]^2)/(3*x^3) + (2*a^2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x]^2)/(3*x) - ArcSech[a*x]^3/(3*x^3)

Rubi [A] time = 0.125027, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$, Rules used = {6285, 5373, 3311, 3296, 2637, 2633}

$$\frac{14a^2 \sqrt{\frac{1-ax}{ax+1}}(ax+1)}{9x} + \frac{2a^2 \sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)^2}{3x} - \frac{4a^2 \operatorname{sech}^{-1}(ax)}{3x} + \frac{2\left(\frac{1-ax}{ax+1}\right)^{3/2}(ax+1)^3}{27x^3} + \frac{\sqrt{\frac{1-ax}{ax+1}}(ax+1)\operatorname{sech}^{-1}(ax)}{3x^3}$$

Antiderivative was successfully verified.

[In] Int[ArcSech[a*x]^3/x^4,x]

[Out] (14*a^2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x))/(9*x) + (2*((1 - a*x)/(1 + a*x))^(3/2)*(1 + a*x)^3)/(27*x^3) - (2*ArcSech[a*x])/(9*x^3) - (4*a^2*ArcSech[a*x])/(3*x) + (Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x]^2)/(3*x^3) + (2*a^2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x)*ArcSech[a*x]^2)/(3*x) - ArcSech[a*x]^3/(3*x^3)

Rule 6285

Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))^ (n_.)*(x_.)^(m_.), x_Symbol] :> -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^(n)*Sech[x]^(m + 1)*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

Rule 5373

Int[Cosh[(a_.) + (b_.)*(x_.)^(n_.)]^(p_.)*(x_.)^(m_.)*Sinh[(a_.) + (b_.)*(x_.)^(n_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Cosh[a + b*x^n]^(p + 1))/(b*n*(p + 1)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cosh[a + b*x^n]^(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]

Rule 3311

Int[((c_.) + (d_.)*(x_.))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_.)])^(n_.), x_Symbol] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Ssin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c + d*x)^(m)*(b*Ssin[e + f*x])^(n - 2), x], x] - Dist[(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Ssin[e + f*x])^n, x], x] - Simp[(b*(c + d*x)^(m)*Cos[e + f*x]*(b*Ssin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

Rule 3296

```
Int[((c_.) + (d_.)*(x_.))^(m_.)*sin[(e_.) + (f_.)*(x_.)], x_Symbol] := -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_.)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^{-1}(ax)^3}{x^4} dx &= -\left(a^3 \operatorname{Subst}\left(\int x^3 \cosh^2(x) \sinh(x) dx, x, \operatorname{sech}^{-1}(ax)\right)\right) \\
&= -\frac{\operatorname{sech}^{-1}(ax)^3}{3x^3} + a^3 \operatorname{Subst}\left(\int x^2 \cosh^3(x) dx, x, \operatorname{sech}^{-1}(ax)\right) \\
&= -\frac{2\operatorname{sech}^{-1}(ax)}{9x^3} + \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)^2}{3x^3} - \frac{\operatorname{sech}^{-1}(ax)^3}{3x^3} + \frac{1}{9}(2a^3) \operatorname{Subst}\left(\int \cosh^3(x) dx, x, \operatorname{sech}^{-1}(ax)\right) \\
&= -\frac{2\operatorname{sech}^{-1}(ax)}{9x^3} + \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)^2}{3x^3} + \frac{2a^2\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)^2}{3x} - \frac{\operatorname{sech}^{-1}(ax)^3}{3x^3} + \\
&= \frac{2a^2\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{9x} + \frac{2\left(\frac{1-ax}{1+ax}\right)^{3/2}(1+ax)^3}{27x^3} - \frac{2\operatorname{sech}^{-1}(ax)}{9x^3} - \frac{4a^2\operatorname{sech}^{-1}(ax)}{3x} + \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)^2}{3x^3} \\
&= \frac{14a^2\sqrt{\frac{1-ax}{1+ax}}(1+ax)}{9x} + \frac{2\left(\frac{1-ax}{1+ax}\right)^{3/2}(1+ax)^3}{27x^3} - \frac{2\operatorname{sech}^{-1}(ax)}{9x^3} - \frac{4a^2\operatorname{sech}^{-1}(ax)}{3x} + \frac{\sqrt{\frac{1-ax}{1+ax}}(1+ax)\operatorname{sech}^{-1}(ax)^2}{3x^3}
\end{aligned}$$

Mathematica [A] time = 0.113122, size = 120, normalized size = 0.67

$$\frac{2\sqrt{\frac{1-ax}{ax+1}}(20a^3x^3 + 20a^2x^2 + ax + 1) + 9\sqrt{\frac{1-ax}{ax+1}}(2a^3x^3 + 2a^2x^2 + ax + 1)\operatorname{sech}^{-1}(ax)^2 - 6(6a^2x^2 + 1)\operatorname{sech}^{-1}(ax) - 9\operatorname{sech}^{-1}(ax)^3}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcSech[a*x]^3/x^4, x]
```

```
[Out] (2*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x + 20*a^2*x^2 + 20*a^3*x^3) - 6*(1 + 6
*a^2*x^2)*ArcSech[a*x] + 9*Sqrt[(1 - a*x)/(1 + a*x)]*(1 + a*x + 2*a^2*x^2 +
2*a^3*x^3)*ArcSech[a*x]^2 - 9*ArcSech[a*x]^3)/(27*x^3)
```

Maple [A] time = 0.229, size = 226, normalized size = 1.3

$$a^3 \left(\frac{(\operatorname{arcsech}(ax))^3(ax-1)(ax+1)}{3x^3a^3} - \frac{(\operatorname{arcsech}(ax))^3}{3ax} + \frac{(\operatorname{arcsech}(ax))^2}{3a^2x^2} \sqrt{-\frac{ax-1}{ax}} \sqrt{\frac{ax+1}{ax}} + \frac{2(\operatorname{arcsech}(ax))^2}{3} \sqrt{\frac{ax+1}{ax}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arcsech(a*x)^3/x^4,x)

[Out] $a^3*(1/3*\text{arcsech}(a*x)^3/a^3/x^3*(a*x-1)*(a*x+1)-1/3*\text{arcsech}(a*x)^3/a/x+1/3*\text{arcsech}(a*x)^2/a^2/x^2*(-(a*x-1)/a/x)^{(1/2)}*((a*x+1)/a/x)^{(1/2)}+2/3*\text{arcsech}(a*x)^2*(-(a*x-1)/a/x)^{(1/2)}*((a*x+1)/a/x)^{(1/2)}+2/9*\text{arcsech}(a*x)*(a*x-1)/a^3/x^3*(a*x+1)-14/9/a/x*\text{arcsech}(a*x)+2/27/a^2/x^2*(-(a*x-1)/a/x)^{(1/2)}*((a*x+1)/a/x)^{(1/2)}+40/27*(-(a*x-1)/a/x)^{(1/2)}*((a*x+1)/a/x)^{(1/2)})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{arsech}(ax)^3}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(a*x)^3/x^4,x, algorithm="maxima")

[Out] integrate(arcsech(a*x)^3/x^4, x)

Fricas [A] time = 1.97532, size = 406, normalized size = 2.27

$$\frac{9(2a^3x^3 + ax)\sqrt{-\frac{a^2x^2-1}{a^2x^2}} \log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)^2 - 9 \log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right)^3 - 6(6a^2x^2 + 1) \log\left(\frac{ax\sqrt{-\frac{a^2x^2-1}{a^2x^2}}+1}{ax}\right) + 2(20a^3x^3 + a}{27x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arcsech(a*x)^3/x^4,x, algorithm="fricas")

[Out] $1/27*(9*(2*a^3*x^3 + a*x)*\text{sqrt}(-(a^2*x^2 - 1)/(a^2*x^2))*\log((a*x*\text{sqrt}(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x))^2 - 9*\log((a*x*\text{sqrt}(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x))^3 - 6*(6*a^2*x^2 + 1)*\log((a*x*\text{sqrt}(-(a^2*x^2 - 1)/(a^2*x^2)) + 1)/(a*x)) + 2*(20*a^3*x^3 + a*x)*\text{sqrt}(-(a^2*x^2 - 1)/(a^2*x^2)))/x^3$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{asech}^3(ax)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(asech(a*x)**3/x**4,x)

[Out] Integral(asech(a*x)**3/x**4, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{arsech}(ax)^3}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arcsech(a*x)^3/x^4,x, algorithm="giac")
```

```
[Out] integrate(arcsech(a*x)^3/x^4, x)
```

3.19 $\int x^6 (a + b \operatorname{sech}^{-1}(cx)) dx$

Optimal. Leaf size=142

$$\frac{1}{7}x^7 (a + b \operatorname{sech}^{-1}(cx)) - \frac{bx^5 \sqrt{1-cx}}{42c^2 \sqrt{\frac{1}{cx+1}}} - \frac{5bx^3 \sqrt{1-cx}}{168c^4 \sqrt{\frac{1}{cx+1}}} - \frac{5bx \sqrt{1-cx}}{112c^6 \sqrt{\frac{1}{cx+1}}} + \frac{5b \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \sin^{-1}(cx)}{112c^7}$$

[Out] $(-5*b*x*\sqrt{1-c*x})/(112*c^6*\sqrt{(1+c*x)^{-1}}) - (5*b*x^3*\sqrt{1-c*x})/(168*c^4*\sqrt{(1+c*x)^{-1}}) - (b*x^5*\sqrt{1-c*x})/(42*c^2*\sqrt{(1+c*x)^{-1}}) + (x^7*(a+b*\operatorname{ArcSech}[c*x]))/7 + (5*b*\sqrt{(1+c*x)^{-1}}*\sqrt{cx+1}*\operatorname{ArcSin}[c*x])/(112*c^7)$

Rubi [A] time = 0.0609944, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6283, 100, 12, 90, 41, 216}

$$\frac{1}{7}x^7 (a + b \operatorname{sech}^{-1}(cx)) - \frac{bx^5 \sqrt{1-cx}}{42c^2 \sqrt{\frac{1}{cx+1}}} - \frac{5bx^3 \sqrt{1-cx}}{168c^4 \sqrt{\frac{1}{cx+1}}} - \frac{5bx \sqrt{1-cx}}{112c^6 \sqrt{\frac{1}{cx+1}}} + \frac{5b \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \sin^{-1}(cx)}{112c^7}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^6*(a + b*\operatorname{ArcSech}[c*x]), x]$

[Out] $(-5*b*x*\sqrt{1-c*x})/(112*c^6*\sqrt{(1+c*x)^{-1}}) - (5*b*x^3*\sqrt{1-c*x})/(168*c^4*\sqrt{(1+c*x)^{-1}}) - (b*x^5*\sqrt{1-c*x})/(42*c^2*\sqrt{(1+c*x)^{-1}}) + (x^7*(a+b*\operatorname{ArcSech}[c*x]))/7 + (5*b*\sqrt{(1+c*x)^{-1}}*\sqrt{cx+1}*\operatorname{ArcSin}[c*x])/(112*c^7)$

Rule 6283

$\operatorname{Int}[(a + \operatorname{ArcSech}(c*x))(b*x)^m, x] \rightarrow \operatorname{Simp}[(d*x)^{m+1}(a + b*\operatorname{ArcSech}(c*x))/(d*(m+1)), x] + \operatorname{Dist}[b*\sqrt{1+c*x}*\sqrt{1/(1+c*x)}, \operatorname{Int}[(d*x)^m/(\sqrt{1-c*x}*\sqrt{1+c*x}), x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, m\}, x \&\& \operatorname{NeQ}[m, -1]$

Rule 100

$\operatorname{Int}[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x] \rightarrow \operatorname{Simp}[(b*(a + b*x)^{m-1}(c + d*x)^{n+1}(e + f*x)^{p+1})/(d*f*(m+n+p+1)), x] + \operatorname{Dist}[1/(d*f*(m+n+p+1)), \operatorname{Int}[(a + b*x)^{m-2}(c + d*x)^n*(e + f*x)^p*\operatorname{Simp}[a^2*d*f*(m+n+p+1) - b*(b*c*e*(m-1) + a*(d*e*(n+1) + c*f*(p+1))) + b*(a*d*f*(2*m+n+p) - b*(d*e*(m+n) + c*f*(m+p))]*x, x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, e, f, n, p\}, x \&\& \operatorname{GtQ}[m, 1] \&\& \operatorname{NeQ}[m+n+p+1, 0] \&\& \operatorname{IntegerQ}[m]$

Rule 12

$\operatorname{Int}(a*(u), x) \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /;$ $\operatorname{FreeQ}[a, x] \&\& \operatorname{!MatchQ}[u, (b)*(v)] /;$ $\operatorname{FreeQ}[b, x]$

Rule 90

$\operatorname{Int}[(a + b*x)^2*(c + d*x)^n*(e + f*x)^p, x] \rightarrow \operatorname{Simp}[(b*(a + b*x)*(c + d*x)^{n+1}(e + f*x)^{p+1})/$

```
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 41

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] := Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned}
\int x^6 (a + b \operatorname{sech}^{-1}(cx)) dx &= \frac{1}{7} x^7 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{7} \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{x^6}{\sqrt{1-cx} \sqrt{1+cx}} dx \\
&= -\frac{bx^5 \sqrt{1-cx}}{42c^2 \sqrt{\frac{1}{1+cx}}} + \frac{1}{7} x^7 (a + b \operatorname{sech}^{-1}(cx)) - \frac{\left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int -\frac{5x^4}{\sqrt{1-cx} \sqrt{1+cx}} dx}{42c^2} \\
&= -\frac{bx^5 \sqrt{1-cx}}{42c^2 \sqrt{\frac{1}{1+cx}}} + \frac{1}{7} x^7 (a + b \operatorname{sech}^{-1}(cx)) + \frac{\left(5b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{x^4}{\sqrt{1-cx} \sqrt{1+cx}} dx}{42c^2} \\
&= -\frac{5bx^3 \sqrt{1-cx}}{168c^4 \sqrt{\frac{1}{1+cx}}} - \frac{bx^5 \sqrt{1-cx}}{42c^2 \sqrt{\frac{1}{1+cx}}} + \frac{1}{7} x^7 (a + b \operatorname{sech}^{-1}(cx)) - \frac{\left(5b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int -\frac{3x^2}{\sqrt{1-cx} \sqrt{1+cx}} dx}{168c^4} \\
&= -\frac{5bx^3 \sqrt{1-cx}}{168c^4 \sqrt{\frac{1}{1+cx}}} - \frac{bx^5 \sqrt{1-cx}}{42c^2 \sqrt{\frac{1}{1+cx}}} + \frac{1}{7} x^7 (a + b \operatorname{sech}^{-1}(cx)) + \frac{\left(5b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{x^2}{\sqrt{1-cx} \sqrt{1+cx}} dx}{56c^4} \\
&= -\frac{5bx \sqrt{1-cx}}{112c^6 \sqrt{\frac{1}{1+cx}}} - \frac{5bx^3 \sqrt{1-cx}}{168c^4 \sqrt{\frac{1}{1+cx}}} - \frac{bx^5 \sqrt{1-cx}}{42c^2 \sqrt{\frac{1}{1+cx}}} + \frac{1}{7} x^7 (a + b \operatorname{sech}^{-1}(cx)) + \frac{\left(5b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{1}{\sqrt{1-cx} \sqrt{1+cx}} dx}{112c^6} \\
&= -\frac{5bx \sqrt{1-cx}}{112c^6 \sqrt{\frac{1}{1+cx}}} - \frac{5bx^3 \sqrt{1-cx}}{168c^4 \sqrt{\frac{1}{1+cx}}} - \frac{bx^5 \sqrt{1-cx}}{42c^2 \sqrt{\frac{1}{1+cx}}} + \frac{1}{7} x^7 (a + b \operatorname{sech}^{-1}(cx)) + \frac{\left(5b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{1}{\sqrt{1-cx} \sqrt{1+cx}} dx}{112c^6} \\
&= -\frac{5bx \sqrt{1-cx}}{112c^6 \sqrt{\frac{1}{1+cx}}} - \frac{5bx^3 \sqrt{1-cx}}{168c^4 \sqrt{\frac{1}{1+cx}}} - \frac{bx^5 \sqrt{1-cx}}{42c^2 \sqrt{\frac{1}{1+cx}}} + \frac{1}{7} x^7 (a + b \operatorname{sech}^{-1}(cx)) + \frac{5b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx}}{112c^6}
\end{aligned}$$

Mathematica [C] time = 0.187581, size = 143, normalized size = 1.01

$$\frac{ax^7}{7} + b \sqrt{\frac{1-cx}{cx+1}} \left(-\frac{x^5}{42c^2} - \frac{5x^4}{168c^3} - \frac{5x^3}{168c^4} - \frac{5x^2}{112c^5} - \frac{5x}{112c^6} - \frac{x^6}{42c} \right) + \frac{5ib \log \left(2 \sqrt{\frac{1-cx}{cx+1}} (cx+1) - 2icx \right)}{112c^7} + \frac{1}{7} bx^7 \operatorname{sech}^{-1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*ArcSech[c*x]), x]
```

[Out] $(a*x^7)/7 + b*\text{Sqrt}[(1 - c*x)/(1 + c*x)]*((-5*x)/(112*c^6) - (5*x^2)/(112*c^5) - (5*x^3)/(168*c^4) - (5*x^4)/(168*c^3) - x^5/(42*c^2) - x^6/(42*c)) + (b*x^7*\text{ArcSech}[c*x])/7 + (((5*I)/112)*b*\text{Log}[(-2*I)*c*x + 2*\text{Sqrt}[(1 - c*x)/(1 + c*x)]]*(1 + c*x))/c^7$

Maple [A] time = 0.217, size = 138, normalized size = 1.

$$\frac{1}{c^7} \left(\frac{c^7 x^7 a}{7} + b \left(\frac{c^7 x^7 \text{arcsech}(cx)}{7} + \frac{cx}{336} \sqrt{\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} \left(-8c^5 x^5 \sqrt{-c^2 x^2 + 1} - 10c^3 x^3 \sqrt{-c^2 x^2 + 1} - 15cx \sqrt{-c^2 x^2 + 1} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^6*(a+b*arcsech(c*x)),x)`

[Out] $1/c^7*(1/7*c^7*x^7*a+b*(1/7*c^7*x^7*\text{arcsech}(c*x)+1/336*(-(c*x-1)/c/x)^{(1/2)}*c*x*((c*x+1)/c/x)^{(1/2)}*(-8*c^5*x^5*(-c^2*x^2+1)^{(1/2)}-10*c^3*x^3*(-c^2*x^2+1)^{(1/2)}-15*c*x*(-c^2*x^2+1)^{(1/2)}+15*\text{arcsin}(c*x))/(-c^2*x^2+1)^{(1/2}))$

Maxima [A] time = 1.49201, size = 182, normalized size = 1.28

$$\frac{1}{7} ax^7 + \frac{1}{336} \left(48x^7 \text{arsech}(cx) - \frac{15\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}} + 40\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}} + 33\sqrt{\frac{1}{c^2x^2}-1}}{c^6\left(\frac{1}{c^2x^2}-1\right)^3 + 3c^6\left(\frac{1}{c^2x^2}-1\right)^2 + 3c^6\left(\frac{1}{c^2x^2}-1\right) + c^6} + \frac{15 \arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^6} \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6*(a+b*arcsech(c*x)),x, algorithm="maxima")`

[Out] $1/7*a*x^7 + 1/336*(48*x^7*\text{arcsech}(c*x) - ((15*(1/(c^2*x^2) - 1)^{(5/2)} + 40*(1/(c^2*x^2) - 1)^{(3/2)} + 33*\text{sqrt}(1/(c^2*x^2) - 1)))/(c^6*(1/(c^2*x^2) - 1)^3 + 3*c^6*(1/(c^2*x^2) - 1)^2 + 3*c^6*(1/(c^2*x^2) - 1) + c^6) + 15*\text{arctan}(\text{sqrt}(1/(c^2*x^2) - 1))/c^6)/c)*b$

Fricas [A] time = 2.33069, size = 406, normalized size = 2.86

$$\frac{48ac^7x^7 - 48bc^7 \log\left(\frac{cx\sqrt{\frac{c^2x^2-1}{c^2x^2}-1}}{x}\right) - 30b \arctan\left(\frac{cx\sqrt{\frac{c^2x^2-1}{c^2x^2}-1}}{cx}\right) + 48(bc^7x^7 - bc^7) \log\left(\frac{cx\sqrt{\frac{c^2x^2-1}{c^2x^2}+1}}{cx}\right) - (8bc^6x^6 + 10bc^7)}{336c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6*(a+b*arcsech(c*x)),x, algorithm="fricas")`

[Out] $1/336*(48*a*c^7*x^7 - 48*b*c^7*\log((c*x*\text{sqrt}(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) - 30*b*\text{arctan}((c*x*\text{sqrt}(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c*x)) + 48*(b*c^7*x^7 - b*c^7)*\log((c*x*\text{sqrt}(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (8*b*c^6*x^6 + 10*b*c^7))$

$$c^6 x^6 + 10 b c^4 x^4 + 15 b^2 c^2 x^2) \sqrt{-(c^2 x^2 - 1)/(c^2 x^2))} / c^7$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^6 (a + b \operatorname{asech}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**6*(a+b*asech(c*x)),x)

[Out] Integral(x**6*(a + b*asech(c*x)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arsech}(cx) + a) x^6 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6*(a+b*arcsech(c*x)),x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)*x^6, x)

3.20 $\int x^5 (a + b \operatorname{sech}^{-1}(cx)) dx$

Optimal. Leaf size=109

$$\frac{1}{6}x^6 (a + b \operatorname{sech}^{-1}(cx)) - \frac{bx^4 \sqrt{1-cx}}{30c^2 \sqrt{\frac{1}{cx+1}}} - \frac{2bx^2 \sqrt{1-cx}}{45c^4 \sqrt{\frac{1}{cx+1}}} - \frac{4b \sqrt{1-cx}}{45c^6 \sqrt{\frac{1}{cx+1}}}$$

[Out] $(-4*b*\operatorname{Sqrt}[1 - c*x])/(45*c^6*\operatorname{Sqrt}[(1 + c*x)^{-1}]) - (2*b*x^2*\operatorname{Sqrt}[1 - c*x])/(45*c^4*\operatorname{Sqrt}[(1 + c*x)^{-1}]) - (b*x^4*\operatorname{Sqrt}[1 - c*x])/(30*c^2*\operatorname{Sqrt}[(1 + c*x)^{-1}]) + (x^6*(a + b*\operatorname{ArcSech}[c*x]))/6$

Rubi [A] time = 0.0466268, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6283, 100, 12, 74}

$$\frac{1}{6}x^6 (a + b \operatorname{sech}^{-1}(cx)) - \frac{bx^4 \sqrt{1-cx}}{30c^2 \sqrt{\frac{1}{cx+1}}} - \frac{2bx^2 \sqrt{1-cx}}{45c^4 \sqrt{\frac{1}{cx+1}}} - \frac{4b \sqrt{1-cx}}{45c^6 \sqrt{\frac{1}{cx+1}}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^5*(a + b*\operatorname{ArcSech}[c*x]), x]$

[Out] $(-4*b*\operatorname{Sqrt}[1 - c*x])/(45*c^6*\operatorname{Sqrt}[(1 + c*x)^{-1}]) - (2*b*x^2*\operatorname{Sqrt}[1 - c*x])/(45*c^4*\operatorname{Sqrt}[(1 + c*x)^{-1}]) - (b*x^4*\operatorname{Sqrt}[1 - c*x])/(30*c^2*\operatorname{Sqrt}[(1 + c*x)^{-1}]) + (x^6*(a + b*\operatorname{ArcSech}[c*x]))/6$

Rule 6283

$\operatorname{Int}[(a + \operatorname{ArcSech}[c*x])*(x)^m, x] \rightarrow \operatorname{Simp}[(d*x)^{m+1}*(a + b*\operatorname{ArcSech}[c*x])/(d*(m+1)), x] + \operatorname{Dist}[(b*\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[1/(1 + c*x)])/(m+1), \operatorname{Int}[(d*x)^m/(\operatorname{Sqrt}[1 - c*x]*\operatorname{Sqrt}[1 + c*x]), x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, m\}, x$ && $\operatorname{NeQ}[m, -1]$

Rule 100

$\operatorname{Int}[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x] \rightarrow \operatorname{Simp}[(b*(a + b*x)^{m-1}*(c + d*x)^{n+1}*(e + f*x)^{p+1})/(d*f*(m+n+p+1)), x] + \operatorname{Dist}[1/(d*f*(m+n+p+1)), \operatorname{Int}[(a + b*x)^{m-2}*(c + d*x)^n*(e + f*x)^p*\operatorname{Simp}[a^2*d*f*(m+n+p+1) - b*(b*c*e*(m-1) + a*(d*e*(n+1) + c*f*(p+1))) + b*(a*d*f*(2*m+n+p) - b*(d*e*(m+n) + c*f*(m+p))]*x, x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, e, f, n, p\}, x$ && $\operatorname{GtQ}[m, 1]$ && $\operatorname{NeQ}[m+n+p+1, 0]$ && $\operatorname{IntegerQ}[m]$

Rule 12

$\operatorname{Int}[a*(u), x] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /;$ $\operatorname{FreeQ}[a, x]$ && $\operatorname{!MatchQ}[u, (b)*(v)] /;$ $\operatorname{FreeQ}[b, x]$

Rule 74

$\operatorname{Int}[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x] \rightarrow \operatorname{Simp}[(b*(c + d*x)^{n+1}*(e + f*x)^{p+1})/(d*f*(n+p+2)), x] /;$ $\operatorname{FreeQ}\{a, b, c, d, e, f, n, p\}, x$ && $\operatorname{NeQ}[n+p+2, 0]$ && $\operatorname{EqQ}[a*d*f*(n+p+2) - b*(d*e*(n+1) + c*f*(p+1)), 0]$

Rubi steps

$$\begin{aligned}
\int x^5 (a + b \operatorname{sech}^{-1}(cx)) dx &= \frac{1}{6} x^6 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{6} \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{x^5}{\sqrt{1-cx} \sqrt{1+cx}} dx \\
&= -\frac{bx^4 \sqrt{1-cx}}{30c^2 \sqrt{\frac{1}{1+cx}}} + \frac{1}{6} x^6 (a + b \operatorname{sech}^{-1}(cx)) - \frac{\left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int -\frac{4x^3}{\sqrt{1-cx} \sqrt{1+cx}} dx}{30c^2} \\
&= -\frac{bx^4 \sqrt{1-cx}}{30c^2 \sqrt{\frac{1}{1+cx}}} + \frac{1}{6} x^6 (a + b \operatorname{sech}^{-1}(cx)) + \frac{\left(2b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{x^3}{\sqrt{1-cx} \sqrt{1+cx}} dx}{15c^2} \\
&= -\frac{2bx^2 \sqrt{1-cx}}{45c^4 \sqrt{\frac{1}{1+cx}}} - \frac{bx^4 \sqrt{1-cx}}{30c^2 \sqrt{\frac{1}{1+cx}}} + \frac{1}{6} x^6 (a + b \operatorname{sech}^{-1}(cx)) - \frac{\left(2b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int -\frac{2x}{\sqrt{1-cx} \sqrt{1+cx}} dx}{45c^4} \\
&= -\frac{2bx^2 \sqrt{1-cx}}{45c^4 \sqrt{\frac{1}{1+cx}}} - \frac{bx^4 \sqrt{1-cx}}{30c^2 \sqrt{\frac{1}{1+cx}}} + \frac{1}{6} x^6 (a + b \operatorname{sech}^{-1}(cx)) + \frac{\left(4b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{x}{\sqrt{1-cx} \sqrt{1+cx}} dx}{45c^4} \\
&= -\frac{4b \sqrt{1-cx}}{45c^6 \sqrt{\frac{1}{1+cx}}} - \frac{2bx^2 \sqrt{1-cx}}{45c^4 \sqrt{\frac{1}{1+cx}}} - \frac{bx^4 \sqrt{1-cx}}{30c^2 \sqrt{\frac{1}{1+cx}}} + \frac{1}{6} x^6 (a + b \operatorname{sech}^{-1}(cx))
\end{aligned}$$

Mathematica [A] time = 0.0840902, size = 97, normalized size = 0.89

$$\frac{ax^6}{6} + b \sqrt{\frac{1-cx}{cx+1}} \left(-\frac{x^4}{30c^2} - \frac{2x^3}{45c^3} - \frac{2x^2}{45c^4} - \frac{4x}{45c^5} - \frac{4}{45c^6} - \frac{x^5}{30c} \right) + \frac{1}{6} bx^6 \operatorname{sech}^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[x^5*(a + b*ArcSech[c*x]),x]

[Out] (a*x^6)/6 + b*Sqrt[(1 - c*x)/(1 + c*x)]*(-4/(45*c^6) - (4*x)/(45*c^5) - (2*x^2)/(45*c^4) - (2*x^3)/(45*c^3) - x^4/(30*c^2) - x^5/(30*c)) + (b*x^6*ArcSech[c*x])/6

Maple [A] time = 0.193, size = 81, normalized size = 0.7

$$\frac{1}{c^6} \left(\frac{c^6 x^6 a}{6} + b \left(\frac{c^6 x^6 \operatorname{arcsech}(cx)}{6} - \frac{cx(3c^4 x^4 + 4c^2 x^2 + 8)}{90} \sqrt{\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(a+b*arcsech(c*x)),x)

[Out] 1/c^6*(1/6*c^6*x^6*a+b*(1/6*c^6*x^6*arcsech(c*x)-1/90*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)*(3*c^4*x^4+4*c^2*x^2+8)))

Maxima [A] time = 0.989576, size = 105, normalized size = 0.96

$$\frac{1}{6} ax^6 + \frac{1}{90} \left(15x^6 \operatorname{ar} \operatorname{sech}(cx) - \frac{3c^4 x^5 \left(\frac{1}{c^2 x^2} - 1 \right)^{\frac{5}{2}} - 10c^2 x^3 \left(\frac{1}{c^2 x^2} - 1 \right)^{\frac{3}{2}} + 15x \sqrt{\frac{1}{c^2 x^2} - 1}}{c^5} \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(a+b*arcsech(c*x)),x, algorithm="maxima")

[Out] $\frac{1}{6}ax^6 + \frac{1}{90}(15x^6\operatorname{arcsech}(cx) - (3c^4x^5(1/(c^2x^2) - 1)^{(5/2)} - 10c^2x^3(1/(c^2x^2) - 1)^{(3/2)} + 15x\sqrt{1/(c^2x^2) - 1}))/c^5)b$

Fricas [A] time = 1.98351, size = 217, normalized size = 1.99

$$\frac{15bc^5x^6 \log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}+1}}{cx}\right) + 15ac^5x^6 - (3bc^4x^5 + 4bc^2x^3 + 8bx)\sqrt{-\frac{c^2x^2-1}{c^2x^2}}}{90c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(a+b*arcsech(c*x)),x, algorithm="fricas")

[Out] $\frac{1}{90}(15bc^5x^6\log((cx\sqrt{-(c^2x^2-1)/(c^2x^2)}+1)/(cx)) + 15ac^5x^6 - (3bc^4x^5 + 4bc^2x^3 + 8bx)\sqrt{-(c^2x^2-1)/(c^2x^2)}))/c^5$

Sympy [A] time = 16.6257, size = 94, normalized size = 0.86

$$\begin{cases} \frac{ax^6}{6} + \frac{bx^6 \operatorname{asech}(cx)}{6} - \frac{bx^4\sqrt{-c^2x^2+1}}{30c^2} - \frac{2bx^2\sqrt{-c^2x^2+1}}{45c^4} - \frac{4b\sqrt{-c^2x^2+1}}{45c^6} & \text{for } c \neq 0 \\ \frac{x^{6(a+\infty b)}}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*(a+b*asech(c*x)),x)

[Out] Piecewise((a*x**6/6 + b*x**6*asech(c*x)/6 - b*x**4*sqrt(-c**2*x**2 + 1)/(30*c**2) - 2*b*x**2*sqrt(-c**2*x**2 + 1)/(45*c**4) - 4*b*sqrt(-c**2*x**2 + 1)/(45*c**6), Ne(c, 0)), (x**6*(a + oo*b)/6, True))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{ar} \operatorname{sech}(cx) + a)x^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(a+b*arcsech(c*x)),x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)*x^5, x)

3.21 $\int x^4 (a + b \operatorname{sech}^{-1}(cx)) dx$

Optimal. Leaf size=110

$$\frac{1}{5}x^5(a + b \operatorname{sech}^{-1}(cx)) - \frac{bx^3\sqrt{1-cx}}{20c^2\sqrt{\frac{1}{cx+1}}} - \frac{3bx\sqrt{1-cx}}{40c^4\sqrt{\frac{1}{cx+1}}} + \frac{3b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sin^{-1}(cx)}{40c^5}$$

[Out] $(-3*b*x*\operatorname{Sqrt}[1 - c*x])/(40*c^4*\operatorname{Sqrt}[(1 + c*x)^{-1}]) - (b*x^3*\operatorname{Sqrt}[1 - c*x])/(20*c^2*\operatorname{Sqrt}[(1 + c*x)^{-1}]) + (x^5*(a + b*\operatorname{ArcSech}[c*x]))/5 + (3*b*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{ArcSin}[c*x])/(40*c^5)$

Rubi [A] time = 0.0400105, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6283, 100, 12, 90, 41, 216}

$$\frac{1}{5}x^5(a + b \operatorname{sech}^{-1}(cx)) - \frac{bx^3\sqrt{1-cx}}{20c^2\sqrt{\frac{1}{cx+1}}} - \frac{3bx\sqrt{1-cx}}{40c^4\sqrt{\frac{1}{cx+1}}} + \frac{3b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sin^{-1}(cx)}{40c^5}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^4*(a + b*\operatorname{ArcSech}[c*x]), x]$

[Out] $(-3*b*x*\operatorname{Sqrt}[1 - c*x])/(40*c^4*\operatorname{Sqrt}[(1 + c*x)^{-1}]) - (b*x^3*\operatorname{Sqrt}[1 - c*x])/(20*c^2*\operatorname{Sqrt}[(1 + c*x)^{-1}]) + (x^5*(a + b*\operatorname{ArcSech}[c*x]))/5 + (3*b*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{ArcSin}[c*x])/(40*c^5)$

Rule 6283

$\operatorname{Int}[(a_.) + \operatorname{ArcSech}[(c_.)*(x_.)]*(b_.)]*((d_.)*(x_.))^{(m_.)}, x_Symbol] \rightarrow \operatorname{Simp}[(d*x)^{(m+1)}*(a + b*\operatorname{ArcSech}[c*x])]/(d*(m+1)), x] + \operatorname{Dist}[(b*\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[1/(1 + c*x)])]/(m+1), \operatorname{Int}[(d*x)^m/(\operatorname{Sqrt}[1 - c*x]*\operatorname{Sqrt}[1 + c*x]), x], x] /; \operatorname{FreeQ}\{a, b, c, d, m\}, x] \&\& \operatorname{NeQ}[m, -1]$

Rule 100

$\operatorname{Int}[(a_.) + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}*((e_.) + (f_.)*(x_.))^{(p_.)}, x_Symbol] \rightarrow \operatorname{Simp}[(b*(a + b*x)^{(m-1)}*(c + d*x)^{(n+1)}*(e + f*x)^{(p+1)})]/(d*f*(m+n+p+1)), x] + \operatorname{Dist}[1/(d*f*(m+n+p+1)), \operatorname{Int}[(a + b*x)^{(m-2)}*(c + d*x)^n*(e + f*x)^p*\operatorname{Simp}[a^2*d*f*(m+n+p+1) - b*(b*c*e*(m-1) + a*(d*e*(n+1) + c*f*(p+1))) + b*(a*d*f*(2*m+n+p) - b*(d*e*(m+n) + c*f*(m+p))]*x, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, n, p\}, x] \&\& \operatorname{GtQ}[m, 1] \&\& \operatorname{NeQ}[m+n+p+1, 0] \&\& \operatorname{IntegerQ}[m]$

Rule 12

$\operatorname{Int}[(a_.)*(u_.), x_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \&\& \operatorname{!MatchQ}[u, (b_.)*(v_.)] /; \operatorname{FreeQ}[b, x]$

Rule 90

$\operatorname{Int}[(a_.) + (b_.)*(x_.))^{2*}*((c_.) + (d_.)*(x_.))^{(n_.)}*((e_.) + (f_.)*(x_.))^{(p_.)}, x_Symbol] \rightarrow \operatorname{Simp}[(b*(a + b*x)*(c + d*x)^{(n+1)}*(e + f*x)^{(p+1)})]/(d*f*(n+p+3)), x] + \operatorname{Dist}[1/(d*f*(n+p+3)), \operatorname{Int}[(c + d*x)^n*(e + f*x)^p*\operatorname{Simp}[a^2*d*f*(n+p+3) - b*(b*c*e + a*(d*e*(n+1) + c*f*(p+1)))] + b$

`*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]`

Rule 41

`Int[((a_) + (b_.)*(x_)^(m_.))*((c_) + (d_.)*(x_)^(m_.), x_Symbol] := Int[(a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))`

Rule 216

`Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]`

Rubi steps

$$\begin{aligned}
 \int x^4 (a + b \operatorname{sech}^{-1}(cx)) dx &= \frac{1}{5} x^5 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{5} \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{x^4}{\sqrt{1-cx}\sqrt{1+cx}} dx \\
 &= -\frac{bx^3\sqrt{1-cx}}{20c^2\sqrt{\frac{1}{1+cx}}} + \frac{1}{5} x^5 (a + b \operatorname{sech}^{-1}(cx)) - \frac{\left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int -\frac{3x^2}{\sqrt{1-cx}\sqrt{1+cx}} dx}{20c^2} \\
 &= -\frac{bx^3\sqrt{1-cx}}{20c^2\sqrt{\frac{1}{1+cx}}} + \frac{1}{5} x^5 (a + b \operatorname{sech}^{-1}(cx)) + \frac{\left(3b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{x^2}{\sqrt{1-cx}\sqrt{1+cx}} dx}{20c^2} \\
 &= -\frac{3bx\sqrt{1-cx}}{40c^4\sqrt{\frac{1}{1+cx}}} - \frac{bx^3\sqrt{1-cx}}{20c^2\sqrt{\frac{1}{1+cx}}} + \frac{1}{5} x^5 (a + b \operatorname{sech}^{-1}(cx)) + \frac{\left(3b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{1}{\sqrt{1-cx}\sqrt{1+cx}} dx}{40c^4} \\
 &= -\frac{3bx\sqrt{1-cx}}{40c^4\sqrt{\frac{1}{1+cx}}} - \frac{bx^3\sqrt{1-cx}}{20c^2\sqrt{\frac{1}{1+cx}}} + \frac{1}{5} x^5 (a + b \operatorname{sech}^{-1}(cx)) + \frac{\left(3b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{1}{\sqrt{1-c^2x^2}} dx}{40c^4} \\
 &= -\frac{3bx\sqrt{1-cx}}{40c^4\sqrt{\frac{1}{1+cx}}} - \frac{bx^3\sqrt{1-cx}}{20c^2\sqrt{\frac{1}{1+cx}}} + \frac{1}{5} x^5 (a + b \operatorname{sech}^{-1}(cx)) + \frac{3b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sin^{-1}(cx)}{40c^5}
 \end{aligned}$$

Mathematica [C] time = 0.118249, size = 123, normalized size = 1.12

$$\frac{ax^5}{5} + b \sqrt{\frac{1-cx}{cx+1}} \left(-\frac{x^3}{20c^2} - \frac{3x^2}{40c^3} - \frac{3x}{40c^4} - \frac{x^4}{20c} \right) + \frac{3ib \log \left(2 \sqrt{\frac{1-cx}{cx+1}} (cx+1) - 2icx \right)}{40c^5} + \frac{1}{5} bx^5 \operatorname{sech}^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[x^4*(a + b*ArcSech[c*x]),x]

[Out] (a*x^5)/5 + b*Sqrt[(1 - c*x)/(1 + c*x)]*((-3*x)/(40*c^4) - (3*x^2)/(40*c^3) - x^3/(20*c^2) - x^4/(20*c)) + (b*x^5*ArcSech[c*x])/5 + (((3*I)/40)*b*Log[(-2*I)*c*x + 2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)]/c^5

Maple [A] time = 0.225, size = 118, normalized size = 1.1

$$\frac{1}{c^5} \left(\frac{c^5 x^5 a}{5} + b \left(\frac{c^5 x^5 \operatorname{arcsech}(cx)}{5} + \frac{cx}{40} \sqrt{\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} \left(-2c^3 x^3 \sqrt{-c^2 x^2 + 1} - 3cx \sqrt{-c^2 x^2 + 1} + 3 \arcsin(cx) \right) \frac{1}{\sqrt{-c^2 x^2 + 1}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(a+b*arcsech(c*x)),x)`

[Out] $1/c^5*(1/5*c^5*x^5*a+b*(1/5*c^5*x^5*arcsech(c*x)+1/40*(-(c*x-1)/c/x)^{(1/2)}*c*x*((c*x+1)/c/x)^{(1/2)}*(-2*c^3*x^3*(-c^2*x^2+1)^{(1/2)}-3*c*x*(-c^2*x^2+1)^{(1/2)}+3*arcsin(c*x))/(-c^2*x^2+1)^{(1/2))}$

Maxima [A] time = 1.51525, size = 143, normalized size = 1.3

$$\frac{1}{5}ax^5 + \frac{1}{40} \left(8x^5 \operatorname{arsech}(cx) - \frac{3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}} + 5\sqrt{\frac{1}{c^2x^2}-1}}{c^4\left(\frac{1}{c^2x^2}-1\right)^2 + 2c^4\left(\frac{1}{c^2x^2}-1\right) + c^4} + \frac{3 \arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^4} \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*(a+b*arcsech(c*x)),x, algorithm="maxima")`

[Out] $1/5*a*x^5 + 1/40*(8*x^5*arcsech(c*x) - ((3*(1/(c^2*x^2) - 1)^{(3/2)} + 5*sqrt(1/(c^2*x^2) - 1))/(c^4*(1/(c^2*x^2) - 1)^2 + 2*c^4*(1/(c^2*x^2) - 1) + c^4) + 3*arctan(sqrt(1/(c^2*x^2) - 1))/c^4)/c)*b$

Fricas [B] time = 2.01139, size = 378, normalized size = 3.44

$$\frac{8ac^5x^5 - 8bc^5 \log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{x}\right) - 6b \arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right) + 8(bc^5x^5 - bc^5) \log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right) - (2bc^4x^4 + 3bc^2x^2)}{40c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*(a+b*arcsech(c*x)),x, algorithm="fricas")`

[Out] $1/40*(8*a*c^5*x^5 - 8*b*c^5*\log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) - 6*b*arctan((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c*x)) + 8*(b*c^5*x^5 - b*c^5)*\log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (2*b*c^4*x^4 + 3*b*c^2*x^2)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/c^5$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 (a + b \operatorname{asech}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*(a+b*asech(c*x)),x)`

[Out] `Integral(x**4*(a + b*asech(c*x)), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arsech}(cx) + a)x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x^4, x)
```

3.22 $\int x^3 (a + b \operatorname{sech}^{-1}(cx)) dx$

Optimal. Leaf size=77

$$\frac{1}{4}x^4 (a + b \operatorname{sech}^{-1}(cx)) - \frac{bx^2\sqrt{1-cx}}{12c^2\sqrt{\frac{1}{cx+1}}} - \frac{b\sqrt{1-cx}}{6c^4\sqrt{\frac{1}{cx+1}}}$$

[Out] $-(b\sqrt{1-cx})/(6c^4\sqrt{(1+cx)^{-1}}) - (bx^2\sqrt{1-cx})/(12c^2\sqrt{(1+cx)^{-1}}) + (x^4(a + b\operatorname{ArcSech}[cx]))/4$

Rubi [A] time = 0.0307868, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6283, 100, 12, 74}

$$\frac{1}{4}x^4 (a + b \operatorname{sech}^{-1}(cx)) - \frac{bx^2\sqrt{1-cx}}{12c^2\sqrt{\frac{1}{cx+1}}} - \frac{b\sqrt{1-cx}}{6c^4\sqrt{\frac{1}{cx+1}}}$$

Antiderivative was successfully verified.

[In] Int[x^3*(a + b*ArcSech[c*x]),x]

[Out] $-(b\sqrt{1-cx})/(6c^4\sqrt{(1+cx)^{-1}}) - (bx^2\sqrt{1-cx})/(12c^2\sqrt{(1+cx)^{-1}}) + (x^4(a + b\operatorname{ArcSech}[cx]))/4$

Rule 6283

Int[((a_.) + ArcSech[(c_.)*(x_)])*(b_.)*((d_.)*(x_))^(m_.), x_Symbol] := Simp[((d*x)^(m+1)*(a + b*ArcSech[c*x]))/(d*(m+1)), x] + Dist[(b*sqrt[1+c*x]*sqrt[1/(1+c*x)])/(m+1), Int[(d*x)^m/(sqrt[1-c*x]*sqrt[1+c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]

Rule 100

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m-1)*(c + d*x)^(n+1)*(e + f*x)^(p+1))/(d*f*(m+n+p+1)), x] + Dist[1/(d*f*(m+n+p+1)), Int[(a + b*x)^(m-2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m+n+p+1) - b*(b*c*e*(m-1) + a*(d*e*(n+1) + c*f*(p+1))) + b*(a*d*f*(2*m+n+p) - b*(d*e*(m+n) + c*f*(m+p)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && GtQ[m, 1] && NeQ[m+n+p+1, 0] && IntegerQ[m]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 74

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[(b*(c + d*x)^(n+1)*(e + f*x)^(p+1))/(d*f*(n+p+2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n+p+2, 0] && EqQ[a*d*f*(n+p+2) - b*(d*e*(n+1) + c*f*(p+1)), 0]

Rubi steps

$$\begin{aligned}
\int x^3 (a + b \operatorname{sech}^{-1}(cx)) dx &= \frac{1}{4} x^4 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{4} \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{x^3}{\sqrt{1-cx} \sqrt{1+cx}} dx \\
&= -\frac{bx^2 \sqrt{1-cx}}{12c^2 \sqrt{\frac{1}{1+cx}}} + \frac{1}{4} x^4 (a + b \operatorname{sech}^{-1}(cx)) - \frac{\left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int -\frac{2x}{\sqrt{1-cx} \sqrt{1+cx}} dx}{12c^2} \\
&= -\frac{bx^2 \sqrt{1-cx}}{12c^2 \sqrt{\frac{1}{1+cx}}} + \frac{1}{4} x^4 (a + b \operatorname{sech}^{-1}(cx)) + \frac{\left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{x}{\sqrt{1-cx} \sqrt{1+cx}} dx}{6c^2} \\
&= -\frac{b \sqrt{1-cx}}{6c^4 \sqrt{\frac{1}{1+cx}}} - \frac{bx^2 \sqrt{1-cx}}{12c^2 \sqrt{\frac{1}{1+cx}}} + \frac{1}{4} x^4 (a + b \operatorname{sech}^{-1}(cx))
\end{aligned}$$

Mathematica [A] time = 0.0767012, size = 77, normalized size = 1.

$$\frac{ax^4}{4} + b \sqrt{\frac{1-cx}{cx+1}} \left(-\frac{x^2}{12c^2} - \frac{x}{6c^3} - \frac{1}{6c^4} - \frac{x^3}{12c} \right) + \frac{1}{4} bx^4 \operatorname{sech}^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[x^3*(a + b*ArcSech[c*x]),x]

[Out] (a*x^4)/4 + b*Sqrt[(1 - c*x)/(1 + c*x)]*(-1/(6*c^4) - x/(6*c^3) - x^2/(12*c^2) - x^3/(12*c)) + (b*x^4*ArcSech[c*x])/4

Maple [A] time = 0.187, size = 72, normalized size = 0.9

$$\frac{1}{c^4} \left(\frac{c^4 x^4 a}{4} + b \left(\frac{c^4 x^4 \operatorname{arcsech}(cx)}{4} - \frac{cx(c^2 x^2 + 2)}{12} \sqrt{-\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(a+b*arcsech(c*x)),x)

[Out] 1/c^4*(1/4*c^4*x^4*a+b*(1/4*c^4*x^4*arcsech(c*x)-1/12*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)*(c^2*x^2+2)))

Maxima [A] time = 1.00829, size = 77, normalized size = 1.

$$\frac{1}{4} ax^4 + \frac{1}{12} \left(3x^4 \operatorname{arsech}(cx) + \frac{c^2 x^3 \left(\frac{1}{c^2 x^2} - 1 \right)^{\frac{3}{2}} - 3x \sqrt{\frac{1}{c^2 x^2} - 1}}{c^3} \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*arcsech(c*x)),x, algorithm="maxima")

[Out] $1/4*a*x^4 + 1/12*(3*x^4*\operatorname{arcsech}(c*x) + (c^2*x^3*(1/(c^2*x^2) - 1))^{(3/2)} - 3*x*\sqrt{1/(c^2*x^2) - 1})/c^3)*b$

Fricas [A] time = 1.98669, size = 193, normalized size = 2.51

$$\frac{3bc^3x^4 \log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right) + 3ac^3x^4 - (bc^2x^3 + 2bx)\sqrt{-\frac{c^2x^2-1}{c^2x^2}}}{12c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(a+b*arcsech(c*x)),x, algorithm="fricas")`

[Out] $1/12*(3*b*c^3*x^4*\log((c*x*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)} + 1)/(c*x)) + 3*a*c^3*x^4 - (b*c^2*x^3 + 2*b*x)*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)})/c^3$

Sympy [A] time = 5.8029, size = 68, normalized size = 0.88

$$\begin{cases} \frac{ax^4}{4} + \frac{bx^4 \operatorname{asech}(cx)}{4} - \frac{bx^2\sqrt{-c^2x^2+1}}{12c^2} - \frac{b\sqrt{-c^2x^2+1}}{6c^4} & \text{for } c \neq 0 \\ \frac{x^4(a+\infty b)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(a+b*asech(c*x)),x)`

[Out] `Piecewise((a*x**4/4 + b*x**4*asech(c*x)/4 - b*x**2*sqrt(-c**2*x**2 + 1)/(12*c**2) - b*sqrt(-c**2*x**2 + 1)/(6*c**4), Ne(c, 0)), (x**4*(a + oo*b)/4, True))`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arsech}(cx) + a)x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(a+b*arcsech(c*x)),x, algorithm="giac")`

[Out] `integrate((b*arcsech(c*x) + a)*x^3, x)`

3.23 $\int x^2 (a + b \operatorname{sech}^{-1}(cx)) dx$

Optimal. Leaf size=78

$$\frac{1}{3}x^3 (a + b \operatorname{sech}^{-1}(cx)) - \frac{bx\sqrt{1-cx}}{6c^2\sqrt{\frac{1}{cx+1}}} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sin^{-1}(cx)}{6c^3}$$

[Out] $-(b*x*\text{Sqrt}[1 - c*x])/(6*c^2*\text{Sqrt}[(1 + c*x)^{-1}]) + (x^3*(a + b*\text{ArcSech}[c*x]))/3 + (b*\text{Sqrt}[(1 + c*x)^{-1}]*\text{Sqrt}[1 + c*x]*\text{ArcSin}[c*x])/(6*c^3)$

Rubi [A] time = 0.0274582, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6283, 90, 41, 216}

$$\frac{1}{3}x^3 (a + b \operatorname{sech}^{-1}(cx)) - \frac{bx\sqrt{1-cx}}{6c^2\sqrt{\frac{1}{cx+1}}} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sin^{-1}(cx)}{6c^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[x^2*(a + b*\text{ArcSech}[c*x]), x]$

[Out] $-(b*x*\text{Sqrt}[1 - c*x])/(6*c^2*\text{Sqrt}[(1 + c*x)^{-1}]) + (x^3*(a + b*\text{ArcSech}[c*x]))/3 + (b*\text{Sqrt}[(1 + c*x)^{-1}]*\text{Sqrt}[1 + c*x]*\text{ArcSin}[c*x])/(6*c^3)$

Rule 6283

$\text{Int}[(a_.) + \text{ArcSech}[c_.*(x_.)]*(b_.)]*((d_.)*(x_.))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[(d*x)^{(m+1)}*(a + b*\text{ArcSech}[c*x])]/(d*(m+1)), x] + \text{Dist}[(b*\text{Sqrt}[1 + c*x]*\text{Sqrt}[1/(1 + c*x)])/(m+1), \text{Int}[(d*x)^m/(\text{Sqrt}[1 - c*x]*\text{Sqrt}[1 + c*x]), x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x] \&\& \text{NeQ}[m, -1]$

Rule 90

$\text{Int}[(a_.) + (b_.)*(x_.))^{2*(c_.) + (d_.)*(x_.)^{(n_.)}*((e_.) + (f_.)*(x_.))^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[(b*(a + b*x)*(c + d*x)^{(n+1)}*(e + f*x)^{(p+1)})/(d*f*(n+p+3)), x] + \text{Dist}[1/(d*f*(n+p+3)), \text{Int}[(c + d*x)^n*(e + f*x)^p*\text{Simp}[a^2*d*f*(n+p+3) - b*(b*c*e + a*(d*e*(n+1) + c*f*(p+1))) + b*(a*d*f*(n+p+4) - b*(d*e*(n+2) + c*f*(p+2)))*x, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n, p\}, x] \&\& \text{NeQ}[n+p+3, 0]$

Rule 41

$\text{Int}[(a_.) + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(m_.)}, x_Symbol] \rightarrow \text{Int}[(a*c + b*d*x^2)^m, x] /; \text{FreeQ}\{a, b, c, d, m\}, x] \&\& \text{EqQ}[b*c + a*d, 0] \&\& (\text{IntegerQ}[m] || (\text{GtQ}[a, 0] \&\& \text{GtQ}[c, 0]))$

Rule 216

$\text{Int}[1/\text{Sqrt}[(a_.) + (b_.)*(x_.)^2], x_Symbol] \rightarrow \text{Simp}[\text{ArcSin}[(\text{Rt}[-b, 2]*x)/\text{Sqrt}[a]]/\text{Rt}[-b, 2], x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{GtQ}[a, 0] \&\& \text{NegQ}[b]$

Rubi steps

$$\begin{aligned}
\int x^2 (a + b \operatorname{sech}^{-1}(cx)) dx &= \frac{1}{3} x^3 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{3} \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{x^2}{\sqrt{1-cx}\sqrt{1+cx}} dx \\
&= -\frac{bx\sqrt{1-cx}}{6c^2\sqrt{\frac{1}{1+cx}}} + \frac{1}{3} x^3 (a + b \operatorname{sech}^{-1}(cx)) + \frac{\left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{1}{\sqrt{1-cx}\sqrt{1+cx}} dx}{6c^2} \\
&= -\frac{bx\sqrt{1-cx}}{6c^2\sqrt{\frac{1}{1+cx}}} + \frac{1}{3} x^3 (a + b \operatorname{sech}^{-1}(cx)) + \frac{\left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{1}{\sqrt{1-c^2x^2}} dx}{6c^2} \\
&= -\frac{bx\sqrt{1-cx}}{6c^2\sqrt{\frac{1}{1+cx}}} + \frac{1}{3} x^3 (a + b \operatorname{sech}^{-1}(cx)) + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sin^{-1}(cx)}{6c^3}
\end{aligned}$$

Mathematica [C] time = 0.0865029, size = 103, normalized size = 1.32

$$\frac{ax^3}{3} + b\sqrt{\frac{1-cx}{cx+1}} \left(-\frac{x}{6c^2} - \frac{x^2}{6c} \right) + \frac{ib \log \left(2\sqrt{\frac{1-cx}{cx+1}}(cx+1) - 2icx \right)}{6c^3} + \frac{1}{3} bx^3 \operatorname{sech}^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a + b*ArcSech[c*x]),x]

[Out] (a*x^3)/3 + b*Sqrt[(1 - c*x)/(1 + c*x)]*(-x/(6*c^2) - x^2/(6*c)) + (b*x^3*ArcSech[c*x])/3 + ((I/6)*b*Log[(-2*I)*c*x + 2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)])/c^3

Maple [A] time = 0.192, size = 96, normalized size = 1.2

$$\frac{1}{c^3} \left(\frac{c^3 x^3 a}{3} + b \left(\frac{c^3 x^3 \operatorname{arcsech}(cx)}{3} + \frac{cx}{6} \sqrt{-\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} \left(-cx\sqrt{-c^2x^2+1} + \arcsin(cx) \right) \frac{1}{\sqrt{-c^2x^2+1}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a+b*arcsech(c*x)),x)

[Out] 1/c^3*(1/3*c^3*x^3*a+b*(1/3*c^3*x^3*arcsech(c*x)+1/6*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)*(-c*x*(-c^2*x^2+1)^(1/2)+arcsin(c*x))/(-c^2*x^2+1)^(1/2)))

Maxima [A] time = 1.53006, size = 99, normalized size = 1.27

$$\frac{1}{3} ax^3 + \frac{1}{6} \left(2x^3 \operatorname{arsech}(cx) - \frac{\frac{\sqrt{\frac{1}{c^2x^2}-1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^2}}{c} \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*arcsech(c*x)),x, algorithm="maxima")

[Out] $\frac{1}{3}ax^3 + \frac{1}{6}(2x^3\operatorname{arcsech}(cx) - (\sqrt{1/(c^2x^2) - 1})/(c^2(1/(c^2x^2) - 1) + c^2) + \arctan(\sqrt{1/(c^2x^2) - 1}))/c^2)/c)*b$

Fricas [B] time = 2.12044, size = 352, normalized size = 4.51

$$\frac{2ac^3x^3 - bc^2x^2\sqrt{-\frac{c^2x^2-1}{c^2x^2}} - 2bc^3\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{x}\right) - 2b\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right) + 2(bc^3x^3 - bc^3)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)}{6c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*arcsech(c*x)),x, algorithm="fricas")

[Out] $\frac{1}{6}(2ac^3x^3 - bc^2x^2\sqrt{-(c^2x^2 - 1)/(c^2x^2)} - 2bc^3\log((cx\sqrt{-(c^2x^2 - 1)/(c^2x^2)} - 1)/x) - 2b\arctan((cx\sqrt{-(c^2x^2 - 1)/(c^2x^2)} - 1)/(cx)) + 2(bc^3x^3 - bc^3)\log((cx\sqrt{-(c^2x^2 - 1)/(c^2x^2)} + 1)/(cx)))/c^3$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2(a + b \operatorname{asech}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(a+b*asech(c*x)),x)

[Out] Integral(x**2*(a + b*asech(c*x)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arsech}(cx) + a)x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*arcsech(c*x)),x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)*x^2, x)

3.24 $\int x \left(a + b \operatorname{sech}^{-1}(cx) \right) dx$

Optimal. Leaf size=45

$$\frac{1}{2}x^2 \left(a + b \operatorname{sech}^{-1}(cx) \right) - \frac{b\sqrt{1-cx}}{2c^2\sqrt{\frac{1}{cx+1}}}$$

[Out] $-(b\sqrt{1-cx})/(2c^2\sqrt{(1+cx)^{-1}}) + (x^2*(a + b\operatorname{ArcSech}[c*x]))/2$

Rubi [A] time = 0.0136379, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {6283, 74}

$$\frac{1}{2}x^2 \left(a + b \operatorname{sech}^{-1}(cx) \right) - \frac{b\sqrt{1-cx}}{2c^2\sqrt{\frac{1}{cx+1}}}$$

Antiderivative was successfully verified.

[In] Int[x*(a + b*ArcSech[c*x]), x]

[Out] $-(b\sqrt{1-cx})/(2c^2\sqrt{(1+cx)^{-1}}) + (x^2*(a + b\operatorname{ArcSech}[c*x]))/2$

Rule 6283

Int[((a_.) + ArcSech[(c_.)*(x_)])*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] := Simp[((d*x)^(m+1)*(a + b*ArcSech[c*x]))/(d*(m+1)), x] + Dist[(b*Sqrt[1+c*x]*Sqrt[1/(1+c*x))]/(m+1), Int[(d*x)^m/(Sqrt[1-c*x]*Sqrt[1+c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]

Rule 74

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n+1)*(e + f*x)^(p+1))/(d*f*(n+p+2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n+p+2, 0] && EqQ[a*d*f*(n+p+2) - b*(d*e*(n+1) + c*f*(p+1)), 0]

Rubi steps

$$\begin{aligned} \int x \left(a + b \operatorname{sech}^{-1}(cx) \right) dx &= \frac{1}{2}x^2 \left(a + b \operatorname{sech}^{-1}(cx) \right) + \frac{1}{2} \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{x}{\sqrt{1-cx}\sqrt{1+cx}} dx \\ &= -\frac{b\sqrt{1-cx}}{2c^2\sqrt{\frac{1}{1+cx}}} + \frac{1}{2}x^2 \left(a + b \operatorname{sech}^{-1}(cx) \right) \end{aligned}$$

Mathematica [A] time = 0.0515405, size = 57, normalized size = 1.27

$$\frac{ax^2}{2} + b \left(-\frac{1}{2c^2} - \frac{x}{2c} \right) \sqrt{\frac{1-cx}{cx+1}} + \frac{1}{2}bx^2 \operatorname{sech}^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + b*ArcSech[c*x]),x]

[Out] (a*x^2)/2 + b*(-1/(2*c^2) - x/(2*c))*Sqrt[(1 - c*x)/(1 + c*x)] + (b*x^2*ArcSech[c*x])/2

Maple [A] time = 0.178, size = 63, normalized size = 1.4

$$\frac{1}{c^2} \left(\frac{c^2 x^2 a}{2} + b \left(\frac{\operatorname{arcsech}(cx) c^2 x^2}{2} - \frac{cx}{2} \sqrt{-\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a+b*arcsech(c*x)),x)

[Out] 1/c^2*(1/2*c^2*x^2*a+b*(1/2*arcsech(c*x)*c^2*x^2-1/2*(-(c*x-1)/c/x)^(1/2)*(c*x+1)/c/x)^(1/2)*c*x)

Maxima [A] time = 0.979394, size = 49, normalized size = 1.09

$$\frac{1}{2} ax^2 + \frac{1}{2} \left(x^2 \operatorname{arosech}(cx) - \frac{x \sqrt{\frac{1}{c^2 x^2} - 1}}{c} \right) b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arcsech(c*x)),x, algorithm="maxima")

[Out] 1/2*a*x^2 + 1/2*(x^2*arcsech(c*x) - x*sqrt(1/(c^2*x^2) - 1)/c)*b

Fricas [B] time = 1.90979, size = 157, normalized size = 3.49

$$\frac{bcx^2 \log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right) + acx^2 - bx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arcsech(c*x)),x, algorithm="fricas")

[Out] 1/2*(b*c*x^2*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + a*c*x^2 - b*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/c

Sympy [A] time = 1.31947, size = 46, normalized size = 1.02

$$\begin{cases} \frac{ax^2}{2} + \frac{bx^2 \operatorname{asech}(cx)}{2} - \frac{b\sqrt{-c^2x^2+1}}{2c^2} & \text{for } c \neq 0 \\ \frac{x^2(a+\infty b)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asech(c*x)),x)
```

```
[Out] Piecewise((a*x**2/2 + b*x**2*asech(c*x)/2 - b*sqrt(-c**2*x**2 + 1)/(2*c**2)
, Ne(c, 0)), (x**2*(a + oo*b)/2, True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{ar} \operatorname{sech}(cx) + a)x dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x, x)
```

3.25 $\int (a + b \operatorname{sech}^{-1}(cx)) dx$

Optimal. Leaf size=40

$$ax + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sin^{-1}(cx)}{c} + bx\operatorname{sech}^{-1}(cx)$$

[Out] a*x + b*x*ArcSech[c*x] + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x])/c

Rubi [A] time = 0.0148849, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {6277, 216}

$$ax + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sin^{-1}(cx)}{c} + bx\operatorname{sech}^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Int[a + b*ArcSech[c*x], x]

[Out] a*x + b*x*ArcSech[c*x] + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x])/c

Rule 6277

Int[ArcSech[(c_.)*(x_)], x_Symbol] := Simp[x*ArcSech[c*x], x] + Dist[Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[1/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[c, x]

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rubi steps

$$\begin{aligned} \int (a + b \operatorname{sech}^{-1}(cx)) dx &= ax + b \int \operatorname{sech}^{-1}(cx) dx \\ &= ax + bx \operatorname{sech}^{-1}(cx) + \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{1}{\sqrt{1-c^2x^2}} dx \\ &= ax + bx \operatorname{sech}^{-1}(cx) + \frac{b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sin^{-1}(cx)}{c} \end{aligned}$$

Mathematica [A] time = 0.0990898, size = 60, normalized size = 1.5

$$ax - \frac{b\sqrt{\frac{1-cx}{cx+1}}\sqrt{1-c^2x^2}\sin^{-1}(cx)}{c(cx-1)} + bx\operatorname{sech}^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[a + b*ArcSech[c*x], x]

[Out] $a*x + b*x*\text{ArcSech}[c*x] - (b*\text{Sqrt}[(1 - c*x)/(1 + c*x)]*\text{Sqrt}[1 - c^2*x^2]*\text{ArcSin}[c*x])/(c*(-1 + c*x))$

Maple [A] time = 0.159, size = 42, normalized size = 1.1

$$ax + bx\text{arcsech}(cx) - \frac{b}{c} \arctan\left(\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(a+b*arcsech(c*x),x)`

[Out] $a*x+b*x*\text{arcsech}(c*x)-b/c*\arctan((-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)})$

Maxima [A] time = 0.972533, size = 42, normalized size = 1.05

$$ax + \frac{\left(cx \text{arsech}(cx) - \arctan\left(\sqrt{\frac{1}{c^2x^2} - 1}\right)\right)b}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a+b*arcsech(c*x),x, algorithm="maxima")`

[Out] $a*x + (c*x*\text{arcsech}(c*x) - \arctan(\text{sqrt}(1/(c^2*x^2) - 1)))*b/c$

Fricas [B] time = 1.99977, size = 262, normalized size = 6.55

$$\frac{acx - bc \log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{x}\right) - 2b \arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right) + (bcx - bc) \log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a+b*arcsech(c*x),x, algorithm="fricas")`

[Out] $(a*c*x - b*c*\log((c*x*\text{sqrt}(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) - 2*b*\arctan((c*x*\text{sqrt}(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c*x)) + (b*c*x - b*c)*\log((c*x*\text{sqrt}(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/c$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \text{asech}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a+b*asech(c*x),x)`

[Out] Integral(a + b*asech(c*x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int b \operatorname{arsech}(cx) + a dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b*arcsech(c*x),x, algorithm="giac")

[Out] integrate(b*arcsech(c*x) + a, x)

$$3.26 \quad \int \frac{a+b\operatorname{sech}^{-1}(cx)}{x} dx$$

Optimal. Leaf size=56

$$\frac{1}{2}b\operatorname{PolyLog}\left(2, -e^{-2\operatorname{sech}^{-1}(cx)}\right) - \frac{(a+b\operatorname{sech}^{-1}(cx))^2}{2b} - \log\left(e^{-2\operatorname{sech}^{-1}(cx)} + 1\right)(a+b\operatorname{sech}^{-1}(cx))$$

[Out] $-(a + b*\operatorname{ArcSech}[c*x])^2/(2*b) - (a + b*\operatorname{ArcSech}[c*x])*Log[1 + E^{(-2*\operatorname{ArcSech}[c*x])}] + (b*\operatorname{PolyLog}[2, -E^{(-2*\operatorname{ArcSech}[c*x])}])/2$

Rubi [A] time = 0.0879521, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6281, 5660, 3718, 2190, 2279, 2391}

$$-\frac{1}{2}b\operatorname{PolyLog}\left(2, -e^{2\operatorname{sech}^{-1}(cx)}\right) + \frac{(a+b\operatorname{sech}^{-1}(cx))^2}{2b} - \log\left(e^{2\operatorname{sech}^{-1}(cx)} + 1\right)(a+b\operatorname{sech}^{-1}(cx))$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b*ArcSech[c*x])/x, x]

[Out] $(a + b*\operatorname{ArcSech}[c*x])^2/(2*b) - (a + b*\operatorname{ArcSech}[c*x])*Log[1 + E^{(2*\operatorname{ArcSech}[c*x])}] - (b*\operatorname{PolyLog}[2, -E^{(2*\operatorname{ArcSech}[c*x])}])/2$

Rule 6281

Int[((a_.) + ArcSech[(c_.)*(x_)])*(b_.)/(x_), x_Symbol] := -Subst[Int[(a + b*ArcCosh[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]

Rule 5660

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.)^n/(x_), x_Symbol] := Subst[Int[(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]

Rule 3718

Int[((c_.) + (d_.)*(x_)^m)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c + d*x)^m * E^(2*(-I*e) + f*fz*x)) / (1 + E^(2*(-I*e) + f*fz*x))], x] /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

Rule 2190

Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^n)*((c_.) + (d_.)*(x_)^m)/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^n), x_Symbol] := Simp[((c + d*x)^m * Log[1 + (b*(F^(g*(e + f*x)))^n)/a]) / (b*f*g*n * Log[F]), x] - Dist[(d*m) / (b*f*g*n * Log[F]), Int[(c + d*x)^(m - 1) * Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^n], x_Symbol] := Dist[1/(d*e*n * Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

```
Int[Log[(c_.)*(d_) + (e_.)*(x_)^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned} \int \frac{a + b \operatorname{sech}^{-1}(cx)}{x} dx &= -\operatorname{Subst} \left(\int \frac{a + b \cosh^{-1}\left(\frac{x}{c}\right)}{x} dx, x, \frac{1}{x} \right) \\ &= -\operatorname{Subst} \left(\int (a + bx) \tanh(x) dx, x, \operatorname{sech}^{-1}(cx) \right) \\ &= \frac{(a + b \operatorname{sech}^{-1}(cx))^2}{2b} - 2 \operatorname{Subst} \left(\int \frac{e^{2x}(a + bx)}{1 + e^{2x}} dx, x, \operatorname{sech}^{-1}(cx) \right) \\ &= \frac{(a + b \operatorname{sech}^{-1}(cx))^2}{2b} - (a + b \operatorname{sech}^{-1}(cx)) \log(1 + e^{2 \operatorname{sech}^{-1}(cx)}) + b \operatorname{Subst} \left(\int \log(1 + e^{2x}) dx, x, \operatorname{sech}^{-1}(cx) \right) \\ &= \frac{(a + b \operatorname{sech}^{-1}(cx))^2}{2b} - (a + b \operatorname{sech}^{-1}(cx)) \log(1 + e^{2 \operatorname{sech}^{-1}(cx)}) + \frac{1}{2} b \operatorname{Subst} \left(\int \frac{\log(1 + x)}{x} dx, x, \operatorname{sech}^{-1}(cx) \right) \\ &= \frac{(a + b \operatorname{sech}^{-1}(cx))^2}{2b} - (a + b \operatorname{sech}^{-1}(cx)) \log(1 + e^{2 \operatorname{sech}^{-1}(cx)}) - \frac{1}{2} b \operatorname{Li}_2 \left(-e^{2 \operatorname{sech}^{-1}(cx)} \right) \end{aligned}$$

Mathematica [A] time = 0.043811, size = 47, normalized size = 0.84

$$\frac{1}{2} b \left(\operatorname{PolyLog} \left(2, -e^{-2 \operatorname{sech}^{-1}(cx)} \right) - \operatorname{sech}^{-1}(cx) \left(\operatorname{sech}^{-1}(cx) + 2 \log \left(e^{-2 \operatorname{sech}^{-1}(cx)} + 1 \right) \right) \right) + a \log(x)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSech[c*x])/x, x]
```

```
[Out] a*Log[x] + (b*(-(ArcSech[c*x]*(ArcSech[c*x] + 2*Log[1 + E^(-2*ArcSech[c*x])])) + PolyLog[2, -E^(-2*ArcSech[c*x])]))/2
```

Maple [A] time = 0.256, size = 100, normalized size = 1.8

$$a \ln(cx) + \frac{b (\operatorname{arcsech}(cx))^2}{2} - b \operatorname{arcsech}(cx) \ln \left(1 + \left(\frac{1}{cx} + \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}} \right)^2 \right) - \frac{b}{2} \operatorname{polylog} \left(2, - \left(\frac{1}{cx} + \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}} \right)^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))/x, x)
```

```
[Out] a*ln(c*x)+1/2*b*arcsech(c*x)^2-b*arcsech(c*x)*ln(1+(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)-1/2*b*polylog(2,-(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b \int \frac{\log \left(\sqrt{\frac{1}{cx} + 1} \sqrt{\frac{1}{cx} - 1} + \frac{1}{cx} \right)}{x} dx + a \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/x,x, algorithm="maxima")

[Out] b*integrate(log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/x, x) + a*log(x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b \operatorname{ar} \operatorname{sech}(cx) + a}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/x,x, algorithm="fricas")

[Out] integral((b*arcsech(c*x) + a)/x, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{asech}(cx)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asech(c*x))/x,x)

[Out] Integral((a + b*asech(c*x))/x, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{ar} \operatorname{sech}(cx) + a}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/x,x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)/x, x)

$$3.27 \quad \int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^2} dx$$

Optimal. Leaf size=40

$$\frac{b\sqrt{1-cx}}{x\sqrt{\frac{1}{cx+1}}} - \frac{a+b\operatorname{sech}^{-1}(cx)}{x}$$

[Out] (b*Sqrt[1 - c*x])/(x*Sqrt[(1 + c*x)^(-1)]) - (a + b*ArcSech[c*x])/x

Rubi [A] time = 0.0201413, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6283, 95}

$$\frac{b\sqrt{1-cx}}{x\sqrt{\frac{1}{cx+1}}} - \frac{a+b\operatorname{sech}^{-1}(cx)}{x}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcSech[c*x])/x^2,x]

[Out] (b*Sqrt[1 - c*x])/(x*Sqrt[(1 + c*x)^(-1)]) - (a + b*ArcSech[c*x])/x

Rule 6283

Int[((a_.) + ArcSech[(c_.)*(x_)])*(b_.))*((d_.)*(x_)^(m_.), x_Symbol] :> Simp[((d*x)^(m + 1)*(a + b*ArcSech[c*x]))/(d*(m + 1)), x] + Dist[(b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)])/ (m + 1), Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]

Rule 95

Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^2} dx &= -\frac{a+b\operatorname{sech}^{-1}(cx)}{x} - \left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx} \right) \int \frac{1}{x^2\sqrt{1-cx}\sqrt{1+cx}} dx \\ &= \frac{b\sqrt{1-cx}}{x\sqrt{\frac{1}{1+cx}}} - \frac{a+b\operatorname{sech}^{-1}(cx)}{x} \end{aligned}$$

Mathematica [A] time = 0.0540285, size = 42, normalized size = 1.05

$$-\frac{a}{x} + b\left(c + \frac{1}{x}\right)\sqrt{\frac{1-cx}{cx+1}} - \frac{b\operatorname{sech}^{-1}(cx)}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcSech[c*x])/x^2,x]

[Out] -(a/x) + b*(c + x^(-1))*Sqrt[(1 - c*x)/(1 + c*x)] - (b*ArcSech[c*x])/x

Maple [A] time = 0.178, size = 58, normalized size = 1.5

$$c \left(-\frac{a}{cx} + b \left(-\frac{\operatorname{arcsech}(cx)}{cx} + \sqrt{-\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))/x^2,x)

[Out] c*(-a/c/x+b*(-1/c/x*arcsech(c*x)+(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)))

Maxima [A] time = 0.981928, size = 43, normalized size = 1.08

$$\left(c \sqrt{\frac{1}{c^2 x^2} - 1} - \frac{\operatorname{arsech}(cx)}{x} \right) b - \frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/x^2,x, algorithm="maxima")

[Out] (c*sqrt(1/(c^2*x^2) - 1) - arcsech(c*x)/x)*b - a/x

Fricas [A] time = 1.96489, size = 138, normalized size = 3.45

$$\frac{bcx \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2}} - b \log \left(\frac{cx \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2}} + 1}{cx} \right) - a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/x^2,x, algorithm="fricas")

[Out] (b*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - b*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - a)/x

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{asech}(cx)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asech(c*x))/x**2,x)

```
[Out] Integral((a + b*asech(c*x))/x**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{ar} \operatorname{sech}(cx) + a}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/x^2, x)
```

3.28 $\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^3} dx$

Optimal. Leaf size=94

$$-\frac{a+b\operatorname{sech}^{-1}(cx)}{2x^2} + \frac{1}{4}bc^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\sqrt{1-cx}\sqrt{cx+1}\right) + \frac{b\sqrt{1-cx}}{4x^2\sqrt{\frac{1}{cx+1}}}$$

[Out] (b*Sqrt[1 - c*x])/(4*x^2*Sqrt[(1 + c*x)^(-1)]) - (a + b*ArcSech[c*x])/(2*x^2) + (b*c^2*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[1 - c*x]*Sqrt[1 + c*x]])/4

Rubi [A] time = 0.0399034, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6283, 103, 12, 92, 208}

$$-\frac{a+b\operatorname{sech}^{-1}(cx)}{2x^2} + \frac{1}{4}bc^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\sqrt{1-cx}\sqrt{cx+1}\right) + \frac{b\sqrt{1-cx}}{4x^2\sqrt{\frac{1}{cx+1}}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcSech[c*x])/x^3,x]

[Out] (b*Sqrt[1 - c*x])/(4*x^2*Sqrt[(1 + c*x)^(-1)]) - (a + b*ArcSech[c*x])/(2*x^2) + (b*c^2*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[1 - c*x]*Sqrt[1 + c*x]])/4

Rule 6283

Int[((a_.) + ArcSech[(c_.)*(x_)])*(b_.)*((d_.)*(x_))^(m_.), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*ArcSech[c*x]))/(d*(m + 1)), x] + Dist[(b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)])/(m + 1), Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]

Rule 103

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 92

Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))), x_Symbol] := Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d*e - f*(b*c + a*d), 0]

Rule 208

$\text{Int}[(a + b \cdot (x^2)^{-1}), x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2] \cdot \text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$

Rubi steps

$$\begin{aligned} \int \frac{a + b \operatorname{sech}^{-1}(cx)}{x^3} dx &= -\frac{a + b \operatorname{sech}^{-1}(cx)}{2x^2} - \frac{1}{2} \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{1}{x^3 \sqrt{1-cx} \sqrt{1+cx}} dx \\ &= \frac{b \sqrt{1-cx}}{4x^2 \sqrt{\frac{1}{1+cx}}} - \frac{a + b \operatorname{sech}^{-1}(cx)}{2x^2} - \frac{1}{4} \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{c^2}{x \sqrt{1-cx} \sqrt{1+cx}} dx \\ &= \frac{b \sqrt{1-cx}}{4x^2 \sqrt{\frac{1}{1+cx}}} - \frac{a + b \operatorname{sech}^{-1}(cx)}{2x^2} - \frac{1}{4} \left(bc^2 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{1}{x \sqrt{1-cx} \sqrt{1+cx}} dx \\ &= \frac{b \sqrt{1-cx}}{4x^2 \sqrt{\frac{1}{1+cx}}} - \frac{a + b \operatorname{sech}^{-1}(cx)}{2x^2} + \frac{1}{4} \left(bc^3 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \operatorname{Subst} \left(\int \frac{1}{c - cx^2} dx, x, \sqrt{1-cx} \sqrt{1+cx} \right) \\ &= \frac{b \sqrt{1-cx}}{4x^2 \sqrt{\frac{1}{1+cx}}} - \frac{a + b \operatorname{sech}^{-1}(cx)}{2x^2} + \frac{1}{4} bc^2 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \tanh^{-1} \left(\sqrt{1-cx} \sqrt{1+cx} \right) \end{aligned}$$

Mathematica [A] time = 0.0683868, size = 117, normalized size = 1.24

$$-\frac{a}{2x^2} - \frac{1}{4} bc^2 \log(x) + \frac{1}{4} bc^2 \log \left(cx \sqrt{\frac{1-cx}{cx+1}} + \sqrt{\frac{1-cx}{cx+1}} + 1 \right) + b \left(\frac{c}{4x} + \frac{1}{4x^2} \right) \sqrt{\frac{1-cx}{cx+1}} - \frac{b \operatorname{sech}^{-1}(cx)}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcSech[c*x])/x^3,x]

[Out] -a/(2*x^2) + b*(1/(4*x^2) + c/(4*x))*Sqrt[(1 - c*x)/(1 + c*x)] - (b*ArcSech[c*x])/(2*x^2) - (b*c^2*Log[x])/4 + (b*c^2*Log[1 + Sqrt[(1 - c*x)/(1 + c*x)]] + c*x*Sqrt[(1 - c*x)/(1 + c*x)])/4

Maple [A] time = 0.183, size = 112, normalized size = 1.2

$$c^2 \left(-\frac{a}{2c^2x^2} + b \left(-\frac{\operatorname{arcsech}(cx)}{2c^2x^2} + \frac{1}{4cx} \sqrt{-\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} \left(\operatorname{Arctanh} \left(\frac{1}{\sqrt{-c^2x^2+1}} \right) c^2x^2 + \sqrt{-c^2x^2+1} \right) \frac{1}{\sqrt{-c^2x^2+1}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))/x^3,x)

[Out] c^2*(-1/2*a/c^2/x^2+b*(-1/2/c^2/x^2*arcsech(c*x)+1/4*(-(c*x-1)/c/x)^(1/2)/c/x*((c*x+1)/c/x)^(1/2)*(arctanh(1/(-c^2*x^2+1)^(1/2))*c^2*x^2+(-c^2*x^2+1)^(1/2)))/(-c^2*x^2+1)^(1/2))

Maxima [A] time = 0.988527, size = 142, normalized size = 1.51

$$-\frac{1}{8}b \left(\frac{2c^4x\sqrt{\frac{1}{c^2x^2}-1}}{c^2x^2\left(\frac{1}{c^2x^2}-1\right)-1} - c^3 \log\left(cx\sqrt{\frac{1}{c^2x^2}-1}+1\right) + c^3 \log\left(cx\sqrt{\frac{1}{c^2x^2}-1}-1\right) \right) + \frac{4 \operatorname{arsech}(cx)}{x^2} - \frac{a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/x^3,x, algorithm="maxima")

[Out] -1/8*b*((2*c^4*x*sqrt(1/(c^2*x^2) - 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) - c^3*log(c*x*sqrt(1/(c^2*x^2) - 1) + 1) + c^3*log(c*x*sqrt(1/(c^2*x^2) - 1) - 1))/c + 4*arcsech(c*x)/x^2) - 1/2*a/x^2

Fricas [A] time = 1.85841, size = 170, normalized size = 1.81

$$\frac{bcx\sqrt{-\frac{c^2x^2-1}{c^2x^2}} + (bc^2x^2 - 2b) \log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right) - 2a}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/x^3,x, algorithm="fricas")

[Out] 1/4*(b*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + (b*c^2*x^2 - 2*b)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - 2*a)/x^2

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{asech}(cx)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asech(c*x))/x**3,x)

[Out] Integral((a + b*asech(c*x))/x**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsech}(cx) + a}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/x^3,x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)/x^3, x)

3.29 $\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^4} dx$

Optimal. Leaf size=77

$$-\frac{a+b\operatorname{sech}^{-1}(cx)}{3x^3} + \frac{2bc^2\sqrt{1-cx}}{9x\sqrt{\frac{1}{cx+1}}} + \frac{b\sqrt{1-cx}}{9x^3\sqrt{\frac{1}{cx+1}}}$$

[Out] (b*Sqrt[1 - c*x])/(9*x^3*Sqrt[(1 + c*x)^(-1)]) + (2*b*c^2*Sqrt[1 - c*x])/(9*x*Sqrt[(1 + c*x)^(-1)]) - (a + b*ArcSech[c*x])/(3*x^3)

Rubi [A] time = 0.0332651, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6283, 103, 12, 95}

$$-\frac{a+b\operatorname{sech}^{-1}(cx)}{3x^3} + \frac{2bc^2\sqrt{1-cx}}{9x\sqrt{\frac{1}{cx+1}}} + \frac{b\sqrt{1-cx}}{9x^3\sqrt{\frac{1}{cx+1}}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcSech[c*x])/x^4, x]

[Out] (b*Sqrt[1 - c*x])/(9*x^3*Sqrt[(1 + c*x)^(-1)]) + (2*b*c^2*Sqrt[1 - c*x])/(9*x*Sqrt[(1 + c*x)^(-1)]) - (a + b*ArcSech[c*x])/(3*x^3)

Rule 6283

```
Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))*((d_.)*(x_.))^(m_.), x_Symbol] :> Simp[
((d*x)^(m + 1)*(a + b*ArcSech[c*x]))/(d*(m + 1)), x] + Dist[(b*Sqrt[1 +
c*x]*Sqrt[1/(1 + c*x)])/(m + 1), Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x]),
x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.)*((e_.) + (f_.)*(x_.))^(p_.), x_Symbol] :> Simp[
(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.)*((e_.) + (f_.)*(x_.))^(p_.), x_Symbol] :> Simp[
(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + b \operatorname{sech}^{-1}(cx)}{x^4} dx &= -\frac{a + b \operatorname{sech}^{-1}(cx)}{3x^3} - \frac{1}{3} \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{1}{x^4 \sqrt{1-cx} \sqrt{1+cx}} dx \\
&= \frac{b \sqrt{1-cx}}{9x^3 \sqrt{\frac{1}{1+cx}}} - \frac{a + b \operatorname{sech}^{-1}(cx)}{3x^3} + \frac{1}{9} \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int -\frac{2c^2}{x^2 \sqrt{1-cx} \sqrt{1+cx}} dx \\
&= \frac{b \sqrt{1-cx}}{9x^3 \sqrt{\frac{1}{1+cx}}} - \frac{a + b \operatorname{sech}^{-1}(cx)}{3x^3} - \frac{1}{9} \left(2bc^2 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{1}{x^2 \sqrt{1-cx} \sqrt{1+cx}} dx \\
&= \frac{b \sqrt{1-cx}}{9x^3 \sqrt{\frac{1}{1+cx}}} + \frac{2bc^2 \sqrt{1-cx}}{9x \sqrt{\frac{1}{1+cx}}} - \frac{a + b \operatorname{sech}^{-1}(cx)}{3x^3}
\end{aligned}$$

Mathematica [A] time = 0.0628587, size = 74, normalized size = 0.96

$$-\frac{a}{3x^3} + b \left(\frac{2c^2}{9x} + \frac{2c^3}{9} + \frac{c}{9x^2} + \frac{1}{9x^3} \right) \sqrt{\frac{1-cx}{cx+1}} - \frac{b \operatorname{sech}^{-1}(cx)}{3x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcSech[c*x])/x^4,x]

[Out] -a/(3*x^3) + b*((2*c^3)/9 + 1/(9*x^3) + c/(9*x^2) + (2*c^2)/(9*x))*Sqrt[(1 - c*x)/(1 + c*x)] - (b*ArcSech[c*x])/(3*x^3)

Maple [A] time = 0.181, size = 77, normalized size = 1.

$$c^3 \left(-\frac{a}{3c^3x^3} + b \left(-\frac{\operatorname{arcsech}(cx)}{3c^3x^3} + \frac{2c^2x^2 + 1}{9c^2x^2} \sqrt{-\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))/x^4,x)

[Out] c^3*(-1/3*a/c^3/x^3+b*(-1/3/c^3/x^3*arcsech(c*x)+1/9*(-(c*x-1)/c/x)^(1/2)/c^2/x^2*((c*x+1)/c/x)^(1/2)*(2*c^2*x^2+1)))

Maxima [A] time = 0.979279, size = 76, normalized size = 0.99

$$\frac{1}{9} b \left(\frac{c^4 \left(\frac{1}{c^2x^2} - 1 \right)^{\frac{3}{2}} + 3c^4 \sqrt{\frac{1}{c^2x^2} - 1}}{c} - \frac{3 \operatorname{ar sech}(cx)}{x^3} \right) - \frac{a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/x^4,x, algorithm="maxima")

[Out] $\frac{1}{9}b\left(\frac{c^4}{c^2x^2} - 1\right)^{3/2} + 3c^4\sqrt{\frac{1}{c^2x^2} - 1} - 3\operatorname{arcsech}(cx)/x^3 - \frac{1}{3}a/x^3$

Fricas [A] time = 1.89021, size = 174, normalized size = 2.26

$$\frac{3b \log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right) - (2bc^3x^3 + bcx)\sqrt{-\frac{c^2x^2-1}{c^2x^2}} + 3a}{9x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsech(c*x))/x^4,x, algorithm="fricas")`

[Out] $-\frac{1}{9}(3b\log((cx\sqrt{-(c^2x^2-1)/(c^2x^2)}+1)/(cx)) - (2b*c^3*x^3 + b*c*x)\sqrt{-(c^2x^2-1)/(c^2x^2)} + 3a)/x^3$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{asech}(cx)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*asech(c*x))/x**4,x)`

[Out] `Integral((a + b*asech(c*x))/x**4, x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsech}(cx) + a}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsech(c*x))/x^4,x, algorithm="giac")`

[Out] `integrate((b*arcsech(c*x) + a)/x^4, x)`

$$3.30 \quad \int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^5} dx$$

Optimal. Leaf size=126

$$-\frac{a+b\operatorname{sech}^{-1}(cx)}{4x^4} + \frac{3bc^2\sqrt{1-cx}}{32x^2\sqrt{\frac{1}{cx+1}}} + \frac{3}{32}bc^4\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\sqrt{1-cx}\sqrt{cx+1}\right) + \frac{b\sqrt{1-cx}}{16x^4\sqrt{\frac{1}{cx+1}}}$$

[Out] (b*Sqrt[1 - c*x])/(16*x^4*Sqrt[(1 + c*x)^(-1)]) + (3*b*c^2*Sqrt[1 - c*x])/(32*x^2*Sqrt[(1 + c*x)^(-1)]) - (a + b*ArcSech[c*x])/(4*x^4) + (3*b*c^4*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[1 - c*x]*Sqrt[1 + c*x]])/32

Rubi [A] time = 0.054912, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6283, 103, 12, 92, 208}

$$-\frac{a+b\operatorname{sech}^{-1}(cx)}{4x^4} + \frac{3bc^2\sqrt{1-cx}}{32x^2\sqrt{\frac{1}{cx+1}}} + \frac{3}{32}bc^4\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\sqrt{1-cx}\sqrt{cx+1}\right) + \frac{b\sqrt{1-cx}}{16x^4\sqrt{\frac{1}{cx+1}}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcSech[c*x])/x^5, x]

[Out] (b*Sqrt[1 - c*x])/(16*x^4*Sqrt[(1 + c*x)^(-1)]) + (3*b*c^2*Sqrt[1 - c*x])/(32*x^2*Sqrt[(1 + c*x)^(-1)]) - (a + b*ArcSech[c*x])/(4*x^4) + (3*b*c^4*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[1 - c*x]*Sqrt[1 + c*x]])/32

Rule 6283

Int[((a_.) + ArcSech[(c_.)*(x_)])*(b_.)*((d_.)*(x_))^(m_.), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*ArcSech[c*x]))/(d*(m + 1)), x] + Dist[(b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)])/(m + 1), Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]

Rule 103

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])

Rule 12

Int[(a_.)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_.)*(v_) /; FreeQ[b, x]]

Rule 92

Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))), x_Symbol] := Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d*e - f*(b*c + a*d), 0]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
 \int \frac{a + b \operatorname{sech}^{-1}(cx)}{x^5} dx &= -\frac{a + b \operatorname{sech}^{-1}(cx)}{4x^4} - \frac{1}{4} \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{1}{x^5 \sqrt{1-cx} \sqrt{1+cx}} dx \\
 &= \frac{b \sqrt{1-cx}}{16x^4 \sqrt{\frac{1}{1+cx}}} - \frac{a + b \operatorname{sech}^{-1}(cx)}{4x^4} + \frac{1}{16} \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int -\frac{3c^2}{x^3 \sqrt{1-cx} \sqrt{1+cx}} dx \\
 &= \frac{b \sqrt{1-cx}}{16x^4 \sqrt{\frac{1}{1+cx}}} - \frac{a + b \operatorname{sech}^{-1}(cx)}{4x^4} - \frac{1}{16} \left(3bc^2 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{1}{x^3 \sqrt{1-cx} \sqrt{1+cx}} dx \\
 &= \frac{b \sqrt{1-cx}}{16x^4 \sqrt{\frac{1}{1+cx}}} + \frac{3bc^2 \sqrt{1-cx}}{32x^2 \sqrt{\frac{1}{1+cx}}} - \frac{a + b \operatorname{sech}^{-1}(cx)}{4x^4} - \frac{1}{32} \left(3bc^2 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{c^2}{x \sqrt{1-cx} \sqrt{1+cx}} dx \\
 &= \frac{b \sqrt{1-cx}}{16x^4 \sqrt{\frac{1}{1+cx}}} + \frac{3bc^2 \sqrt{1-cx}}{32x^2 \sqrt{\frac{1}{1+cx}}} - \frac{a + b \operatorname{sech}^{-1}(cx)}{4x^4} - \frac{1}{32} \left(3bc^4 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{1}{x \sqrt{1-cx} \sqrt{1+cx}} dx \\
 &= \frac{b \sqrt{1-cx}}{16x^4 \sqrt{\frac{1}{1+cx}}} + \frac{3bc^2 \sqrt{1-cx}}{32x^2 \sqrt{\frac{1}{1+cx}}} - \frac{a + b \operatorname{sech}^{-1}(cx)}{4x^4} + \frac{1}{32} \left(3bc^5 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \operatorname{Subst} \left(\int \frac{1}{c - cx^2} dx \right) \\
 &= \frac{b \sqrt{1-cx}}{16x^4 \sqrt{\frac{1}{1+cx}}} + \frac{3bc^2 \sqrt{1-cx}}{32x^2 \sqrt{\frac{1}{1+cx}}} - \frac{a + b \operatorname{sech}^{-1}(cx)}{4x^4} + \frac{3}{32} bc^4 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \tanh^{-1} \left(\sqrt{1-cx} \sqrt{1+cx} \right)
 \end{aligned}$$

Mathematica [A] time = 0.0949213, size = 137, normalized size = 1.09

$$-\frac{a}{4x^4} + b \left(\frac{3c^2}{32x^2} + \frac{3c^3}{32x} + \frac{c}{16x^3} + \frac{1}{16x^4} \right) \sqrt{\frac{1-cx}{cx+1}} - \frac{3}{32} bc^4 \log(x) + \frac{3}{32} bc^4 \log \left(cx \sqrt{\frac{1-cx}{cx+1}} + \sqrt{\frac{1-cx}{cx+1}} + 1 \right) - \frac{b \operatorname{sech}^{-1}(cx)}{4x^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcSech[c*x])/x^5, x]

[Out] -a/(4*x^4) + b*(1/(16*x^4) + c/(16*x^3) + (3*c^2)/(32*x^2) + (3*c^3)/(32*x)) * Sqrt[(1 - c*x)/(1 + c*x)] - (b*ArcSech[c*x])/(4*x^4) - (3*b*c^4*Log[x])/32 + (3*b*c^4*Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]])/32

Maple [A] time = 0.182, size = 135, normalized size = 1.1

$$c^4 \left(-\frac{a}{4c^4x^4} + b \left(-\frac{\operatorname{arcsech}(cx)}{4c^4x^4} + \frac{1}{32c^3x^3} \sqrt{\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} \left(3 \operatorname{Arctanh} \left(\frac{1}{\sqrt{-c^2x^2+1}} \right) c^4x^4 + 3 \sqrt{-c^2x^2+1} c^2x^2 + 2 \sqrt{-c^2x^2+1} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))/x^5, x)

[Out] c^4*(-1/4*a/c^4/x^4+b*(-1/4/c^4/x^4*arcsech(c*x)+1/32*(-(c*x-1)/c/x)^(1/2)/c^3/x^3*((c*x+1)/c/x)^(1/2)*(3*arctanh(1/(-c^2*x^2+1)^(1/2))*c^4*x^4+3*(-c^2*x^2+1)^(1/2)*c^2*x^2+2*(-c^2*x^2+1)^(1/2)*c*x))/32

$$2*x^2+1)^{(1/2)}*c^2*x^2+2*(-c^2*x^2+1)^{(1/2))/(-c^2*x^2+1)^{(1/2))}$$

Maxima [A] time = 0.988312, size = 198, normalized size = 1.57

$$\frac{1}{64} b \left(\frac{3c^5 \log\left(cx\sqrt{\frac{1}{c^2x^2}-1}+1\right) - 3c^5 \log\left(cx\sqrt{\frac{1}{c^2x^2}-1}-1\right) - \frac{2\left(3c^8x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}} - 5c^6x\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^4x^4\left(\frac{1}{c^2x^2}-1\right)^2 - 2c^2x^2\left(\frac{1}{c^2x^2}-1\right)+1}}{c} - \frac{16 \operatorname{arsech}(cx)}{x^4} \right) - \frac{a}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/x^5,x, algorithm="maxima")

[Out] 1/64*b*((3*c^5*log(c*x*sqrt(1/(c^2*x^2) - 1) + 1) - 3*c^5*log(c*x*sqrt(1/(c^2*x^2) - 1) - 1) - 2*(3*c^8*x^3*(1/(c^2*x^2) - 1)^(3/2) - 5*c^6*x*sqrt(1/(c^2*x^2) - 1)))/(c^4*x^4*(1/(c^2*x^2) - 1)^2 - 2*c^2*x^2*(1/(c^2*x^2) - 1) + 1))/c - 16*arcsech(c*x)/x^4) - 1/4*a/x^4

Fricas [A] time = 1.90141, size = 198, normalized size = 1.57

$$\frac{(3bc^4x^4 - 8b) \log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right) + (3bc^3x^3 + 2bcx)\sqrt{-\frac{c^2x^2-1}{c^2x^2}} - 8a}{32x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/x^5,x, algorithm="fricas")

[Out] 1/32*((3*b*c^4*x^4 - 8*b)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + (3*b*c^3*x^3 + 2*b*c*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 8*a)/x^4

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{asech}(cx)}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asech(c*x))/x**5,x)

[Out] Integral((a + b*asech(c*x))/x**5, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsech}(cx) + a}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/x^5, x)
```


3.31 $\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^6} dx$

Optimal. Leaf size=109

$$-\frac{a+b\operatorname{sech}^{-1}(cx)}{5x^5} + \frac{4bc^2\sqrt{1-cx}}{75x^3\sqrt{\frac{1}{cx+1}}} + \frac{8bc^4\sqrt{1-cx}}{75x\sqrt{\frac{1}{cx+1}}} + \frac{b\sqrt{1-cx}}{25x^5\sqrt{\frac{1}{cx+1}}}$$

[Out] (b*Sqrt[1 - c*x])/(25*x^5*Sqrt[(1 + c*x)^(-1)]) + (4*b*c^2*Sqrt[1 - c*x])/(75*x^3*Sqrt[(1 + c*x)^(-1)]) + (8*b*c^4*Sqrt[1 - c*x])/(75*x*Sqrt[(1 + c*x)^(-1)]) - (a + b*ArcSech[c*x])/(5*x^5)

Rubi [A] time = 0.0503686, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6283, 103, 12, 95}

$$-\frac{a+b\operatorname{sech}^{-1}(cx)}{5x^5} + \frac{4bc^2\sqrt{1-cx}}{75x^3\sqrt{\frac{1}{cx+1}}} + \frac{8bc^4\sqrt{1-cx}}{75x\sqrt{\frac{1}{cx+1}}} + \frac{b\sqrt{1-cx}}{25x^5\sqrt{\frac{1}{cx+1}}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcSech[c*x])/x^6, x]

[Out] (b*Sqrt[1 - c*x])/(25*x^5*Sqrt[(1 + c*x)^(-1)]) + (4*b*c^2*Sqrt[1 - c*x])/(75*x^3*Sqrt[(1 + c*x)^(-1)]) + (8*b*c^4*Sqrt[1 - c*x])/(75*x*Sqrt[(1 + c*x)^(-1)]) - (a + b*ArcSech[c*x])/(5*x^5)

Rule 6283

Int[((a_.) + ArcSech[(c_.)*(x_)])*(b_.)*((d_.)*(x_))^(m_.), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*ArcSech[c*x]))/(d*(m + 1)), x] + Dist[(b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)])/(m + 1), Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]

Rule 103

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 95

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*

$c*f*(n + 1) + b*d*e*(p + 1), 0] \&\& \text{NeQ}[m, -1]$

Rubi steps

$$\begin{aligned}
 \int \frac{a + b \operatorname{sech}^{-1}(cx)}{x^6} dx &= -\frac{a + b \operatorname{sech}^{-1}(cx)}{5x^5} - \frac{1}{5} \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{1}{x^6 \sqrt{1-cx} \sqrt{1+cx}} dx \\
 &= \frac{b \sqrt{1-cx}}{25x^5 \sqrt{\frac{1}{1+cx}}} - \frac{a + b \operatorname{sech}^{-1}(cx)}{5x^5} + \frac{1}{25} \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int -\frac{4c^2}{x^4 \sqrt{1-cx} \sqrt{1+cx}} dx \\
 &= \frac{b \sqrt{1-cx}}{25x^5 \sqrt{\frac{1}{1+cx}}} - \frac{a + b \operatorname{sech}^{-1}(cx)}{5x^5} - \frac{1}{25} \left(4bc^2 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{1}{x^4 \sqrt{1-cx} \sqrt{1+cx}} dx \\
 &= \frac{b \sqrt{1-cx}}{25x^5 \sqrt{\frac{1}{1+cx}}} + \frac{4bc^2 \sqrt{1-cx}}{75x^3 \sqrt{\frac{1}{1+cx}}} - \frac{a + b \operatorname{sech}^{-1}(cx)}{5x^5} + \frac{1}{75} \left(4bc^2 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int -\frac{2c^2}{x^2 \sqrt{1-cx} \sqrt{1+cx}} dx \\
 &= \frac{b \sqrt{1-cx}}{25x^5 \sqrt{\frac{1}{1+cx}}} + \frac{4bc^2 \sqrt{1-cx}}{75x^3 \sqrt{\frac{1}{1+cx}}} - \frac{a + b \operatorname{sech}^{-1}(cx)}{5x^5} - \frac{1}{75} \left(8bc^4 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{1}{x^2 \sqrt{1-cx} \sqrt{1+cx}} dx \\
 &= \frac{b \sqrt{1-cx}}{25x^5 \sqrt{\frac{1}{1+cx}}} + \frac{4bc^2 \sqrt{1-cx}}{75x^3 \sqrt{\frac{1}{1+cx}}} + \frac{8bc^4 \sqrt{1-cx}}{75x \sqrt{\frac{1}{1+cx}}} - \frac{a + b \operatorname{sech}^{-1}(cx)}{5x^5}
 \end{aligned}$$

Mathematica [A] time = 0.0860176, size = 94, normalized size = 0.86

$$-\frac{a}{5x^5} + b \left(\frac{4c^3}{75x^2} + \frac{4c^2}{75x^3} + \frac{8c^4}{75x} + \frac{8c^5}{75} + \frac{c}{25x^4} + \frac{1}{25x^5} \right) \sqrt{\frac{1-cx}{cx+1}} - \frac{b \operatorname{sech}^{-1}(cx)}{5x^5}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcSech[c*x])/x^6,x]

[Out] -a/(5*x^5) + b*((8*c^5)/75 + 1/(25*x^5) + c/(25*x^4) + (4*c^2)/(75*x^3) + (4*c^3)/(75*x^2) + (8*c^4)/(75*x))*Sqrt[(1 - c*x)/(1 + c*x)] - (b*ArcSech[c*x])/(5*x^5)

Maple [A] time = 0.18, size = 85, normalized size = 0.8

$$c^5 \left(-\frac{a}{5c^5x^5} + b \left(-\frac{\operatorname{arcsech}(cx)}{5c^5x^5} + \frac{8c^4x^4 + 4c^2x^2 + 3}{75c^4x^4} \sqrt{-\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))/x^6,x)

[Out] c^5*(-1/5*a/c^5/x^5+b*(-1/5/c^5/x^5*arcsech(c*x)+1/75*(-(c*x-1)/c/x)^(1/2)/c^4/x^4*((c*x+1)/c/x)^(1/2)*(8*c^4*x^4+4*c^2*x^2+3)))

Maxima [A] time = 0.99591, size = 99, normalized size = 0.91

$$\frac{1}{75} b \left(\frac{3 c^6 \left(\frac{1}{c^2 x^2} - 1 \right)^{\frac{5}{2}} + 10 c^6 \left(\frac{1}{c^2 x^2} - 1 \right)^{\frac{3}{2}} + 15 c^6 \sqrt{\frac{1}{c^2 x^2} - 1}}{c} - \frac{15 \operatorname{arsech}(cx)}{x^5} \right) - \frac{a}{5 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/x^6,x, algorithm="maxima")

[Out] 1/75*b*((3*c^6*(1/(c^2*x^2) - 1)^(5/2) + 10*c^6*(1/(c^2*x^2) - 1)^(3/2) + 15*c^6*sqrt(1/(c^2*x^2) - 1))/c - 15*arcsech(c*x)/x^5) - 1/5*a/x^5

Fricas [A] time = 1.87304, size = 200, normalized size = 1.83

$$\frac{15 b \log \left(\frac{c x \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2} + 1}}{c x} \right) - (8 b c^5 x^5 + 4 b c^3 x^3 + 3 b c x) \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2}} + 15 a}{75 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/x^6,x, algorithm="fricas")

[Out] -1/75*(15*b*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (8*b*c^5*x^5 + 4*b*c^3*x^3 + 3*b*c*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 15*a)/x^5

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{asech}(cx)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asech(c*x))/x**6,x)

[Out] Integral((a + b*asech(c*x))/x**6, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsech}(cx) + a}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/x^6,x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)/x^6, x)

3.32 $\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^7} dx$

Optimal. Leaf size=158

$$-\frac{a+b\operatorname{sech}^{-1}(cx)}{6x^6} + \frac{5bc^4\sqrt{1-cx}}{96x^2\sqrt{\frac{1}{cx+1}}} + \frac{5bc^2\sqrt{1-cx}}{144x^4\sqrt{\frac{1}{cx+1}}} + \frac{5}{96}bc^6\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\sqrt{1-cx}\sqrt{cx+1}\right) + \frac{b\sqrt{1-cx}}{36x^6\sqrt{\frac{1}{cx+1}}}$$

[Out] (b*Sqrt[1 - c*x])/(36*x^6*Sqrt[(1 + c*x)^(-1)]) + (5*b*c^2*Sqrt[1 - c*x])/(144*x^4*Sqrt[(1 + c*x)^(-1)]) + (5*b*c^4*Sqrt[1 - c*x])/(96*x^2*Sqrt[(1 + c*x)^(-1)]) - (a + b*ArcSech[c*x])/(6*x^6) + (5*b*c^6*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[1 - c*x]*Sqrt[1 + c*x]])/96

Rubi [A] time = 0.0728699, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6283, 103, 12, 92, 208}

$$-\frac{a+b\operatorname{sech}^{-1}(cx)}{6x^6} + \frac{5bc^4\sqrt{1-cx}}{96x^2\sqrt{\frac{1}{cx+1}}} + \frac{5bc^2\sqrt{1-cx}}{144x^4\sqrt{\frac{1}{cx+1}}} + \frac{5}{96}bc^6\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\sqrt{1-cx}\sqrt{cx+1}\right) + \frac{b\sqrt{1-cx}}{36x^6\sqrt{\frac{1}{cx+1}}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcSech[c*x])/x^7, x]

[Out] (b*Sqrt[1 - c*x])/(36*x^6*Sqrt[(1 + c*x)^(-1)]) + (5*b*c^2*Sqrt[1 - c*x])/(144*x^4*Sqrt[(1 + c*x)^(-1)]) + (5*b*c^4*Sqrt[1 - c*x])/(96*x^2*Sqrt[(1 + c*x)^(-1)]) - (a + b*ArcSech[c*x])/(6*x^6) + (5*b*c^6*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[1 - c*x]*Sqrt[1 + c*x]])/96

Rule 6283

Int[((a_.) + ArcSech[(c_.)*(x_)])*(b_.)*((d_.)*(x_))^(m_.), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*ArcSech[c*x])/(d*(m + 1)), x] + Dist[(b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)])/(m + 1), Int[(d*x)^m/(Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1]

Rule 103

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 92

Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))), x_Symbol] := Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],

$x, \text{Sqrt}[a + b*x]*\text{Sqrt}[c + d*x]], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{EqQ}[2*b*d*e - f*(b*c + a*d), 0]$

Rule 208

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] :> \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}\{a, b\}, x\} \&\& \text{NegQ}[a/b]$

Rubi steps

$$\begin{aligned} \int \frac{a + b \operatorname{sech}^{-1}(cx)}{x^7} dx &= -\frac{a + b \operatorname{sech}^{-1}(cx)}{6x^6} - \frac{1}{6} \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{1}{x^7 \sqrt{1-cx} \sqrt{1+cx}} dx \\ &= \frac{b \sqrt{1-cx}}{36x^6 \sqrt{\frac{1}{1+cx}}} - \frac{a + b \operatorname{sech}^{-1}(cx)}{6x^6} + \frac{1}{36} \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int -\frac{5c^2}{x^5 \sqrt{1-cx} \sqrt{1+cx}} dx \\ &= \frac{b \sqrt{1-cx}}{36x^6 \sqrt{\frac{1}{1+cx}}} - \frac{a + b \operatorname{sech}^{-1}(cx)}{6x^6} - \frac{1}{36} \left(5bc^2 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{1}{x^5 \sqrt{1-cx} \sqrt{1+cx}} dx \\ &= \frac{b \sqrt{1-cx}}{36x^6 \sqrt{\frac{1}{1+cx}}} + \frac{5bc^2 \sqrt{1-cx}}{144x^4 \sqrt{\frac{1}{1+cx}}} - \frac{a + b \operatorname{sech}^{-1}(cx)}{6x^6} + \frac{1}{144} \left(5bc^2 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int -\frac{3c}{x^3 \sqrt{1-cx} \sqrt{1+cx}} dx \\ &= \frac{b \sqrt{1-cx}}{36x^6 \sqrt{\frac{1}{1+cx}}} + \frac{5bc^2 \sqrt{1-cx}}{144x^4 \sqrt{\frac{1}{1+cx}}} - \frac{a + b \operatorname{sech}^{-1}(cx)}{6x^6} - \frac{1}{48} \left(5bc^4 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{1}{x^3 \sqrt{1-cx} \sqrt{1+cx}} dx \\ &= \frac{b \sqrt{1-cx}}{36x^6 \sqrt{\frac{1}{1+cx}}} + \frac{5bc^2 \sqrt{1-cx}}{144x^4 \sqrt{\frac{1}{1+cx}}} + \frac{5bc^4 \sqrt{1-cx}}{96x^2 \sqrt{\frac{1}{1+cx}}} - \frac{a + b \operatorname{sech}^{-1}(cx)}{6x^6} - \frac{1}{96} \left(5bc^4 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{1}{x \sqrt{1-cx} \sqrt{1+cx}} dx \\ &= \frac{b \sqrt{1-cx}}{36x^6 \sqrt{\frac{1}{1+cx}}} + \frac{5bc^2 \sqrt{1-cx}}{144x^4 \sqrt{\frac{1}{1+cx}}} + \frac{5bc^4 \sqrt{1-cx}}{96x^2 \sqrt{\frac{1}{1+cx}}} - \frac{a + b \operatorname{sech}^{-1}(cx)}{6x^6} - \frac{1}{96} \left(5bc^6 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{1}{x \sqrt{1-cx} \sqrt{1+cx}} dx \\ &= \frac{b \sqrt{1-cx}}{36x^6 \sqrt{\frac{1}{1+cx}}} + \frac{5bc^2 \sqrt{1-cx}}{144x^4 \sqrt{\frac{1}{1+cx}}} + \frac{5bc^4 \sqrt{1-cx}}{96x^2 \sqrt{\frac{1}{1+cx}}} - \frac{a + b \operatorname{sech}^{-1}(cx)}{6x^6} + \frac{1}{96} \left(5bc^7 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{1}{x \sqrt{1-cx} \sqrt{1+cx}} dx \\ &= \frac{b \sqrt{1-cx}}{36x^6 \sqrt{\frac{1}{1+cx}}} + \frac{5bc^2 \sqrt{1-cx}}{144x^4 \sqrt{\frac{1}{1+cx}}} + \frac{5bc^4 \sqrt{1-cx}}{96x^2 \sqrt{\frac{1}{1+cx}}} - \frac{a + b \operatorname{sech}^{-1}(cx)}{6x^6} + \frac{5}{96} bc^6 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \operatorname{arctanh}\left(\frac{x \sqrt{1-cx}}{\sqrt{1+cx}}\right) \end{aligned}$$

Mathematica [A] time = 0.142301, size = 157, normalized size = 0.99

$$-\frac{a}{6x^6} + b \left(\frac{5c^4}{96x^2} + \frac{5c^3}{144x^3} + \frac{5c^2}{144x^4} + \frac{5c^5}{96x} + \frac{c}{36x^5} + \frac{1}{36x^6} \right) \sqrt{\frac{1-cx}{cx+1}} - \frac{5}{96} bc^6 \log(x) + \frac{5}{96} bc^6 \log \left(cx \sqrt{\frac{1-cx}{cx+1}} + \sqrt{1-cx} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcSech[c*x])/x^7, x]

[Out] $-\frac{a}{(6*x^6)} + \frac{b*(1/(36*x^6) + c/(36*x^5) + (5*c^2)/(144*x^4) + (5*c^3)/(144*x^3) + (5*c^4)/(96*x^2) + (5*c^5)/(96*x))*\text{Sqrt}[(1 - c*x)/(1 + c*x)] - \frac{b*\text{ArcSech}[c*x]}{(6*x^6)} - \frac{(5*b*c^6*\text{Log}[x])}{96} + \frac{(5*b*c^6*\text{Log}[1 + \text{Sqrt}[(1 - c*x)/(1 + c*x)]) + c*x*\text{Sqrt}[(1 - c*x)/(1 + c*x)])}{96}$

Maple [A] time = 0.189, size = 155, normalized size = 1.

$$c^6 \left(-\frac{a}{6c^6x^6} + b \left(-\frac{\operatorname{arcsech}(cx)}{6c^6x^6} + \frac{1}{288c^5x^5} \sqrt{\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} \left(15 \operatorname{Artanh} \left(\frac{1}{\sqrt{-c^2x^2+1}} \right) c^6x^6 + 15c^4x^4\sqrt{-c^2x^2+1} + 10c^2x^2\sqrt{-c^2x^2+1} + 1 \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))/x^7,x)

[Out] $c^6 * (-1/6 * a / c^6 / x^6 + b * (-1/6 / c^6 / x^6 * \operatorname{arcsech}(c * x) + 1/288 * (-c * x - 1) / c / x)^{(1/2)} / c^5 / x^5 * ((c * x + 1) / c / x)^{(1/2)} * (15 * \operatorname{arctanh}(1 / (-c^2 * x^2 + 1)^{(1/2)}) * c^6 * x^6 + 15 * c^4 * x^4 * (-c^2 * x^2 + 1)^{(1/2)} + 10 * (-c^2 * x^2 + 1)^{(1/2)} * c^2 * x^2 + 8 * (-c^2 * x^2 + 1)^{(1/2)}) / (-c^2 * x^2 + 1)^{(1/2)})$

Maxima [A] time = 1.01455, size = 250, normalized size = 1.58

$$\frac{1}{576} b \left(\frac{15c^7 \log\left(cx\sqrt{\frac{1}{c^2x^2}-1}+1\right) - 15c^7 \log\left(cx\sqrt{\frac{1}{c^2x^2}-1}-1\right) - \frac{2\left(15c^{12}x^5\left(\frac{1}{c^2x^2}-1\right)^2 - 40c^{10}x^3\left(\frac{1}{c^2x^2}-1\right)^3 + 33c^8x\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^6x^6\left(\frac{1}{c^2x^2}-1\right)^3 - 3c^4x^4\left(\frac{1}{c^2x^2}-1\right)^2 + 3c^2x^2\left(\frac{1}{c^2x^2}-1\right) - 1}}{c} - 96 \operatorname{arcsech}(cx)/x^6 - 1/6 * a / x^6 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/x^7,x, algorithm="maxima")

[Out] $1/576 * b * ((15 * c^7 * \log(cx * \sqrt{1/(c^2 * x^2) - 1}) + 1) - 15 * c^7 * \log(cx * \sqrt{1/(c^2 * x^2) - 1}) - 1) - 2 * (15 * c^{12} * x^5 * (1/(c^2 * x^2) - 1)^{(5/2)} - 40 * c^{10} * x^3 * (1/(c^2 * x^2) - 1)^{(3/2)} + 33 * c^8 * x * \sqrt{1/(c^2 * x^2) - 1}) / (c^6 * x^6 * (1/(c^2 * x^2) - 1)^3 - 3 * c^4 * x^4 * (1/(c^2 * x^2) - 1)^2 + 3 * c^2 * x^2 * (1/(c^2 * x^2) - 1) - 1) / c - 96 * \operatorname{arcsech}(c * x) / x^6 - 1/6 * a / x^6$

Fricas [A] time = 1.83268, size = 227, normalized size = 1.44

$$\frac{3(5bc^6x^6 - 16b) \log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}+1}}{cx}\right) + (15bc^5x^5 + 10bc^3x^3 + 8bcx)\sqrt{-\frac{c^2x^2-1}{c^2x^2}} - 48a}{288x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/x^7,x, algorithm="fricas")

[Out] $1/288 * (3 * (5 * b * c^6 * x^6 - 16 * b) * \log((c * x * \sqrt{-(c^2 * x^2 - 1)/(c^2 * x^2)}) + 1) / (c * x)) + (15 * b * c^5 * x^5 + 10 * b * c^3 * x^3 + 8 * b * c * x) * \sqrt{-(c^2 * x^2 - 1)/(c^2 * x^2)} - 48 * a) / x^6$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{asech}(cx)}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asech(c*x))/x**7,x)

[Out] Integral((a + b*asech(c*x))/x**7, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsech}(cx) + a}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/x^7,x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)/x^7, x)

3.33 $\int x^3 \left(a + b \operatorname{sech}^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=124

$$-\frac{bx^2\sqrt{\frac{1-cx}{cx+1}}(cx+1)(a+b\operatorname{sech}^{-1}(cx))}{6c^2} - \frac{b\sqrt{\frac{1-cx}{cx+1}}(cx+1)(a+b\operatorname{sech}^{-1}(cx))}{3c^4} + \frac{1}{4}x^4(a+b\operatorname{sech}^{-1}(cx))^2 - \frac{b^2x^2}{12c^2} - \frac{b^2\log(x)}{3c^4}$$

[Out] $-(b^2*x^2)/(12*c^2) - (b*\sqrt{(1-c*x)/(1+c*x)}*(1+c*x)*(a+b*\operatorname{ArcSech}[c*x]))/(3*c^4) - (b*x^2*\sqrt{(1-c*x)/(1+c*x)}*(1+c*x)*(a+b*\operatorname{ArcSech}[c*x]))/(6*c^2) + (x^4*(a+b*\operatorname{ArcSech}[c*x])^2)/4 - (b^2*\log[x])/(3*c^4)$

Rubi [A] time = 0.119395, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {6285, 5451, 4185, 4184, 3475}

$$-\frac{bx^2\sqrt{\frac{1-cx}{cx+1}}(cx+1)(a+b\operatorname{sech}^{-1}(cx))}{6c^2} - \frac{b\sqrt{\frac{1-cx}{cx+1}}(cx+1)(a+b\operatorname{sech}^{-1}(cx))}{3c^4} + \frac{1}{4}x^4(a+b\operatorname{sech}^{-1}(cx))^2 - \frac{b^2x^2}{12c^2} - \frac{b^2\log(x)}{3c^4}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^3*(a + b*\operatorname{ArcSech}[c*x])^2, x]$

[Out] $-(b^2*x^2)/(12*c^2) - (b*\sqrt{(1-c*x)/(1+c*x)}*(1+c*x)*(a+b*\operatorname{ArcSech}[c*x]))/(3*c^4) - (b*x^2*\sqrt{(1-c*x)/(1+c*x)}*(1+c*x)*(a+b*\operatorname{ArcSech}[c*x]))/(6*c^2) + (x^4*(a+b*\operatorname{ArcSech}[c*x])^2)/4 - (b^2*\log[x])/(3*c^4)$

Rule 6285

$\operatorname{Int}[(a_.) + \operatorname{ArcSech}(c_.*(x_.))*(b_.)]^{(n_.)}*(x_.)^{(m_.)}, x_Symbol] :> -\operatorname{Dist}[(c^{(m+1)})^{(-1)}, \operatorname{Subst}[\operatorname{Int}[(a + b*x)^n*\operatorname{Sech}[x]^{(m+1)}*\operatorname{Tanh}[x], x], x, \operatorname{ArcSech}[c*x]], x] /; \operatorname{FreeQ}\{a, b, c\}, x \ \&\& \operatorname{IntegerQ}[n] \ \&\& \operatorname{IntegerQ}[m] \ \&\& (\operatorname{GtQ}[n, 0] \ || \operatorname{LtQ}[m, -1])$

Rule 5451

$\operatorname{Int}[(c_.) + (d_.)*(x_.)]^{(m_.)}*\operatorname{Sech}[(a_.) + (b_.)*(x_.)]^{(n_.)}*\operatorname{Tanh}[(a_.) + (b_.)*(x_.)]^{(p_.)}, x_Symbol] :> -\operatorname{Simp}[(c + d*x)^m*\operatorname{Sech}[a + b*x]^n/(b*n), x] + \operatorname{Dist}[(d*m)/(b*n), \operatorname{Int}[(c + d*x)^{m-1}*\operatorname{Sech}[a + b*x]^n, x], x] /; \operatorname{FreeQ}\{a, b, c, d, n\}, x \ \&\& \operatorname{EqQ}[p, 1] \ \&\& \operatorname{GtQ}[m, 0]$

Rule 4185

$\operatorname{Int}[(\operatorname{csc}(e_.) + (f_.)*(x_.))*(b_.)]^{(n_.)}*((c_.) + (d_.)*(x_.)), x_Symbol] :> -\operatorname{Simp}[(b^2*(c + d*x)*\operatorname{Cot}[e + f*x]*(b*\operatorname{Csc}[e + f*x])^{(n-2)})/(f*(n-1)), x] + (\operatorname{Dist}[(b^2*(n-2))/(n-1), \operatorname{Int}[(c + d*x)*(b*\operatorname{Csc}[e + f*x])^{(n-2)}, x], x] - \operatorname{Simp}[(b^2*d*(b*\operatorname{Csc}[e + f*x])^{(n-2)})/(f^2*(n-1)*(n-2)), x]) /; \operatorname{FreeQ}\{b, c, d, e, f\}, x \ \&\& \operatorname{GtQ}[n, 1] \ \&\& \operatorname{NeQ}[n, 2]$

Rule 4184

$\operatorname{Int}[\operatorname{csc}(e_.) + (f_.)*(x_.)]^2*((c_.) + (d_.)*(x_.)]^{(m_.)}, x_Symbol] :> -\operatorname{Simp}[(c + d*x)^m*\operatorname{Cot}[e + f*x]/f, x] + \operatorname{Dist}[(d*m)/f, \operatorname{Int}[(c + d*x)^{m-1}*\operatorname{Cot}[e + f*x], x], x] /; \operatorname{FreeQ}\{c, d, e, f\}, x \ \&\& \operatorname{GtQ}[m, 0]$

Rule 3475

`Int[tan[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\begin{aligned} \int x^3 (a + b \operatorname{sech}^{-1}(cx))^2 dx &= -\frac{\operatorname{Subst}\left(\int (a + bx)^2 \operatorname{sech}^4(x) \tanh(x) dx, x, \operatorname{sech}^{-1}(cx)\right)}{c^4} \\ &= \frac{1}{4} x^4 (a + b \operatorname{sech}^{-1}(cx))^2 - \frac{b \operatorname{Subst}\left(\int (a + bx) \operatorname{sech}^4(x) dx, x, \operatorname{sech}^{-1}(cx)\right)}{2c^4} \\ &= -\frac{b^2 x^2}{12c^2} - \frac{bx^2 \sqrt{\frac{1-cx}{1+cx}} (1+cx) (a + b \operatorname{sech}^{-1}(cx))}{6c^2} + \frac{1}{4} x^4 (a + b \operatorname{sech}^{-1}(cx))^2 - \frac{b \operatorname{Subst}\left(\int (a + bx) \operatorname{sech}^4(x) dx, x, \operatorname{sech}^{-1}(cx)\right)}{2c^4} \\ &= -\frac{b^2 x^2}{12c^2} - \frac{b \sqrt{\frac{1-cx}{1+cx}} (1+cx) (a + b \operatorname{sech}^{-1}(cx))}{3c^4} - \frac{bx^2 \sqrt{\frac{1-cx}{1+cx}} (1+cx) (a + b \operatorname{sech}^{-1}(cx))}{6c^2} + \frac{1}{4} x^4 (a + b \operatorname{sech}^{-1}(cx))^2 - \frac{b \operatorname{Subst}\left(\int (a + bx) \operatorname{sech}^4(x) dx, x, \operatorname{sech}^{-1}(cx)\right)}{2c^4} \\ &= -\frac{b^2 x^2}{12c^2} - \frac{b \sqrt{\frac{1-cx}{1+cx}} (1+cx) (a + b \operatorname{sech}^{-1}(cx))}{3c^4} - \frac{bx^2 \sqrt{\frac{1-cx}{1+cx}} (1+cx) (a + b \operatorname{sech}^{-1}(cx))}{6c^2} + \frac{1}{4} x^4 (a + b \operatorname{sech}^{-1}(cx))^2 - \frac{b \operatorname{Subst}\left(\int (a + bx) \operatorname{sech}^4(x) dx, x, \operatorname{sech}^{-1}(cx)\right)}{2c^4} \end{aligned}$$

Mathematica [A] time = 0.330303, size = 212, normalized size = 1.71

$$\frac{-3a^2c^4x^4 + 2abc^3x^3\sqrt{\frac{1-cx}{cx+1}} + 2abc^2x^2\sqrt{\frac{1-cx}{cx+1}} + 2b\operatorname{sech}^{-1}(cx)\left(b\sqrt{\frac{1-cx}{cx+1}}(c^3x^3 + c^2x^2 + 2cx + 2) - 3ac^4x^4\right) + 4abcx\sqrt{\frac{1-cx}{cx+1}}}{12c^4}$$

Antiderivative was successfully verified.

`[In] Integrate[x^3*(a + b*ArcSech[c*x])^2,x]`

`[Out] -(b^2*c^2*x^2 - 3*a^2*c^4*x^4 + 4*a*b*Sqrt[(1 - c*x)/(1 + c*x)] + 4*a*b*c*x*Sqrt[(1 - c*x)/(1 + c*x)] + 2*a*b*c^2*x^2*Sqrt[(1 - c*x)/(1 + c*x)] + 2*a*b*c^3*x^3*Sqrt[(1 - c*x)/(1 + c*x)] + 2*b*(-3*a*c^4*x^4 + b*Sqrt[(1 - c*x)/(1 + c*x)])*(2 + 2*c*x + c^2*x^2 + c^3*x^3))*ArcSech[c*x] - 3*b^2*c^4*x^4*ArcSech[c*x]^2 + 4*b^2*Log[x])/(12*c^4)`

Maple [B] time = 0.29, size = 264, normalized size = 2.1

$$\frac{a^2x^4}{4} - \frac{b^2\operatorname{arcsech}(cx)}{3c^4} + \frac{b^2(\operatorname{arcsech}(cx))^2x^4}{4} - \frac{b^2\operatorname{arcsech}(cx)x^3}{6c} \sqrt{\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} - \frac{b^2\operatorname{arcsech}(cx)x}{3c^3} \sqrt{\frac{cx-1}{cx}}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(x^3*(a+b*arcsech(c*x))^2,x)`

`[Out] 1/4*a^2*x^4-1/3/c^4*b^2*arcsech(c*x)+1/4*b^2*arcsech(c*x)^2*x^4-1/6/c*b^2*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*arcsech(c*x)*x^3-1/3/c^3*b^2*arcsech(c*x)*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*x-1/12*b^2*x^2/c^2+1/3/c^4*b^2*ln(1+(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)+1/2*a*b*x^4*arcsech(c*x)-1/6/c*a*b*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*x^3-1/3/c^3*a*b*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4} a^2 x^4 + \frac{1}{6} \left(3 x^4 \operatorname{arosech}(cx) + \frac{c^2 x^3 \left(\frac{1}{c^2 x^2} - 1 \right)^{\frac{3}{2}} - 3 x \sqrt{\frac{1}{c^2 x^2} - 1}}{c^3} \right) ab + b^2 \int x^3 \log \left(\sqrt{\frac{1}{cx} + 1} \sqrt{\frac{1}{cx} - 1} + \frac{1}{cx} \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*arcsech(c*x))^2,x, algorithm="maxima")

[Out] 1/4*a^2*x^4 + 1/6*(3*x^4*arcsech(c*x) + (c^2*x^3*(1/(c^2*x^2) - 1)^(3/2) - 3*x*sqrt(1/(c^2*x^2) - 1))/c^3)*a*b + b^2*integrate(x^3*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))^2, x)

Fricas [B] time = 2.16148, size = 525, normalized size = 4.23

$$3 b^2 c^4 x^4 \log \left(\frac{cx \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2} + 1}}{cx} \right)^2 + 3 a^2 c^4 x^4 - 6 abc^4 \log \left(\frac{cx \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2} - 1}}{x} \right) - b^2 c^2 x^2 - 4 b^2 \log(x) + 2 \left(3 abc^4 x^4 - 3 abc^4 - (b^2 c^3 x^3) \right)$$

12 c^4

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*arcsech(c*x))^2,x, algorithm="fricas")

[Out] 1/12*(3*b^2*c^4*x^4*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x))^2 + 3*a^2*c^4*x^4 - 6*a*b*c^4*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) - b^2*c^2*x^2 - 4*b^2*log(x) + 2*(3*a*b*c^4*x^4 - 3*a*b*c^4 - (b^2*c^3*x^3 + 2*b^2*c*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - 2*(a*b*c^3*x^3 + 2*a*b*c*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/c^4

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 (a + b \operatorname{asech}(cx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(a+b*asech(c*x))**2,x)

[Out] Integral(x**3*(a + b*asech(c*x))**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arsech}(cx) + a)^2 x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*arcsech(c*x))^2,x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)^2*x^3, x)

3.34 $\int x^2 (a + b \operatorname{sech}^{-1}(cx))^2 dx$

Optimal. Leaf size=140

$$\frac{ib^2 \operatorname{PolyLog}\left(2, -ie^{\operatorname{sech}^{-1}(cx)}\right)}{3c^3} - \frac{ib^2 \operatorname{PolyLog}\left(2, ie^{\operatorname{sech}^{-1}(cx)}\right)}{3c^3} - \frac{bx\sqrt{\frac{1-cx}{cx+1}}(cx+1)(a + b \operatorname{sech}^{-1}(cx))}{3c^2} - \frac{2b \tan^{-1}\left(e^{\operatorname{sech}^{-1}(cx)}\right)}{3c^2}$$

[Out] $-(b^2 x)/(3c^2) - (b x \sqrt{(1 - cx)/(1 + cx)} (1 + cx) (a + b \operatorname{ArcSech}[cx]))/(3c^2) + (x^3 (a + b \operatorname{ArcSech}[cx])^2)/3 - (2 b (a + b \operatorname{ArcSech}[cx]) \operatorname{ArcTan}[E^{\operatorname{ArcSech}[cx]}])/(3c^3) + ((I/3) b^2 \operatorname{PolyLog}[2, (-I) E^{\operatorname{ArcSech}[cx]}])/c^3 - ((I/3) b^2 \operatorname{PolyLog}[2, I E^{\operatorname{ArcSech}[cx]}])/c^3$

Rubi [A] time = 0.12288, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6285, 5451, 4185, 4180, 2279, 2391}

$$\frac{ib^2 \operatorname{PolyLog}\left(2, -ie^{\operatorname{sech}^{-1}(cx)}\right)}{3c^3} - \frac{ib^2 \operatorname{PolyLog}\left(2, ie^{\operatorname{sech}^{-1}(cx)}\right)}{3c^3} - \frac{bx\sqrt{\frac{1-cx}{cx+1}}(cx+1)(a + b \operatorname{sech}^{-1}(cx))}{3c^2} - \frac{2b \tan^{-1}\left(e^{\operatorname{sech}^{-1}(cx)}\right)}{3c^2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^2 (a + b \operatorname{ArcSech}[cx])^2, x]$

[Out] $-(b^2 x)/(3c^2) - (b x \sqrt{(1 - cx)/(1 + cx)} (1 + cx) (a + b \operatorname{ArcSech}[cx]))/(3c^2) + (x^3 (a + b \operatorname{ArcSech}[cx])^2)/3 - (2 b (a + b \operatorname{ArcSech}[cx]) \operatorname{ArcTan}[E^{\operatorname{ArcSech}[cx]}])/(3c^3) + ((I/3) b^2 \operatorname{PolyLog}[2, (-I) E^{\operatorname{ArcSech}[cx]}])/c^3 - ((I/3) b^2 \operatorname{PolyLog}[2, I E^{\operatorname{ArcSech}[cx]}])/c^3$

Rule 6285

$\operatorname{Int}[(a + \operatorname{ArcSech}[c x])^n (x)^m, x_Symbol] \rightarrow -\operatorname{Dist}[(c^{m+1})^{-1}, \operatorname{Subst}[\operatorname{Int}[(a + b x)^n \operatorname{Sech}[x]^{m+1} \operatorname{Tanh}[x], x], x, \operatorname{ArcSech}[c x]], x] /; \operatorname{FreeQ}\{a, b, c, x\} \ \&\& \operatorname{IntegerQ}[n] \ \&\& \operatorname{IntegerQ}[m] \ \&\& (\operatorname{GtQ}[n, 0] \ \|\ \operatorname{LtQ}[m, -1])$

Rule 5451

$\operatorname{Int}[(c + d x)^m \operatorname{Sech}[a + b x]^n \operatorname{Tanh}[a + b x]^p, x_Symbol] \rightarrow -\operatorname{Simp}[(c + d x)^m \operatorname{Sech}[a + b x]^n / (b n), x] + \operatorname{Dist}[(d m) / (b n), \operatorname{Int}[(c + d x)^{m-1} \operatorname{Sech}[a + b x]^n, x], x] /; \operatorname{FreeQ}\{a, b, c, d, n\}, x \ \&\& \operatorname{EqQ}[p, 1] \ \&\& \operatorname{GtQ}[m, 0]$

Rule 4185

$\operatorname{Int}[(c \operatorname{csc}[e + f x] + (d x)) (b x)^n, x_Symbol] \rightarrow -\operatorname{Simp}[(b^2 (c + d x) \operatorname{Cot}[e + f x] (b \operatorname{Csc}[e + f x])^{n-2}) / (f (n-1)), x] + (\operatorname{Dist}[(b^2 (n-2)) / (n-1), \operatorname{Int}[(c + d x) (b \operatorname{Csc}[e + f x])^{n-2}, x], x] - \operatorname{Simp}[(b^2 d (b \operatorname{Csc}[e + f x])^{n-2}) / (f^2 (n-1) (n-2)), x]) /; \operatorname{FreeQ}\{b, c, d, e, f\}, x \ \&\& \operatorname{GtQ}[n, 1] \ \&\& \operatorname{NeQ}[n, 2]$

Rule 4180

$\operatorname{Int}[\operatorname{csc}[e + \operatorname{Pi} k + (d x)] (b x)^n, x_Symbol] \rightarrow \operatorname{Simp}[(-2 (c + d x)^m \operatorname{ArcTanh}[E^{-(I e) + f f z x}] / E^{(I k \operatorname{Pi})}) / (f f z I), x] + (-\operatorname{Dist}[(d m) / (f f z I), \operatorname{Int}[(c + d x)^{m-1} \operatorname{Log}[1$

- E^{-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^{-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x)] /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]}}

Rule 2279

Int[Log[(a_) + (b_)*((F_)^(e_)*((c_) + (d_)*(x_)))]^(n_)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rubi steps

$$\int x^2 (a + b \operatorname{sech}^{-1}(cx))^2 dx = -\frac{\operatorname{Subst}\left(\int (a + bx)^2 \operatorname{sech}^3(x) \tanh(x) dx, x, \operatorname{sech}^{-1}(cx)\right)}{c^3}$$

$$= \frac{1}{3}x^3 (a + b \operatorname{sech}^{-1}(cx))^2 - \frac{(2b) \operatorname{Subst}\left(\int (a + bx) \operatorname{sech}^3(x) dx, x, \operatorname{sech}^{-1}(cx)\right)}{3c^3}$$

$$= -\frac{b^2x}{3c^2} - \frac{bx\sqrt{\frac{1-cx}{1+cx}}(1+cx)(a + b \operatorname{sech}^{-1}(cx))}{3c^2} + \frac{1}{3}x^3 (a + b \operatorname{sech}^{-1}(cx))^2 - \frac{b \operatorname{Subst}\left(\int (a + bx) \operatorname{sech}^3(x) dx, x, \operatorname{sech}^{-1}(cx)\right)}{3c^3}$$

$$= -\frac{b^2x}{3c^2} - \frac{bx\sqrt{\frac{1-cx}{1+cx}}(1+cx)(a + b \operatorname{sech}^{-1}(cx))}{3c^2} + \frac{1}{3}x^3 (a + b \operatorname{sech}^{-1}(cx))^2 - \frac{2b(a + b \operatorname{sech}^{-1}(cx))}{3c^2}$$

$$= -\frac{b^2x}{3c^2} - \frac{bx\sqrt{\frac{1-cx}{1+cx}}(1+cx)(a + b \operatorname{sech}^{-1}(cx))}{3c^2} + \frac{1}{3}x^3 (a + b \operatorname{sech}^{-1}(cx))^2 - \frac{2b(a + b \operatorname{sech}^{-1}(cx))}{3c^2}$$

$$= -\frac{b^2x}{3c^2} - \frac{bx\sqrt{\frac{1-cx}{1+cx}}(1+cx)(a + b \operatorname{sech}^{-1}(cx))}{3c^2} + \frac{1}{3}x^3 (a + b \operatorname{sech}^{-1}(cx))^2 - \frac{2b(a + b \operatorname{sech}^{-1}(cx))}{3c^2}$$

Mathematica [A] time = 1.15445, size = 224, normalized size = 1.6

$$\frac{1}{3} \left(\frac{b^2 \left(i \operatorname{PolyLog}\left(2, -ie^{-\operatorname{sech}^{-1}(cx)}\right) - i \operatorname{PolyLog}\left(2, ie^{-\operatorname{sech}^{-1}(cx)}\right) + c^3 x^3 \operatorname{sech}^{-1}(cx)^2 - cx - cx \sqrt{\frac{1-cx}{cx+1}}(cx+1) \operatorname{sech}^{-1}(cx) + i \right)}{c^3} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2*(a + b*ArcSech[c*x])^2,x]

[Out] (a^2*x^3 + a*b*(2*x^3*ArcSech[c*x] - (Sqrt[(1 - c*x)/(1 + c*x)]*(-(c*x) + c^3*x^3 + Sqrt[1 - c^2*x^2]*ArcSin[c*x]))/(c^3*(-1 + c*x))) + (b^2*(-(c*x) - c*x*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*ArcSech[c*x] + c^3*x^3*ArcSech[c*x]^2 + I*ArcSech[c*x]*Log[1 - I/E^ArcSech[c*x]] - I*ArcSech[c*x]*Log[1 + I/E^ArcSech[c*x]] + I*PolyLog[2, (-I)/E^ArcSech[c*x]] - I*PolyLog[2, I/E^ArcSech[c*x]]))/c^3)/3

Maple [A] time = 0.278, size = 372, normalized size = 2.7

$$\frac{x^3 a^2}{3} - \frac{b^2 \operatorname{arcsech}(cx) x^2}{3c} \sqrt{\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} + \frac{x^3 b^2 (\operatorname{arcsech}(cx))^2}{3} - \frac{b^2 x}{3c^2} + \frac{i}{3} \frac{b^2 \operatorname{arcsech}(cx)}{c^3} \ln \left(1 + i \left(\frac{1}{cx} + \sqrt{-1 + \frac{1}{cx}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a+b*arcsech(c*x))^2,x)`

[Out] $\frac{1}{3}x^3a^2 - \frac{1}{3}cb^2 \left(-\frac{c*x-1}{c/x} \right)^{1/2} \left(\frac{c*x+1}{c/x} \right)^{1/2} \operatorname{arcsech}(c*x) * x^2 + \frac{1}{3}x^3b^2 \operatorname{arcsech}(c*x)^2 - \frac{1}{3}b^2x/c^2 + \frac{1}{3}I/c^3b^2 \operatorname{arcsech}(c*x) * \ln \left(1 + I \left(\frac{1}{c/x} + (-1 + 1/c/x)^{1/2} \right) \left(1 + 1/c/x \right)^{1/2} \right) - \frac{1}{3}I/c^3b^2 \operatorname{arcsech}(c*x) * \ln \left(1 - I \left(\frac{1}{c/x} + (-1 + 1/c/x)^{1/2} \right) \left(1 + 1/c/x \right)^{1/2} \right) + \frac{1}{3}I/c^3b^2 \operatorname{dilog} \left(1 + I \left(\frac{1}{c/x} + (-1 + 1/c/x)^{1/2} \right) \left(1 + 1/c/x \right)^{1/2} \right) - \frac{1}{3}I/c^3b^2 \operatorname{dilog} \left(1 - I \left(\frac{1}{c/x} + (-1 + 1/c/x)^{1/2} \right) \left(1 + 1/c/x \right)^{1/2} \right) + \frac{2}{3}a*b*x^3 \operatorname{arcsech}(c*x) - \frac{1}{3}c*a*b \left(-\frac{c*x-1}{c/x} \right)^{1/2} * x^2 \left(\frac{c*x+1}{c/x} \right)^{1/2} + \frac{1}{3}c^2*a*b \left(-\frac{c*x-1}{c/x} \right)^{1/2} * x \left(\frac{c*x+1}{c/x} \right)^{1/2} / \left(-c^2*x^2+1 \right)^{1/2} * \arcsin(c*x)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}a^2x^3 + \frac{1}{3} \left(2x^3 \operatorname{arsech}(cx) - \frac{\frac{\sqrt{\frac{1}{c^2x^2}-1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^2}}{c} \right) ab + b^2 \int x^2 \log \left(\sqrt{\frac{1}{cx} + 1} \sqrt{\frac{1}{cx} - 1} + \frac{1}{cx} \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arcsech(c*x))^2,x, algorithm="maxima")`

[Out] $\frac{1}{3}a^2x^3 + \frac{1}{3}(2x^3 \operatorname{arcsech}(c*x) - (\sqrt{1/(c^2*x^2) - 1}/(c^2*(1/(c^2*x^2) - 1) + c^2) + \arctan(\sqrt{1/(c^2*x^2) - 1})/c^2)/c)*a*b + b^2 \operatorname{integrate}(x^2 * \log(\sqrt{1/(c*x) + 1} * \sqrt{1/(c*x) - 1} + 1/(c*x))^2, x)$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(b^2x^2 \operatorname{arsech}(cx)^2 + 2abx^2 \operatorname{arsech}(cx) + a^2x^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arcsech(c*x))^2,x, algorithm="fricas")`

[Out] `integral(b^2*x^2*arcsech(c*x)^2 + 2*a*b*x^2*arcsech(c*x) + a^2*x^2, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 (a + b \operatorname{asech}(cx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(a+b*asech(c*x))**2,x)`

[Out] `Integral(x**2*(a + b*asech(c*x))**2, x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{ar} \operatorname{sech}(cx) + a)^2 x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsech(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)^2*x^2, x)
```

3.35 $\int x \left(a + b \operatorname{sech}^{-1}(cx) \right)^2 dx$

Optimal. Leaf size=65

$$-\frac{b\sqrt{\frac{1-cx}{cx+1}}(cx+1)(a+b\operatorname{sech}^{-1}(cx))}{c^2} + \frac{1}{2}x^2(a+b\operatorname{sech}^{-1}(cx))^2 - \frac{b^2 \log(x)}{c^2}$$

[Out] $-\left(\frac{b\sqrt{(1-cx)/(1+cx)}(1+cx)(a+b\operatorname{ArcSech}[cx])}{c^2}\right) + \frac{x^2(a+b\operatorname{ArcSech}[cx])^2}{2} - \frac{b^2\log[x]}{c^2}$

Rubi [A] time = 0.0752577, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6285, 5451, 4184, 3475}

$$-\frac{b\sqrt{\frac{1-cx}{cx+1}}(cx+1)(a+b\operatorname{sech}^{-1}(cx))}{c^2} + \frac{1}{2}x^2(a+b\operatorname{sech}^{-1}(cx))^2 - \frac{b^2 \log(x)}{c^2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x(a+b\operatorname{ArcSech}[cx])^2, x]$

[Out] $-\left(\frac{b\sqrt{(1-cx)/(1+cx)}(1+cx)(a+b\operatorname{ArcSech}[cx])}{c^2}\right) + \frac{x^2(a+b\operatorname{ArcSech}[cx])^2}{2} - \frac{b^2\log[x]}{c^2}$

Rule 6285

$\operatorname{Int}[(a + \operatorname{ArcSech}[c \cdot x])^n (x)^m, x_Symbol] \rightarrow -\operatorname{Dist}[(c^{m+1})^{-1}, \operatorname{Subst}[\operatorname{Int}[(a + b \cdot x)^n \operatorname{Sech}[x]^{m+1} \operatorname{Tanh}[x], x], x, \operatorname{ArcSech}[c \cdot x]], x] /; \operatorname{FreeQ}\{a, b, c, x\} \ \&\& \ \operatorname{IntegerQ}[n] \ \&\& \ \operatorname{IntegerQ}[m] \ \&\& \ (\operatorname{GtQ}[n, 0] \ || \ \operatorname{LtQ}[m, -1])$

Rule 5451

$\operatorname{Int}[(c + d \cdot x)^m \operatorname{Sech}[a + b \cdot x]^n \operatorname{Tanh}[a + b \cdot x]^p, x_Symbol] \rightarrow -\operatorname{Simp}[(c + d \cdot x)^m \operatorname{Sech}[a + b \cdot x]^n / (b \cdot n), x] + \operatorname{Dist}[(d \cdot m) / (b \cdot n), \operatorname{Int}[(c + d \cdot x)^{m-1} \operatorname{Sech}[a + b \cdot x]^n, x], x] /; \operatorname{FreeQ}\{a, b, c, d, n\}, x \ \&\& \ \operatorname{EqQ}[p, 1] \ \&\& \ \operatorname{GtQ}[m, 0]$

Rule 4184

$\operatorname{Int}[\csc[e + f \cdot x]^2 (c + d \cdot x)^m, x_Symbol] \rightarrow -\operatorname{Simp}[(c + d \cdot x)^m \operatorname{Cot}[e + f \cdot x] / f, x] + \operatorname{Dist}[(d \cdot m) / f, \operatorname{Int}[(c + d \cdot x)^{m-1} \operatorname{Cot}[e + f \cdot x], x], x] /; \operatorname{FreeQ}\{c, d, e, f, x\} \ \&\& \ \operatorname{GtQ}[m, 0]$

Rule 3475

$\operatorname{Int}[\tan[c + d \cdot x], x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[\operatorname{Cos}[c + d \cdot x], x]] / d, x] /; \operatorname{FreeQ}\{c, d\}, x]$

Rubi steps

$$\begin{aligned}
\int x(a + b \operatorname{sech}^{-1}(cx))^2 dx &= -\frac{\operatorname{Subst}\left(\int (a + bx)^2 \operatorname{sech}^2(x) \tanh(x) dx, x, \operatorname{sech}^{-1}(cx)\right)}{c^2} \\
&= \frac{1}{2}x^2(a + b \operatorname{sech}^{-1}(cx))^2 - \frac{b \operatorname{Subst}\left(\int (a + bx) \operatorname{sech}^2(x) dx, x, \operatorname{sech}^{-1}(cx)\right)}{c^2} \\
&= -\frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)(a + b \operatorname{sech}^{-1}(cx))}{c^2} + \frac{1}{2}x^2(a + b \operatorname{sech}^{-1}(cx))^2 + \frac{b^2 \operatorname{Subst}\left(\int \tanh(x) dx, x, \operatorname{sech}^{-1}(cx)\right)}{c^2} \\
&= -\frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)(a + b \operatorname{sech}^{-1}(cx))}{c^2} + \frac{1}{2}x^2(a + b \operatorname{sech}^{-1}(cx))^2 - \frac{b^2 \log(x)}{c^2}
\end{aligned}$$

Mathematica [A] time = 0.208577, size = 112, normalized size = 1.72

$$\frac{a\left(ac^2x^2 - 2b\sqrt{\frac{1-cx}{cx+1}}(cx+1)\right) - 2b \operatorname{sech}^{-1}(cx)\left(b\sqrt{\frac{1-cx}{cx+1}}(cx+1) - ac^2x^2\right) + b^2c^2x^2 \operatorname{sech}^{-1}(cx)^2 - 2b^2 \log(cx)}{2c^2}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + b*ArcSech[c*x])^2,x]

[Out] (a*(a*c^2*x^2 - 2*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)) - 2*b*(-(a*c^2*x^2) + b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))*ArcSech[c*x] + b^2*c^2*x^2*ArcSech[c*x]^2 - 2*b^2*Log[c*x])/(2*c^2)

Maple [B] time = 0.253, size = 168, normalized size = 2.6

$$\frac{a^2x^2}{2} - \frac{b^2 \operatorname{arcsech}(cx)}{c^2} - \frac{b^2 \operatorname{arcsech}(cx)x}{c} \sqrt{\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} + \frac{x^2b^2(\operatorname{arcsech}(cx))^2}{2} + \frac{b^2}{c^2} \ln\left(1 + \left(\frac{1}{cx} + \sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a+b*arcsech(c*x))^2,x)

[Out] 1/2*a^2*x^2-1/c^2*b^2*arcsech(c*x)-1/c*b^2*arcsech(c*x)*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*x+1/2*x^2*b^2*arcsech(c*x)^2+1/c^2*b^2*ln(1+(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)+a*b*arcsech(c*x)*x^2-1/c*a*b*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*x

Maxima [A] time = 1.01994, size = 113, normalized size = 1.74

$$\frac{1}{2}b^2x^2 \operatorname{arsh}(cx)^2 + \frac{1}{2}a^2x^2 + \left(x^2 \operatorname{arsh}(cx) - \frac{x\sqrt{\frac{1}{c^2x^2} - 1}}{c}\right)ab - \left(\frac{x\sqrt{\frac{1}{c^2x^2} - 1} \operatorname{arsh}(cx)}{c} + \frac{\log(x)}{c^2}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arcsech(c*x))^2,x, algorithm="maxima")

[Out] 1/2*b^2*x^2*arcsech(c*x)^2 + 1/2*a^2*x^2 + (x^2*arcsech(c*x) - x*sqrt(1/(c^2*x^2) - 1)/c)*a*b - (x*sqrt(1/(c^2*x^2) - 1)*arcsech(c*x)/c + log(x)/c^2)*

b^2

Fricas [B] time = 2.06572, size = 446, normalized size = 6.86

$$\frac{b^2 c^2 x^2 \log\left(\frac{cx\sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2}} + 1}{cx}\right)^2 + a^2 c^2 x^2 - 2 abc^2 \log\left(\frac{cx\sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2}} - 1}{x}\right) - 2 abcx\sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2}} - 2 b^2 \log(x) + 2\left(abc^2 x^2 - b^2 cx\sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2}}\right)}{2 c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arcsech(c*x))^2,x, algorithm="fricas")

[Out] 1/2*(b^2*c^2*x^2*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x))^2 + a^2*c^2*x^2 - 2*a*b*c^2*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) - 2*a*b*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 2*b^2*log(x) + 2*(a*b*c^2*x^2 - b^2*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - a*b*c^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/c^2

Sympy [A] time = 3.94712, size = 99, normalized size = 1.52

$$\begin{cases} \frac{a^2 x^2}{2} + abx^2 \operatorname{asech}(cx) - \frac{ab\sqrt{-c^2 x^2 + 1}}{c^2} + \frac{b^2 x^2 \operatorname{asech}^2(cx)}{2} - \frac{b^2 \sqrt{-c^2 x^2 + 1} \operatorname{asech}(cx)}{c^2} - \frac{b^2 \log(x)}{c^2} & \text{for } c \neq 0 \\ \frac{x^2(a+ob)^2}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*asech(c*x))**2,x)

[Out] Piecewise((a**2*x**2/2 + a*b*x**2*asech(c*x) - a*b*sqrt(-c**2*x**2 + 1)/c**2 + b**2*x**2*asech(c*x)**2/2 - b**2*sqrt(-c**2*x**2 + 1)*asech(c*x)/c**2 - b**2*log(x)/c**2, Ne(c, 0)), (x**2*(a + oo*b)**2/2, True))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arsech}(cx) + a)^2 x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arcsech(c*x))^2,x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)^2*x, x)

3.36 $\int (a + b \operatorname{sech}^{-1}(cx))^2 dx$

Optimal. Leaf size=78

$$\frac{2ib^2 \operatorname{PolyLog}\left(2, -ie^{\operatorname{sech}^{-1}(cx)}\right)}{c} - \frac{2ib^2 \operatorname{PolyLog}\left(2, ie^{\operatorname{sech}^{-1}(cx)}\right)}{c} + x(a + b \operatorname{sech}^{-1}(cx))^2 - \frac{4b \tan^{-1}\left(e^{\operatorname{sech}^{-1}(cx)}\right)(a + b \operatorname{sech}^{-1}(cx))}{c}$$

```
[Out] x*(a + b*ArcSech[c*x])^2 - (4*b*(a + b*ArcSech[c*x])*ArcTan[E^ArcSech[c*x]]
)/c + ((2*I)*b^2*PolyLog[2, (-I)*E^ArcSech[c*x]])/c - ((2*I)*b^2*PolyLog[2,
I*E^ArcSech[c*x]])/c
```

Rubi [A] time = 0.0702562, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6279, 5451, 4180, 2279, 2391}

$$\frac{2ib^2 \operatorname{PolyLog}\left(2, -ie^{\operatorname{sech}^{-1}(cx)}\right)}{c} - \frac{2ib^2 \operatorname{PolyLog}\left(2, ie^{\operatorname{sech}^{-1}(cx)}\right)}{c} + x(a + b \operatorname{sech}^{-1}(cx))^2 - \frac{4b \tan^{-1}\left(e^{\operatorname{sech}^{-1}(cx)}\right)(a + b \operatorname{sech}^{-1}(cx))}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])^2, x]
```

```
[Out] x*(a + b*ArcSech[c*x])^2 - (4*b*(a + b*ArcSech[c*x])*ArcTan[E^ArcSech[c*x]]
)/c + ((2*I)*b^2*PolyLog[2, (-I)*E^ArcSech[c*x]])/c - ((2*I)*b^2*PolyLog[2,
I*E^ArcSech[c*x]])/c
```

Rule 6279

```
Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))^n_, x_Symbol] :> -Dist[c^(-1), Su
bst[Int[(a + b*x)^n*Sech[x]*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a,
b, c, n}, x] && IGtQ[n, 0]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_.))^m_.*Sech[(a_.) + (b_.)*(x_.)]^n_.*Tanh[(a_.) +
(b_.)*(x_.)]^p_., x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n),
x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_.)]*((c_.) + (d_.)*(x_
))^m_., x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
- E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_.) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_.))))^n_.], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rubi steps

$$\begin{aligned} \int (a + b \operatorname{sech}^{-1}(cx))^2 dx &= -\frac{\operatorname{Subst}\left(\int (a + bx)^2 \operatorname{sech}(x) \tanh(x) dx, x, \operatorname{sech}^{-1}(cx)\right)}{c} \\ &= x(a + b \operatorname{sech}^{-1}(cx))^2 - \frac{(2b) \operatorname{Subst}\left(\int (a + bx) \operatorname{sech}(x) dx, x, \operatorname{sech}^{-1}(cx)\right)}{c} \\ &= x(a + b \operatorname{sech}^{-1}(cx))^2 - \frac{4b(a + b \operatorname{sech}^{-1}(cx)) \tan^{-1}\left(e^{\operatorname{sech}^{-1}(cx)}\right)}{c} + \frac{(2ib^2) \operatorname{Subst}\left(\int \log(1 - \frac{1}{e^{\operatorname{sech}^{-1}(cx)}}) dx, x, \operatorname{sech}^{-1}(cx)\right)}{c} \\ &= x(a + b \operatorname{sech}^{-1}(cx))^2 - \frac{4b(a + b \operatorname{sech}^{-1}(cx)) \tan^{-1}\left(e^{\operatorname{sech}^{-1}(cx)}\right)}{c} + \frac{(2ib^2) \operatorname{Subst}\left(\int \frac{\log(1 - ix)}{x} dx, x, \operatorname{sech}^{-1}(cx)\right)}{c} \\ &= x(a + b \operatorname{sech}^{-1}(cx))^2 - \frac{4b(a + b \operatorname{sech}^{-1}(cx)) \tan^{-1}\left(e^{\operatorname{sech}^{-1}(cx)}\right)}{c} + \frac{2ib^2 \operatorname{Li}_2\left(-ie^{\operatorname{sech}^{-1}(cx)}\right)}{c} \end{aligned}$$

Mathematica [A] time = 0.224079, size = 126, normalized size = 1.62

$$\frac{ib^2 \left(2 \operatorname{PolyLog}\left(2, -ie^{-\operatorname{sech}^{-1}(cx)}\right) - 2 \operatorname{PolyLog}\left(2, ie^{-\operatorname{sech}^{-1}(cx)}\right) + \operatorname{sech}^{-1}(cx) \left(-icx \operatorname{sech}^{-1}(cx) + 2 \log\left(1 - ie^{-\operatorname{sech}^{-1}(cx)}\right) \right) \right)}{c}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*ArcSech[c*x])^2, x]

[Out] a^2*x + (2*a*b*(c*x*ArcSech[c*x] - 2*ArcTan[Tanh[ArcSech[c*x]/2]]))/c + (I*b^2*(ArcSech[c*x]*((-I)*c*x*ArcSech[c*x] + 2*Log[1 - I/E^ArcSech[c*x]] - 2*Log[1 + I/E^ArcSech[c*x]]) + 2*PolyLog[2, (-I)/E^ArcSech[c*x]] - 2*PolyLog[2, I/E^ArcSech[c*x]]))/c

Maple [A] time = 0.221, size = 250, normalized size = 3.2

$$xb^2 (\operatorname{arcsech}(cx))^2 + \frac{2i \operatorname{arcsech}(cx) b^2}{c} \ln\left(1 + i\left(\frac{1}{cx} + \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}\right)\right) - \frac{2i \operatorname{arcsech}(cx) b^2}{c} \ln\left(1 - i\left(\frac{1}{cx} + \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))^2, x)

[Out] x*b^2*arcsech(c*x)^2+2*I/c*ln(1+I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))) *arcsech(c*x)*b^2-2*I/c*ln(1-I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))) *arcsech(c*x)*b^2+2*x*a*b*arcsech(c*x)+2*I/c*dilog(1+I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))) *b^2-2*I/c*dilog(1-I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))) *b^2+a^2*x-2/c*arctan((-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))*a*b

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(x \log\left(\sqrt{cx+1}\sqrt{-cx+1}+1\right)^2 - \int -\frac{c^2x^2 \log(c)^2 + (c^2x^2 - 1) \log(x)^2 + (c^2x^2 \log(c)^2 + (c^2x^2 - 1) \log(x)^2 - \log(c)^2)}{c} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))^2,x, algorithm="maxima")

[Out] $(x \log(\sqrt{c*x + 1}) \sqrt{-c*x + 1} + 1)^2 - \text{integrate}(- (c^2*x^2 \log(c)^2 + (c^2*x^2 - 1) \log(x)^2 + (c^2*x^2 \log(c)^2 + (c^2*x^2 - 1) \log(x)^2 - \log(c)^2 + 2*(c^2*x^2 \log(c) - \log(c)) \log(x)) \sqrt{c*x + 1} \sqrt{-c*x + 1} - 2*(c^2*x^2 \log(c) + (c^2*x^2 (\log(c) + 1) + (c^2*x^2 - 1) \log(x) - \log(c)) \sqrt{c*x + 1} \sqrt{-c*x + 1} + (c^2*x^2 - 1) \log(x) - \log(c)) \log(\sqrt{c*x + 1}) \sqrt{-c*x + 1} + 1) - \log(c)^2 + 2*(c^2*x^2 \log(c) - \log(c)) \log(x)) / (c^2*x^2 + (c^2*x^2 - 1) \sqrt{c*x + 1} \sqrt{-c*x + 1} - 1), x) * b^2 + a^2*x + 2*(c*x*arcsech(c*x) - \arctan(\sqrt{1/(c^2*x^2) - 1})) * a*b/c$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}(b^2 \operatorname{arsech}(cx)^2 + 2ab \operatorname{arsech}(cx) + a^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))^2,x, algorithm="fricas")

[Out] integral(b^2*arcsech(c*x)^2 + 2*a*b*arcsech(c*x) + a^2, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{asech}(cx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asech(c*x))**2,x)

[Out] Integral((a + b*asech(c*x))**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arsech}(cx) + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))^2,x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)^2, x)

$$3.37 \quad \int \frac{(a + b \operatorname{sech}^{-1}(cx))^2}{x} dx$$

Optimal. Leaf size=83

$$-b \operatorname{PolyLog}\left(2, -e^{2 \operatorname{sech}^{-1}(cx)}\right) (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{2} b^2 \operatorname{PolyLog}\left(3, -e^{2 \operatorname{sech}^{-1}(cx)}\right) + \frac{(a + b \operatorname{sech}^{-1}(cx))^3}{3b} - \log\left(e^{2 \operatorname{sech}^{-1}(cx)}\right)$$

[Out] (a + b*ArcSech[c*x])^3/(3*b) - (a + b*ArcSech[c*x])^2*Log[1 + E^(2*ArcSech[c*x])] - b*(a + b*ArcSech[c*x])*PolyLog[2, -E^(2*ArcSech[c*x])] + (b^2*PolyLog[3, -E^(2*ArcSech[c*x])])/2

Rubi [A] time = 0.125281, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6285, 3718, 2190, 2531, 2282, 6589}

$$-b \operatorname{PolyLog}\left(2, -e^{2 \operatorname{sech}^{-1}(cx)}\right) (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{2} b^2 \operatorname{PolyLog}\left(3, -e^{2 \operatorname{sech}^{-1}(cx)}\right) + \frac{(a + b \operatorname{sech}^{-1}(cx))^3}{3b} - \log\left(e^{2 \operatorname{sech}^{-1}(cx)}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcSech[c*x])^2/x, x]

[Out] (a + b*ArcSech[c*x])^3/(3*b) - (a + b*ArcSech[c*x])^2*Log[1 + E^(2*ArcSech[c*x])] - b*(a + b*ArcSech[c*x])*PolyLog[2, -E^(2*ArcSech[c*x])] + (b^2*PolyLog[3, -E^(2*ArcSech[c*x])])/2

Rule 6285

Int[((a_.) + ArcSech[(c_.)*(x_)])*(b_.)]^(n_.)*(x_)^(m_.), x_Symbol] :> -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

Rule 3718

Int[((c_.) + (d_.)*(x_)^(m_.))*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x))]/(1 + E^(2*(-I*e) + f*fz*x)), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

Rule 2190

Int[(((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_)^(m_.)))/((a_.) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)), x_Symbol] :> Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2531

Int[Log[1 + (e_.)*((F_)^(c_.)*((a_.) + (b_.)*(x_)))^(n_.)]*((f_.) + (g_.)*(x_)^(m_.)), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)])/ (b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f

, g, n}, x] && GtQ[m, 0]

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned} \int \frac{(a + b \operatorname{sech}^{-1}(cx))^2}{x} dx &= -\operatorname{Subst} \left(\int (a + bx)^2 \tanh(x) dx, x, \operatorname{sech}^{-1}(cx) \right) \\ &= \frac{(a + b \operatorname{sech}^{-1}(cx))^3}{3b} - 2 \operatorname{Subst} \left(\int \frac{e^{2x}(a + bx)^2}{1 + e^{2x}} dx, x, \operatorname{sech}^{-1}(cx) \right) \\ &= \frac{(a + b \operatorname{sech}^{-1}(cx))^3}{3b} - (a + b \operatorname{sech}^{-1}(cx))^2 \log(1 + e^{2 \operatorname{sech}^{-1}(cx)}) + (2b) \operatorname{Subst} \left(\int (a + bx) \log(1 + e^{2x}) dx, x, \operatorname{sech}^{-1}(cx) \right) \\ &= \frac{(a + b \operatorname{sech}^{-1}(cx))^3}{3b} - (a + b \operatorname{sech}^{-1}(cx))^2 \log(1 + e^{2 \operatorname{sech}^{-1}(cx)}) - b(a + b \operatorname{sech}^{-1}(cx)) \operatorname{Li}_2(-e^{-2 \operatorname{sech}^{-1}(cx)}) \\ &= \frac{(a + b \operatorname{sech}^{-1}(cx))^3}{3b} - (a + b \operatorname{sech}^{-1}(cx))^2 \log(1 + e^{2 \operatorname{sech}^{-1}(cx)}) - b(a + b \operatorname{sech}^{-1}(cx)) \operatorname{Li}_2(-e^{-2 \operatorname{sech}^{-1}(cx)}) \\ &= \frac{(a + b \operatorname{sech}^{-1}(cx))^3}{3b} - (a + b \operatorname{sech}^{-1}(cx))^2 \log(1 + e^{2 \operatorname{sech}^{-1}(cx)}) - b(a + b \operatorname{sech}^{-1}(cx)) \operatorname{Li}_2(-e^{-2 \operatorname{sech}^{-1}(cx)}) \end{aligned}$$

Mathematica [A] time = 0.156747, size = 116, normalized size = 1.4

$$ab \left(\operatorname{PolyLog} \left(2, -e^{-2 \operatorname{sech}^{-1}(cx)} \right) - \operatorname{sech}^{-1}(cx) \left(\operatorname{sech}^{-1}(cx) + 2 \log \left(e^{-2 \operatorname{sech}^{-1}(cx)} + 1 \right) \right) \right) + b^2 \left(\operatorname{sech}^{-1}(cx) \operatorname{PolyLog} \left(2, -e^{-2 \operatorname{sech}^{-1}(cx)} \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*ArcSech[c*x])^2/x,x]

[Out] a^2*Log[c*x] + a*b*(-(ArcSech[c*x]*(ArcSech[c*x] + 2*Log[1 + E^(-2*ArcSech[c*x])])) + PolyLog[2, -E^(-2*ArcSech[c*x])]) + b^2*(-ArcSech[c*x]^3/3 - ArcSech[c*x]^2*Log[1 + E^(-2*ArcSech[c*x])] + ArcSech[c*x]*PolyLog[2, -E^(-2*ArcSech[c*x])] + PolyLog[3, -E^(-2*ArcSech[c*x])]/2)

Maple [A] time = 0.237, size = 250, normalized size = 3.

$$a^2 \ln(cx) + \frac{b^2 (\operatorname{arcsech}(cx))^3}{3} - b^2 (\operatorname{arcsech}(cx))^2 \ln \left(1 + \left(\frac{1}{cx} + \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}} \right)^2 \right) - b^2 \operatorname{arcsech}(cx) \operatorname{polylog} \left(2, - \left(\frac{1}{cx} + \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}} \right)^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))^2/x,x)

[Out] $a^2 \ln(cx) + 1/3 b^2 \operatorname{arcsech}(cx)^3 - b^2 \operatorname{arcsech}(cx)^2 \ln(1 + (1/cx + (-1 + 1/cx)^{1/2})^{1/2} * (1 + 1/cx)^{1/2})^2 - b^2 \operatorname{arcsech}(cx) \operatorname{polylog}(2, -(1/cx + (-1 + 1/cx)^{1/2})^{1/2} * (1 + 1/cx)^{1/2})^2 + 1/2 b^2 \operatorname{polylog}(3, -(1/cx + (-1 + 1/cx)^{1/2})^{1/2} * (1 + 1/cx)^{1/2})^2 - 2 a b \operatorname{arcsech}(cx) \ln(1 + (1/cx + (-1 + 1/cx)^{1/2})^{1/2} * (1 + 1/cx)^{1/2})^2 + a b \operatorname{arcsech}(cx)^2 - a b \operatorname{polylog}(2, -(1/cx + (-1 + 1/cx)^{1/2})^{1/2} * (1 + 1/cx)^{1/2})^2$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a^2 \log(x) + \int \frac{b^2 \log\left(\sqrt{\frac{1}{cx} + 1} \sqrt{\frac{1}{cx} - 1} + \frac{1}{cx}\right)^2}{x} + \frac{2 ab \log\left(\sqrt{\frac{1}{cx} + 1} \sqrt{\frac{1}{cx} - 1} + \frac{1}{cx}\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))^2/x,x, algorithm="maxima")

[Out] $a^2 \log(x) + \operatorname{integrate}(b^2 \log(\sqrt{1/(cx) + 1} \sqrt{1/(cx) - 1} + 1/(cx))^2/x + 2 a b \log(\sqrt{1/(cx) + 1} \sqrt{1/(cx) - 1} + 1/(cx))/x, x)$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b^2 \operatorname{arsech}(cx)^2 + 2 ab \operatorname{arsech}(cx) + a^2}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))^2/x,x, algorithm="fricas")

[Out] $\operatorname{integral}((b^2 \operatorname{arcsech}(cx)^2 + 2 a b \operatorname{arcsech}(cx) + a^2)/x, x)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{asech}(cx))^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asech(c*x))**2/x,x)

[Out] $\operatorname{Integral}((a + b \operatorname{asech}(cx))**2/x, x)$

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsech}(cx) + a)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)^2/x, x)
```


$$3.38 \quad \int \frac{\left(a + b \operatorname{sech}^{-1}(cx)\right)^2}{x^2} dx$$

Optimal. Leaf size=61

$$\frac{2b\sqrt{\frac{1-cx}{cx+1}}(cx+1)(a+b\operatorname{sech}^{-1}(cx))}{x} - \frac{(a+b\operatorname{sech}^{-1}(cx))^2}{x} - \frac{2b^2}{x}$$

[Out] $(-2*b^2)/x + (2*b*sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x]))/x - (a + b*ArcSech[c*x])^2/x$

Rubi [A] time = 0.0699774, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {6285, 3296, 2638}

$$\frac{2b\sqrt{\frac{1-cx}{cx+1}}(cx+1)(a+b\operatorname{sech}^{-1}(cx))}{x} - \frac{(a+b\operatorname{sech}^{-1}(cx))^2}{x} - \frac{2b^2}{x}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcSech[c*x])^2/x^2, x]

[Out] $(-2*b^2)/x + (2*b*sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x]))/x - (a + b*ArcSech[c*x])^2/x$

Rule 6285

Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))^n_.*(x_)^m_.], x_Symbol] :> -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

Rule 3296

Int[((c_.) + (d_.)*(x_.))^m_.*sin[(e_.) + (f_.)*(x_.)], x_Symbol] :> -Simp[(c + d*x)^m*cos[e + f*x]/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_.)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \operatorname{sech}^{-1}(cx))^2}{x^2} dx &= -\left(c \operatorname{Subst} \left(\int (a + bx)^2 \sinh(x) dx, x, \operatorname{sech}^{-1}(cx) \right) \right) \\
&= -\frac{(a + b \operatorname{sech}^{-1}(cx))^2}{x} + (2bc) \operatorname{Subst} \left(\int (a + bx) \cosh(x) dx, x, \operatorname{sech}^{-1}(cx) \right) \\
&= \frac{2b\sqrt{\frac{1-cx}{1+cx}}(1+cx)(a + b \operatorname{sech}^{-1}(cx))}{x} - \frac{(a + b \operatorname{sech}^{-1}(cx))^2}{x} - (2b^2c) \operatorname{Subst} \left(\int \sinh(x) dx, x, \operatorname{sech}^{-1}(cx) \right) \\
&= -\frac{2b^2}{x} + \frac{2b\sqrt{\frac{1-cx}{1+cx}}(1+cx)(a + b \operatorname{sech}^{-1}(cx))}{x} - \frac{(a + b \operatorname{sech}^{-1}(cx))^2}{x}
\end{aligned}$$

Mathematica [A] time = 0.228817, size = 87, normalized size = 1.43

$$\frac{a^2 - 2ab\sqrt{\frac{1-cx}{cx+1}}(cx+1) - 2b \operatorname{sech}^{-1}(cx) \left(b\sqrt{\frac{1-cx}{cx+1}}(cx+1) - a \right) + b^2 \operatorname{sech}^{-1}(cx)^2 + 2b^2}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcSech[c*x])^2/x^2, x]

[Out] -((a^2 + 2*b^2 - 2*a*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) - 2*b*(-a + b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))*ArcSech[c*x] + b^2*ArcSech[c*x]^2)/x)

Maple [B] time = 0.217, size = 124, normalized size = 2.

$$c \left(-\frac{a^2}{cx} + b^2 \left(-\frac{(\operatorname{arcsech}(cx))^2}{cx} + 2 \operatorname{arcsech}(cx) \sqrt{-\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} - 2 \frac{1}{cx} \right) + 2ab \left(-\frac{\operatorname{arcsech}(cx)}{cx} + \sqrt{-\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))^2/x^2, x)

[Out] c*(-a^2/c/x+b^2*(-arcsech(c*x)^2/c/x+2*arcsech(c*x)*(-(c*x-1)/c/x)^(1/2))*((c*x+1)/c/x)^(1/2)-2/c/x)+2*a*b*(-1/c/x*arcsech(c*x)+(-(c*x-1)/c/x)^(1/2))*((c*x+1)/c/x)^(1/2))

Maxima [A] time = 1.02864, size = 105, normalized size = 1.72

$$2 \left(c \sqrt{\frac{1}{c^2 x^2} - 1} - \frac{\operatorname{arosech}(cx)}{x} \right) ab + 2 \left(c \sqrt{\frac{1}{c^2 x^2} - 1} \operatorname{arosech}(cx) - \frac{1}{x} \right) b^2 - \frac{b^2 \operatorname{arosech}(cx)^2}{x} - \frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))^2/x^2, x, algorithm="maxima")

[Out] 2*(c*sqrt(1/(c^2*x^2) - 1) - arcsech(c*x)/x)*a*b + 2*(c*sqrt(1/(c^2*x^2) - 1)*arcsech(c*x) - 1/x)*b^2 - b^2*arcsech(c*x)^2/x - a^2/x

Fricas [B] time = 1.65977, size = 301, normalized size = 4.93

$$\frac{2 abcx\sqrt{-\frac{c^2x^2-1}{c^2x^2}} - b^2 \log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)^2 - a^2 - 2b^2 + 2\left(b^2cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}} - ab\right) \log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))^2/x^2,x, algorithm="fricas")

[Out] (2*a*b*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - b^2*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x))^2 - a^2 - 2*b^2 + 2*(b^2*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - a*b)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/x

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{asech}(cx))^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asech(c*x))**2/x**2,x)

[Out] Integral((a + b*asech(c*x))**2/x**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsech}(cx) + a)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))^2/x^2,x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)^2/x^2, x)

$$3.39 \quad \int \frac{(a + b \operatorname{sech}^{-1}(cx))^2}{x^3} dx$$

Optimal. Leaf size=118

$$-\frac{1}{2}abc^2 \operatorname{sech}^{-1}(cx) + \frac{b\sqrt{\frac{1-cx}{cx+1}}(cx+1)(a+b \operatorname{sech}^{-1}(cx))}{2x^2} - \frac{(1-cx)(cx+1)(a+b \operatorname{sech}^{-1}(cx))^2}{2x^2} - \frac{1}{4}b^2c^2 \operatorname{sech}^{-1}(cx)^2 - \frac{b^2}{2}c^2$$

[Out] $-(b^2*(1 - c*x)*(1 + c*x))/(4*x^2) - (a*b*c^2*ArcSech[c*x])/2 - (b^2*c^2*ArcSech[c*x]^2)/4 + (b*sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x]))/(2*x^2) - ((1 - c*x)*(1 + c*x)*(a + b*ArcSech[c*x])^2)/(2*x^2)$

Rubi [A] time = 0.0847385, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {6285, 5446, 3310}

$$-\frac{1}{2}abc^2 \operatorname{sech}^{-1}(cx) + \frac{b\sqrt{\frac{1-cx}{cx+1}}(cx+1)(a+b \operatorname{sech}^{-1}(cx))}{2x^2} - \frac{(1-cx)(cx+1)(a+b \operatorname{sech}^{-1}(cx))^2}{2x^2} - \frac{1}{4}b^2c^2 \operatorname{sech}^{-1}(cx)^2 - \frac{b^2}{2}c^2$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcSech[c*x])^2/x^3,x]

[Out] $-(b^2*(1 - c*x)*(1 + c*x))/(4*x^2) - (a*b*c^2*ArcSech[c*x])/2 - (b^2*c^2*ArcSech[c*x]^2)/4 + (b*sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x]))/(2*x^2) - ((1 - c*x)*(1 + c*x)*(a + b*ArcSech[c*x])^2)/(2*x^2)$

Rule 6285

Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))^n*(x_)^m, x_Symbol] :> -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

Rule 5446

Int[Cosh[(a_.) + (b_.)*(x_.)]*((c_.) + (d_.)*(x_.))^m*Sinh[(a_.) + (b_.)*(x_.)]^n, x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n + 1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rule 3310

Int[((c_.) + (d_.)*(x_.))*((b_.)*sin[(e_.) + (f_.)*(x_.)])^n, x_Symbol] :> Simp[(d*(b*Sine[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c + d*x)*(b*Sine[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b*Sine[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \operatorname{sech}^{-1}(cx))^2}{x^3} dx &= -\left(c^2 \operatorname{Subst}\left(\int (a + bx)^2 \cosh(x) \sinh(x) dx, x, \operatorname{sech}^{-1}(cx)\right)\right) \\
&= -\frac{(1 - cx)(1 + cx)(a + b \operatorname{sech}^{-1}(cx))^2}{2x^2} + (bc^2) \operatorname{Subst}\left(\int (a + bx) \sinh^2(x) dx, x, \operatorname{sech}^{-1}(cx)\right) \\
&= -\frac{b^2(1 - cx)(1 + cx)}{4x^2} + \frac{b\sqrt{\frac{1 - cx}{1 + cx}}(1 + cx)(a + b \operatorname{sech}^{-1}(cx))}{2x^2} - \frac{(1 - cx)(1 + cx)(a + b \operatorname{sech}^{-1}(cx))^2}{2x^2} \\
&= -\frac{b^2(1 - cx)(1 + cx)}{4x^2} - \frac{1}{2}abc^2 \operatorname{sech}^{-1}(cx) - \frac{1}{4}b^2c^2 \operatorname{sech}^{-1}(cx)^2 + \frac{b\sqrt{\frac{1 - cx}{1 + cx}}(1 + cx)(a + b \operatorname{sech}^{-1}(cx))}{2x^2}
\end{aligned}$$

Mathematica [A] time = 0.163408, size = 183, normalized size = 1.55

$$\frac{-2a^2 - 2abc^2x^2 \log(x) + 2abc^2x^2 \log\left(cx\sqrt{\frac{1 - cx}{cx + 1}} + \sqrt{\frac{1 - cx}{cx + 1}} + 1\right) + 2ab\sqrt{\frac{1 - cx}{cx + 1}} + 2abcx\sqrt{\frac{1 - cx}{cx + 1}} + 2b \operatorname{sech}^{-1}(cx)\left(b\sqrt{\frac{1 - cx}{cx + 1}}(cx\sqrt{\frac{1 - cx}{cx + 1}} + \sqrt{\frac{1 - cx}{cx + 1}} + 1)\right)}{4x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcSech[c*x])^2/x^3,x]

[Out] $(-2a^2 - b^2 + 2ab\sqrt{\frac{1 - cx}{1 + cx}} + 2abcx\sqrt{\frac{1 - cx}{1 + cx}} + 2b \operatorname{sech}^{-1}(cx)\left(b\sqrt{\frac{1 - cx}{1 + cx}}(cx\sqrt{\frac{1 - cx}{1 + cx}} + \sqrt{\frac{1 - cx}{1 + cx}} + 1)\right) + b^2(-2 + c^2x^2)\operatorname{ArcSech}[cx]^2 - 2ab\sqrt{\frac{1 - cx}{1 + cx}} - 2abcx\sqrt{\frac{1 - cx}{1 + cx}} - 2b \operatorname{sech}^{-1}(cx)\left(b\sqrt{\frac{1 - cx}{1 + cx}}(cx\sqrt{\frac{1 - cx}{1 + cx}} + \sqrt{\frac{1 - cx}{1 + cx}} + 1)\right))/4x^2$

Maple [A] time = 0.236, size = 192, normalized size = 1.6

$$c^2 \left(-\frac{a^2}{2c^2x^2} + b^2 \left(-\frac{(\operatorname{arcsech}(cx))^2}{2c^2x^2} + \frac{\operatorname{arcsech}(cx)}{2cx} \sqrt{\frac{cx - 1}{cx}} \sqrt{\frac{cx + 1}{cx}} + \frac{(\operatorname{arcsech}(cx))^2}{4} - \frac{1}{4c^2x^2} \right) + 2ab \left(-\frac{1}{2} \frac{\operatorname{arcsech}(cx)}{cx} + \frac{\operatorname{arcsech}(cx)}{cx} \sqrt{\frac{cx - 1}{cx}} \sqrt{\frac{cx + 1}{cx}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))^2/x^3,x)

[Out] $c^2 \left(-\frac{1}{2} \frac{a^2}{c^2x^2} + b^2 \left(-\frac{1}{2} \frac{(\operatorname{arcsech}(cx))^2}{c^2x^2} + \frac{\operatorname{arcsech}(cx)}{2cx} \sqrt{\frac{cx - 1}{cx}} \sqrt{\frac{cx + 1}{cx}} + \frac{(\operatorname{arcsech}(cx))^2}{4} - \frac{1}{4c^2x^2} \right) + 2ab \left(-\frac{1}{2} \frac{\operatorname{arcsech}(cx)}{cx} + \frac{\operatorname{arcsech}(cx)}{cx} \sqrt{\frac{cx - 1}{cx}} \sqrt{\frac{cx + 1}{cx}} \right) \right)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{4}ab \left(\frac{2c^4x\sqrt{\frac{1}{c^2x^2}-1}}{c^2x^2\left(\frac{1}{c^2x^2}-1\right)-1} - c^3 \log\left(cx\sqrt{\frac{1}{c^2x^2}-1} + 1\right) + c^3 \log\left(cx\sqrt{\frac{1}{c^2x^2}-1} - 1\right) + \frac{4 \operatorname{arosech}(cx)}{x^2} \right) + b^2 \int \frac{\log\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}\right)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))^2/x^3,x, algorithm="maxima")

[Out] $-1/4*a*b*((2*c^4*x*\sqrt{1/(c^2*x^2)} - 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) - c^3*\log(c*x*\sqrt{1/(c^2*x^2)} - 1) + 1) + c^3*\log(c*x*\sqrt{1/(c^2*x^2)} - 1) - 1)/c + 4*arcsech(c*x)/x^2 + b^2*\int(\log(\sqrt{1/(c*x)} + 1)*\sqrt{1/(c*x)} - 1) + 1/(c*x))^2/x^3, x) - 1/2*a^2/x^2$

Fricas [A] time = 1.61809, size = 355, normalized size = 3.01

$$\frac{2abcx\sqrt{-\frac{c^2x^2-1}{c^2x^2}} + (b^2c^2x^2 - 2b^2)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)^2 - 2a^2 - b^2 + 2\left(abc^2x^2 + b^2cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}} - 2ab\right)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))^2/x^3,x, algorithm="fricas")

[Out] $1/4*(2*a*b*c*x*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)} + (b^2*c^2*x^2 - 2*b^2)*\log((c*x*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)} + 1)/(c*x))^2 - 2*a^2 - b^2 + 2*(a*b*c^2*x^2 + b^2*c*x*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)} - 2*a*b)*\log((c*x*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)} + 1)/(c*x)))/x^2$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{asech}(cx))^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asech(c*x))^2/x**3,x)

[Out] Integral((a + b*asech(c*x))^2/x**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsech}(cx) + a)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))^2/x^3,x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)^2/x^3, x)

$$3.40 \quad \int \frac{(a + b \operatorname{sech}^{-1}(cx))^2}{x^4} dx$$

Optimal. Leaf size=122

$$\frac{4bc^2 \sqrt{\frac{1-cx}{cx+1}}(cx+1)(a+b \operatorname{sech}^{-1}(cx))}{9x} + \frac{2b \sqrt{\frac{1-cx}{cx+1}}(cx+1)(a+b \operatorname{sech}^{-1}(cx))}{9x^3} - \frac{(a+b \operatorname{sech}^{-1}(cx))^2}{3x^3} - \frac{4b^2c^2}{9x} - \frac{2b^2}{27x^3}$$

[Out] $(-2*b^2)/(27*x^3) - (4*b^2*c^2)/(9*x) + (2*b* \operatorname{Sqrt}[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b* \operatorname{ArcSech}[c*x]))/(9*x^3) + (4*b*c^2* \operatorname{Sqrt}[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b* \operatorname{ArcSech}[c*x]))/(9*x) - (a + b* \operatorname{ArcSech}[c*x])^2/(3*x^3)$

Rubi [A] time = 0.101873, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {6285, 5447, 3310, 3296, 2638}

$$\frac{4bc^2 \sqrt{\frac{1-cx}{cx+1}}(cx+1)(a+b \operatorname{sech}^{-1}(cx))}{9x} + \frac{2b \sqrt{\frac{1-cx}{cx+1}}(cx+1)(a+b \operatorname{sech}^{-1}(cx))}{9x^3} - \frac{(a+b \operatorname{sech}^{-1}(cx))^2}{3x^3} - \frac{4b^2c^2}{9x} - \frac{2b^2}{27x^3}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + b* \operatorname{ArcSech}[c*x])^2/x^4, x]$

[Out] $(-2*b^2)/(27*x^3) - (4*b^2*c^2)/(9*x) + (2*b* \operatorname{Sqrt}[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b* \operatorname{ArcSech}[c*x]))/(9*x^3) + (4*b*c^2* \operatorname{Sqrt}[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b* \operatorname{ArcSech}[c*x]))/(9*x) - (a + b* \operatorname{ArcSech}[c*x])^2/(3*x^3)$

Rule 6285

$\operatorname{Int}[(a + \operatorname{ArcSech}[c*x])^2/x^4, x] \rightarrow -\operatorname{Dist}[(c^{m+1})^{-1}, \operatorname{Subst}[\operatorname{Int}[(a + b*x)^n * \operatorname{Sech}[x]^{m+1} * \operatorname{Tanh}[x], x], x, \operatorname{ArcSech}[c*x]], x] /;$ $\operatorname{FreeQ}\{a, b, c, x\} \ \&\& \ \operatorname{IntegerQ}[n] \ \&\& \ \operatorname{IntegerQ}[m] \ \&\& \ (\operatorname{GtQ}[n, 0] \ || \ \operatorname{LtQ}[m, -1])$

Rule 5447

$\operatorname{Int}[\operatorname{Cosh}[a + b*x]^{n+1} * \operatorname{Sinh}[a + b*x]^m, x] \rightarrow \operatorname{Simp}[(c + d*x)^m * \operatorname{Cosh}[a + b*x]^{n+1} / (b*(n+1)), x] - \operatorname{Dist}[(d*m)/(b*(n+1)), \operatorname{Int}[(c + d*x)^{m-1} * \operatorname{Cosh}[a + b*x]^{n+1}, x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, n, x\} \ \&\& \ \operatorname{IGtQ}[m, 0] \ \&\& \ \operatorname{NeQ}[n, -1]$

Rule 3310

$\operatorname{Int}[(c + d*x)^n * \sin[e + f*x], x] \rightarrow \operatorname{Simp}[(d*(b*\sin[e + f*x])^n)/(f^2*n^2), x] + (\operatorname{Dist}[(b^2*(n-1))/n, \operatorname{Int}[(c + d*x)*(b*\sin[e + f*x])^{n-2}, x], x] - \operatorname{Simp}[(b*(c + d*x)*\cos[e + f*x]*(b*\sin[e + f*x])^{n-1})/(f*n), x]) /;$ $\operatorname{FreeQ}\{b, c, d, e, f, x\} \ \&\& \ \operatorname{GtQ}[n, 1]$

Rule 3296

$\operatorname{Int}[(c + d*x)^m * \sin[e + f*x], x] \rightarrow -\operatorname{Simp}[(c + d*x)^m * \cos[e + f*x]/f, x] + \operatorname{Dist}[(d*m)/f, \operatorname{Int}[(c + d*x)^{m-1} * \cos[e + f*x], x], x] /;$ $\operatorname{FreeQ}\{c, d, e, f, x\} \ \&\& \ \operatorname{GtQ}[m, 0]$

Rule 2638

`Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\begin{aligned} \int \frac{(a + b \operatorname{sech}^{-1}(cx))^2}{x^4} dx &= -\left(c^3 \operatorname{Subst}\left(\int (a + bx)^2 \cosh^2(x) \sinh(x) dx, x, \operatorname{sech}^{-1}(cx)\right)\right) \\ &= -\frac{(a + b \operatorname{sech}^{-1}(cx))^2}{3x^3} + \frac{1}{3}(2bc^3) \operatorname{Subst}\left(\int (a + bx) \cosh^3(x) dx, x, \operatorname{sech}^{-1}(cx)\right) \\ &= -\frac{2b^2}{27x^3} + \frac{2b\sqrt{\frac{1-cx}{1+cx}}(1+cx)(a + b \operatorname{sech}^{-1}(cx))}{9x^3} - \frac{(a + b \operatorname{sech}^{-1}(cx))^2}{3x^3} + \frac{1}{9}(4bc^3) \operatorname{Subst}\left(\int (a + bx) \cosh^3(x) dx, x, \operatorname{sech}^{-1}(cx)\right) \\ &= -\frac{2b^2}{27x^3} + \frac{2b\sqrt{\frac{1-cx}{1+cx}}(1+cx)(a + b \operatorname{sech}^{-1}(cx))}{9x^3} + \frac{4bc^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)(a + b \operatorname{sech}^{-1}(cx))}{9x} - \frac{(a + b \operatorname{sech}^{-1}(cx))^2}{3x^3} \\ &= -\frac{2b^2}{27x^3} - \frac{4b^2c^2}{9x} + \frac{2b\sqrt{\frac{1-cx}{1+cx}}(1+cx)(a + b \operatorname{sech}^{-1}(cx))}{9x^3} + \frac{4bc^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)(a + b \operatorname{sech}^{-1}(cx))}{9x} \end{aligned}$$

Mathematica [A] time = 0.249804, size = 134, normalized size = 1.1

$$\frac{-9a^2 + 6ab\sqrt{\frac{1-cx}{cx+1}}(2c^3x^3 + 2c^2x^2 + cx + 1) + 6b \operatorname{sech}^{-1}(cx) \left(b\sqrt{\frac{1-cx}{cx+1}}(2c^3x^3 + 2c^2x^2 + cx + 1) - 3a\right) - 2b^2(6c^2x^2 + 1) - 9(a + b \operatorname{sech}^{-1}(cx))^2}{27x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcSech[c*x])^2/x^4, x]

[Out] $(-9a^2 - 2b^2(1 + 6c^2x^2) + 6ab\sqrt{\frac{1-cx}{cx+1}}(2c^3x^3 + 2c^2x^2 + cx + 1) + 6b \operatorname{sech}^{-1}(cx) \left(b\sqrt{\frac{1-cx}{cx+1}}(2c^3x^3 + 2c^2x^2 + cx + 1) - 3a\right) - 2b^2(6c^2x^2 + 1) - 9(a + b \operatorname{sech}^{-1}(cx))^2)/(27x^3)$

Maple [B] time = 0.225, size = 226, normalized size = 1.9

$$c^3 \left(-\frac{a^2}{3c^3x^3} + b^2 \left(\frac{(\operatorname{arcsech}(cx))^2(cx-1)(cx+1)}{3c^3x^3} - \frac{(\operatorname{arcsech}(cx))^2}{3cx} + \frac{2 \operatorname{arcsech}(cx)}{9c^2x^2} \sqrt{\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} + \frac{4 \operatorname{arcsech}(cx)}{9} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))^2/x^4, x)

[Out] $c^3 \left(-\frac{1}{3} \frac{a^2}{c^3x^3} + b^2 \left(\frac{1}{3} \frac{\operatorname{arcsech}(cx)^2}{c^3x^3} (cx-1)(cx+1) - \frac{1}{3} \frac{\operatorname{arcsech}(cx)^2}{cx} + \frac{2}{9} \frac{\operatorname{arcsech}(cx)}{c^2x^2} \sqrt{\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} + \frac{4}{9} \operatorname{arcsech}(cx) \right) \right)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2}{9} ab \left(\frac{c^4 \left(\frac{1}{c^2 x^2} - 1 \right)^{\frac{3}{2}} + 3 c^4 \sqrt{\frac{1}{c^2 x^2} - 1} - 3 \operatorname{arsech}(cx)}{c} - \frac{3 \operatorname{arsech}(cx)}{x^3} \right) + b^2 \int \frac{\log \left(\sqrt{\frac{1}{cx} + 1} \sqrt{\frac{1}{cx} - 1} + \frac{1}{cx} \right)^2}{x^4} dx - \frac{a^2}{3 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))^2/x^4,x, algorithm="maxima")

[Out] 2/9*a*b*((c^4*(1/(c^2*x^2) - 1)^(3/2) + 3*c^4*sqrt(1/(c^2*x^2) - 1))/c - 3*arcsech(c*x)/x^3) + b^2*integrate(log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))^2/x^4, x) - 1/3*a^2/x^3

Fricas [A] time = 1.65368, size = 392, normalized size = 3.21

$$\frac{12 b^2 c^2 x^2 + 9 b^2 \log \left(\frac{cx \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2} + 1}}{cx} \right)^2 + 9 a^2 + 2 b^2 + 6 \left(3 ab - (2 b^2 c^3 x^3 + b^2 cx) \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2}} \right) \log \left(\frac{cx \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2} + 1}}{cx} \right) - 6 (2 a^2 + b^2)}{27 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))^2/x^4,x, algorithm="fricas")

[Out] -1/27*(12*b^2*c^2*x^2 + 9*b^2*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x))^2 + 9*a^2 + 2*b^2 + 6*(3*a*b - (2*b^2*c^3*x^3 + b^2*c*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - 6*(2*a*b*c^3*x^3 + a*b*c*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/x^3

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{asech}(cx))^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asech(c*x))**2/x**4,x)

[Out] Integral((a + b*asech(c*x))**2/x**4, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsech}(cx) + a)^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))^2/x^4,x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)^2/x^4, x)

$$3.41 \quad \int \frac{(a+b\operatorname{sech}^{-1}(cx))^2}{x^5} dx$$

Optimal. Leaf size=151

$$\frac{3bc^2\sqrt{\frac{1-cx}{cx+1}}(cx+1)(a+b\operatorname{sech}^{-1}(cx))}{16x^2} + \frac{3}{16}abc^4\operatorname{sech}^{-1}(cx) - \frac{(a+b\operatorname{sech}^{-1}(cx))^2}{4x^4} + \frac{b\sqrt{\frac{1-cx}{cx+1}}(cx+1)(a+b\operatorname{sech}^{-1}(cx))}{8x^4} -$$

[Out] $-b^2/(32*x^4) - (3*b^2*c^2)/(32*x^2) + (3*a*b*c^4*ArcSech[c*x])/16 + (3*b^2*c^4*ArcSech[c*x]^2)/32 + (b*sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x]))/(8*x^4) + (3*b*c^2*sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x]))/(16*x^2) - (a + b*ArcSech[c*x])^2/(4*x^4)$

Rubi [A] time = 0.119685, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {6285, 5447, 3310}

$$\frac{3bc^2\sqrt{\frac{1-cx}{cx+1}}(cx+1)(a+b\operatorname{sech}^{-1}(cx))}{16x^2} + \frac{3}{16}abc^4\operatorname{sech}^{-1}(cx) - \frac{(a+b\operatorname{sech}^{-1}(cx))^2}{4x^4} + \frac{b\sqrt{\frac{1-cx}{cx+1}}(cx+1)(a+b\operatorname{sech}^{-1}(cx))}{8x^4} -$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcSech[c*x])^2/x^5,x]

[Out] $-b^2/(32*x^4) - (3*b^2*c^2)/(32*x^2) + (3*a*b*c^4*ArcSech[c*x])/16 + (3*b^2*c^4*ArcSech[c*x]^2)/32 + (b*sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x]))/(8*x^4) + (3*b*c^2*sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x]))/(16*x^2) - (a + b*ArcSech[c*x])^2/(4*x^4)$

Rule 6285

Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))^n*(x_)^m, x_Symbol] :> -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

Rule 5447

Int[Cosh[(a_.) + (b_.)*(x_.)]^n*((c_.) + (d_.)*(x_.))^m*Sinh[(a_.) + (b_.)*(x_.)], x_Symbol] :> Simp[((c + d*x)^m*Cosh[a + b*x]^(n + 1))/(b*(n + 1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rule 3310

Int[((c_.) + (d_.)*(x_.))*((b_.)*sin[(e_.) + (f_.)*(x_.)])^n, x_Symbol] :> Simp[(d*(b*sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c + d*x)*(b*sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b*sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \operatorname{sech}^{-1}(cx))^2}{x^5} dx &= -\left(c^4 \operatorname{Subst}\left(\int (a + bx)^2 \cosh^3(x) \sinh(x) dx, x, \operatorname{sech}^{-1}(cx)\right)\right) \\
&= -\frac{(a + b \operatorname{sech}^{-1}(cx))^2}{4x^4} + \frac{1}{2}(bc^4) \operatorname{Subst}\left(\int (a + bx) \cosh^4(x) dx, x, \operatorname{sech}^{-1}(cx)\right) \\
&= -\frac{b^2}{32x^4} + \frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)(a + b \operatorname{sech}^{-1}(cx))}{8x^4} - \frac{(a + b \operatorname{sech}^{-1}(cx))^2}{4x^4} + \frac{1}{8}(3bc^4) \operatorname{Subst}\left(\int (a + bx) \cosh^4(x) dx, x, \operatorname{sech}^{-1}(cx)\right) \\
&= -\frac{b^2}{32x^4} - \frac{3b^2c^2}{32x^2} + \frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)(a + b \operatorname{sech}^{-1}(cx))}{8x^4} + \frac{3bc^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)(a + b \operatorname{sech}^{-1}(cx))}{16x^2} \\
&= -\frac{b^2}{32x^4} - \frac{3b^2c^2}{32x^2} + \frac{3}{16}abc^4\operatorname{sech}^{-1}(cx) + \frac{3}{32}b^2c^4\operatorname{sech}^{-1}(cx)^2 + \frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)(a + b \operatorname{sech}^{-1}(cx))}{8x^4}
\end{aligned}$$

Mathematica [A] time = 0.259171, size = 268, normalized size = 1.77

$$\frac{-8a^2 + 6abc^3x^3\sqrt{\frac{1-cx}{cx+1}} + 6abc^2x^2\sqrt{\frac{1-cx}{cx+1}} - 6abc^4x^4\log(x) + 6abc^4x^4\log\left(cx\sqrt{\frac{1-cx}{cx+1}} + \sqrt{\frac{1-cx}{cx+1}} + 1\right) + 2b\operatorname{sech}^{-1}(cx)\left(b\sqrt{\frac{1-cx}{cx+1}} + 1\right)}{32x^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcSech[c*x])^2/x^5, x]

[Out] $(-8a^2 - b^2 - 3b^2c^2x^2 + 4ab\sqrt{(1-cx)/(1+cx)} + 4ab^2c^2x^2\sqrt{(1-cx)/(1+cx)} + 6ab^2c^2x^2\sqrt{(1-cx)/(1+cx)} + 6ab^2c^3x^3\sqrt{(1-cx)/(1+cx)} + 2b(-8a + b\sqrt{(1-cx)/(1+cx)}) * (2 + 2cx + 3c^2x^2 + 3c^3x^3)) * \operatorname{ArcSech}[cx] + b^2(-8 + 3c^4x^4) * \operatorname{ArcSech}[cx]^2 - 6ab^2c^4x^4 * \log[x] + 6ab^2c^4x^4 * \log[1 + \sqrt{(1-cx)/(1+cx)}] + cx\sqrt{(1-cx)/(1+cx)}) / (32x^4)$

Maple [B] time = 0.233, size = 298, normalized size = 2.

$$c^4 \left(-\frac{a^2}{4c^4x^4} + b^2 \left(\frac{(\operatorname{arcsech}(cx))^2 (cx-1)(cx+1)}{4c^4x^4} - \frac{(\operatorname{arcsech}(cx))^2}{4c^2x^2} + \frac{\operatorname{arcsech}(cx)}{8c^3x^3} \sqrt{\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} + \frac{3 \operatorname{arcsech}(cx)}{16cx} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))^2/x^5, x)

[Out] $c^4 * (-1/4 * a^2 / c^4 / x^4 + b^2 * (1/4 * \operatorname{arcsech}(c*x)^2 / c^4 / x^4 * (c*x-1) * (c*x+1) - 1/4 / c^2 / x^2 * \operatorname{arcsech}(c*x)^2 + 1/8 * \operatorname{arcsech}(c*x) / c^3 / x^3 * (-c*x-1) / c * x^{(1/2)} * ((c*x+1) / c * x)^{(1/2)} + 3/16 * \operatorname{arcsech}(c*x) / c * x * (-c*x-1) / c * x^{(1/2)} * ((c*x+1) / c * x)^{(1/2)} + 3/32 * \operatorname{arcsech}(c*x)^2 + 1/32 * (c*x-1) / c^4 / x^4 * (c*x+1) - 1/8 / c^2 / x^2 + 2 * a * b * (-1/4 / c^4 / x^4 * \operatorname{arcsech}(c*x) + 1/32 * (-c*x-1) / c * x^{(1/2)} / c^3 / x^3 * ((c*x+1) / c * x)^{(1/2)} * (3 * \operatorname{arctanh}(1 / (-c^2 * x^2 + 1)^{(1/2)})) * c^4 * x^4 + 3 * (-c^2 * x^2 + 1)^{(1/2)} * c^2 * x^2 + 2 * (-c^2 * x^2 + 1)^{(1/2)}) / (-c^2 * x^2 + 1)^{(1/2)})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{32} ab \left(\frac{3c^5 \log\left(cx\sqrt{\frac{1}{c^2x^2}-1}+1\right) - 3c^5 \log\left(cx\sqrt{\frac{1}{c^2x^2}-1}-1\right) - \frac{2\left(3c^8x^3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}} - 5c^6x\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^4x^4\left(\frac{1}{c^2x^2}-1\right)^2 - 2c^2x^2\left(\frac{1}{c^2x^2}-1\right)+1}}{c} - \frac{16 \operatorname{arsech}(cx)}{x^4} \right) + b^2 \int \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))^2/x^5,x, algorithm="maxima")

[Out] 1/32*a*b*((3*c^5*log(c*x*sqrt(1/(c^2*x^2) - 1) + 1) - 3*c^5*log(c*x*sqrt(1/(c^2*x^2) - 1) - 1) - 2*(3*c^8*x^3*(1/(c^2*x^2) - 1)^(3/2) - 5*c^6*x*sqrt(1/(c^2*x^2) - 1)))/(c^4*x^4*(1/(c^2*x^2) - 1)^2 - 2*c^2*x^2*(1/(c^2*x^2) - 1) + 1))/c - 16*arcsech(c*x)/x^4) + b^2*integrate(log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))^2/x^5, x) - 1/4*a^2/x^4

Fricas [A] time = 1.66952, size = 439, normalized size = 2.91

$$\frac{3b^2c^2x^2 - (3b^2c^4x^4 - 8b^2) \log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}+1}}{cx}\right)^2 + 8a^2 + b^2 - 2\left(3abc^4x^4 - 8ab + (3b^2c^3x^3 + 2b^2cx)\sqrt{-\frac{c^2x^2-1}{c^2x^2}}\right) \log\left(\frac{cx}{\dots}\right)}{32x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))^2/x^5,x, algorithm="fricas")

[Out] -1/32*(3*b^2*c^2*x^2 - (3*b^2*c^4*x^4 - 8*b^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x))^2 + 8*a^2 + b^2 - 2*(3*a*b*c^4*x^4 - 8*a*b + (3*b^2*c^3*x^3 + 2*b^2*c*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - 2*(3*a*b*c^3*x^3 + 2*a*b*c*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/x^4

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{asech}(cx))^2}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asech(c*x))**2/x**5,x)

[Out] Integral((a + b*asech(c*x))**2/x**5, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsech}(cx) + a)^2}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^2/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)^2/x^5, x)
```

3.42 $\int x^3 \left(a + b \operatorname{sech}^{-1}(cx) \right)^3 dx$

Optimal. Leaf size=223

$$\frac{b^3 \operatorname{PolyLog}\left(2, -e^{2\operatorname{sech}^{-1}(cx)}\right)}{2c^4} - \frac{b^2 x^2 \left(a + b \operatorname{sech}^{-1}(cx)\right)}{4c^2} + \frac{b^2 \log\left(e^{2\operatorname{sech}^{-1}(cx)} + 1\right) \left(a + b \operatorname{sech}^{-1}(cx)\right)}{c^4} - \frac{bx^2 \sqrt{\frac{1-cx}{cx+1}}(cx+1)}{4c^4}$$

[Out] (b^3*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(4*c^4) - (b^2*x^2*(a + b*ArcSech[c*x]))/(4*c^2) - (b*(a + b*ArcSech[c*x])^2)/(2*c^4) - (b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x])^2)/(2*c^4) - (b*x^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x])^2)/(4*c^2) + (x^4*(a + b*ArcSech[c*x])^3)/4 + (b^2*(a + b*ArcSech[c*x])*Log[1 + E^(2*ArcSech[c*x])])/c^4 + (b^3*PolyLog[2, -E^(2*ArcSech[c*x])])/(2*c^4)

Rubi [A] time = 0.239406, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {6285, 5451, 4186, 3767, 8, 4184, 3718, 2190, 2279, 2391}

$$\frac{b^3 \operatorname{PolyLog}\left(2, -e^{2\operatorname{sech}^{-1}(cx)}\right)}{2c^4} - \frac{b^2 x^2 \left(a + b \operatorname{sech}^{-1}(cx)\right)}{4c^2} + \frac{b^2 \log\left(e^{2\operatorname{sech}^{-1}(cx)} + 1\right) \left(a + b \operatorname{sech}^{-1}(cx)\right)}{c^4} - \frac{bx^2 \sqrt{\frac{1-cx}{cx+1}}(cx+1)}{4c^4}$$

Antiderivative was successfully verified.

[In] Int[x^3*(a + b*ArcSech[c*x])^3,x]

[Out] (b^3*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(4*c^4) - (b^2*x^2*(a + b*ArcSech[c*x]))/(4*c^2) - (b*(a + b*ArcSech[c*x])^2)/(2*c^4) - (b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x])^2)/(2*c^4) - (b*x^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x])^2)/(4*c^2) + (x^4*(a + b*ArcSech[c*x])^3)/4 + (b^2*(a + b*ArcSech[c*x])*Log[1 + E^(2*ArcSech[c*x])])/c^4 + (b^3*PolyLog[2, -E^(2*ArcSech[c*x])])/(2*c^4)

Rule 6285

Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))^ (n_.)*(x_.)^(m_.), x_Symbol] :> -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

Rule 5451

Int[((c_.) + (d_.)*(x_.))^(m_.)*Sech[(a_.) + (b_.)*(x_.)]^(n_.)*Tanh[(a_.) + (b_.)*(x_.)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n), x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]

Rule 4186

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^ (n_.)*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m - 1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 4184

Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := -Simp[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 3718

Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x))/(1 + E^(2*(-I*e) + f*fz*x)), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

Rule 2190

Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_) + (b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rubi steps

$$\begin{aligned} \int x^3 (a + b \operatorname{sech}^{-1}(cx))^3 dx &= -\frac{\operatorname{Subst}\left(\int (a + bx)^3 \operatorname{sech}^4(x) \tanh(x) dx, x, \operatorname{sech}^{-1}(cx)\right)}{c^4} \\ &= \frac{1}{4}x^4 (a + b \operatorname{sech}^{-1}(cx))^3 - \frac{(3b) \operatorname{Subst}\left(\int (a + bx)^2 \operatorname{sech}^4(x) dx, x, \operatorname{sech}^{-1}(cx)\right)}{4c^4} \\ &= -\frac{b^2 x^2 (a + b \operatorname{sech}^{-1}(cx))}{4c^2} - \frac{bx^2 \sqrt{\frac{1-cx}{1+cx}} (1+cx) (a + b \operatorname{sech}^{-1}(cx))^2}{4c^2} + \frac{1}{4}x^4 (a + b \operatorname{sech}^{-1}(cx))^3 \\ &= -\frac{b^2 x^2 (a + b \operatorname{sech}^{-1}(cx))}{4c^2} - \frac{b \sqrt{\frac{1-cx}{1+cx}} (1+cx) (a + b \operatorname{sech}^{-1}(cx))^2}{2c^4} - \frac{bx^2 \sqrt{\frac{1-cx}{1+cx}} (1+cx) (a + b \operatorname{sech}^{-1}(cx))^2}{4c^2} \\ &= \frac{b^3 \sqrt{\frac{1-cx}{1+cx}} (1+cx)}{4c^4} - \frac{b^2 x^2 (a + b \operatorname{sech}^{-1}(cx))}{4c^2} - \frac{b (a + b \operatorname{sech}^{-1}(cx))^2}{2c^4} - \frac{b \sqrt{\frac{1-cx}{1+cx}} (1+cx) (a + b \operatorname{sech}^{-1}(cx))^2}{2c^4} \\ &= \frac{b^3 \sqrt{\frac{1-cx}{1+cx}} (1+cx)}{4c^4} - \frac{b^2 x^2 (a + b \operatorname{sech}^{-1}(cx))}{4c^2} - \frac{b (a + b \operatorname{sech}^{-1}(cx))^2}{2c^4} - \frac{b \sqrt{\frac{1-cx}{1+cx}} (1+cx) (a + b \operatorname{sech}^{-1}(cx))^2}{2c^4} \\ &= \frac{b^3 \sqrt{\frac{1-cx}{1+cx}} (1+cx)}{4c^4} - \frac{b^2 x^2 (a + b \operatorname{sech}^{-1}(cx))}{4c^2} - \frac{b (a + b \operatorname{sech}^{-1}(cx))^2}{2c^4} - \frac{b \sqrt{\frac{1-cx}{1+cx}} (1+cx) (a + b \operatorname{sech}^{-1}(cx))^2}{2c^4} \\ &= \frac{b^3 \sqrt{\frac{1-cx}{1+cx}} (1+cx)}{4c^4} - \frac{b^2 x^2 (a + b \operatorname{sech}^{-1}(cx))}{4c^2} - \frac{b (a + b \operatorname{sech}^{-1}(cx))^2}{2c^4} - \frac{b \sqrt{\frac{1-cx}{1+cx}} (1+cx) (a + b \operatorname{sech}^{-1}(cx))^2}{2c^4} \end{aligned}$$

Mathematica [A] time = 1.55825, size = 315, normalized size = 1.41

$$b^3 \left(-2 \operatorname{PolyLog}\left(2, -e^{-2 \operatorname{sech}^{-1}(cx)}\right) - c^2 x^2 \sqrt{\frac{1-cx}{cx+1}} (cx+1) \operatorname{sech}^{-1}(cx)^2 - c^2 x^2 \operatorname{sech}^{-1}(cx) + \sqrt{\frac{1-cx}{cx+1}} (cx+1) - 2 \sqrt{\frac{1-cx}{cx+1}} (cx+1) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^3*(a + b*ArcSech[c*x])^3,x]

[Out] (a^3*c^4*x^4 + b^3*c^4*x^4*ArcSech[c*x]^3 + a^2*b*(-(Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(2 + c^2*x^2)) + 3*c^4*x^4*ArcSech[c*x]) - a*b^2*(c^2*x^2 + 2*Sqrt[(1 - c*x)/(1 + c*x)]*(2 + 2*c*x + c^2*x^2 + c^3*x^3)*ArcSech[c*x] - 3*c^4*x^4*ArcSech[c*x]^2 - 4*Log[1/(c*x)]) + b^3*(Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) - c^2*x^2*ArcSech[c*x] - 2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*ArcSech[c*x]^2 - c^2*x^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*ArcSech[c*x]^2 + 2*ArcSech[c*x]*(ArcSech[c*x] + 2*Log[1 + E^(-2*ArcSech[c*x])])) - 2*PolyLog[2, -E^(-2*ArcSech[c*x])]))/(4*c^4)

Maple [B] time = 0.385, size = 546, normalized size = 2.5

$$\frac{x^4 a^3}{4} + \frac{b^3 (\operatorname{arcsech}(cx))^3 x^4}{4} - \frac{b^3 (\operatorname{arcsech}(cx))^2 x^3}{4c} \sqrt{\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} - \frac{b^3 (\operatorname{arcsech}(cx))^2 x}{2c^3} \sqrt{\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} - \frac{b^3 a}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(a+b*arcsech(c*x))^3,x)

[Out] 1/4*x^4*a^3+1/4*b^3*arcsech(c*x)^3*x^4-1/4/c*b^3*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*arcsech(c*x)^2*x^3-1/2/c^3*b^3*arcsech(c*x)^2*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*x-1/4/c^2*b^3*arcsech(c*x)*x^2+1/4/c^3*b^3*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*arcsech(c*x)

$$\begin{aligned} & x-1)/c/x)^{(1/2)}*((c*x+1)/c/x)^{(1/2)}*x^{-1/2}/c^4*b^3*\operatorname{arcsech}(c*x)^2-1/4/c^4*b^3 \\ & +1/c^4*b^3*\operatorname{arcsech}(c*x)*\ln(1+(1/c/x+(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2}))^2)+1 \\ & /2*b^3*\operatorname{polylog}(2,-(1/c/x+(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2}))^2)/c^4-1/c^4*a*b \\ & ^2*\operatorname{arcsech}(c*x)+3/4*a*b^2*\operatorname{arcsech}(c*x)^2*x^4-1/2/c*a*b^2*(-(c*x-1)/c/x)^{(1/2)} \\ & *(c*x+1)/c/x)^{(1/2)}*\operatorname{arcsech}(c*x)*x^3-1/c^3*a*b^2*(-(c*x-1)/c/x)^{(1/2)}*((\\ & c*x+1)/c/x)^{(1/2)}*\operatorname{arcsech}(c*x)*x^{-1/4}/c^2*x^2*a*b^2+1/c^4*a*b^2*\ln(1+(1/c/x+ \\ & (-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2}))^2)+3/4*a^2*b*x^4*\operatorname{arcsech}(c*x)-1/4/c*a^2*b \\ & *(-(c*x-1)/c/x)^{(1/2)}*((c*x+1)/c/x)^{(1/2)}*x^3-1/2/c^3*a^2*b*(-(c*x-1)/c/x)^{(\\ & 1/2)}*x*((c*x+1)/c/x)^{(1/2)} \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4} a^3 x^4 + \frac{1}{4} \left(3 x^4 \operatorname{arsech}(cx) + \frac{c^2 x^3 \left(\frac{1}{c^2 x^2} - 1 \right)^{\frac{3}{2}} - 3 x \sqrt{\frac{1}{c^2 x^2} - 1}}{c^3} \right) a^2 b + \int b^3 x^3 \log \left(\sqrt{\frac{1}{cx} + 1} \sqrt{\frac{1}{cx} - 1} + \frac{1}{cx} \right)^3 + 3 a b^2 x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*arcsech(c*x))^3,x, algorithm="maxima")

[Out] 1/4*a^3*x^4 + 1/4*(3*x^4*arcsech(c*x) + (c^2*x^3*(1/(c^2*x^2) - 1)^(3/2) - 3*x*sqrt(1/(c^2*x^2) - 1))/c^3)*a^2*b + integrate(b^3*x^3*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))^3 + 3*a*b^2*x^3*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(b^3 x^3 \operatorname{arsech}(cx)^3 + 3 a b^2 x^3 \operatorname{arsech}(cx)^2 + 3 a^2 b x^3 \operatorname{arsech}(cx) + a^3 x^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*arcsech(c*x))^3,x, algorithm="fricas")

[Out] integral(b^3*x^3*arcsech(c*x)^3 + 3*a*b^2*x^3*arcsech(c*x)^2 + 3*a^2*b*x^3*arcsech(c*x) + a^3*x^3, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^3 (a + b \operatorname{asech}(cx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(a+b*asech(c*x))**3,x)

[Out] Integral(x**3*(a + b*asech(c*x))**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arsech}(cx) + a)^3 x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsech(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)^3*x^3, x)
```

3.43 $\int x^2 \left(a + b \operatorname{sech}^{-1}(cx) \right)^3 dx$

Optimal. Leaf size=242

$$\frac{ib^2 \operatorname{PolyLog}\left(2, -ie^{\operatorname{sech}^{-1}(cx)}\right) \left(a + b \operatorname{sech}^{-1}(cx)\right)}{c^3} - \frac{ib^2 \operatorname{PolyLog}\left(2, ie^{\operatorname{sech}^{-1}(cx)}\right) \left(a + b \operatorname{sech}^{-1}(cx)\right)}{c^3} - \frac{ib^3 \operatorname{PolyLog}\left(3, -ie^{\operatorname{sech}^{-1}(cx)}\right) \left(a + b \operatorname{sech}^{-1}(cx)\right)}{c^3}$$

```
[Out] -((b^2*x*(a + b*ArcSech[c*x]))/c^2) - (b*x*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x])^2)/(2*c^2) + (x^3*(a + b*ArcSech[c*x])^3)/3 - (b*(a + b*ArcSech[c*x])^2*ArcTan[E^ArcSech[c*x]])/c^3 + (b^3*ArcTan[(Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(c*x))]/c^3 + (I*b^2*(a + b*ArcSech[c*x])*PolyLog[2, (-I)*E^ArcSech[c*x]])/c^3 - (I*b^2*(a + b*ArcSech[c*x])*PolyLog[2, I*E^ArcSech[c*x]])/c^3 - (I*b^3*PolyLog[3, (-I)*E^ArcSech[c*x]])/c^3 + (I*b^3*PolyLog[3, I*E^ArcSech[c*x]])/c^3
```

Rubi [A] time = 0.195349, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6285, 5451, 4186, 3770, 4180, 2531, 2282, 6589}

$$\frac{ib^2 \operatorname{PolyLog}\left(2, -ie^{\operatorname{sech}^{-1}(cx)}\right) \left(a + b \operatorname{sech}^{-1}(cx)\right)}{c^3} - \frac{ib^2 \operatorname{PolyLog}\left(2, ie^{\operatorname{sech}^{-1}(cx)}\right) \left(a + b \operatorname{sech}^{-1}(cx)\right)}{c^3} - \frac{ib^3 \operatorname{PolyLog}\left(3, -ie^{\operatorname{sech}^{-1}(cx)}\right) \left(a + b \operatorname{sech}^{-1}(cx)\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcSech[c*x])^3,x]
```

```
[Out] -((b^2*x*(a + b*ArcSech[c*x]))/c^2) - (b*x*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x])^2)/(2*c^2) + (x^3*(a + b*ArcSech[c*x])^3)/3 - (b*(a + b*ArcSech[c*x])^2*ArcTan[E^ArcSech[c*x]])/c^3 + (b^3*ArcTan[(Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(c*x))]/c^3 + (I*b^2*(a + b*ArcSech[c*x])*PolyLog[2, (-I)*E^ArcSech[c*x]])/c^3 - (I*b^2*(a + b*ArcSech[c*x])*PolyLog[2, I*E^ArcSech[c*x]])/c^3 - (I*b^3*PolyLog[3, (-I)*E^ArcSech[c*x]])/c^3 + (I*b^3*PolyLog[3, I*E^ArcSech[c*x]])/c^3
```

Rule 6285

```
Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))^ (n_.)*(x_.)^(m_.), x_Symbol] :> -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_.))^(m_.)*Sech[(a_.) + (b_.)*(x_.)]^(n_.)*Tanh[(a_.) + (b_.)*(x_.)]^(p_.), x_Symbol] :> -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n), x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sech[a + b*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^ (n_.)*((c_.) + (d_.)*(x_.))^(m_.), x_Symbol] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
```

```
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m - 1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]] /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-I*e) + f*fz*x)/E^(I*k*Pi)]/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-I*e) + f*fz*x)/E^(I*k*Pi)], x], x] /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m - 1)*PolyLog[2, -(e*(F^(c*(a + b*x))))^n]], x], x] /; FreeQ[{F, a, b, c, e, f, g, n}, x] && GtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x], Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; FunctionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))* (F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int x^2 (a + b \operatorname{sech}^{-1}(cx))^3 dx &= -\frac{\operatorname{Subst}\left(\int (a + bx)^3 \operatorname{sech}^3(x) \tanh(x) dx, x, \operatorname{sech}^{-1}(cx)\right)}{c^3} \\
&= \frac{1}{3} x^3 (a + b \operatorname{sech}^{-1}(cx))^3 - \frac{b \operatorname{Subst}\left(\int (a + bx)^2 \operatorname{sech}^3(x) dx, x, \operatorname{sech}^{-1}(cx)\right)}{c^3} \\
&= -\frac{b^2 x (a + b \operatorname{sech}^{-1}(cx))}{c^2} - \frac{bx \sqrt{\frac{1-cx}{1+cx}} (1+cx) (a + b \operatorname{sech}^{-1}(cx))^2}{2c^2} + \frac{1}{3} x^3 (a + b \operatorname{sech}^{-1}(cx))^3 \\
&= -\frac{b^2 x (a + b \operatorname{sech}^{-1}(cx))}{c^2} - \frac{bx \sqrt{\frac{1-cx}{1+cx}} (1+cx) (a + b \operatorname{sech}^{-1}(cx))^2}{2c^2} + \frac{1}{3} x^3 (a + b \operatorname{sech}^{-1}(cx))^3 \\
&= -\frac{b^2 x (a + b \operatorname{sech}^{-1}(cx))}{c^2} - \frac{bx \sqrt{\frac{1-cx}{1+cx}} (1+cx) (a + b \operatorname{sech}^{-1}(cx))^2}{2c^2} + \frac{1}{3} x^3 (a + b \operatorname{sech}^{-1}(cx))^3 \\
&= -\frac{b^2 x (a + b \operatorname{sech}^{-1}(cx))}{c^2} - \frac{bx \sqrt{\frac{1-cx}{1+cx}} (1+cx) (a + b \operatorname{sech}^{-1}(cx))^2}{2c^2} + \frac{1}{3} x^3 (a + b \operatorname{sech}^{-1}(cx))^3 \\
&= -\frac{b^2 x (a + b \operatorname{sech}^{-1}(cx))}{c^2} - \frac{bx \sqrt{\frac{1-cx}{1+cx}} (1+cx) (a + b \operatorname{sech}^{-1}(cx))^2}{2c^2} + \frac{1}{3} x^3 (a + b \operatorname{sech}^{-1}(cx))^3
\end{aligned}$$

Mathematica [A] time = 1.01831, size = 440, normalized size = 1.82

$$-6ab^2 \left(-i \operatorname{PolyLog}\left(2, -ie^{-\operatorname{sech}^{-1}(cx)}\right) + i \operatorname{PolyLog}\left(2, ie^{-\operatorname{sech}^{-1}(cx)}\right) - c^3 x^3 \operatorname{sech}^{-1}(cx)^2 + cx + cx \sqrt{\frac{1-cx}{cx+1}} (cx+1) \operatorname{sech}^{-1}(cx) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2*(a + b*ArcSech[c*x])^3,x]

[Out] $(2a^3c^3x^3 - 3a^2bcx\sqrt{(1-cx)/(1+cx)}(1+cx) + 6a^2b^2c^3x^3\operatorname{ArcSech}[cx] + (3I)a^2b\operatorname{Log}[(-2I)cx + 2\sqrt{(1-cx)/(1+cx)}](1+cx) - 6ab^2(cx + cx\sqrt{(1-cx)/(1+cx)}(1+cx)\operatorname{ArcSech}[cx] - c^3x^3\operatorname{ArcSech}[cx]^2 - I\operatorname{ArcSech}[cx]\operatorname{Log}[1 - I/E^{\operatorname{ArcSech}[cx]}] + I\operatorname{ArcSech}[cx]\operatorname{Log}[1 + I/E^{\operatorname{ArcSech}[cx]}] - I\operatorname{PolyLog}[2, (-I)/E^{\operatorname{ArcSech}[cx]}] + I\operatorname{PolyLog}[2, I/E^{\operatorname{ArcSech}[cx]}]) - b^3(6cx\operatorname{ArcSech}[cx] + 3cx\sqrt{(1-cx)/(1+cx)}(1+cx)\operatorname{ArcSech}[cx]^2 - 2c^3x^3\operatorname{ArcSech}[cx]^3 - (3I)((-4I)\operatorname{ArcTan}[\operatorname{Tanh}[\operatorname{ArcSech}[cx]/2]] + \operatorname{ArcSech}[cx]^2\operatorname{Log}[1 - I/E^{\operatorname{ArcSech}[cx]}] - \operatorname{ArcSech}[cx]^2\operatorname{Log}[1 + I/E^{\operatorname{ArcSech}[cx]}] + 2\operatorname{ArcSech}[cx]\operatorname{PolyLog}[2, (-I)/E^{\operatorname{ArcSech}[cx]}] - 2\operatorname{ArcSech}[cx]\operatorname{PolyLog}[2, I/E^{\operatorname{ArcSech}[cx]}] + 2\operatorname{PolyLog}[3, (-I)/E^{\operatorname{ArcSech}[cx]}] - 2\operatorname{PolyLog}[3, I/E^{\operatorname{ArcSech}[cx]}])))/(6c^3)$

Maple [F] time = 0.49, size = 0, normalized size = 0.

$$\int x^2 (a + b \operatorname{arcsech}(cx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a+b*arcsech(c*x))^3,x)

[Out] $\int x^2 (a + b \operatorname{arcsech}(cx))^3 dx$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3} a^3 x^3 + \frac{1}{2} \left(2x^3 \operatorname{arsech}(cx) - \frac{\frac{\sqrt{\frac{1}{c^2 x^2} - 1}}{c^2} + \frac{\arctan\left(\sqrt{\frac{1}{c^2 x^2} - 1}\right)}{c^2}}{c} \right) a^2 b + \int b^3 x^2 \log\left(\sqrt{\frac{1}{cx} + 1} \sqrt{\frac{1}{cx} - 1} + \frac{1}{cx}\right)^3 + 3 ab^2 x^2 \log$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arcsech(c*x))^3,x, algorithm="maxima")`

[Out] $\frac{1}{3} a^3 x^3 + \frac{1}{2} (2x^3 \operatorname{arcsech}(cx) - (\sqrt{1/(c^2 x^2)} - 1)/(c^2 (1/(c^2 x^2) - 1) + c^2) + \arctan(\sqrt{1/(c^2 x^2)} - 1)/c^2)/c) a^2 b + \int (b^3 x^2 \log(\sqrt{1/(cx)} + 1) \sqrt{1/(cx)} - 1 + 1/(cx))^3 + 3 a b^2 x^2 \log(\sqrt{1/(cx)} + 1) \sqrt{1/(cx)} - 1 + 1/(cx))^2, x)$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\int (b^3 x^2 \operatorname{arsech}(cx)^3 + 3 ab^2 x^2 \operatorname{arsech}(cx)^2 + 3 a^2 b x^2 \operatorname{arsech}(cx) + a^3 x^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arcsech(c*x))^3,x, algorithm="fricas")`

[Out] $\int (b^3 x^2 \operatorname{arcsech}(cx)^3 + 3 a b^2 x^2 \operatorname{arcsech}(cx)^2 + 3 a^2 b x^2 \operatorname{arcsech}(cx) + a^3 x^2, x)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 (a + b \operatorname{asech}(cx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(a+b*asech(c*x))**3,x)`

[Out] `Integral(x**2*(a + b*asech(c*x))**3, x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arsech}(cx) + a)^3 x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arcsech(c*x))^3,x, algorithm="giac")`

[Out] `integrate((b*arcsech(c*x) + a)^3*x^2, x)`

3.44 $\int x \left(a + b \operatorname{sech}^{-1}(cx) \right)^3 dx$

Optimal. Leaf size=126

$$\frac{3b^3 \operatorname{PolyLog}\left(2, -e^{2\operatorname{sech}^{-1}(cx)}\right)}{2c^2} + \frac{3b^2 \log\left(e^{2\operatorname{sech}^{-1}(cx)} + 1\right) \left(a + b \operatorname{sech}^{-1}(cx)\right)}{c^2} - \frac{3b \sqrt{\frac{1-cx}{cx+1}} (cx+1) \left(a + b \operatorname{sech}^{-1}(cx)\right)^2}{2c^2}$$

[Out] $(-3*b*(a + b*\operatorname{ArcSech}[c*x])^2)/(2*c^2) - (3*b*\sqrt{(1 - c*x)/(1 + c*x)}*(1 + c*x)*(a + b*\operatorname{ArcSech}[c*x])^2)/(2*c^2) + (x^2*(a + b*\operatorname{ArcSech}[c*x])^3)/2 + (3*b^2*(a + b*\operatorname{ArcSech}[c*x])*Log[1 + E^{(2*\operatorname{ArcSech}[c*x])}])/c^2 + (3*b^3*\operatorname{PolyLog}[2, -E^{(2*\operatorname{ArcSech}[c*x])}])/(2*c^2)$

Rubi [A] time = 0.156583, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6285, 5451, 4184, 3718, 2190, 2279, 2391}

$$\frac{3b^3 \operatorname{PolyLog}\left(2, -e^{2\operatorname{sech}^{-1}(cx)}\right)}{2c^2} + \frac{3b^2 \log\left(e^{2\operatorname{sech}^{-1}(cx)} + 1\right) \left(a + b \operatorname{sech}^{-1}(cx)\right)}{c^2} - \frac{3b \sqrt{\frac{1-cx}{cx+1}} (cx+1) \left(a + b \operatorname{sech}^{-1}(cx)\right)^2}{2c^2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x*(a + b*\operatorname{ArcSech}[c*x])^3, x]$

[Out] $(-3*b*(a + b*\operatorname{ArcSech}[c*x])^2)/(2*c^2) - (3*b*\sqrt{(1 - c*x)/(1 + c*x)}*(1 + c*x)*(a + b*\operatorname{ArcSech}[c*x])^2)/(2*c^2) + (x^2*(a + b*\operatorname{ArcSech}[c*x])^3)/2 + (3*b^2*(a + b*\operatorname{ArcSech}[c*x])*Log[1 + E^{(2*\operatorname{ArcSech}[c*x])}])/c^2 + (3*b^3*\operatorname{PolyLog}[2, -E^{(2*\operatorname{ArcSech}[c*x])}])/(2*c^2)$

Rule 6285

$\operatorname{Int}[(a + \operatorname{ArcSech}(c*x))*(b*x)^n*(x)^m, x_Symbol] \rightarrow -\operatorname{Dist}[(c^{m+1})^{-1}, \operatorname{Subst}[\operatorname{Int}[(a + b*x)^n*\operatorname{Sech}[x]^{m+1}*\operatorname{Tanh}[x], x], x, \operatorname{ArcSech}[c*x]], x] /; \operatorname{FreeQ}\{a, b, c, x\} \ \&\& \ \operatorname{IntegerQ}[n] \ \&\& \ \operatorname{IntegerQ}[m] \ \&\& \ (\operatorname{GtQ}[n, 0] \ || \ \operatorname{LtQ}[m, -1])$

Rule 5451

$\operatorname{Int}[(c + d*x)^m*\operatorname{Sech}(a + b*x)^n*\operatorname{Tanh}(a + b*x)^p, x_Symbol] \rightarrow -\operatorname{Simp}[(c + d*x)^m*\operatorname{Sech}[a + b*x]^n/(b*n), x] + \operatorname{Dist}[(d*m)/(b*n), \operatorname{Int}[(c + d*x)^{m-1}*\operatorname{Sech}[a + b*x]^n, x], x] /; \operatorname{FreeQ}\{a, b, c, d, n\}, x \ \&\& \ \operatorname{EqQ}[p, 1] \ \&\& \ \operatorname{GtQ}[m, 0]$

Rule 4184

$\operatorname{Int}[\operatorname{csc}(e + f*x)^2*(c + d*x)^m, x_Symbol] \rightarrow -\operatorname{Simp}[(c + d*x)^m*\operatorname{Cot}[e + f*x]/f, x] + \operatorname{Dist}[(d*m)/f, \operatorname{Int}[(c + d*x)^{m-1}*\operatorname{Cot}[e + f*x], x], x] /; \operatorname{FreeQ}\{c, d, e, f, x\} \ \&\& \ \operatorname{GtQ}[m, 0]$

Rule 3718

$\operatorname{Int}[(c + d*x)^m*\operatorname{tan}(e + \operatorname{Complex}[0, fz]*f*x), x_Symbol] \rightarrow -\operatorname{Simp}[(c + d*x)^{m+1}/(d*(m+1)), x] + \operatorname{Dist}[2*I, \operatorname{Int}[(c + d*x)^m*\operatorname{E}^{(2*(-I*e) + f*fz*x})}/(1 + \operatorname{E}^{(2*(-I*e) + f*fz*x})}), x], x] /; \operatorname{FreeQ}\{c, d, e, f, fz, x\} \ \&\& \ \operatorname{IGtQ}[m, 0]$

Rule 2190

```
Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_)))^(n_))], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_)*(d_) + (e_)*(x_)^(n_)]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned} \int x (a + b \operatorname{sech}^{-1}(cx))^3 dx &= -\frac{\operatorname{Subst}\left(\int (a + bx)^3 \operatorname{sech}^2(x) \tanh(x) dx, x, \operatorname{sech}^{-1}(cx)\right)}{c^2} \\ &= \frac{1}{2} x^2 (a + b \operatorname{sech}^{-1}(cx))^3 - \frac{(3b) \operatorname{Subst}\left(\int (a + bx)^2 \operatorname{sech}^2(x) dx, x, \operatorname{sech}^{-1}(cx)\right)}{2c^2} \\ &= -\frac{3b \sqrt{\frac{1-cx}{1+cx}} (1+cx) (a + b \operatorname{sech}^{-1}(cx))^2}{2c^2} + \frac{1}{2} x^2 (a + b \operatorname{sech}^{-1}(cx))^3 + \frac{(3b^2) \operatorname{Subst}\left(\int (a + bx) \operatorname{sech}^2(x) dx, x, \operatorname{sech}^{-1}(cx)\right)}{2c^2} \\ &= -\frac{3b (a + b \operatorname{sech}^{-1}(cx))^2}{2c^2} - \frac{3b \sqrt{\frac{1-cx}{1+cx}} (1+cx) (a + b \operatorname{sech}^{-1}(cx))^2}{2c^2} + \frac{1}{2} x^2 (a + b \operatorname{sech}^{-1}(cx))^3 - \frac{(3b^2) \operatorname{Subst}\left(\int (a + bx) \operatorname{sech}^2(x) dx, x, \operatorname{sech}^{-1}(cx)\right)}{2c^2} \\ &= -\frac{3b (a + b \operatorname{sech}^{-1}(cx))^2}{2c^2} - \frac{3b \sqrt{\frac{1-cx}{1+cx}} (1+cx) (a + b \operatorname{sech}^{-1}(cx))^2}{2c^2} + \frac{1}{2} x^2 (a + b \operatorname{sech}^{-1}(cx))^3 - \frac{(3b^2) \operatorname{Subst}\left(\int (a + bx) \operatorname{sech}^2(x) dx, x, \operatorname{sech}^{-1}(cx)\right)}{2c^2} \\ &= -\frac{3b (a + b \operatorname{sech}^{-1}(cx))^2}{2c^2} - \frac{3b \sqrt{\frac{1-cx}{1+cx}} (1+cx) (a + b \operatorname{sech}^{-1}(cx))^2}{2c^2} + \frac{1}{2} x^2 (a + b \operatorname{sech}^{-1}(cx))^3 - \frac{(3b^2) \operatorname{Subst}\left(\int (a + bx) \operatorname{sech}^2(x) dx, x, \operatorname{sech}^{-1}(cx)\right)}{2c^2} \\ &= -\frac{3b (a + b \operatorname{sech}^{-1}(cx))^2}{2c^2} - \frac{3b \sqrt{\frac{1-cx}{1+cx}} (1+cx) (a + b \operatorname{sech}^{-1}(cx))^2}{2c^2} + \frac{1}{2} x^2 (a + b \operatorname{sech}^{-1}(cx))^3 - \frac{(3b^2) \operatorname{Subst}\left(\int (a + bx) \operatorname{sech}^2(x) dx, x, \operatorname{sech}^{-1}(cx)\right)}{2c^2} \end{aligned}$$

Mathematica [A] time = 0.867704, size = 219, normalized size = 1.74

$$\frac{-3b^3 \operatorname{PolyLog}\left(2, -e^{-2 \operatorname{sech}^{-1}(cx)}\right) + a \left(a \left(ac^2 x^2 - 3b \sqrt{\frac{1-cx}{cx+1}} (cx+1) \right) + 6b^2 \log\left(\frac{1}{cx}\right) \right) - 3b^2 \operatorname{sech}^{-1}(cx)^2 \left(b \left(cx \sqrt{\frac{1-cx}{cx+1}} + \sqrt{\frac{1-cx}{cx+1}} \right) \right)}{2c^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x*(a + b*ArcSech[c*x])^3, x]

[Out] (-3*b^2*(-(a*c^2*x^2) + b*(-1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]))*ArcSech[c*x]^2 + b^3*c^2*x^2*ArcSech[c*x]^3 + 3*b*ArcSech[c*x]*(a*(a*c^2*x^2 - 2*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)) + 2*b^2*Log[1 + E^(-2*ArcSech[c*x])]) + a*(a*(a*c^2*x^2 - 3*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)) + 6*b^2*Log[1/(c*x)]) - 3*b^3*PolyLog[2, -E^(-2*ArcSech[c*x])])/(2*c^2)

Maple [B] time = 0.302, size = 343, normalized size = 2.7

$$\frac{x^2 a^3}{2} - \frac{3 b^3 (\operatorname{arcsech}(cx))^2 x}{2c} \sqrt{\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} + \frac{x^2 b^3 (\operatorname{arcsech}(cx))^3}{2} - \frac{3 b^3 (\operatorname{arcsech}(cx))^2}{2c^2} + 3 \frac{b^3 \operatorname{arcsech}(cx)}{c^2} \ln$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a+b*arcsech(c*x))^3,x)

[Out] $\frac{1}{2}x^2a^3 - \frac{3}{2} \frac{b^3 \operatorname{arcsech}(cx)^2 x}{c} \sqrt{\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} + \frac{x^2 b^3 \operatorname{arcsech}(cx)^3}{2} - \frac{3 b^3 \operatorname{arcsech}(cx)^2}{2c^2} + 3 \frac{b^3 \operatorname{arcsech}(cx)}{c^2} \ln(1 + \frac{1}{c/x + (-1 + 1/c/x)^{1/2}} \frac{(1 + 1/c/x)^{1/2}}{(1 + 1/c/x + (-1 + 1/c/x)^{1/2})^2}) + \frac{3}{2} b^3 \operatorname{polylog}(2, -\frac{1}{c/x + (-1 + 1/c/x)^{1/2}} \frac{(1 + 1/c/x)^{1/2}}{(1 + 1/c/x + (-1 + 1/c/x)^{1/2})^2}) - \frac{3}{c^2} a^2 b^2 \operatorname{arcsech}(cx) - \frac{3}{c} a^2 b^2 \sqrt{\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} \operatorname{arcsech}(cx) + \frac{3}{2} x^2 a^2 b^2 \operatorname{arcsech}(cx)^2 + \frac{3}{c^2} a^2 b^2 \ln(1 + \frac{1}{c/x + (-1 + 1/c/x)^{1/2}} \frac{(1 + 1/c/x)^{1/2}}{(1 + 1/c/x + (-1 + 1/c/x)^{1/2})^2}) + \frac{3}{2} x^2 a^2 b^2 \operatorname{arcsech}(cx) - \frac{3}{2} \frac{a^2 b^2}{c} \sqrt{\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} x$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3}{2} ab^2 x^2 \operatorname{arsech}(cx)^2 + \frac{1}{2} a^3 x^2 + \frac{3}{2} \left(x^2 \operatorname{arsech}(cx) - \frac{x \sqrt{\frac{1}{c^2 x^2} - 1}}{c} \right) a^2 b - 3 \left(\frac{x \sqrt{\frac{1}{c^2 x^2} - 1} \operatorname{arsech}(cx)}{c} + \frac{\log(x)}{c^2} \right) ab^2 + b^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arcsech(c*x))^3,x, algorithm="maxima")

[Out] $\frac{3}{2} a^2 b^2 x^2 \operatorname{arcsech}(cx)^2 + \frac{1}{2} a^3 x^2 + \frac{3}{2} (x^2 \operatorname{arcsech}(cx) - x \sqrt{\frac{1}{c^2 x^2} - 1}) a^2 b - 3 (x \sqrt{\frac{1}{c^2 x^2} - 1} \operatorname{arcsech}(cx) / c + \log(x) / c^2) a^2 b^2 + b^3 \operatorname{integrate}(x \log(\sqrt{\frac{1}{c^2 x^2} + 1}) \sqrt{\frac{1}{c^2 x^2} - 1} + \frac{1}{c^2 x^2})^3, x$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(b^3 x \operatorname{arsech}(cx)^3 + 3 ab^2 x \operatorname{arsech}(cx)^2 + 3 a^2 b x \operatorname{arsech}(cx) + a^3 x, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arcsech(c*x))^3,x, algorithm="fricas")

[Out] $\operatorname{integral}(b^3 x \operatorname{arcsech}(cx)^3 + 3 a^2 b^2 x \operatorname{arcsech}(cx)^2 + 3 a^2 b x \operatorname{arcsech}(cx) + a^3 x, x)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x (a + b \operatorname{asech}(cx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asech(c*x))**3,x)
```

```
[Out] Integral(x*(a + b*asech(c*x))**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arsech}(cx) + a)^3 x dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsech(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)^3*x, x)
```

3.45 $\int (a + b \operatorname{sech}^{-1}(cx))^3 dx$

Optimal. Leaf size=140

$$\frac{6ib^2 \operatorname{PolyLog}\left(2, -ie^{\operatorname{sech}^{-1}(cx)}\right)(a + b \operatorname{sech}^{-1}(cx))}{c} - \frac{6ib^2 \operatorname{PolyLog}\left(2, ie^{\operatorname{sech}^{-1}(cx)}\right)(a + b \operatorname{sech}^{-1}(cx))}{c} - \frac{6ib^3 \operatorname{PolyLog}\left(3, (-I)E^{\operatorname{sech}^{-1}(cx)}\right)}{c}$$

```
[Out] x*(a + b*ArcSech[c*x])^3 - (6*b*(a + b*ArcSech[c*x])^2*ArcTan[E^ArcSech[c*x]])/c + ((6*I)*b^2*(a + b*ArcSech[c*x])*PolyLog[2, (-I)*E^ArcSech[c*x]])/c - ((6*I)*b^2*(a + b*ArcSech[c*x])*PolyLog[2, I*E^ArcSech[c*x]])/c - ((6*I)*b^3*PolyLog[3, (-I)*E^ArcSech[c*x]])/c + ((6*I)*b^3*PolyLog[3, I*E^ArcSech[c*x]])/c
```

Rubi [A] time = 0.111417, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$, Rules used = {6279, 5451, 4180, 2531, 2282, 6589}

$$\frac{6ib^2 \operatorname{PolyLog}\left(2, -ie^{\operatorname{sech}^{-1}(cx)}\right)(a + b \operatorname{sech}^{-1}(cx))}{c} - \frac{6ib^2 \operatorname{PolyLog}\left(2, ie^{\operatorname{sech}^{-1}(cx)}\right)(a + b \operatorname{sech}^{-1}(cx))}{c} - \frac{6ib^3 \operatorname{PolyLog}\left(3, (-I)E^{\operatorname{sech}^{-1}(cx)}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])^3, x]
```

```
[Out] x*(a + b*ArcSech[c*x])^3 - (6*b*(a + b*ArcSech[c*x])^2*ArcTan[E^ArcSech[c*x]])/c + ((6*I)*b^2*(a + b*ArcSech[c*x])*PolyLog[2, (-I)*E^ArcSech[c*x]])/c - ((6*I)*b^2*(a + b*ArcSech[c*x])*PolyLog[2, I*E^ArcSech[c*x]])/c - ((6*I)*b^3*PolyLog[3, (-I)*E^ArcSech[c*x]])/c + ((6*I)*b^3*PolyLog[3, I*E^ArcSech[c*x]])/c
```

Rule 6279

```
Int[((a_.) + ArcSech[(c_.)*(x_)])*(b_.))^n_, x_Symbol] := -Dist[c^(-1), Subst[Int[(a + b*x)^n*Sech[x]*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[n, 0]
```

Rule 5451

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) + (b_.)*(x_)]^(n_.)*Tanh[(a_.) + (b_.)*(x_)]^(p_.), x_Symbol] := -Simp[((c + d*x)^m*Sech[a + b*x]^n)/(b*n), x] + Dist[(d*m)/(b*n), Int[(c + d*x)^(m-1)*Sech[a + b*x]^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] := Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m-1)*Log[1 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m-1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] := -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x))))]
```

```

)))^n)]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]

```

Rule 2282

```

Int[u_, x_Symbol] := With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

```

Rule 6589

```

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]

```

Rubi steps

$$\begin{aligned}
\int (a + b \operatorname{sech}^{-1}(cx))^3 dx &= -\frac{\operatorname{Subst}\left(\int (a + bx)^3 \operatorname{sech}(x) \tanh(x) dx, x, \operatorname{sech}^{-1}(cx)\right)}{c} \\
&= x(a + b \operatorname{sech}^{-1}(cx))^3 - \frac{(3b) \operatorname{Subst}\left(\int (a + bx)^2 \operatorname{sech}(x) dx, x, \operatorname{sech}^{-1}(cx)\right)}{c} \\
&= x(a + b \operatorname{sech}^{-1}(cx))^3 - \frac{6b(a + b \operatorname{sech}^{-1}(cx))^2 \tan^{-1}\left(e^{\operatorname{sech}^{-1}(cx)}\right)}{c} + \frac{(6ib^2) \operatorname{Subst}\left(\int (a + bx) dx, x, \operatorname{sech}^{-1}(cx)\right)}{c} \\
&= x(a + b \operatorname{sech}^{-1}(cx))^3 - \frac{6b(a + b \operatorname{sech}^{-1}(cx))^2 \tan^{-1}\left(e^{\operatorname{sech}^{-1}(cx)}\right)}{c} + \frac{6ib^2(a + b \operatorname{sech}^{-1}(cx)) \operatorname{Li}_2\left(-e^{-\operatorname{sech}^{-1}(cx)}\right)}{c} \\
&= x(a + b \operatorname{sech}^{-1}(cx))^3 - \frac{6b(a + b \operatorname{sech}^{-1}(cx))^2 \tan^{-1}\left(e^{\operatorname{sech}^{-1}(cx)}\right)}{c} + \frac{6ib^2(a + b \operatorname{sech}^{-1}(cx)) \operatorname{Li}_2\left(-e^{-\operatorname{sech}^{-1}(cx)}\right)}{c} \\
&= x(a + b \operatorname{sech}^{-1}(cx))^3 - \frac{6b(a + b \operatorname{sech}^{-1}(cx))^2 \tan^{-1}\left(e^{\operatorname{sech}^{-1}(cx)}\right)}{c} + \frac{6ib^2(a + b \operatorname{sech}^{-1}(cx)) \operatorname{Li}_2\left(-e^{-\operatorname{sech}^{-1}(cx)}\right)}{c}
\end{aligned}$$

Mathematica [B] time = 0.408751, size = 282, normalized size = 2.01

$$\frac{3iab^2 \left(2 \operatorname{PolyLog}\left(2, -ie^{-\operatorname{sech}^{-1}(cx)}\right) - 2 \operatorname{PolyLog}\left(2, ie^{-\operatorname{sech}^{-1}(cx)}\right) + \operatorname{sech}^{-1}(cx) \left(-icx \operatorname{sech}^{-1}(cx) + 2 \log\left(1 - ie^{-\operatorname{sech}^{-1}(cx)}\right) \right) \right)}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSech[c*x])^3, x]
```

```
[Out] a^3*x + 3*a^2*b*x*ArcSech[c*x] - (3*a^2*b*ArcTan[(c*x*Sqrt[(1 - c*x)/(1 + c
*x))]/(-1 + c*x)]/c + ((3*I)*a*b^2*(ArcSech[c*x]*((-I)*c*x*ArcSech[c*x] +
2*Log[1 - I/E^ArcSech[c*x]] - 2*Log[1 + I/E^ArcSech[c*x]]) + 2*PolyLog[2, (
-I)/E^ArcSech[c*x]] - 2*PolyLog[2, I/E^ArcSech[c*x]]))/c + (b^3*(c*x*ArcSec
h[c*x]^3 - (3*I)*(-(ArcSech[c*x]^2*(Log[1 - I/E^ArcSech[c*x]] - Log[1 + I/E
^ArcSech[c*x]])) - 2*ArcSech[c*x]*(PolyLog[2, (-I)/E^ArcSech[c*x]] - PolyLo
g[2, I/E^ArcSech[c*x]]) - 2*(PolyLog[3, (-I)/E^ArcSech[c*x]] - PolyLog[3, I
/E^ArcSech[c*x]]))))/c

```

Maple [F] time = 0.329, size = 0, normalized size = 0.

$$\int (a + b \operatorname{arcsech}(cx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))^3,x)

[Out] int((a+b*arcsech(c*x))^3,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))^3,x, algorithm="maxima")

[Out] $b^3 x \log(\sqrt{cx+1} \sqrt{-cx+1} + 1)^3 + a^3 x + 3(c x \operatorname{arcsech}(c x) - \arctan(\sqrt{1/(c^2 x^2) - 1})) a^2 b/c - \int (b^3 \log(c)^3 - 3 a b^2 \log(c)^2 - (b^3 c^2 x^2 - b^3) \log(x)^3 - (b^3 c^2 \log(c)^3 - 3 a b^2 c^2 \log(c)^2) x^2 + 3(b^3 \log(c) - a b^2 - (b^3 c^2 \log(c) - a b^2 c^2) x^2 + (b^3 \log(c) - a b^2 - (b^3 c^2 (\log(c) + 1) - a b^2 c^2) x^2 - (b^3 c^2 x^2 - b^3) \log(x)) \sqrt{cx+1} \sqrt{-cx+1} - (b^3 c^2 x^2 - b^3) \log(x)) \log(\sqrt{cx+1} \sqrt{-cx+1} + 1)^2 + 3(b^3 \log(c) - a b^2 - (b^3 c^2 \log(c) - a b^2 c^2) x^2) \log(x)^2 + (b^3 \log(c)^3 - 3 a b^2 \log(c)^2 - (b^3 c^2 x^2 - b^3) \log(x)^3 - (b^3 c^2 \log(c)^3 - 3 a b^2 c^2 \log(c)^2) x^2 + 3(b^3 \log(c) - a b^2 - (b^3 c^2 \log(c) - a b^2 c^2) x^2) \log(x)^2 + 3(b^3 \log(c)^2 - 2 a b^2 \log(c) - (b^3 c^2 \log(c)^2 - 2 a b^2 c^2 \log(c)) x^2) \log(x) \sqrt{cx+1} \sqrt{-cx+1} - 3(b^3 \log(c)^2 - 2 a b^2 \log(c) - (b^3 c^2 \log(c)^2 - 2 a b^2 c^2 \log(c)) x^2 - (b^3 c^2 x^2 - b^3) \log(x)^2 + (b^3 \log(c)^2 - 2 a b^2 \log(c) - (b^3 c^2 \log(c)^2 - 2 a b^2 c^2 \log(c)) x^2 - (b^3 c^2 x^2 - b^3) \log(x)^2 + 2(b^3 \log(c) - a b^2 - (b^3 c^2 \log(c) - a b^2 c^2) x^2) \log(x)) \sqrt{cx+1} \sqrt{-cx+1} + 2(b^3 \log(c) - a b^2 - (b^3 c^2 \log(c) - a b^2 c^2) x^2) \log(x)) \log(\sqrt{cx+1} \sqrt{-cx+1} + 1) + 3(b^3 \log(c)^2 - 2 a b^2 \log(c) - (b^3 c^2 \log(c)^2 - 2 a b^2 c^2 \log(c)) x^2) \log(x)) / (c^2 x^2 + (c^2 x^2 - 1) \sqrt{cx+1} \sqrt{-cx+1} - 1), x$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}(b^3 \operatorname{arsh}(cx)^3 + 3 a b^2 \operatorname{arsh}(cx)^2 + 3 a^2 b \operatorname{arsh}(cx) + a^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))^3,x, algorithm="fricas")

[Out] integral(b^3*arcsech(c*x)^3 + 3*a*b^2*arcsech(c*x)^2 + 3*a^2*b*arcsech(c*x) + a^3, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{asech}(cx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asech(c*x))**3,x)

[Out] Integral((a + b*asech(c*x))**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arsech}(cx) + a)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))^3,x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)^3, x)

$$3.46 \quad \int \frac{(a + b \operatorname{sech}^{-1}(cx))^3}{x} dx$$

Optimal. Leaf size=114

$$\frac{3}{2}b^2 \operatorname{PolyLog}\left(3, -e^{2\operatorname{sech}^{-1}(cx)}\right)(a + b \operatorname{sech}^{-1}(cx)) - \frac{3}{2}b \operatorname{PolyLog}\left(2, -e^{2\operatorname{sech}^{-1}(cx)}\right)(a + b \operatorname{sech}^{-1}(cx))^2 - \frac{3}{4}b^3 \operatorname{PolyLog}\left(4, -e^{2\operatorname{sech}^{-1}(cx)}\right)(a + b \operatorname{sech}^{-1}(cx))^3$$

[Out] (a + b*ArcSech[c*x])^4/(4*b) - (a + b*ArcSech[c*x])^3*Log[1 + E^(2*ArcSech[c*x])] - (3*b*(a + b*ArcSech[c*x])^2*PolyLog[2, -E^(2*ArcSech[c*x])])/2 + (3*b^2*(a + b*ArcSech[c*x])*PolyLog[3, -E^(2*ArcSech[c*x])])/2 - (3*b^3*PolyLog[4, -E^(2*ArcSech[c*x])])/4

Rubi [A] time = 0.14476, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6285, 3718, 2190, 2531, 6609, 2282, 6589}

$$\frac{3}{2}b^2 \operatorname{PolyLog}\left(3, -e^{2\operatorname{sech}^{-1}(cx)}\right)(a + b \operatorname{sech}^{-1}(cx)) - \frac{3}{2}b \operatorname{PolyLog}\left(2, -e^{2\operatorname{sech}^{-1}(cx)}\right)(a + b \operatorname{sech}^{-1}(cx))^2 - \frac{3}{4}b^3 \operatorname{PolyLog}\left(4, -e^{2\operatorname{sech}^{-1}(cx)}\right)(a + b \operatorname{sech}^{-1}(cx))^3$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcSech[c*x])^3/x, x]

[Out] (a + b*ArcSech[c*x])^4/(4*b) - (a + b*ArcSech[c*x])^3*Log[1 + E^(2*ArcSech[c*x])] - (3*b*(a + b*ArcSech[c*x])^2*PolyLog[2, -E^(2*ArcSech[c*x])])/2 + (3*b^2*(a + b*ArcSech[c*x])*PolyLog[3, -E^(2*ArcSech[c*x])])/2 - (3*b^3*PolyLog[4, -E^(2*ArcSech[c*x])])/4

Rule 6285

Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))^n*(x_)^m, x_Symbol] :> -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

Rule 3718

Int[((c_.) + (d_.)*(x_)^m)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c + d*x)^m*E^(2*(-I*e) + f*fz*x))/(1 + E^(2*(-I*e) + f*fz*x)), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

Rule 2190

Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^n)*((c_.) + (d_.)*(x_)^m)/((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^n), x_Symbol] :> Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2531

Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^n]*((f_.) + (g_.)*(x_)^m), x_Symbol] :> -Simp[(f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x]

```

)))^n)]/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]

```

Rule 6609

```

Int[((e_.) + (f_.)*(x_.))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_.)))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^(
m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
d, e, f, n, p}, x] && GtQ[m, 0]

```

Rule 2282

```

Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_)] /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_) [v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]

```

Rule 6589

```

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_.))^(p_.)]/((d_.) + (e_.)*(x_.)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]

```

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \operatorname{sech}^{-1}(cx))^3}{x} dx &= -\operatorname{Subst}\left(\int (a + bx)^3 \tanh(x) dx, x, \operatorname{sech}^{-1}(cx)\right) \\
&= \frac{(a + b \operatorname{sech}^{-1}(cx))^4}{4b} - 2 \operatorname{Subst}\left(\int \frac{e^{2x}(a + bx)^3}{1 + e^{2x}} dx, x, \operatorname{sech}^{-1}(cx)\right) \\
&= \frac{(a + b \operatorname{sech}^{-1}(cx))^4}{4b} - (a + b \operatorname{sech}^{-1}(cx))^3 \log\left(1 + e^{2 \operatorname{sech}^{-1}(cx)}\right) + (3b) \operatorname{Subst}\left(\int (a + bx)^2 \log\right) \\
&= \frac{(a + b \operatorname{sech}^{-1}(cx))^4}{4b} - (a + b \operatorname{sech}^{-1}(cx))^3 \log\left(1 + e^{2 \operatorname{sech}^{-1}(cx)}\right) - \frac{3}{2}b (a + b \operatorname{sech}^{-1}(cx))^2 \operatorname{Li}_2\left(\right) \\
&= \frac{(a + b \operatorname{sech}^{-1}(cx))^4}{4b} - (a + b \operatorname{sech}^{-1}(cx))^3 \log\left(1 + e^{2 \operatorname{sech}^{-1}(cx)}\right) - \frac{3}{2}b (a + b \operatorname{sech}^{-1}(cx))^2 \operatorname{Li}_2\left(\right) \\
&= \frac{(a + b \operatorname{sech}^{-1}(cx))^4}{4b} - (a + b \operatorname{sech}^{-1}(cx))^3 \log\left(1 + e^{2 \operatorname{sech}^{-1}(cx)}\right) - \frac{3}{2}b (a + b \operatorname{sech}^{-1}(cx))^2 \operatorname{Li}_2\left(\right) \\
&= \frac{(a + b \operatorname{sech}^{-1}(cx))^4}{4b} - (a + b \operatorname{sech}^{-1}(cx))^3 \log\left(1 + e^{2 \operatorname{sech}^{-1}(cx)}\right) - \frac{3}{2}b (a + b \operatorname{sech}^{-1}(cx))^2 \operatorname{Li}_2\left(\right)
\end{aligned}$$

Mathematica [A] time = 0.229202, size = 182, normalized size = 1.6

$$\frac{1}{4} \left(6b^2 \operatorname{PolyLog}\left(3, -e^{-2 \operatorname{sech}^{-1}(cx)}\right) (a + b \operatorname{sech}^{-1}(cx)) + 6b \operatorname{PolyLog}\left(2, -e^{-2 \operatorname{sech}^{-1}(cx)}\right) (a + b \operatorname{sech}^{-1}(cx))^2 + 3b^3 \operatorname{PolyLog}\left(1, -e^{-2 \operatorname{sech}^{-1}(cx)}\right) (a + b \operatorname{sech}^{-1}(cx))^3 \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcSech[c*x])^3/x, x]
```



```
[Out] (-6*a^2*b*ArcSech[c*x]^2 - 4*a*b^2*ArcSech[c*x]^3 - b^3*ArcSech[c*x]^4 - 12
*a^2*b*ArcSech[c*x]*Log[1 + E^(-2*ArcSech[c*x])] - 12*a*b^2*ArcSech[c*x]^2*
Log[1 + E^(-2*ArcSech[c*x])] - 4*b^3*ArcSech[c*x]^3*Log[1 + E^(-2*ArcSech[c
*x])]) + 4*a^3*Log[c*x] + 6*b*(a + b*ArcSech[c*x])^2*PolyLog[2, -E^(-2*ArcSe
ch[c*x])] + 6*b^2*(a + b*ArcSech[c*x])*PolyLog[3, -E^(-2*ArcSech[c*x])] + 3
*b^3*PolyLog[4, -E^(-2*ArcSech[c*x])])/4
```

Maple [B] time = 0.272, size = 454, normalized size = 4.

$$a^3 \ln(cx) + \frac{b^3 (\operatorname{arcsech}(cx))^4}{4} - b^3 (\operatorname{arcsech}(cx))^3 \ln\left(1 + \left(\frac{1}{cx} + \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}\right)^2\right) - \frac{3b^3 (\operatorname{arcsech}(cx))^2}{2} \operatorname{polylog}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))^3/x,x)
```

```
[Out] a^3*ln(c*x)+1/4*b^3*arcsech(c*x)^4-b^3*arcsech(c*x)^3*ln(1+(1/c/x+(-1+1/c/x
)^(1/2)*(1+1/c/x)^(1/2))^2)-3/2*b^3*arcsech(c*x)^2*polylog(2,-(1/c/x+(-1+1/
c/x)^(1/2)*(1+1/c/x)^(1/2))^2)+3/2*b^3*arcsech(c*x)*polylog(3,-(1/c/x+(-1+1
/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)-3/4*b^3*polylog(4,-(1/c/x+(-1+1/c/x)^(1/2)*
(1+1/c/x)^(1/2))^2)+a*b^2*arcsech(c*x)^3-3*a*b^2*arcsech(c*x)^2*ln(1+(1/c/x
+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)-3*a*b^2*arcsech(c*x)*polylog(2,-(1/c/
x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)+3/2*a*b^2*polylog(3,-(1/c/x+(-1+1/c/
x)^(1/2)*(1+1/c/x)^(1/2))^2)+3/2*a^2*b*arcsech(c*x)^2-3*a^2*b*arcsech(c*x)*
ln(1+(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)-3/2*a^2*b*polylog(2,-(1/c/
x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$a^3 \log(x) + \int \frac{b^3 \log\left(\sqrt{\frac{1}{cx} + 1} \sqrt{\frac{1}{cx} - 1} + \frac{1}{cx}\right)^3}{x} + \frac{3ab^2 \log\left(\sqrt{\frac{1}{cx} + 1} \sqrt{\frac{1}{cx} - 1} + \frac{1}{cx}\right)^2}{x} + \frac{3a^2b \log\left(\sqrt{\frac{1}{cx} + 1} \sqrt{\frac{1}{cx} - 1} + \frac{1}{cx}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^3/x,x, algorithm="maxima")
```

```
[Out] a^3*log(x) + integrate(b^3*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x
))^3/x + 3*a*b^2*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))^2/x + 3
*a^2*b*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b^3 \operatorname{arsech}(cx)^3 + 3ab^2 \operatorname{arsech}(cx)^2 + 3a^2b \operatorname{arsech}(cx) + a^3}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*arcsech(c*x)^3 + 3*a*b^2*arcsech(c*x)^2 + 3*a^2*b*arcsech(c*x
) + a^3)/x, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{asech}(cx))^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))**3/x,x)
```

```
[Out] Integral((a + b*asech(c*x))**3/x, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsech}(cx) + a)^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)^3/x, x)
```

$$3.47 \quad \int \frac{(a + b \operatorname{sech}^{-1}(cx))^3}{x^2} dx$$

Optimal. Leaf size=102

$$\frac{6b^2(a + b \operatorname{sech}^{-1}(cx))}{x} + \frac{3b\sqrt{\frac{1-cx}{cx+1}}(cx+1)(a + b \operatorname{sech}^{-1}(cx))^2}{x} - \frac{(a + b \operatorname{sech}^{-1}(cx))^3}{x} + \frac{6b^3\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{x}$$

[Out] (6*b^3*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/x - (6*b^2*(a + b*ArcSech[c*x])/x + (3*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x])^2)/x - (a + b*ArcSech[c*x])^3/x

Rubi [A] time = 0.102711, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {6285, 3296, 2637}

$$\frac{6b^2(a + b \operatorname{sech}^{-1}(cx))}{x} + \frac{3b\sqrt{\frac{1-cx}{cx+1}}(cx+1)(a + b \operatorname{sech}^{-1}(cx))^2}{x} - \frac{(a + b \operatorname{sech}^{-1}(cx))^3}{x} + \frac{6b^3\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{x}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcSech[c*x])^3/x^2, x]

[Out] (6*b^3*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/x - (6*b^2*(a + b*ArcSech[c*x])/x + (3*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x])^2)/x - (a + b*ArcSech[c*x])^3/x

Rule 6285

Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))^n_.*(x_.)^m_.], x_Symbol] := -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

Rule 3296

Int[((c_.) + (d_.)*(x_.))^m_.*sin[(e_.) + (f_.)*(x_.)], x_Symbol] := -Simp[((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \frac{(a + b \operatorname{sech}^{-1}(cx))^3}{x^2} dx &= -\left(c \operatorname{Subst} \left(\int (a + bx)^3 \sinh(x) dx, x, \operatorname{sech}^{-1}(cx) \right)\right) \\
&= -\frac{(a + b \operatorname{sech}^{-1}(cx))^3}{x} + (3bc) \operatorname{Subst} \left(\int (a + bx)^2 \cosh(x) dx, x, \operatorname{sech}^{-1}(cx) \right) \\
&= \frac{3b \sqrt{\frac{1-cx}{1+cx}} (1+cx) (a + b \operatorname{sech}^{-1}(cx))^2}{x} - \frac{(a + b \operatorname{sech}^{-1}(cx))^3}{x} - (6b^2c) \operatorname{Subst} \left(\int (a + bx) \sinh(x) dx, x, \operatorname{sech}^{-1}(cx) \right) \\
&= -\frac{6b^2 (a + b \operatorname{sech}^{-1}(cx))}{x} + \frac{3b \sqrt{\frac{1-cx}{1+cx}} (1+cx) (a + b \operatorname{sech}^{-1}(cx))^2}{x} - \frac{(a + b \operatorname{sech}^{-1}(cx))^3}{x} + (6b^3c) \operatorname{Subst} \left(\int \sinh(x) dx, x, \operatorname{sech}^{-1}(cx) \right) \\
&= \frac{6b^3 \sqrt{\frac{1-cx}{1+cx}} (1+cx)}{x} - \frac{6b^2 (a + b \operatorname{sech}^{-1}(cx))}{x} + \frac{3b \sqrt{\frac{1-cx}{1+cx}} (1+cx) (a + b \operatorname{sech}^{-1}(cx))^2}{x} - \frac{(a + b \operatorname{sech}^{-1}(cx))^3}{x}
\end{aligned}$$

Mathematica [A] time = 0.321964, size = 165, normalized size = 1.62

$$\frac{3b \operatorname{sech}^{-1}(cx) \left(a^2 - 2ab \sqrt{\frac{1-cx}{cx+1}} (cx+1) + 2b^2 \right) - 3a^2 b \sqrt{\frac{1-cx}{cx+1}} (cx+1) + a^3 - 3b^2 \operatorname{sech}^{-1}(cx)^2 \left(b \sqrt{\frac{1-cx}{cx+1}} (cx+1) - a \right) + 6ab^2}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcSech[c*x])^3/x^2,x]

[Out] -((a^3 + 6*a*b^2 - 3*a^2*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) - 6*b^3*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + 3*b*(a^2 + 2*b^2 - 2*a*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))*ArcSech[c*x] - 3*b^2*(-a + b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))*ArcSech[c*x]^2 + b^3*ArcSech[c*x]^3)/x)

Maple [B] time = 0.252, size = 227, normalized size = 2.2

$$c \left(-\frac{a^3}{cx} + b^3 \left(-\frac{(\operatorname{arcsech}(cx))^3}{cx} + 3 (\operatorname{arcsech}(cx))^2 \sqrt{-\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} - 6 \frac{\operatorname{arcsech}(cx)}{cx} + 6 \sqrt{-\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} \right) + 3 ab^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))^3/x^2,x)

[Out] c*(-a^3/c/x+b^3*(-arcsech(c*x)^3/c/x+3*arcsech(c*x)^2*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)-6/c/x*arcsech(c*x)+6*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2))+3*a*b^2*(-arcsech(c*x)^2/c/x+2*arcsech(c*x)*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)-2/c/x)+3*a^2*b*(-1/c/x*arcsech(c*x)+(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)))

Maxima [A] time = 1.03547, size = 194, normalized size = 1.9

$$-\frac{b^3 \operatorname{arosech}(cx)^3}{x} + 3 \left(c \sqrt{\frac{1}{c^2 x^2}} - 1 - \frac{\operatorname{arosech}(cx)}{x} \right) a^2 b + 6 \left(c \sqrt{\frac{1}{c^2 x^2}} - 1 \operatorname{arosech}(cx) - \frac{1}{x} \right) ab^2 + 3 \left(c \sqrt{\frac{1}{c^2 x^2}} - 1 \operatorname{arosech}(cx) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))^3/x^2,x, algorithm="maxima")

[Out] $-b^3 \operatorname{arcsech}(cx)^3/x + 3(c\sqrt{1/(c^2x^2)} - 1) - \operatorname{arcsech}(cx)/x) * a^2 * b + 6(c\sqrt{1/(c^2x^2)} - 1) * \operatorname{arcsech}(cx) - 1/x) * a * b^2 + 3(c\sqrt{1/(c^2x^2)} - 1) * \operatorname{arcsech}(cx)^2 + 2c\sqrt{1/(c^2x^2)} - 1) - 2 * \operatorname{arcsech}(cx)/x) * b^3 - 3 * a * b^2 * \operatorname{arcsech}(cx)^2/x - a^3/x$

Fricas [B] time = 1.68919, size = 485, normalized size = 4.75

$$b^3 \log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)^3 - 3(a^2b + 2b^3)cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}} + a^3 + 6ab^2 - 3\left(b^3cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}} - ab^2\right) \log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)^2 - 3\left(2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))^3/x^2,x, algorithm="fricas")

[Out] $-(b^3 \log((cx\sqrt{-(c^2x^2-1)/(c^2x^2)} + 1)/(cx))^3 - 3(a^2b + 2b^3) * cx\sqrt{-(c^2x^2-1)/(c^2x^2)} + a^3 + 6ab^2 - 3(b^3 * cx\sqrt{-(c^2x^2-1)/(c^2x^2)} - ab^2) * \log((cx\sqrt{-(c^2x^2-1)/(c^2x^2)} + 1)/(cx))^2 - 3(2ab^2 * cx\sqrt{-(c^2x^2-1)/(c^2x^2)} - a^2b - 2b^3) * \log((cx\sqrt{-(c^2x^2-1)/(c^2x^2)} + 1)/(cx)))/x$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{asech}(cx))^3}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asech(c*x))**3/x**2,x)

[Out] Integral((a + b*asech(c*x))**3/x**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsech}(cx) + a)^3}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))^3/x^2,x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)^3/x^2, x)

$$3.48 \quad \int \frac{(a + b \operatorname{sech}^{-1}(cx))^3}{x^3} dx$$

Optimal. Leaf size=163

$$-\frac{3b^2(1-cx)(cx+1)(a+b\operatorname{sech}^{-1}(cx))}{4x^2} - \frac{1}{4}c^2(a+b\operatorname{sech}^{-1}(cx))^3 + \frac{3b\sqrt{\frac{1-cx}{cx+1}}(cx+1)(a+b\operatorname{sech}^{-1}(cx))^2}{4x^2} - \frac{(1-cx)(cx+1)}{4x^2}$$

[Out] (3*b^3*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(8*x^2) - (3*b^3*c^2*ArcSech[c*x])/8 - (3*b^2*(1 - c*x)*(1 + c*x)*(a + b*ArcSech[c*x]))/(4*x^2) + (3*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x])^2)/(4*x^2) - (c^2*(a + b*ArcSech[c*x])^3)/4 - ((1 - c*x)*(1 + c*x)*(a + b*ArcSech[c*x])^3)/(2*x^2)

Rubi [A] time = 0.116223, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6285, 5446, 3311, 32, 2635, 8}

$$-\frac{3b^2(1-cx)(cx+1)(a+b\operatorname{sech}^{-1}(cx))}{4x^2} - \frac{1}{4}c^2(a+b\operatorname{sech}^{-1}(cx))^3 + \frac{3b\sqrt{\frac{1-cx}{cx+1}}(cx+1)(a+b\operatorname{sech}^{-1}(cx))^2}{4x^2} - \frac{(1-cx)(cx+1)}{4x^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcSech[c*x])^3/x^3,x]

[Out] (3*b^3*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(8*x^2) - (3*b^3*c^2*ArcSech[c*x])/8 - (3*b^2*(1 - c*x)*(1 + c*x)*(a + b*ArcSech[c*x]))/(4*x^2) + (3*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x])^2)/(4*x^2) - (c^2*(a + b*ArcSech[c*x])^3)/4 - ((1 - c*x)*(1 + c*x)*(a + b*ArcSech[c*x])^3)/(2*x^2)

Rule 6285

Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))^ (n_.)*(x_.)^(m_.), x_Symbol] :> -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

Rule 5446

Int[Cosh[(a_.) + (b_.)*(x_.)]*((c_.) + (d_.)*(x_.))^(m_.)*Sinh[(a_.) + (b_.)*(x_.)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sinh[a + b*x]^(n + 1))/(b*(n + 1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rule 3311

Int[((c_.) + (d_.)*(x_.))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sinh[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sinh[e + f*x])^(n - 2), x], x] - Dist[(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sinh[e + f*x])^n, x], x] - Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sinh[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

Rule 32

Int[((a_.) + (b_.)*(x_.))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_.)])^(n_), x_Symbol] := -Simp[(b*cos[c + d*x] * (b*sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned} \int \frac{(a + b \operatorname{sech}^{-1}(cx))^3}{x^3} dx &= -\left(c^2 \operatorname{Subst}\left(\int (a + bx)^3 \cosh(x) \sinh(x) dx, x, \operatorname{sech}^{-1}(cx)\right)\right) \\ &= -\frac{(1 - cx)(1 + cx)(a + b \operatorname{sech}^{-1}(cx))^3}{2x^2} + \frac{1}{2}(3bc^2) \operatorname{Subst}\left(\int (a + bx)^2 \sinh^2(x) dx, x, \operatorname{sech}^{-1}(cx)\right) \\ &= -\frac{3b^2(1 - cx)(1 + cx)(a + b \operatorname{sech}^{-1}(cx))}{4x^2} + \frac{3b\sqrt{\frac{1 - cx}{1 + cx}}(1 + cx)(a + b \operatorname{sech}^{-1}(cx))^2}{4x^2} - \frac{(1 - cx)}{4x^2} \\ &= \frac{3b^3\sqrt{\frac{1 - cx}{1 + cx}}(1 + cx)}{8x^2} - \frac{3b^2(1 - cx)(1 + cx)(a + b \operatorname{sech}^{-1}(cx))}{4x^2} + \frac{3b\sqrt{\frac{1 - cx}{1 + cx}}(1 + cx)(a + b \operatorname{sech}^{-1}(cx))^2}{4x^2} \\ &= \frac{3b^3\sqrt{\frac{1 - cx}{1 + cx}}(1 + cx)}{8x^2} - \frac{3}{8}b^3c^2\operatorname{sech}^{-1}(cx) - \frac{3b^2(1 - cx)(1 + cx)(a + b \operatorname{sech}^{-1}(cx))}{4x^2} + \frac{3b\sqrt{\frac{1 - cx}{1 + cx}}(1 + cx)(a + b \operatorname{sech}^{-1}(cx))^2}{4x^2} \end{aligned}$$

Mathematica [A] time = 0.46125, size = 245, normalized size = 1.5

$$-3bc^2x^2(2a^2 + b^2)\log(x) + 3bc^2x^2(2a^2 + b^2)\log\left(cx\sqrt{\frac{1 - cx}{cx + 1}} + \sqrt{\frac{1 - cx}{cx + 1}} + 1\right) + 3b(2a^2 + b^2)\sqrt{\frac{1 - cx}{cx + 1}}(cx + 1) - 6b\operatorname{sech}^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcSech[c*x])^3/x^3, x]

[Out] $(-4a^3 - 6ab^2 + 3b(2a^2 + b^2)\sqrt{(1 - cx)/(1 + cx)})(1 + cx) - 6b(2a^2 + b^2 - 2ab\sqrt{(1 - cx)/(1 + cx)})(1 + cx)\operatorname{ArcSech}[cx] + 6b^2(b\sqrt{(1 - cx)/(1 + cx)}(1 + cx) + a(-2 + c^2x^2))\operatorname{ArcSech}[cx]^2 + 2b^3(-2 + c^2x^2)\operatorname{ArcSech}[cx]^3 - 3b(2a^2 + b^2)c^2x^2\operatorname{Log}[x] + 3b(2a^2 + b^2)c^2x^2\operatorname{Log}[1 + \sqrt{(1 - cx)/(1 + cx)}] + c^2x^2\sqrt{(1 - cx)/(1 + cx)}(1 + cx)\operatorname{Log}[1 + \sqrt{(1 - cx)/(1 + cx)}])/(8x^2)$

Maple [B] time = 0.274, size = 321, normalized size = 2.

$$c^2\left(-\frac{a^3}{2c^2x^2} + b^3\left(-\frac{(\operatorname{arcsech}(cx))^3}{2c^2x^2} + \frac{3(\operatorname{arcsech}(cx))^2}{4cx}\sqrt{\frac{cx - 1}{cx}}\sqrt{\frac{cx + 1}{cx}} + \frac{(\operatorname{arcsech}(cx))^3}{4} - \frac{3\operatorname{arcsech}(cx)}{4c^2x^2} + \frac{3}{8c}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arcsech(c*x))^3/x^3,x)`

[Out] $c^2*(-1/2*a^3/c^2/x^2+b^3*(-1/2/c^2/x^2*arcsech(c*x)^3+3/4*arcsech(c*x)^2/c/x*(-(c*x-1)/c/x)^{(1/2)}*((c*x+1)/c/x)^{(1/2)}+1/4*arcsech(c*x)^3-3/4/c^2/x^2*arcsech(c*x)+3/8/c/x*(-(c*x-1)/c/x)^{(1/2)}*((c*x+1)/c/x)^{(1/2)}+3/8*arcsech(c*x))+3*a*b^2*(-1/2/c^2/x^2*arcsech(c*x)^2+1/2*arcsech(c*x)/c/x*(-(c*x-1)/c/x)^{(1/2)}*((c*x+1)/c/x)^{(1/2)}+1/4*arcsech(c*x)^2-1/4/c^2/x^2)+3*a^2*b*(-1/2/c^2/x^2*arcsech(c*x)+1/4*(-(c*x-1)/c/x)^{(1/2)}/c/x*((c*x+1)/c/x)^{(1/2)}*(arctanh(1/(-c^2*x^2+1)^{(1/2)})*c^2*x^2+(-c^2*x^2+1)^{(1/2)})/(-c^2*x^2+1)^{(1/2))}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{3}{8}a^2b \left(\frac{2c^4x\sqrt{\frac{1}{c^2x^2}-1}}{c^2x^2\left(\frac{1}{c^2x^2}-1\right)-1} - c^3 \log\left(cx\sqrt{\frac{1}{c^2x^2}-1}+1\right) + c^3 \log\left(cx\sqrt{\frac{1}{c^2x^2}-1}-1\right) + \frac{4 \operatorname{arsech}(cx)}{x^2} \right) - \frac{a^3}{2x^2} + \int \frac{b^3 \log\left(\sqrt{\frac{1}{cx}+1}\right)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsech(c*x))^3/x^3,x, algorithm="maxima")`

[Out] $-3/8*a^2*b*((2*c^4*x*\sqrt{1/(c^2*x^2)-1})/(c^2*x^2*(1/(c^2*x^2)-1)-1) - c^3*\log(c*x*\sqrt{1/(c^2*x^2)-1}+1) + c^3*\log(c*x*\sqrt{1/(c^2*x^2)-1}-1))/c + 4*arcsech(c*x)/x^2) - 1/2*a^3/x^2 + integrate(b^3*\log(\sqrt{1/(c*x)+1})*\sqrt{1/(c*x)-1} + 1/(c*x))^3/x^3 + 3*a*b^2*\log(\sqrt{1/(c*x)+1})*\sqrt{1/(c*x)-1} + 1/(c*x))^2/x^3, x)$

Fricas [A] time = 1.64249, size = 583, normalized size = 3.58

$$\frac{2(b^3c^2x^2 - 2b^3)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)^3 + 3(2a^2b + b^3)cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}} - 4a^3 - 6ab^2 + 6\left(ab^2c^2x^2 + b^3cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}} - 2ab^2\right)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right)}{8x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsech(c*x))^3/x^3,x, algorithm="fricas")`

[Out] $1/8*(2*(b^3*c^2*x^2 - 2*b^3)*\log((c*x*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)} + 1)/(c*x))^3 + 3*(2*a^2*b + b^3)*c*x*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)} - 4*a^3 - 6*a*b^2 + 6*(a*b^2*c^2*x^2 + b^3*c*x*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)} - 2*a*b^2)*\log((c*x*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)} + 1)/(c*x))^2 + 3*(4*a*b^2*c*x*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)} + (2*a^2*b + b^3)*c^2*x^2 - 4*a^2*b - 2*b^3)*\log((c*x*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)} + 1)/(c*x)))/x^2$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{asech}(cx))^3}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((a+b*asech(c*x))**3/x**3,x)
```

```
[Out] Integral((a + b*asech(c*x))**3/x**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{ar} \operatorname{sech}(cx) + a)^3}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))^3/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)^3/x^3, x)
```

$$3.49 \quad \int \frac{(a + b \operatorname{sech}^{-1}(cx))^3}{x^4} dx$$

Optimal. Leaf size=213

$$-\frac{4b^2c^2(a + b \operatorname{sech}^{-1}(cx))}{3x} - \frac{2b^2(a + b \operatorname{sech}^{-1}(cx))}{9x^3} + \frac{2bc^2\sqrt{\frac{1-cx}{cx+1}}(cx+1)(a + b \operatorname{sech}^{-1}(cx))^2}{3x} + \frac{b\sqrt{\frac{1-cx}{cx+1}}(cx+1)(a + b \operatorname{sech}^{-1}(cx))}{3x^3}$$

[Out] (14*b^3*c^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(9*x) + (2*b^3*((1 - c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3)/(27*x^3) - (2*b^2*(a + b*ArcSech[c*x]))/(9*x^3) - (4*b^2*c^2*(a + b*ArcSech[c*x]))/(3*x) + (b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x])^2)/(3*x^3) + (2*b*c^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x])^2)/(3*x) - (a + b*ArcSech[c*x])^3/(3*x^3)

Rubi [A] time = 0.166402, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6285, 5447, 3311, 3296, 2637, 2633}

$$-\frac{4b^2c^2(a + b \operatorname{sech}^{-1}(cx))}{3x} - \frac{2b^2(a + b \operatorname{sech}^{-1}(cx))}{9x^3} + \frac{2bc^2\sqrt{\frac{1-cx}{cx+1}}(cx+1)(a + b \operatorname{sech}^{-1}(cx))^2}{3x} + \frac{b\sqrt{\frac{1-cx}{cx+1}}(cx+1)(a + b \operatorname{sech}^{-1}(cx))}{3x^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcSech[c*x])^3/x^4, x]

[Out] (14*b^3*c^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(9*x) + (2*b^3*((1 - c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3)/(27*x^3) - (2*b^2*(a + b*ArcSech[c*x]))/(9*x^3) - (4*b^2*c^2*(a + b*ArcSech[c*x]))/(3*x) + (b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x])^2)/(3*x^3) + (2*b*c^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x])^2)/(3*x) - (a + b*ArcSech[c*x])^3/(3*x^3)

Rule 6285

Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))^n*(x_)^m, x_Symbol] :> -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

Rule 5447

Int[Cosh[(a_.) + (b_.)*(x_.)]^n*((c_.) + (d_.)*(x_.))^m*Sinh[(a_.) + (b_.)*(x_.)], x_Symbol] :> Simp[((c + d*x)^m*Cosh[a + b*x]^(n + 1))/(b*(n + 1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rule 3311

Int[((c_.) + (d_.)*(x_.))^m*((b_.)*sin[(e_.) + (f_.)*(x_.)])^n, x_Symbol] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Ssin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Ssin[e + f*x])^(n - 2), x], x] - Dist[(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Ssin[e + f*x])^n, x], x] - Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Ssin[e + f*x])^(n - 1))/(f*n), x]) /;

FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

Rule 3296

Int[((c_.) + (d_.)*(x_.))^(m_.)*sin[(e_.) + (f_.)*(x_.)], x_Symbol] := -Simp[
((c + d*x)^m*cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]

Rule 2633

Int[sin[(c_.) + (d_.)*(x_.)]^(n_.), x_Symbol] := -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]

Rubi steps

$$\begin{aligned} \int \frac{(a + b \operatorname{sech}^{-1}(cx))^3}{x^4} dx &= -\left(c^3 \operatorname{Subst}\left(\int (a + bx)^3 \cosh^2(x) \sinh(x) dx, x, \operatorname{sech}^{-1}(cx)\right)\right) \\ &= -\frac{(a + b \operatorname{sech}^{-1}(cx))^3}{3x^3} + (bc^3) \operatorname{Subst}\left(\int (a + bx)^2 \cosh^3(x) dx, x, \operatorname{sech}^{-1}(cx)\right) \\ &= -\frac{2b^2(a + b \operatorname{sech}^{-1}(cx))}{9x^3} + \frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)(a + b \operatorname{sech}^{-1}(cx))^2}{3x^3} - \frac{(a + b \operatorname{sech}^{-1}(cx))^3}{3x^3} + \frac{1}{3} \\ &= -\frac{2b^2(a + b \operatorname{sech}^{-1}(cx))}{9x^3} + \frac{b\sqrt{\frac{1-cx}{1+cx}}(1+cx)(a + b \operatorname{sech}^{-1}(cx))^2}{3x^3} + \frac{2bc^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)(a + b \operatorname{sech}^{-1}(cx))}{3x} \\ &= \frac{2b^3c^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{9x} + \frac{2b^3\left(\frac{1-cx}{1+cx}\right)^{3/2}(1+cx)^3}{27x^3} - \frac{2b^2(a + b \operatorname{sech}^{-1}(cx))}{9x^3} - \frac{4b^2c^2(a + b \operatorname{sech}^{-1}(cx))}{3x} \\ &= \frac{14b^3c^2\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{9x} + \frac{2b^3\left(\frac{1-cx}{1+cx}\right)^{3/2}(1+cx)^3}{27x^3} - \frac{2b^2(a + b \operatorname{sech}^{-1}(cx))}{9x^3} - \frac{4b^2c^2(a + b \operatorname{sech}^{-1}(cx))}{3x} \end{aligned}$$

Mathematica [A] time = 0.382663, size = 256, normalized size = 1.2

$$-3b \operatorname{sech}^{-1}(cx) \left(9a^2 - 6ab\sqrt{\frac{1-cx}{cx+1}}(2c^3x^3 + 2c^2x^2 + cx + 1) + 2b^2(6c^2x^2 + 1)\right) + 9a^2b\sqrt{\frac{1-cx}{cx+1}}(2c^3x^3 + 2c^2x^2 + cx + 1) -$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcSech[c*x])^3/x^4, x]

[Out] (-9*a^3 - 6*a*b^2*(1 + 6*c^2*x^2) + 9*a^2*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x + 2*c^2*x^2 + 2*c^3*x^3) + 2*b^3*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x + 2*0*c^2*x^2 + 20*c^3*x^3) - 3*b*(9*a^2 + 2*b^2*(1 + 6*c^2*x^2) - 6*a*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x + 2*c^2*x^2 + 2*c^3*x^3))*ArcSech[c*x] + 9*b^2*(-3*a + b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x + 2*c^2*x^2 + 2*c^3*x^3))*ArcSech[c*x]^2 - 9*b^3*ArcSech[c*x]^3)/(27*x^3)

Maple [B] time = 0.274, size = 455, normalized size = 2.1

$$c^3 \left(-\frac{a^3}{3c^3x^3} + b^3 \left(\frac{(\operatorname{arcsech}(cx))^3 (cx-1)(cx+1)}{3c^3x^3} - \frac{(\operatorname{arcsech}(cx))^3}{3cx} + \frac{(\operatorname{arcsech}(cx))^2}{3c^2x^2} \sqrt{\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} + \frac{2(\operatorname{arcsech}(cx))}{3} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))^3/x^4,x)

[Out] $c^3 \cdot (-1/3 \cdot a^3 / c^3 / x^3 + b^3 \cdot (1/3 \cdot \operatorname{arcsech}(c \cdot x)^3 / c^3 / x^3 \cdot (c \cdot x - 1) \cdot (c \cdot x + 1) - 1/3 \cdot a \cdot \operatorname{arcsech}(c \cdot x)^3 / c / x + 1/3 \cdot \operatorname{arcsech}(c \cdot x)^2 / c^2 / x^2 \cdot (- (c \cdot x - 1) / c / x)^{(1/2)} \cdot ((c \cdot x + 1) / c / x)^{(1/2)} + 2/3 \cdot \operatorname{arcsech}(c \cdot x)^2 \cdot (- (c \cdot x - 1) / c / x)^{(1/2)} \cdot ((c \cdot x + 1) / c / x)^{(1/2)} + 2/9 \cdot \operatorname{arcsech}(c \cdot x) \cdot (c \cdot x - 1) / c^3 / x^3 \cdot (c \cdot x + 1) - 14/9 / c / x \cdot \operatorname{arcsech}(c \cdot x) + 2/27 / c^2 / x^2 \cdot (- (c \cdot x - 1) / c / x)^{(1/2)} \cdot ((c \cdot x + 1) / c / x)^{(1/2)} + 40/27 \cdot (- (c \cdot x - 1) / c / x)^{(1/2)} \cdot ((c \cdot x + 1) / c / x)^{(1/2)}) + 3 \cdot a \cdot b^2 \cdot (1/3 \cdot \operatorname{arcsech}(c \cdot x)^2 / c^3 / x^3 \cdot (c \cdot x - 1) \cdot (c \cdot x + 1) - 1/3 \cdot \operatorname{arcsech}(c \cdot x)^2 / c / x + 2/9 \cdot \operatorname{arcsech}(c \cdot x) / c^2 / x^2 \cdot (- (c \cdot x - 1) / c / x)^{(1/2)} \cdot ((c \cdot x + 1) / c / x)^{(1/2)} + 4/9 \cdot \operatorname{arcsech}(c \cdot x) \cdot (- (c \cdot x - 1) / c / x)^{(1/2)} \cdot ((c \cdot x + 1) / c / x)^{(1/2)} + 2/27 \cdot (c \cdot x - 1) / c^3 / x^3 \cdot (c \cdot x + 1) - 14/27 / c / x) + 3 \cdot a^2 \cdot b \cdot (-1/3 / c^3 / x^3 \cdot \operatorname{arcsech}(c \cdot x) + 1/9 \cdot (- (c \cdot x - 1) / c / x)^{(1/2)} / c^2 / x^2 \cdot ((c \cdot x + 1) / c / x)^{(1/2)} \cdot (2 \cdot c^2 \cdot x^2 + 1))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3} a^2 b \left(\frac{c^4 \left(\frac{1}{c^2 x^2} - 1 \right)^{\frac{3}{2}} + 3 c^4 \sqrt{\frac{1}{c^2 x^2} - 1}}{c} - \frac{3 \operatorname{arcsch}(cx)}{x^3} \right) - \frac{a^3}{3 x^3} + \int \frac{b^3 \log \left(\sqrt{\frac{1}{cx} + 1} \sqrt{\frac{1}{cx} - 1} + \frac{1}{cx} \right)^3}{x^4} + \frac{3 a b^2 \log \left(\sqrt{\frac{1}{cx} + 1} \sqrt{\frac{1}{cx} - 1} + \frac{1}{cx} \right)}{x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))^3/x^4,x, algorithm="maxima")

[Out] $1/3 \cdot a^2 \cdot b \cdot ((c^4 \cdot (1/(c^2 \cdot x^2) - 1))^{(3/2)} + 3 \cdot c^4 \cdot \sqrt{1/(c^2 \cdot x^2) - 1}) / c - 3 \cdot \operatorname{arcsech}(c \cdot x) / x^3 - 1/3 \cdot a^3 / x^3 + \operatorname{integrate}(b^3 \cdot \log(\sqrt{1/(c \cdot x) + 1}) \cdot \sqrt{1/(c \cdot x) - 1} + 1/(c \cdot x))^3 / x^4 + 3 \cdot a \cdot b^2 \cdot \log(\sqrt{1/(c \cdot x) + 1}) \cdot \sqrt{1/(c \cdot x) - 1} + 1/(c \cdot x))^2 / x^4, x)$

Fricas [A] time = 1.78489, size = 649, normalized size = 3.05

$$36 a b^2 c^2 x^2 + 9 b^3 \log \left(\frac{c x \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2} + 1}}{c x} \right)^3 + 9 a^3 + 6 a b^2 + 9 \left(3 a b^2 - (2 b^3 c^3 x^3 + b^3 c x) \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2}} \right) \log \left(\frac{c x \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2} + 1}}{c x} \right)^2 + 3 \left(12 b^3 c^2 x^2 + 9 a^2 b + 2 b^3 - 6 (2 a b^2 c^3 x^3 + a b^2 c x) \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2}} \right) \log \left(\frac{c x \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2} + 1}}{c x} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))^3/x^4,x, algorithm="fricas")

[Out] $-1/27 \cdot (36 \cdot a \cdot b^2 \cdot c^2 \cdot x^2 + 9 \cdot b^3 \cdot \log((c \cdot x \cdot \sqrt{-(c^2 \cdot x^2 - 1)/(c^2 \cdot x^2)} + 1)/(c \cdot x))^3 + 9 \cdot a^3 + 6 \cdot a \cdot b^2 + 9 \cdot (3 \cdot a \cdot b^2 - (2 \cdot b^3 \cdot c^3 \cdot x^3 + b^3 \cdot c \cdot x) \cdot \sqrt{-(c^2 \cdot x^2 - 1)/(c^2 \cdot x^2)})) \cdot \log((c \cdot x \cdot \sqrt{-(c^2 \cdot x^2 - 1)/(c^2 \cdot x^2)} + 1)/(c \cdot x))^2 + 3 \cdot (12 \cdot b^3 \cdot c^2 \cdot x^2 + 9 \cdot a^2 \cdot b + 2 \cdot b^3 - 6 \cdot (2 \cdot a \cdot b^2 \cdot c^3 \cdot x^3 + a \cdot b^2 \cdot c \cdot x) \cdot \sqrt{-(c^2 \cdot x^2 - 1)/(c^2 \cdot x^2)})) \cdot \log((c \cdot x \cdot \sqrt{-(c^2 \cdot x^2 - 1)/(c^2 \cdot x^2)} + 1)/(c \cdot x))$

$+ 1)/(c*x)) - (2*(9*a^2*b + 20*b^3)*c^3*x^3 + (9*a^2*b + 2*b^3)*c*x)*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)))/x^3$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{asech}(cx))^3}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asech(c*x))**3/x**4,x)

[Out] Integral((a + b*asech(c*x))**3/x**4, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsech}(cx) + a)^3}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))^3/x^4,x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)^3/x^4, x)

$$3.50 \quad \int \frac{(a + b \operatorname{sech}^{-1}(cx))^3}{x^5} dx$$

Optimal. Leaf size=242

$$-\frac{9b^2c^2(a + b \operatorname{sech}^{-1}(cx))}{32x^2} - \frac{3b^2(a + b \operatorname{sech}^{-1}(cx))}{32x^4} + \frac{9bc^2\sqrt{\frac{1-cx}{cx+1}}(cx+1)(a + b \operatorname{sech}^{-1}(cx))^2}{32x^2} + \frac{3}{32}c^4(a + b \operatorname{sech}^{-1}(cx))^3$$

[Out] (3*b^3*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(128*x^4) + (45*b^3*c^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(256*x^2) + (45*b^3*c^4*ArcSech[c*x])/256 - (3*b^2*(a + b*ArcSech[c*x]))/(32*x^4) - (9*b^2*c^2*(a + b*ArcSech[c*x]))/(32*x^2) + (3*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x])^2)/(16*x^4) + (9*b*c^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x])^2)/(32*x^2) + (3*c^4*(a + b*ArcSech[c*x])^3)/32 - (a + b*ArcSech[c*x])^3/(4*x^4)

Rubi [A] time = 0.196284, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6285, 5447, 3311, 32, 2635, 8}

$$-\frac{9b^2c^2(a + b \operatorname{sech}^{-1}(cx))}{32x^2} - \frac{3b^2(a + b \operatorname{sech}^{-1}(cx))}{32x^4} + \frac{9bc^2\sqrt{\frac{1-cx}{cx+1}}(cx+1)(a + b \operatorname{sech}^{-1}(cx))^2}{32x^2} + \frac{3}{32}c^4(a + b \operatorname{sech}^{-1}(cx))^3$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcSech[c*x])^3/x^5, x]

[Out] (3*b^3*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(128*x^4) + (45*b^3*c^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(256*x^2) + (45*b^3*c^4*ArcSech[c*x])/256 - (3*b^2*(a + b*ArcSech[c*x]))/(32*x^4) - (9*b^2*c^2*(a + b*ArcSech[c*x]))/(32*x^2) + (3*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x])^2)/(16*x^4) + (9*b*c^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcSech[c*x])^2)/(32*x^2) + (3*c^4*(a + b*ArcSech[c*x])^3)/32 - (a + b*ArcSech[c*x])^3/(4*x^4)

Rule 6285

Int[((a_.) + ArcSech[(c_.)*(x_)])*(b_.))^n*(x_)^m, x_Symbol] :> -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

Rule 5447

Int[Cosh[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)], x_Symbol] :> Simp[((c + d*x)^m*Cosh[a + b*x]^(n + 1))/(b*(n + 1)), x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cosh[a + b*x]^(n + 1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]

Rule 3311

Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sine[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sine[e + f*x])^(n - 2), x], x] - Dist[(d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sine[e + f*x])^n, x], x]

- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x] /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x] * (b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned} \int \frac{(a + b \operatorname{sech}^{-1}(cx))^3}{x^5} dx &= -\left(c^4 \operatorname{Subst}\left(\int (a + bx)^3 \cosh^3(x) \sinh(x) dx, x, \operatorname{sech}^{-1}(cx)\right)\right) \\ &= -\frac{(a + b \operatorname{sech}^{-1}(cx))^3}{4x^4} + \frac{1}{4} (3bc^4) \operatorname{Subst}\left(\int (a + bx)^2 \cosh^4(x) dx, x, \operatorname{sech}^{-1}(cx)\right) \\ &= -\frac{3b^2 (a + b \operatorname{sech}^{-1}(cx))}{32x^4} + \frac{3b \sqrt{\frac{1-cx}{1+cx}} (1+cx) (a + b \operatorname{sech}^{-1}(cx))^2}{16x^4} - \frac{(a + b \operatorname{sech}^{-1}(cx))^3}{4x^4} + \dots \\ &= \frac{3b^3 \sqrt{\frac{1-cx}{1+cx}} (1+cx)}{128x^4} - \frac{3b^2 (a + b \operatorname{sech}^{-1}(cx))}{32x^4} - \frac{9b^2 c^2 (a + b \operatorname{sech}^{-1}(cx))}{32x^2} + \frac{3b \sqrt{\frac{1-cx}{1+cx}} (1+cx)}{32x^2} \\ &= \frac{3b^3 \sqrt{\frac{1-cx}{1+cx}} (1+cx)}{128x^4} + \frac{45b^3 c^2 \sqrt{\frac{1-cx}{1+cx}} (1+cx)}{256x^2} - \frac{3b^2 (a + b \operatorname{sech}^{-1}(cx))}{32x^4} - \frac{9b^2 c^2 (a + b \operatorname{sech}^{-1}(cx))}{32x^2} \\ &= \frac{3b^3 \sqrt{\frac{1-cx}{1+cx}} (1+cx)}{128x^4} + \frac{45b^3 c^2 \sqrt{\frac{1-cx}{1+cx}} (1+cx)}{256x^2} + \frac{45}{256} b^3 c^4 \operatorname{sech}^{-1}(cx) - \frac{3b^2 (a + b \operatorname{sech}^{-1}(cx))}{32x^4} \end{aligned}$$

Mathematica [A] time = 0.713887, size = 332, normalized size = 1.37

$$3b \sqrt{\frac{1-cx}{cx+1}} (cx+1) (8a^2 (3c^2 x^2 + 2) + b^2 (15c^2 x^2 + 2)) - 9bc^4 x^4 (8a^2 + 5b^2) \log(x) + 9bc^4 x^4 (8a^2 + 5b^2) \log\left(cx \sqrt{\frac{1-cx}{cx+1}} + \dots\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcSech[c*x])^3/x^5, x]

[Out] (-8*a*(8*a^2 + 3*b^2) - 72*a*b^2*c^2*x^2 + 3*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(8*a^2*(2 + 3*c^2*x^2) + b^2*(2 + 15*c^2*x^2)) - 24*b*(8*a^2 + b^2*(1 + 3*c^2*x^2) - 2*a*b*Sqrt[(1 - c*x)/(1 + c*x)]*(2 + 2*c*x + 3*c^2*x^2 + 3*c^3*x^3))*ArcSech[c*x] + 24*b^2*(b*Sqrt[(1 - c*x)/(1 + c*x)]*(2 + 2*c*x + 3*c^2*x^2 + 3*c^3*x^3) + a*(-8 + 3*c^4*x^4))*ArcSech[c*x]^2 + 8*b^3*(-8 + 3*c^4*x^4)*ArcSech[c*x]^3 - 9*b*(8*a^2 + 5*b^2)*c^4*x^4*Log[x] + 9*b*(8*a^2 + 5*b^2)*c^4*x^4*Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]])/(256*x^4)

Maple [B] time = 0.277, size = 553, normalized size = 2.3

$$c^4 \left(-\frac{a^3}{4c^4x^4} + b^3 \left(\frac{(\operatorname{arcsech}(cx))^3 (cx-1)(cx+1)}{4c^4x^4} - \frac{(\operatorname{arcsech}(cx))^3}{4c^2x^2} + \frac{3(\operatorname{arcsech}(cx))^2}{16c^3x^3} \sqrt{-\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} + \frac{9(\operatorname{arcsech}(cx))}{32} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))^3/x^5,x)

[Out] $c^4 * (-1/4 * a^3 / c^4 / x^4 + b^3 * (1/4 * \operatorname{arcsech}(c*x)^3 / c^4 / x^4 * (c*x-1) * (c*x+1) - 1/4 / c^2 / x^2 * \operatorname{arcsech}(c*x)^3 + 3/16 * \operatorname{arcsech}(c*x)^2 / c^3 / x^3 * (-c*x-1) / c / x^{1/2} * ((c*x+1) / c / x)^{1/2} + 9/32 * \operatorname{arcsech}(c*x)^2 / c / x * (-c*x-1) / c / x^{1/2} * ((c*x+1) / c / x)^{1/2} + 3/32 * \operatorname{arcsech}(c*x)^3 + 3/32 * \operatorname{arcsech}(c*x) * (c*x-1) / c^4 / x^4 * (c*x+1) + 3/128 / c^3 / x^3 * (-c*x-1) / c / x^{1/2} * ((c*x+1) / c / x)^{1/2} + 45/256 / c / x * (-c*x-1) / c / x^{1/2} * ((c*x+1) / c / x)^{1/2} + 45/256 * \operatorname{arcsech}(c*x) - 3/8 / c^2 / x^2 * \operatorname{arcsech}(c*x)) + 3 * a * b^2 * (1/4 * \operatorname{arcsech}(c*x)^2 / c^4 / x^4 * (c*x-1) * (c*x+1) - 1/4 / c^2 / x^2 * \operatorname{arcsech}(c*x)^2 + 1/8 * \operatorname{arcsech}(c*x) / c^3 / x^3 * (-c*x-1) / c / x^{1/2} * ((c*x+1) / c / x)^{1/2} + 3/16 * \operatorname{arcsech}(c*x) / c / x * (-c*x-1) / c / x^{1/2} * ((c*x+1) / c / x)^{1/2} + 3/32 * \operatorname{arcsech}(c*x)^2 + 1/32 * (c*x-1) / c^4 / x^4 * (c*x+1) - 1/8 / c^2 / x^2) + 3 * a^2 * b * (-1/4 / c^4 / x^4 * \operatorname{arcsech}(c*x) + 1/32 * (-c*x-1) / c / x^{1/2} / c^3 / x^3 * ((c*x+1) / c / x)^{1/2} * (3 * \operatorname{arctanh}(1 / (-c^2 * x^2 + 1)^{1/2})) * c^4 * x^4 + 3 * (-c^2 * x^2 + 1)^{1/2} * c^2 * x^2 + 2 * (-c^2 * x^2 + 1)^{1/2} / (-c^2 * x^2 + 1)^{1/2}))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{3}{64} a^2 b \left(\frac{3c^5 \log\left(cx \sqrt{\frac{1}{c^2x^2} - 1} + 1\right) - 3c^5 \log\left(cx \sqrt{\frac{1}{c^2x^2} - 1} - 1\right) - \frac{2\left(3c^8x^3\left(\frac{1}{c^2x^2} - 1\right)^{\frac{3}{2}} - 5c^6x \sqrt{\frac{1}{c^2x^2} - 1}\right)}{c^4x^4\left(\frac{1}{c^2x^2} - 1\right)^2 - 2c^2x^2\left(\frac{1}{c^2x^2} - 1\right) + 1}}{c} - \frac{16 \operatorname{arcsch}(cx)}{x^4} \right) - \frac{a^3}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))^3/x^5,x, algorithm="maxima")

[Out] $3/64 * a^2 * b * ((3 * c^5 * \log(c*x * \sqrt{1/(c^2*x^2) - 1} + 1) - 3 * c^5 * \log(c*x * \sqrt{1/(c^2*x^2) - 1} - 1) - 2 * (3 * c^8 * x^3 * (1/(c^2*x^2) - 1)^{3/2} - 5 * c^6 * x * \sqrt{1/(c^2*x^2) - 1})) / (c^4 * x^4 * (1/(c^2*x^2) - 1)^2 - 2 * c^2 * x^2 * (1/(c^2*x^2) - 1) + 1)) / c - 16 * \operatorname{arcsech}(c*x) / x^4 - 1/4 * a^3 / x^4 + \operatorname{integrate}(b^3 * \log(\sqrt{1/(c*x) + 1} * \sqrt{1/(c*x) - 1} + 1 / (c*x))^{3/2} / x^5 + 3 * a * b^2 * \log(\sqrt{1/(c*x) + 1} * \sqrt{1/(c*x) - 1} + 1 / (c*x))^{2/2} / x^5, x)$

Fricas [A] time = 1.63898, size = 756, normalized size = 3.12

$$72ab^2c^2x^2 - 8(3b^3c^4x^4 - 8b^3) \log\left(\frac{cx \sqrt{-\frac{c^2x^2-1}{c^2x^2}} + 1}{cx}\right)^3 + 64a^3 + 24ab^2 - 24\left(3ab^2c^4x^4 - 8ab^2 + (3b^3c^3x^3 + 2b^3cx) \sqrt{-\frac{c^2}{c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))^3/x^5,x, algorithm="fricas")

[Out]
$$-1/256*(72*a*b^2*c^2*x^2 - 8*(3*b^3*c^4*x^4 - 8*b^3)*\log((c*x*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)} + 1)/(c*x))^3 + 64*a^3 + 24*a*b^2 - 24*(3*a*b^2*c^4*x^4 - 8*a*b^2 + (3*b^3*c^3*x^3 + 2*b^3*c*x)*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)}))*\log((c*x*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)} + 1)/(c*x))^2 - 3*(3*(8*a^2*b + 5*b^3)*c^4*x^4 - 24*b^3*c^2*x^2 - 64*a^2*b - 8*b^3 + 16*(3*a*b^2*c^3*x^3 + 2*a*b^2*c*x)*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)}))*\log((c*x*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)} + 1)/(c*x)) - 3*(3*(8*a^2*b + 5*b^3)*c^3*x^3 + 2*(8*a^2*b + b^3)*c*x)*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)})/x^4$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{asech}(cx))^3}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asech(c*x))**3/x**5,x)

[Out] Integral((a + b*asech(c*x))**3/x**5, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsech}(cx) + a)^3}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))^3/x^5,x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)^3/x^5, x)

$$3.51 \quad \int \frac{x}{a+b\operatorname{sech}^{-1}(cx)} dx$$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{x}{a+b\operatorname{sech}^{-1}(cx)}, x\right)$$

[Out] Unintegrable[x/(a + b*ArcSech[c*x]), x]

Rubi [A] time = 0.016564, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{x}{a+b\operatorname{sech}^{-1}(cx)} dx$$

Verification is Not applicable to the result.

[In] Int[x/(a + b*ArcSech[c*x]), x]

[Out] Defer[Int][x/(a + b*ArcSech[c*x]), x]

Rubi steps

$$\int \frac{x}{a+b\operatorname{sech}^{-1}(cx)} dx = \int \frac{x}{a+b\operatorname{sech}^{-1}(cx)} dx$$

Mathematica [A] time = 2.2104, size = 0, normalized size = 0.

$$\int \frac{x}{a+b\operatorname{sech}^{-1}(cx)} dx$$

Verification is Not applicable to the result.

[In] Integrate[x/(a + b*ArcSech[c*x]), x]

[Out] Integrate[x/(a + b*ArcSech[c*x]), x]

Maple [A] time = 0.736, size = 0, normalized size = 0.

$$\int \frac{x}{a+b\operatorname{arcsech}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+b*arcsech(c*x)), x)

[Out] int(x/(a+b*arcsech(c*x)), x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{b \operatorname{ar} \operatorname{sech}(cx) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*arcsech(c*x)),x, algorithm="maxima")

[Out] integrate(x/(b*arcsech(c*x) + a), x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{x}{b \operatorname{ar} \operatorname{sech}(cx) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*arcsech(c*x)),x, algorithm="fricas")

[Out] integral(x/(b*arcsech(c*x) + a), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{a + b \operatorname{asech}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*asech(c*x)),x)

[Out] Integral(x/(a + b*asech(c*x)), x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{b \operatorname{ar} \operatorname{sech}(cx) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*arcsech(c*x)),x, algorithm="giac")

[Out] integrate(x/(b*arcsech(c*x) + a), x)

$$3.52 \quad \int \frac{1}{a+b\operatorname{sech}^{-1}(cx)} dx$$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{1}{a+b\operatorname{sech}^{-1}(cx)}, x\right)$$

[Out] Unintegrable[(a + b*ArcSech[c*x])^(-1), x]

Rubi [A] time = 0.0071896, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{a+b\operatorname{sech}^{-1}(cx)} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*ArcSech[c*x])^(-1), x]

[Out] Defer[Int] [(a + b*ArcSech[c*x])^(-1), x]

Rubi steps

$$\int \frac{1}{a+b\operatorname{sech}^{-1}(cx)} dx = \int \frac{1}{a+b\operatorname{sech}^{-1}(cx)} dx$$

Mathematica [A] time = 0.033105, size = 0, normalized size = 0.

$$\int \frac{1}{a+b\operatorname{sech}^{-1}(cx)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*ArcSech[c*x])^(-1), x]

[Out] Integrate[(a + b*ArcSech[c*x])^(-1), x]

Maple [A] time = 0.367, size = 0, normalized size = 0.

$$\int (a + b\operatorname{arcsech}(cx))^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*arcsech(c*x)), x)

[Out] int(1/(a+b*arcsech(c*x)), x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b \operatorname{arsech}(cx) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*arcsech(c*x)),x, algorithm="maxima")

[Out] integrate(1/(b*arcsech(c*x) + a), x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{b \operatorname{arsech}(cx) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*arcsech(c*x)),x, algorithm="fricas")

[Out] integral(1/(b*arcsech(c*x) + a), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a + b \operatorname{asech}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*asech(c*x)),x)

[Out] Integral(1/(a + b*asech(c*x)), x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b \operatorname{arsech}(cx) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*arcsech(c*x)),x, algorithm="giac")

[Out] integrate(1/(b*arcsech(c*x) + a), x)

$$3.53 \quad \int \frac{1}{x(a+b\operatorname{sech}^{-1}(cx))} dx$$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{1}{x(a+b\operatorname{sech}^{-1}(cx))}, x\right)$$

[Out] Unintegrable[1/(x*(a + b*ArcSech[c*x])), x]

Rubi [A] time = 0.0271964, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{x(a+b\operatorname{sech}^{-1}(cx))} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x*(a + b*ArcSech[c*x])), x]

[Out] Defer[Int][1/(x*(a + b*ArcSech[c*x])), x]

Rubi steps

$$\int \frac{1}{x(a+b\operatorname{sech}^{-1}(cx))} dx = \int \frac{1}{x(a+b\operatorname{sech}^{-1}(cx))} dx$$

Mathematica [A] time = 0.293543, size = 0, normalized size = 0.

$$\int \frac{1}{x(a+b\operatorname{sech}^{-1}(cx))} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x*(a + b*ArcSech[c*x])), x]

[Out] Integrate[1/(x*(a + b*ArcSech[c*x])), x]

Maple [A] time = 0.298, size = 0, normalized size = 0.

$$\int \frac{1}{x(a+b\operatorname{arcsech}(cx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a+b*arcsech(c*x)), x)

[Out] int(1/x/(a+b*arcsech(c*x)), x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{arsech}(cx) + a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*arcsech(c*x)),x, algorithm="maxima")

[Out] integrate(1/((b*arcsech(c*x) + a)*x), x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{bx \operatorname{arsech}(cx) + ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*arcsech(c*x)),x, algorithm="fricas")

[Out] integral(1/(b*x*arcsech(c*x) + a*x), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x(a + b \operatorname{asech}(cx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*asech(c*x)),x)

[Out] Integral(1/(x*(a + b*asech(c*x))), x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{arsech}(cx) + a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*arcsech(c*x)),x, algorithm="giac")

[Out] integrate(1/((b*arcsech(c*x) + a)*x), x)

$$3.54 \quad \int \frac{1}{x^2(a+b\operatorname{sech}^{-1}(cx))} dx$$

Optimal. Leaf size=46

$$\frac{c \sinh\left(\frac{a}{b}\right) \operatorname{Chi}\left(\frac{a}{b} + \operatorname{sech}^{-1}(cx)\right)}{b} - \frac{c \cosh\left(\frac{a}{b}\right) \operatorname{Shi}\left(\frac{a}{b} + \operatorname{sech}^{-1}(cx)\right)}{b}$$

[Out] (c*CoshIntegral[a/b + ArcSech[c*x]]*Sinh[a/b])/b - (c*Cosh[a/b]*SinhIntegral[a/b + ArcSech[c*x]])/b

Rubi [A] time = 0.110401, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6285, 3303, 3298, 3301}

$$\frac{c \sinh\left(\frac{a}{b}\right) \operatorname{Chi}\left(\frac{a}{b} + \operatorname{sech}^{-1}(cx)\right)}{b} - \frac{c \cosh\left(\frac{a}{b}\right) \operatorname{Shi}\left(\frac{a}{b} + \operatorname{sech}^{-1}(cx)\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(a + b*ArcSech[c*x])),x]

[Out] (c*CoshIntegral[a/b + ArcSech[c*x]]*Sinh[a/b])/b - (c*Cosh[a/b]*SinhIntegral[a/b + ArcSech[c*x]])/b

Rule 6285

Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))^ (n_.)*(x_)^(m_.), x_Symbol] :> -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{x^2 (a + b \operatorname{sech}^{-1}(cx))} dx &= - \left(c \operatorname{Subst} \left(\int \frac{\sinh(x)}{a + bx} dx, x, \operatorname{sech}^{-1}(cx) \right) \right) \\ &= - \left(\left(c \cosh \left(\frac{a}{b} \right) \right) \operatorname{Subst} \left(\int \frac{\sinh \left(\frac{a}{b} + x \right)}{a + bx} dx, x, \operatorname{sech}^{-1}(cx) \right) \right) + \left(c \sinh \left(\frac{a}{b} \right) \right) \operatorname{Subst} \left(\int \frac{\cosh \left(\frac{a}{b} + x \right)}{a + bx} dx, x, \operatorname{sech}^{-1}(cx) \right) \\ &= \frac{c \operatorname{Chi} \left(\frac{a}{b} + \operatorname{sech}^{-1}(cx) \right) \sinh \left(\frac{a}{b} \right)}{b} - \frac{c \cosh \left(\frac{a}{b} \right) \operatorname{Shi} \left(\frac{a}{b} + \operatorname{sech}^{-1}(cx) \right)}{b} \end{aligned}$$

Mathematica [A] time = 0.0747712, size = 43, normalized size = 0.93

$$\frac{c \left(\sinh \left(\frac{a}{b} \right) \operatorname{Chi} \left(\frac{a}{b} + \operatorname{sech}^{-1}(cx) \right) - \cosh \left(\frac{a}{b} \right) \operatorname{Shi} \left(\frac{a}{b} + \operatorname{sech}^{-1}(cx) \right) \right)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(a + b*ArcSech[c*x])),x]

[Out] (c*(CoshIntegral[a/b + ArcSech[c*x]]*Sinh[a/b] - Cosh[a/b]*SinhIntegral[a/b + ArcSech[c*x]]))/b

Maple [A] time = 0.211, size = 54, normalized size = 1.2

$$c \left(-\frac{1}{2b} e^{\frac{a}{b}} \operatorname{Ei} \left(1, \frac{a}{b} + \operatorname{arcsech}(cx) \right) + \frac{1}{2b} e^{-\frac{a}{b}} \operatorname{Ei} \left(1, -\operatorname{arcsech}(cx) - \frac{a}{b} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(a+b*arcsech(c*x)),x)

[Out] c*(-1/2/b*exp(a/b)*Ei(1,a/b+arcsech(c*x))+1/2/b*exp(-a/b)*Ei(1,-arcsech(c*x)-a/b))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{ar} \operatorname{sech}(cx) + a)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b*arcsech(c*x)),x, algorithm="maxima")

[Out] integrate(1/((b*arcsech(c*x) + a)*x^2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left(\frac{1}{bx^2 \operatorname{ar} \operatorname{sech}(cx) + ax^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*x^2*arcsech(c*x) + a*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (a + b \operatorname{arsech}(cx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*asech(c*x)),x)
```

```
[Out] Integral(1/(x**2*(a + b*asech(c*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{arsech}(cx) + a)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*arcsech(c*x) + a)*x^2), x)
```

$$3.55 \quad \int \frac{1}{x^3(a+b\operatorname{sech}^{-1}(cx))} dx$$

Optimal. Leaf size=63

$$\frac{c^2 \sinh\left(\frac{2a}{b}\right) \operatorname{Chi}\left(\frac{2a}{b} + 2\operatorname{sech}^{-1}(cx)\right)}{2b} - \frac{c^2 \cosh\left(\frac{2a}{b}\right) \operatorname{Shi}\left(\frac{2a}{b} + 2\operatorname{sech}^{-1}(cx)\right)}{2b}$$

[Out] (c^2*CoshIntegral[(2*a)/b + 2*ArcSech[c*x]]*Sinh[(2*a)/b])/(2*b) - (c^2*CoshIntegral[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSech[c*x]])/(2*b)

Rubi [A] time = 0.141606, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6285, 5448, 12, 3303, 3298, 3301}

$$\frac{c^2 \sinh\left(\frac{2a}{b}\right) \operatorname{Chi}\left(\frac{2a}{b} + 2\operatorname{sech}^{-1}(cx)\right)}{2b} - \frac{c^2 \cosh\left(\frac{2a}{b}\right) \operatorname{Shi}\left(\frac{2a}{b} + 2\operatorname{sech}^{-1}(cx)\right)}{2b}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(a + b*ArcSech[c*x])),x]

[Out] (c^2*CoshIntegral[(2*a)/b + 2*ArcSech[c*x]]*Sinh[(2*a)/b])/(2*b) - (c^2*CoshIntegral[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSech[c*x]])/(2*b)

Rule 6285

Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))^(n_.)*(x_)^(m_.), x_Symbol] :> -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

Rule 5448

Int[Cosh[(a_.) + (b_.)*(x_.)]^(p_.)*((c_.) + (d_.)*(x_.))^(m_.)*Sinh[(a_.) + (b_.)*(x_.)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a + b*x]^(n*p), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f}

, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{x^3 (a + b \operatorname{sech}^{-1}(cx))} dx &= - \left(c^2 \operatorname{Subst} \left(\int \frac{\cosh(x) \sinh(x)}{a + bx} dx, x, \operatorname{sech}^{-1}(cx) \right) \right) \\ &= - \left(c^2 \operatorname{Subst} \left(\int \frac{\sinh(2x)}{2(a + bx)} dx, x, \operatorname{sech}^{-1}(cx) \right) \right) \\ &= - \left(\frac{1}{2} c^2 \operatorname{Subst} \left(\int \frac{\sinh(2x)}{a + bx} dx, x, \operatorname{sech}^{-1}(cx) \right) \right) \\ &= - \left(\frac{1}{2} \left(c^2 \cosh \left(\frac{2a}{b} \right) \right) \operatorname{Subst} \left(\int \frac{\sinh \left(\frac{2a}{b} + 2x \right)}{a + bx} dx, x, \operatorname{sech}^{-1}(cx) \right) \right) + \frac{1}{2} \left(c^2 \sinh \left(\frac{2a}{b} \right) \right) \operatorname{Subst} \left(\int \frac{1}{a + bx} dx, x, \operatorname{sech}^{-1}(cx) \right) \\ &= \frac{c^2 \operatorname{Chi} \left(\frac{2a}{b} + 2 \operatorname{sech}^{-1}(cx) \right) \sinh \left(\frac{2a}{b} \right)}{2b} - \frac{c^2 \cosh \left(\frac{2a}{b} \right) \operatorname{Shi} \left(\frac{2a}{b} + 2 \operatorname{sech}^{-1}(cx) \right)}{2b} \end{aligned}$$

Mathematica [A] time = 0.0761899, size = 56, normalized size = 0.89

$$\frac{c^2 \left(\sinh \left(\frac{2a}{b} \right) \operatorname{Chi} \left(\frac{2a}{b} + 2 \operatorname{sech}^{-1}(cx) \right) - \cosh \left(\frac{2a}{b} \right) \operatorname{Shi} \left(\frac{2a}{b} + 2 \operatorname{sech}^{-1}(cx) \right) \right)}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(a + b*ArcSech[c*x])),x]

[Out] (c^2*(CoshIntegral[(2*a)/b + 2*ArcSech[c*x]]*Sinh[(2*a)/b] - Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSech[c*x]]))/(2*b)

Maple [A] time = 0.207, size = 60, normalized size = 1.

$$c^2 \left(-\frac{1}{4b} e^{2\frac{a}{b}} \operatorname{Ei} \left(1, 2\frac{a}{b} + 2 \operatorname{arcsech}(cx) \right) + \frac{1}{4b} e^{-2\frac{a}{b}} \operatorname{Ei} \left(1, -2 \operatorname{arcsech}(cx) - 2\frac{a}{b} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(a+b*arcsech(c*x)),x)

[Out] c^2*(-1/4/b*exp(2*a/b)*Ei(1,2*a/b+2*arcsech(c*x))+1/4/b*exp(-2*a/b)*Ei(1,-2*arcsech(c*x)-2*a/b))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{ar} \operatorname{sech}(cx) + a)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b*arcsech(c*x)),x, algorithm="maxima")

[Out] integrate(1/((b*arcsech(c*x) + a)*x^3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{bx^3 \operatorname{ar}\operatorname{sech}(cx) + ax^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b*arcsech(c*x)),x, algorithm="fricas")

[Out] integral(1/(b*x^3*arcsech(c*x) + a*x^3), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 (a + b \operatorname{asech}(cx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(a+b*asech(c*x)), x)

[Out] Integral(1/(x**3*(a + b*asech(c*x))), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{ar}\operatorname{sech}(cx) + a)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b*arcsech(c*x)),x, algorithm="giac")

[Out] integrate(1/((b*arcsech(c*x) + a)*x^3), x)

$$3.56 \quad \int \frac{1}{x^4(a+b\operatorname{sech}^{-1}(cx))} dx$$

Optimal. Leaf size=117

$$\frac{c^3 \sinh\left(\frac{a}{b}\right) \operatorname{Chi}\left(\frac{a}{b} + \operatorname{sech}^{-1}(cx)\right)}{4b} + \frac{c^3 \sinh\left(\frac{3a}{b}\right) \operatorname{Chi}\left(\frac{3a}{b} + 3\operatorname{sech}^{-1}(cx)\right)}{4b} - \frac{c^3 \cosh\left(\frac{a}{b}\right) \operatorname{Shi}\left(\frac{a}{b} + \operatorname{sech}^{-1}(cx)\right)}{4b} - \frac{c^3 \cosh\left(\frac{3a}{b}\right) \operatorname{Shi}\left(\frac{3a}{b} + 3\operatorname{sech}^{-1}(cx)\right)}{4b}$$

[Out] (c^3*CoshIntegral[a/b + ArcSech[c*x]]*Sinh[a/b])/(4*b) + (c^3*CoshIntegral[(3*a)/b + 3*ArcSech[c*x]]*Sinh[(3*a)/b])/(4*b) - (c^3*Cosh[a/b]*SinhIntegral[a/b + ArcSech[c*x]])/(4*b) - (c^3*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSech[c*x]])/(4*b)

Rubi [A] time = 0.236169, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {6285, 5448, 3303, 3298, 3301}

$$\frac{c^3 \sinh\left(\frac{a}{b}\right) \operatorname{Chi}\left(\frac{a}{b} + \operatorname{sech}^{-1}(cx)\right)}{4b} + \frac{c^3 \sinh\left(\frac{3a}{b}\right) \operatorname{Chi}\left(\frac{3a}{b} + 3\operatorname{sech}^{-1}(cx)\right)}{4b} - \frac{c^3 \cosh\left(\frac{a}{b}\right) \operatorname{Shi}\left(\frac{a}{b} + \operatorname{sech}^{-1}(cx)\right)}{4b} - \frac{c^3 \cosh\left(\frac{3a}{b}\right) \operatorname{Shi}\left(\frac{3a}{b} + 3\operatorname{sech}^{-1}(cx)\right)}{4b}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(a + b*ArcSech[c*x])),x]

[Out] (c^3*CoshIntegral[a/b + ArcSech[c*x]]*Sinh[a/b])/(4*b) + (c^3*CoshIntegral[(3*a)/b + 3*ArcSech[c*x]]*Sinh[(3*a)/b])/(4*b) - (c^3*Cosh[a/b]*SinhIntegral[a/b + ArcSech[c*x]])/(4*b) - (c^3*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSech[c*x]])/(4*b)

Rule 6285

Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))^ (n_.)*(x_.)^(m_.), x_Symbol] :> -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

Rule 5448

Int[Cosh[(a_.) + (b_.)*(x_.)]^(p_.)*((c_.) + (d_.)*(x_.))^(m_.)*Sinh[(a_.) + (b_.)*(x_.)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a + b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x^4 (a + b \operatorname{sech}^{-1}(cx))} dx &= - \left(c^3 \operatorname{Subst} \left(\int \frac{\cosh^2(x) \sinh(x)}{a + bx} dx, x, \operatorname{sech}^{-1}(cx) \right) \right) \\ &= - \left(c^3 \operatorname{Subst} \left(\int \left(\frac{\sinh(x)}{4(a + bx)} + \frac{\sinh(3x)}{4(a + bx)} \right) dx, x, \operatorname{sech}^{-1}(cx) \right) \right) \\ &= - \left(\frac{1}{4} c^3 \operatorname{Subst} \left(\int \frac{\sinh(x)}{a + bx} dx, x, \operatorname{sech}^{-1}(cx) \right) \right) - \frac{1}{4} c^3 \operatorname{Subst} \left(\int \frac{\sinh(3x)}{a + bx} dx, x, \operatorname{sech}^{-1}(cx) \right) \\ &= - \left(\frac{1}{4} \left(c^3 \cosh \left(\frac{a}{b} \right) \right) \operatorname{Subst} \left(\int \frac{\sinh \left(\frac{a}{b} + x \right)}{a + bx} dx, x, \operatorname{sech}^{-1}(cx) \right) \right) - \frac{1}{4} \left(c^3 \cosh \left(\frac{3a}{b} \right) \right) \operatorname{Subst} \left(\int \frac{\sinh \left(\frac{3a}{b} + x \right)}{a + bx} dx, x, \operatorname{sech}^{-1}(cx) \right) \\ &= \frac{c^3 \operatorname{Chi} \left(\frac{a}{b} + \operatorname{sech}^{-1}(cx) \right) \sinh \left(\frac{a}{b} \right)}{4b} + \frac{c^3 \operatorname{Chi} \left(\frac{3a}{b} + 3 \operatorname{sech}^{-1}(cx) \right) \sinh \left(\frac{3a}{b} \right)}{4b} - \frac{c^3 \cosh \left(\frac{a}{b} \right) \operatorname{Shi} \left(\frac{a}{b} + \operatorname{sech}^{-1}(cx) \right)}{4b} - \frac{c^3 \cosh \left(\frac{3a}{b} \right) \operatorname{Shi} \left(\frac{3a}{b} + 3 \operatorname{sech}^{-1}(cx) \right)}{4b} \end{aligned}$$

Mathematica [A] time = 0.155581, size = 91, normalized size = 0.78

$$\frac{c^3 \left(\sinh \left(\frac{a}{b} \right) \left(-\operatorname{Chi} \left(\frac{a}{b} + \operatorname{sech}^{-1}(cx) \right) \right) - \sinh \left(\frac{3a}{b} \right) \operatorname{Chi} \left(3 \left(\frac{a}{b} + \operatorname{sech}^{-1}(cx) \right) \right) + \cosh \left(\frac{a}{b} \right) \operatorname{Shi} \left(\frac{a}{b} + \operatorname{sech}^{-1}(cx) \right) + \cosh \left(\frac{3a}{b} \right) \operatorname{Shi} \left(\frac{3a}{b} + 3 \operatorname{sech}^{-1}(cx) \right) \right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*ArcSech[c*x])), x]
```

```
[Out] -(c^3*(-(CoshIntegral[a/b + ArcSech[c*x]]*Sinh[a/b]) - CoshIntegral[3*(a/b + ArcSech[c*x]]*Sinh[(3*a)/b] + Cosh[a/b]*SinhIntegral[a/b + ArcSech[c*x]] + Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSech[c*x])]))/(4*b)
```

Maple [A] time = 0.234, size = 110, normalized size = 0.9

$$c^3 \left(-\frac{1}{8b} e^{3\frac{a}{b}} \operatorname{Ei} \left(1, 3\frac{a}{b} + 3 \operatorname{arcsech}(cx) \right) - \frac{1}{8b} e^{\frac{a}{b}} \operatorname{Ei} \left(1, \frac{a}{b} + \operatorname{arcsech}(cx) \right) + \frac{1}{8b} e^{-\frac{a}{b}} \operatorname{Ei} \left(1, -\operatorname{arcsech}(cx) - \frac{a}{b} \right) + \frac{1}{8b} e^{-3\frac{a}{b}} \operatorname{Ei} \left(1, -3\operatorname{arcsech}(cx) - 3\frac{a}{b} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a+b*arcsech(c*x)), x)
```

```
[Out] c^3*(-1/8/b*exp(3*a/b)*Ei(1,3*a/b+3*arcsech(c*x))-1/8/b*exp(a/b)*Ei(1,a/b+arcsech(c*x))+1/8/b*exp(-a/b)*Ei(1,-arcsech(c*x)-a/b)+1/8/b*exp(-3*a/b)*Ei(1,-3*arcsech(c*x)-3*a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{ar} \operatorname{sech}(cx) + a)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a+b*arcsech(c*x)),x, algorithm="maxima")

[Out] integrate(1/((b*arcsech(c*x) + a)*x^4), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{bx^4 \operatorname{ar} \operatorname{sech}(cx) + ax^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a+b*arcsech(c*x)),x, algorithm="fricas")

[Out] integral(1/(b*x^4*arcsech(c*x) + a*x^4), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 (a + b \operatorname{asech}(cx))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(a+b*asech(c*x)),x)

[Out] Integral(1/(x**4*(a + b*asech(c*x))), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{ar} \operatorname{sech}(cx) + a)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a+b*arcsech(c*x)),x, algorithm="giac")

[Out] integrate(1/((b*arcsech(c*x) + a)*x^4), x)

$$3.57 \quad \int \frac{x}{\left(a + b \operatorname{sech}^{-1}(cx)\right)^2} dx$$

Optimal. Leaf size=14

$$\text{Unintegrable} \left(\frac{x}{\left(a + b \operatorname{sech}^{-1}(cx)\right)^2}, x \right)$$

[Out] Unintegrable[x/(a + b*ArcSech[c*x])^2, x]

Rubi [A] time = 0.0156607, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{x}{\left(a + b \operatorname{sech}^{-1}(cx)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[x/(a + b*ArcSech[c*x])^2, x]

[Out] Defer[Int][x/(a + b*ArcSech[c*x])^2, x]

Rubi steps

$$\int \frac{x}{\left(a + b \operatorname{sech}^{-1}(cx)\right)^2} dx = \int \frac{x}{\left(a + b \operatorname{sech}^{-1}(cx)\right)^2} dx$$

Mathematica [A] time = 16.3804, size = 0, normalized size = 0.

$$\int \frac{x}{\left(a + b \operatorname{sech}^{-1}(cx)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[x/(a + b*ArcSech[c*x])^2, x]

[Out] Integrate[x/(a + b*ArcSech[c*x])^2, x]

Maple [A] time = 0.677, size = 0, normalized size = 0.

$$\int \frac{x}{\left(a + b \operatorname{arcsech}(cx)\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+b*arcsech(c*x))^2, x)

[Out] $\int (x/(a+b*\operatorname{arcsech}(c*x))^2, x)$

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{(c^2x^3 - x)\sqrt{cx + 1}\sqrt{-cx + 1}x + (c^2x^3 - x)x}{(b^2c^2 \log(c) - abc^2)x^2 - b^2 \log(c) - (b^2 \log(c) + b^2 \log(x) - ab)\sqrt{cx + 1}\sqrt{-cx + 1} + ab - (b^2c^2x^2 - \sqrt{cx + 1}\sqrt{-cx + 1}b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a+b*arcsech(c*x))^2,x, algorithm="maxima")`

[Out] $-\left(\frac{(c^2x^3 - x)\sqrt{cx + 1}\sqrt{-cx + 1}x + (c^2x^3 - x)x}{(b^2c^2 \log(c) - abc^2)x^2 - b^2 \log(c) - (b^2 \log(c) + b^2 \log(x) - abc^2)\sqrt{cx + 1}\sqrt{-cx + 1} + ab - (b^2c^2x^2 - \sqrt{cx + 1}\sqrt{-cx + 1}b^2)}\right) + \int \left(\frac{2(2c^2x^2 - 1)(cx + 1)(cx - 1)x + (3c^4x^4 - 8c^2x^2 + 4)\sqrt{cx + 1}\sqrt{-cx + 1}x + 2(c^4x^4 - 2c^2x^2 + 1)x}{(b^2c^4 \log(c) - abc^4)x^4 - (b^2 \log(c) + b^2 \log(x) - abc^2)(cx + 1)(cx - 1) - 2(b^2c^2 \log(c) - abc^2)x^2 + b^2 \log(c) - 2((b^2c^2 \log(c) - abc^2)x^2 - b^2 \log(c) + abc^2)\sqrt{cx + 1}\sqrt{-cx + 1} - abc - (b^2c^4x^4 - 2b^2c^2x^2 - (cx + 1)(cx - 1)b^2 - 2(b^2c^2x^2 - b^2)\sqrt{cx + 1}\sqrt{-cx + 1} + b^2)\log(\sqrt{cx + 1}\sqrt{-cx + 1} + 1) + (b^2c^4x^4 - 2b^2c^2x^2 + b^2)\log(x)} \right), x$

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{x}{b^2 \operatorname{arsech}(cx)^2 + 2ab \operatorname{arsech}(cx) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a+b*arcsech(c*x))^2,x, algorithm="fricas")`

[Out] $\operatorname{integral}(x/(b^2*\operatorname{arcsech}(c*x)^2 + 2*a*b*\operatorname{arcsech}(c*x) + a^2), x)$

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(a + b \operatorname{asech}(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a+b*asech(c*x))**2,x)`

[Out] `Integral(x/(a + b*asech(c*x))**2, x)`

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(b \operatorname{arsech}(cx) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arcsech(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(x/(b*arcsech(c*x) + a)^2, x)
```

$$3.58 \quad \int \frac{1}{(a+b\operatorname{sech}^{-1}(cx))^2} dx$$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{1}{(a+b\operatorname{sech}^{-1}(cx))^2}, x\right)$$

[Out] Unintegrable[(a + b*ArcSech[c*x])^(-2), x]

Rubi [A] time = 0.0067468, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{(a+b\operatorname{sech}^{-1}(cx))^2} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*ArcSech[c*x])^(-2), x]

[Out] Defer[Int] [(a + b*ArcSech[c*x])^(-2), x]

Rubi steps

$$\int \frac{1}{(a+b\operatorname{sech}^{-1}(cx))^2} dx = \int \frac{1}{(a+b\operatorname{sech}^{-1}(cx))^2} dx$$

Mathematica [A] time = 8.59317, size = 0, normalized size = 0.

$$\int \frac{1}{(a+b\operatorname{sech}^{-1}(cx))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*ArcSech[c*x])^(-2), x]

[Out] Integrate[(a + b*ArcSech[c*x])^(-2), x]

Maple [A] time = 0.3, size = 0, normalized size = 0.

$$\int (a + b\operatorname{arcsech}(cx))^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*arcsech(c*x))^2, x)

[Out] $\int (1/(a+b*\operatorname{arcsech}(c*x))^2, x)$

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{c^2x^3 + (c^2x^3 - x)\sqrt{cx+1}\sqrt{-cx+1} - x}{(b^2c^2 \log(c) - abc^2)x^2 - b^2 \log(c) - (b^2 \log(c) + b^2 \log(x) - ab)\sqrt{cx+1}\sqrt{-cx+1} + ab - (b^2c^2x^2 - \sqrt{cx+1}\sqrt{-cx+1})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{integrate}(1/(a+b*\operatorname{arcsech}(c*x))^2, x, \operatorname{algorithm}="maxima")$

[Out] $-(c^2*x^3 + (c^2*x^3 - x)*\sqrt{c*x + 1}*\sqrt{-c*x + 1} - x)/((b^2*c^2*\log(c) - a*b*c^2)*x^2 - b^2*\log(c) - (b^2*\log(c) + b^2*\log(x) - a*b)*\sqrt{c*x + 1}*\sqrt{-c*x + 1} + a*b - (b^2*c^2*x^2 - \sqrt{c*x + 1}*\sqrt{-c*x + 1})*b^2 - b^2)*\log(\sqrt{c*x + 1}*\sqrt{-c*x + 1} + 1) + (b^2*c^2*x^2 - b^2)*\log(x)) + \operatorname{integrate}((c^4*x^4 - 2*c^2*x^2 + (3*c^2*x^2 - 1)*(c*x + 1)*(c*x - 1) + (2*c^4*x^4 - 5*c^2*x^2 + 2)*\sqrt{c*x + 1}*\sqrt{-c*x + 1} + 1)/((b^2*c^4*\log(c) - a*b*c^4)*x^4 - (b^2*\log(c) + b^2*\log(x) - a*b)*(c*x + 1)*(c*x - 1) - 2*(b^2*c^2*\log(c) - a*b*c^2)*x^2 + b^2*\log(c) - 2*((b^2*c^2*\log(c) - a*b*c^2)*x^2 - b^2*\log(c) + a*b + (b^2*c^2*x^2 - b^2)*\log(x))*\sqrt{c*x + 1}*\sqrt{-c*x + 1} - a*b - (b^2*c^4*x^4 - 2*b^2*c^2*x^2 - (c*x + 1)*(c*x - 1)*b^2 - 2*(b^2*c^2*x^2 - b^2)*\sqrt{c*x + 1}*\sqrt{-c*x + 1} + b^2)*\log(\sqrt{c*x + 1}*\sqrt{-c*x + 1} + 1) + (b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*\log(x)), x)$

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{b^2 \operatorname{arsech}(cx)^2 + 2ab \operatorname{arsech}(cx) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{integrate}(1/(a+b*\operatorname{arcsech}(c*x))^2, x, \operatorname{algorithm}="fricas")$

[Out] $\operatorname{integral}(1/(b^2*\operatorname{arcsech}(c*x))^2 + 2*a*b*\operatorname{arcsech}(c*x) + a^2), x)$

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b \operatorname{asech}(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{integrate}(1/(a+b*\operatorname{asech}(c*x))**2, x)$

[Out] $\operatorname{Integral}((a + b*\operatorname{asech}(c*x))**(-2), x)$

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{arsech}(cx) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arcsech(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)^(-2), x)
```

$$3.59 \quad \int \frac{1}{x(a+b\operatorname{sech}^{-1}(cx))^2} dx$$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{1}{x(a+b\operatorname{sech}^{-1}(cx))^2}, x\right)$$

[Out] Unintegrable[1/(x*(a + b*ArcSech[c*x])^2), x]

Rubi [A] time = 0.0253484, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{x(a+b\operatorname{sech}^{-1}(cx))^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x*(a + b*ArcSech[c*x])^2), x]

[Out] Defer[Int][1/(x*(a + b*ArcSech[c*x])^2), x]

Rubi steps

$$\int \frac{1}{x(a+b\operatorname{sech}^{-1}(cx))^2} dx = \int \frac{1}{x(a+b\operatorname{sech}^{-1}(cx))^2} dx$$

Mathematica [A] time = 4.99292, size = 0, normalized size = 0.

$$\int \frac{1}{x(a+b\operatorname{sech}^{-1}(cx))^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x*(a + b*ArcSech[c*x])^2), x]

[Out] Integrate[1/(x*(a + b*ArcSech[c*x])^2), x]

Maple [A] time = 0.286, size = 0, normalized size = 0.

$$\int \frac{1}{x(a+b\operatorname{arcsech}(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a+b*arcsech(c*x))^2, x)

[Out] $\int (1/x/(a+b*\operatorname{arcsech}(c*x))^2, x)$

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{c^2x^3 + (c^2x^3 - x)\sqrt{cx+1}\sqrt{-cx+1} - x}{(b^2c^2x^2 - b^2)x \log(x) - (b^2x \log(x) + (b^2 \log(c) - ab)x)\sqrt{cx+1}\sqrt{-cx+1} + ((b^2c^2 \log(c) - abc^2)x^2 - b^2 \log(c) + ab)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{integrate}(1/x/(a+b*\operatorname{arcsech}(c*x))^2, x, \text{algorithm}="maxima")$

[Out] $-(c^2*x^3 + (c^2*x^3 - x)*\sqrt{c*x + 1}*\sqrt{-c*x + 1} - x)/((b^2*c^2*x^2 - b^2)*x*\log(x) - (b^2*x*\log(x) + (b^2*\log(c) - a*b)*x)*\sqrt{c*x + 1}*\sqrt{-c*x + 1} + ((b^2*c^2*\log(c) - a*b*c^2)*x^2 - b^2*\log(c) + a*b)*x + (\sqrt{c*x + 1}*\sqrt{-c*x + 1}*b^2*x - (b^2*c^2*x^2 - b^2)*x)*\log(\sqrt{c*x + 1}*\sqrt{-c*x + 1} + 1)) + \operatorname{integrate}(-2*(c*x + 1)*(c*x - 1)*c^2*x^2 + (c^4*x^4 - 2*c^2*x^2)*\sqrt{c*x + 1}*\sqrt{-c*x + 1})/((b^2*x*\log(x) + (b^2*\log(c) - a*b)*x)*(c*x + 1)*(c*x - 1) - (b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*x*\log(x) + 2*((b^2*c^2*x^2 - b^2)*x*\log(x) + ((b^2*c^2*\log(c) - a*b*c^2)*x^2 - b^2*\log(c) + a*b)*x)*\sqrt{c*x + 1}*\sqrt{-c*x + 1} - ((b^2*c^4*\log(c) - a*b*c^4)*x^4 - 2*(b^2*c^2*\log(c) - a*b*c^2)*x^2 + b^2*\log(c) - a*b)*x - ((c*x + 1)*(c*x - 1)*b^2*x + 2*(b^2*c^2*x^2 - b^2)*\sqrt{c*x + 1}*\sqrt{-c*x + 1})*x - (b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*x)*\log(\sqrt{c*x + 1}*\sqrt{-c*x + 1} + 1)), x)$

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{b^2x \operatorname{arsech}(cx)^2 + 2abx \operatorname{arsech}(cx) + a^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{integrate}(1/x/(a+b*\operatorname{arcsech}(c*x))^2, x, \text{algorithm}="fricas")$

[Out] $\operatorname{integral}(1/(b^2*x*\operatorname{arcsech}(c*x)^2 + 2*a*b*x*\operatorname{arcsech}(c*x) + a^2*x), x)$

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x(a + b \operatorname{asech}(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{integrate}(1/x/(a+b*\operatorname{asech}(c*x))^2, x)$

[Out] $\operatorname{Integral}(1/(x*(a + b*\operatorname{asech}(c*x))^2), x)$

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{arsech}(cx) + a)^2 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arcsech(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*arcsech(c*x) + a)^2*x), x)
```

$$3.60 \quad \int \frac{1}{x^2 \left(a + b \operatorname{sech}^{-1}(cx) \right)^2} dx$$

Optimal. Leaf size=86

$$-\frac{c \cosh\left(\frac{a}{b}\right) \operatorname{Chi}\left(\frac{a}{b} + \operatorname{sech}^{-1}(cx)\right)}{b^2} + \frac{c \sinh\left(\frac{a}{b}\right) \operatorname{Shi}\left(\frac{a}{b} + \operatorname{sech}^{-1}(cx)\right)}{b^2} + \frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{bx \left(a + b \operatorname{sech}^{-1}(cx) \right)}$$

[Out] (Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(b*x*(a + b*ArcSech[c*x])) - (c*Cosh[a/b]*CoshIntegral[a/b + ArcSech[c*x]])/b^2 + (c*Sinh[a/b]*SinhIntegral[a/b + ArcSech[c*x]])/b^2

Rubi [A] time = 0.144917, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {6285, 3297, 3303, 3298, 3301}

$$-\frac{c \cosh\left(\frac{a}{b}\right) \operatorname{Chi}\left(\frac{a}{b} + \operatorname{sech}^{-1}(cx)\right)}{b^2} + \frac{c \sinh\left(\frac{a}{b}\right) \operatorname{Shi}\left(\frac{a}{b} + \operatorname{sech}^{-1}(cx)\right)}{b^2} + \frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{bx \left(a + b \operatorname{sech}^{-1}(cx) \right)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(a + b*ArcSech[c*x])^2),x]

[Out] (Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(b*x*(a + b*ArcSech[c*x])) - (c*Cosh[a/b]*CoshIntegral[a/b + ArcSech[c*x]])/b^2 + (c*Sinh[a/b]*SinhIntegral[a/b + ArcSech[c*x]])/b^2

Rule 6285

Int[((a_.) + ArcSech[(c_.)*(x_)])*(b_.))^ (n_.)*(x_)^(m_.), x_Symbol] :> -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

Rule 3297

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x^2 (a + b \operatorname{sech}^{-1}(cx))^2} dx &= - \left(c \operatorname{Subst} \left(\int \frac{\sinh(x)}{(a + bx)^2} dx, x, \operatorname{sech}^{-1}(cx) \right) \right) \\ &= \frac{\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{bx(a+b\operatorname{sech}^{-1}(cx))} - \frac{c \operatorname{Subst} \left(\int \frac{\cosh(x)}{a+bx} dx, x, \operatorname{sech}^{-1}(cx) \right)}{b} \\ &= \frac{\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{bx(a+b\operatorname{sech}^{-1}(cx))} - \frac{(c \cosh(\frac{a}{b})) \operatorname{Subst} \left(\int \frac{\cosh(\frac{a}{b}+x)}{a+bx} dx, x, \operatorname{sech}^{-1}(cx) \right)}{b} + \frac{(c \sinh(\frac{a}{b})) \operatorname{Subst} \left(\int \frac{\sinh(\frac{a}{b}+x)}{a+bx} dx, x, \operatorname{sech}^{-1}(cx) \right)}{b} \\ &= \frac{\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{bx(a+b\operatorname{sech}^{-1}(cx))} - \frac{c \cosh(\frac{a}{b}) \operatorname{Chi}(\frac{a}{b} + \operatorname{sech}^{-1}(cx))}{b^2} + \frac{c \sinh(\frac{a}{b}) \operatorname{Shi}(\frac{a}{b} + \operatorname{sech}^{-1}(cx))}{b^2} \end{aligned}$$

Mathematica [A] time = 0.333992, size = 82, normalized size = 0.95

$$\frac{-c \cosh\left(\frac{a}{b}\right) \operatorname{Chi}\left(\frac{a}{b} + \operatorname{sech}^{-1}(cx)\right) + c \sinh\left(\frac{a}{b}\right) \operatorname{Shi}\left(\frac{a}{b} + \operatorname{sech}^{-1}(cx)\right) + \frac{b\sqrt{\frac{1-cx}{1+cx}}(cx+1)}{x(a+b\operatorname{sech}^{-1}(cx))}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*ArcSech[c*x])^2), x]
```

```
[Out] ((b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(x*(a + b*ArcSech[c*x])) - c*Cosh[a/b]*CoshIntegral[a/b + ArcSech[c*x]] + c*Sinh[a/b]*SinhIntegral[a/b + ArcSech[c*x]])/b^2
```

Maple [A] time = 0.258, size = 164, normalized size = 1.9

$$c \left(\frac{1}{2xbc(a + b \operatorname{arcsech}(cx))} \left(\sqrt{-\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} cx - 1 \right) + \frac{1}{2b^2} e^{\frac{a}{b}} \operatorname{Ei} \left(1, \frac{a}{b} + \operatorname{arcsech}(cx) \right) + \frac{1}{2xbc(a + b \operatorname{arcsech}(cx))} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*arcsech(c*x))^2, x)
```

```
[Out] c*(1/2*((-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*c*x-1)/c/x/b/(a+b*arcsech(c*x))+1/2/b^2*exp(a/b)*Ei(1, a/b+arcsech(c*x))+1/2/b*((-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*c*x+1)/c/x/(a+b*arcsech(c*x))+1/2/b^2*exp(-a/b)*Ei(1, -arcsech(c*x)-a/b))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{c^2x^3 + (c^2x^3 - x)\sqrt{cx + 1}\sqrt{-cx + 1} - x}{(b^2c^2x^2 - b^2)x^2 \log(x) + ((b^2c^2 \log(c) - abc^2)x^2 - b^2 \log(c) + ab)x^2 - (b^2x^2 \log(x) + (b^2 \log(c) - ab)x^2)\sqrt{cx + 1}\sqrt{-cx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b*arcsech(c*x))^2,x, algorithm="maxima")

[Out] $-(c^2x^3 + (c^2x^3 - x)\sqrt{cx + 1}\sqrt{-cx + 1} - x)/((b^2c^2x^2 - b^2)x^2 \log(x) + ((b^2c^2 \log(c) - abc^2)x^2 - b^2 \log(c) + ab)x^2 - (b^2x^2 \log(x) + (b^2 \log(c) - ab)x^2)\sqrt{cx + 1}\sqrt{-cx}) + (\sqrt{cx + 1}\sqrt{-cx + 1}b^2x^2 - (b^2c^2x^2 - b^2)x^2)\log(\sqrt{cx + 1}\sqrt{-cx + 1} + 1) + \int (-(c^4x^4 - 2c^2x^2 - (c^2x^2 + 1)(cx + 1)(cx - 1) - (c^2x^2 - 2)\sqrt{cx + 1}\sqrt{-cx + 1} + 1)/(b^2c^4x^4 - 2b^2c^2x^2 + b^2)x^2 \log(x) - (b^2x^2 \log(x) + (b^2 \log(c) - abc^2)x^2)(cx + 1)(cx - 1) + ((b^2c^4 \log(c) - abc^4)x^4 - 2(b^2c^2 \log(c) - abc^2)x^2 + b^2 \log(c) - abc^2)x^2 - 2((b^2c^2x^2 - b^2)x^2 \log(x) + ((b^2c^2 \log(c) - abc^2)x^2 - b^2 \log(c) + abc^2)x^2)\sqrt{cx + 1}\sqrt{-cx + 1} + ((cx + 1)(cx - 1)b^2x^2 + 2(b^2c^2x^2 - b^2)\sqrt{cx + 1}\sqrt{-cx + 1})x^2 - (b^2c^4x^4 - 2b^2c^2x^2 + b^2)x^2)\log(\sqrt{cx + 1}\sqrt{-cx + 1} + 1), x)$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b^2x^2 \operatorname{arsech}(cx)^2 + 2abx^2 \operatorname{arsech}(cx) + a^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b*arcsech(c*x))^2,x, algorithm="fricas")

[Out] integral(1/(b^2*x^2*arcsech(c*x)^2 + 2*a*b*x^2*arcsech(c*x) + a^2*x^2), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (a + b \operatorname{asech}(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(a+b*asech(c*x))**2,x)

[Out] Integral(1/(x**2*(a + b*asech(c*x))**2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{arsech}(cx) + a)^2 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arcsech(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*arcsech(c*x) + a)^2*x^2), x)
```

$$3.61 \quad \int \frac{1}{x^3 \left(a + b \operatorname{sech}^{-1}(cx) \right)^2} dx$$

Optimal. Leaf size=85

$$-\frac{c^2 \cosh\left(\frac{2a}{b}\right) \operatorname{Chi}\left(\frac{2a}{b} + 2 \operatorname{sech}^{-1}(cx)\right)}{b^2} + \frac{c^2 \sinh\left(\frac{2a}{b}\right) \operatorname{Shi}\left(\frac{2a}{b} + 2 \operatorname{sech}^{-1}(cx)\right)}{b^2} + \frac{c^2 \sinh\left(2 \operatorname{sech}^{-1}(cx)\right)}{2b \left(a + b \operatorname{sech}^{-1}(cx) \right)}$$

[Out] $-\left(\frac{c^2 \operatorname{Cosh}\left[\frac{2a}{b}\right] \operatorname{CoshIntegral}\left[\frac{2a}{b} + 2 \operatorname{ArcSech}[c*x]\right]}{b^2}\right) + \left(\frac{c^2 \operatorname{Sinh}\left[2 \operatorname{ArcSech}[c*x]\right]}{2*b*(a + b*\operatorname{ArcSech}[c*x])}\right) + \left(\frac{c^2 \operatorname{Sinh}\left[\frac{2a}{b}\right] \operatorname{SinhIntegral}\left[\frac{2a}{b} + 2 \operatorname{ArcSech}[c*x]\right]}{b^2}\right)$

Rubi [A] time = 0.164571, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6285, 5448, 12, 3297, 3303, 3298, 3301}

$$-\frac{c^2 \cosh\left(\frac{2a}{b}\right) \operatorname{Chi}\left(\frac{2a}{b} + 2 \operatorname{sech}^{-1}(cx)\right)}{b^2} + \frac{c^2 \sinh\left(\frac{2a}{b}\right) \operatorname{Shi}\left(\frac{2a}{b} + 2 \operatorname{sech}^{-1}(cx)\right)}{b^2} + \frac{c^2 \sinh\left(2 \operatorname{sech}^{-1}(cx)\right)}{2b \left(a + b \operatorname{sech}^{-1}(cx) \right)}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}\left[1/\left(x^3*(a + b*\operatorname{ArcSech}[c*x])^2\right), x\right]$

[Out] $-\left(\frac{c^2 \operatorname{Cosh}\left[\frac{2a}{b}\right] \operatorname{CoshIntegral}\left[\frac{2a}{b} + 2 \operatorname{ArcSech}[c*x]\right]}{b^2}\right) + \left(\frac{c^2 \operatorname{Sinh}\left[2 \operatorname{ArcSech}[c*x]\right]}{2*b*(a + b*\operatorname{ArcSech}[c*x])}\right) + \left(\frac{c^2 \operatorname{Sinh}\left[\frac{2a}{b}\right] \operatorname{SinhIntegral}\left[\frac{2a}{b} + 2 \operatorname{ArcSech}[c*x]\right]}{b^2}\right)$

Rule 6285

$\operatorname{Int}\left[\left((a_.) + \operatorname{ArcSech}\left[(c_.)*(x_.)]*(b_.)\right)^{(n_.)}*(x_.)^{(m_.)}, x_Symbol\right] :> -\operatorname{Dist}\left[\left(c^{(m+1)}\right)^{-1}, \operatorname{Subst}\left[\operatorname{Int}\left[(a + b*x)^n \operatorname{Sech}[x]^{(m+1)} \operatorname{Tanh}[x], x\right], x, \operatorname{ArcSech}[c*x]\right], x\right] /;$ $\operatorname{FreeQ}\{a, b, c, x\} \ \&\& \ \operatorname{IntegerQ}[n] \ \&\& \ \operatorname{IntegerQ}[m] \ \&\& \ (\operatorname{GtQ}[n, 0] \ || \ \operatorname{LtQ}[m, -1])$

Rule 5448

$\operatorname{Int}\left[\operatorname{Cosh}\left[(a_.) + (b_.)*(x_.)\right]^{(p_.)}*\left((c_.) + (d_.)*(x_.)\right)^{(m_.)}*\operatorname{Sinh}\left[(a_.) + (b_.)*(x_.)\right]^{(n_.)}, x_Symbol\right] :> \operatorname{Int}\left[\operatorname{ExpandTrigReduce}\left[(c + d*x)^m, \operatorname{Sinh}[a + b*x]^{(n)}*\operatorname{Cosh}[a + b*x]^p, x\right], x\right] /;$ $\operatorname{FreeQ}\{a, b, c, d, m, x\} \ \&\& \ \operatorname{IGtQ}[n, 0] \ \& \ \operatorname{IGtQ}[p, 0]$

Rule 12

$\operatorname{Int}\left[(a_)*(u_), x_Symbol\right] :> \operatorname{Dist}\left[a, \operatorname{Int}[u, x], x\right] /;$ $\operatorname{FreeQ}[a, x] \ \&\& \ !\operatorname{MatchQ}[u, (b_)*(v_)] /;$ $\operatorname{FreeQ}[b, x]$

Rule 3297

$\operatorname{Int}\left[\left((c_.) + (d_.)*(x_.)\right)^{(m_.)}*\sin\left[(e_.) + (f_.)*(x_.)\right], x_Symbol\right] :> \operatorname{Simp}\left[\left((c + d*x)^{(m+1)}*\sin[e + f*x]\right)/(d*(m+1)), x\right] - \operatorname{Dist}\left[f/(d*(m+1)), \operatorname{Int}\left[(c + d*x)^{(m+1)}*\cos[e + f*x], x\right], x\right] /;$ $\operatorname{FreeQ}\{c, d, e, f, x\} \ \&\& \ \operatorname{LtQ}[m, -1]$

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x^3 (a + b \operatorname{sech}^{-1}(cx))^2} dx &= - \left(c^2 \operatorname{Subst} \left(\int \frac{\cosh(x) \sinh(x)}{(a + bx)^2} dx, x, \operatorname{sech}^{-1}(cx) \right) \right) \\ &= - \left(c^2 \operatorname{Subst} \left(\int \frac{\sinh(2x)}{2(a + bx)^2} dx, x, \operatorname{sech}^{-1}(cx) \right) \right) \\ &= - \left(\frac{1}{2} c^2 \operatorname{Subst} \left(\int \frac{\sinh(2x)}{(a + bx)^2} dx, x, \operatorname{sech}^{-1}(cx) \right) \right) \\ &= \frac{c^2 \sinh(2 \operatorname{sech}^{-1}(cx))}{2b(a + b \operatorname{sech}^{-1}(cx))} - \frac{c^2 \operatorname{Subst} \left(\int \frac{\cosh(2x)}{a + bx} dx, x, \operatorname{sech}^{-1}(cx) \right)}{b} \\ &= \frac{c^2 \sinh(2 \operatorname{sech}^{-1}(cx))}{2b(a + b \operatorname{sech}^{-1}(cx))} - \frac{\left(c^2 \cosh\left(\frac{2a}{b}\right) \right) \operatorname{Subst} \left(\int \frac{\cosh\left(\frac{2a}{b} + 2x\right)}{a + bx} dx, x, \operatorname{sech}^{-1}(cx) \right)}{b} + \frac{c^2 \cosh\left(\frac{2a}{b}\right) \operatorname{Chi}\left(\frac{2a}{b} + 2 \operatorname{sech}^{-1}(cx)\right)}{b^2} \\ &= - \frac{c^2 \cosh\left(\frac{2a}{b}\right) \operatorname{Chi}\left(\frac{2a}{b} + 2 \operatorname{sech}^{-1}(cx)\right)}{b^2} + \frac{c^2 \sinh(2 \operatorname{sech}^{-1}(cx))}{2b(a + b \operatorname{sech}^{-1}(cx))} + \frac{c^2 \sinh\left(\frac{2a}{b}\right) \operatorname{Shi}\left(\frac{2a}{b} + 2 \operatorname{sech}^{-1}(cx)\right)}{b^2} \end{aligned}$$

Mathematica [A] time = 0.335204, size = 92, normalized size = 1.08

$$\frac{c^2 \left(-\cosh\left(\frac{2a}{b}\right) \operatorname{Chi}\left(2\left(\frac{a}{b} + \operatorname{sech}^{-1}(cx)\right)\right) + c^2 \sinh\left(\frac{2a}{b}\right) \operatorname{Shi}\left(2\left(\frac{a}{b} + \operatorname{sech}^{-1}(cx)\right)\right) + \frac{b \sqrt{\frac{1-cx}{cx+1}} (cx+1)}{x^2 (a+b \operatorname{sech}^{-1}(cx))} \right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*ArcSech[c*x])^2), x]
```

```
[Out] ((b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(x^2*(a + b*ArcSech[c*x])) - c^2*C
osh[(2*a)/b]*CoshIntegral[2*(a/b + ArcSech[c*x])] + c^2*Sinh[(2*a)/b]*SinhI
ntegral[2*(a/b + ArcSech[c*x])])/b^2
```

Maple [B] time = 0.237, size = 186, normalized size = 2.2

$$c^2 \left(\frac{1}{4c^2x^2(a + b \operatorname{arcsech}(cx))b} \left(2 \sqrt{\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} cx + c^2x^2 - 2 \right) + \frac{1}{2b^2} e^{2\frac{a}{b}} \operatorname{Ei}\left(1, 2\frac{a}{b} + 2 \operatorname{arcsech}(cx)\right) - \frac{1}{4c^2x^2(a + b \operatorname{arcsech}(cx))b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^3/(a+b*arcsech(c*x))^2,x)`

[Out] $c^2*(1/4*(2*(-(c*x-1)/c/x)^{(1/2)}*((c*x+1)/c/x)^{(1/2)}*c*x+c^2*x^2-2)/c^2/x^2/(a+b*arcsech(c*x))/b+1/2/b^2*exp(2*a/b)*Ei(1,2*a/b+2*arcsech(c*x))-1/4/b*(c^2*x^2-2*(-(c*x-1)/c/x)^{(1/2)}*((c*x+1)/c/x)^{(1/2)}*c*x)/c^2/x^2/(a+b*arcsech(c*x))+1/2/b^2*exp(-2*a/b)*Ei(1,-2*arcsech(c*x)-2*a/b)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{c^2x^3 + (c^2x^3 - x)\sqrt{cx + 1}\sqrt{-cx + 1} - x}{(b^2c^2x^2 - b^2)x^3 \log(x) + ((b^2c^2 \log(c) - abc^2)x^2 - b^2 \log(c) + ab)x^3 - (b^2x^3 \log(x) + (b^2 \log(c) - ab)x^3)\sqrt{cx + 1}\sqrt{-cx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/(a+b*arcsech(c*x))^2,x, algorithm="maxima")`

[Out] $-(c^2*x^3 + (c^2*x^3 - x)*sqrt(c*x + 1)*sqrt(-c*x + 1) - x)/((b^2*c^2*x^2 - b^2)*x^3*log(x) + ((b^2*c^2*log(c) - a*b*c^2)*x^2 - b^2*log(c) + a*b)*x^3 - (b^2*x^3*log(x) + (b^2*log(c) - a*b)*x^3)*sqrt(c*x + 1)*sqrt(-c*x + 1) + (sqrt(c*x + 1)*sqrt(-c*x + 1)*b^2*x^3 - (b^2*c^2*x^2 - b^2)*x^3)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)) + integrate(-(2*c^4*x^4 - 4*c^2*x^2 - 2*(c*x + 1)*(c*x - 1) + (c^4*x^4 - 4*c^2*x^2 + 4)*sqrt(c*x + 1)*sqrt(-c*x + 1) + 2)/(b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*x^3*log(x) + ((b^2*c^4*log(c) - a*b*c^4)*x^4 - 2*(b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*log(c) - a*b)*x^3 - (b^2*x^3*log(x) + (b^2*log(c) - a*b)*x^3)*(c*x + 1)*(c*x - 1) - 2*((b^2*c^2*x^2 - b^2)*x^3*log(x) + ((b^2*c^2*log(c) - a*b*c^2)*x^2 - b^2*log(c) + a*b)*x^3)*sqrt(c*x + 1)*sqrt(-c*x + 1) + ((c*x + 1)*(c*x - 1)*b^2*x^3 + 2*(b^2*c^2*x^2 - b^2)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^3 - (b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*x^3)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)), x)$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b^2x^3 \operatorname{arsech}(cx)^2 + 2abx^3 \operatorname{arsech}(cx) + a^2x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^3/(a+b*arcsech(c*x))^2,x, algorithm="fricas")`

[Out] `integral(1/(b^2*x^3*arcsech(c*x)^2 + 2*a*b*x^3*arcsech(c*x) + a^2*x^3), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 (a + b \operatorname{asech}(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/(a+b*asech(c*x))**2,x)`

[Out] Integral(1/(x**3*(a + b*asech(c*x))**2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{arsech}(cx) + a)^2 x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b*arcsech(c*x))^2,x, algorithm="giac")

[Out] integrate(1/((b*arcsech(c*x) + a)^2*x^3), x)

$$3.62 \quad \int \frac{1}{x^4 (a + b \operatorname{sech}^{-1}(cx))^2} dx$$

Optimal. Leaf size=190

$$\frac{c^3 \cosh\left(\frac{a}{b}\right) \operatorname{Chi}\left(\frac{a}{b} + \operatorname{sech}^{-1}(cx)\right)}{4b^2} - \frac{3c^3 \cosh\left(\frac{3a}{b}\right) \operatorname{Chi}\left(\frac{3a}{b} + 3\operatorname{sech}^{-1}(cx)\right)}{4b^2} + \frac{c^3 \sinh\left(\frac{a}{b}\right) \operatorname{Shi}\left(\frac{a}{b} + \operatorname{sech}^{-1}(cx)\right)}{4b^2} + \frac{3c^3 \operatorname{si}}{4b^2}$$

[Out] (c^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(4*b*x*(a + b*ArcSech[c*x])) - (c^3*Cosh[a/b]*CoshIntegral[a/b + ArcSech[c*x]])/(4*b^2) - (3*c^3*Cosh[(3*a)/b]*CoshIntegral[(3*a)/b + 3*ArcSech[c*x]])/(4*b^2) + (c^3*Sinh[3*ArcSech[c*x]])/(4*b*(a + b*ArcSech[c*x])) + (c^3*Sinh[a/b]*SinhIntegral[a/b + ArcSech[c*x]])/(4*b^2) + (3*c^3*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSech[c*x]])/(4*b^2)

Rubi [A] time = 0.294862, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6285, 5448, 3297, 3303, 3298, 3301}

$$\frac{c^3 \cosh\left(\frac{a}{b}\right) \operatorname{Chi}\left(\frac{a}{b} + \operatorname{sech}^{-1}(cx)\right)}{4b^2} - \frac{3c^3 \cosh\left(\frac{3a}{b}\right) \operatorname{Chi}\left(\frac{3a}{b} + 3\operatorname{sech}^{-1}(cx)\right)}{4b^2} + \frac{c^3 \sinh\left(\frac{a}{b}\right) \operatorname{Shi}\left(\frac{a}{b} + \operatorname{sech}^{-1}(cx)\right)}{4b^2} + \frac{3c^3 \operatorname{si}}{4b^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(a + b*ArcSech[c*x])^2),x]

[Out] (c^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(4*b*x*(a + b*ArcSech[c*x])) - (c^3*Cosh[a/b]*CoshIntegral[a/b + ArcSech[c*x]])/(4*b^2) - (3*c^3*Cosh[(3*a)/b]*CoshIntegral[(3*a)/b + 3*ArcSech[c*x]])/(4*b^2) + (c^3*Sinh[3*ArcSech[c*x]])/(4*b*(a + b*ArcSech[c*x])) + (c^3*Sinh[a/b]*SinhIntegral[a/b + ArcSech[c*x]])/(4*b^2) + (3*c^3*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSech[c*x]])/(4*b^2)

Rule 6285

Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))^ (n_.)*(x_.)^(m_.), x_Symbol] :> -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

Rule 5448

Int[Cosh[(a_.) + (b_.)*(x_.)]^(p_.)*((c_.) + (d_.)*(x_.))^(m_.)*Sinh[(a_.) + (b_.)*(x_.)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a + b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3297

Int[((c_.) + (d_.)*(x_.))^(m_.)*sin[(e_.) + (f_.)*(x_.)], x_Symbol] :> Simp[(c + d*x)^(m + 1)*Sin[e + f*x]/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x^4 (a + b \operatorname{sech}^{-1}(cx))^2} dx &= - \left(c^3 \operatorname{Subst} \left(\int \frac{\cosh^2(x) \sinh(x)}{(a + bx)^2} dx, x, \operatorname{sech}^{-1}(cx) \right) \right) \\ &= - \left(c^3 \operatorname{Subst} \left(\int \left(\frac{\sinh(x)}{4(a + bx)^2} + \frac{\sinh(3x)}{4(a + bx)^2} \right) dx, x, \operatorname{sech}^{-1}(cx) \right) \right) \\ &= - \left(\frac{1}{4} c^3 \operatorname{Subst} \left(\int \frac{\sinh(x)}{(a + bx)^2} dx, x, \operatorname{sech}^{-1}(cx) \right) \right) - \frac{1}{4} c^3 \operatorname{Subst} \left(\int \frac{\sinh(3x)}{(a + bx)^2} dx, x, \operatorname{sech}^{-1}(cx) \right) \\ &= \frac{c^2 \sqrt{\frac{1-cx}{1+cx}} (1+cx)}{4bx (a + b \operatorname{sech}^{-1}(cx))} + \frac{c^3 \sinh(3 \operatorname{sech}^{-1}(cx))}{4b (a + b \operatorname{sech}^{-1}(cx))} - \frac{c^3 \operatorname{Subst} \left(\int \frac{\cosh(x)}{a+bx} dx, x, \operatorname{sech}^{-1}(cx) \right)}{4b} \\ &= \frac{c^2 \sqrt{\frac{1-cx}{1+cx}} (1+cx)}{4bx (a + b \operatorname{sech}^{-1}(cx))} + \frac{c^3 \sinh(3 \operatorname{sech}^{-1}(cx))}{4b (a + b \operatorname{sech}^{-1}(cx))} - \frac{(c^3 \cosh(\frac{a}{b})) \operatorname{Subst} \left(\int \frac{\cosh(\frac{a}{b} + x)}{a+bx} dx \right)}{4b} \\ &= \frac{c^2 \sqrt{\frac{1-cx}{1+cx}} (1+cx)}{4bx (a + b \operatorname{sech}^{-1}(cx))} - \frac{c^3 \cosh(\frac{a}{b}) \operatorname{Chi}(\frac{a}{b} + \operatorname{sech}^{-1}(cx))}{4b^2} - \frac{3c^3 \cosh(\frac{3a}{b}) \operatorname{Chi}(\frac{3a}{b} + \operatorname{sech}^{-1}(cx))}{4b^2} \end{aligned}$$

Mathematica [A] time = 0.623285, size = 170, normalized size = 0.89

$$c^3 \left(-\cosh\left(\frac{a}{b}\right) \operatorname{Chi}\left(\frac{a}{b} + \operatorname{sech}^{-1}(cx)\right) - 3c^3 \cosh\left(\frac{3a}{b}\right) \operatorname{Chi}\left(3\left(\frac{a}{b} + \operatorname{sech}^{-1}(cx)\right)\right) + c^3 \sinh\left(\frac{a}{b}\right) \operatorname{Shi}\left(\frac{a}{b} + \operatorname{sech}^{-1}(cx)\right) + \dots \right)$$

$$4b^2$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*ArcSech[c*x])^2), x]
```

```
[Out] ((4*b*Sqrt[(1 - c*x)/(1 + c*x)])/(x^3*(a + b*ArcSech[c*x])) + (4*b*c*Sqrt[(1 - c*x)/(1 + c*x)]/(x^2*(a + b*ArcSech[c*x])) - c^3*Cosh[a/b]*CoshIntegral[a/b + ArcSech[c*x]] - 3*c^3*Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcSech[c*x])] + c^3*Sinh[a/b]*SinhIntegral[a/b + ArcSech[c*x]] + 3*c^3*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSech[c*x])])/(4*b^2)
```

Maple [B] time = 0.273, size = 420, normalized size = 2.2

$$c^3 \left(-\frac{1}{8x^3bc^3(a + b \operatorname{arcsech}(cx))} \left(\sqrt{\frac{cx+1}{cx}} \sqrt{\frac{cx-1}{cx}} c^3 x^3 - 4 \sqrt{\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} cx - 3c^2 x^2 + 4 \right) + \frac{3}{8b^2} e^{3\frac{a}{b}} \operatorname{Ei} \left(1, 3\frac{a}{b} + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^4/(a+b*arcsech(c*x))^2,x)`

[Out] $c^3 * (-1/8 * (((c*x+1)/c/x)^{(1/2)} * (-c*x-1)/c/x)^{(1/2)} * c^3 * x^3 - 4 * (-c*x-1)/c/x)^{(1/2)} * ((c*x+1)/c/x)^{(1/2)} * c*x - 3*c^2*x^2 + 4 / c^3/x^3/b / (a+b*arcsech(c*x)) + 3/8/b^2 * \exp(3*a/b) * \operatorname{Ei}(1, 3*a/b + 3*arcsech(c*x)) + 1/8 * ((-c*x-1)/c/x)^{(1/2)} * ((c*x+1)/c/x)^{(1/2)} * c*x - 1/c/x/b / (a+b*arcsech(c*x)) + 1/8/b^2 * \exp(a/b) * \operatorname{Ei}(1, a/b + arcsech(c*x)) + 1/8/b * ((-c*x-1)/c/x)^{(1/2)} * ((c*x+1)/c/x)^{(1/2)} * c*x + 1/c/x / (a+b*arcsech(c*x)) + 1/8/b^2 * \exp(-a/b) * \operatorname{Ei}(1, -arcsech(c*x) - a/b) - 1/8/b * ((c*x+1)/c/x)^{(1/2)} * (-c*x-1)/c/x)^{(1/2)} * c^3 * x^3 - 4 * (-c*x-1)/c/x)^{(1/2)} * ((c*x+1)/c/x)^{(1/2)} * c*x + 3*c^2*x^2 - 4 / c^3/x^3 / (a+b*arcsech(c*x)) + 3/8/b^2 * \exp(-3*a/b) * \operatorname{Ei}(1, -3*arcsech(c*x) - 3*a/b)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{c^2 x^3 + (c^2 x^3 - x) \sqrt{cx + 1} \sqrt{-cx + 1} - x}{(b^2 c^2 x^2 - b^2) x^4 \log(x) + ((b^2 c^2 \log(c) - abc^2) x^2 - b^2 \log(c) + ab) x^4 - (b^2 x^4 \log(x) + (b^2 \log(c) - ab) x^4) \sqrt{cx + 1} \sqrt{-cx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^4/(a+b*arcsech(c*x))^2,x, algorithm="maxima")`

[Out] $-(c^2 * x^3 + (c^2 * x^3 - x) * \operatorname{sqrt}(c*x + 1) * \operatorname{sqrt}(-c*x + 1) - x) / ((b^2 * c^2 * x^2 - b^2) * x^4 * \log(x) + ((b^2 * c^2 * \log(c) - a * b * c^2) * x^2 - b^2 * \log(c) + a * b) * x^4 - (b^2 * x^4 * \log(x) + (b^2 * \log(c) - a * b) * x^4) * \operatorname{sqrt}(c*x + 1) * \operatorname{sqrt}(-c*x + 1) + (\operatorname{sqrt}(c*x + 1) * \operatorname{sqrt}(-c*x + 1) * b^2 * x^4 - (b^2 * c^2 * x^2 - b^2) * x^4) * \log(\operatorname{sqrt}(c*x + 1) * \operatorname{sqrt}(-c*x + 1) + 1)) - \operatorname{integrate}((3 * c^4 * x^4 - 6 * c^2 * x^2 + (c^2 * x^2 - 3) * (c*x + 1) * (c*x - 1) + (2 * c^4 * x^4 - 7 * c^2 * x^2 + 6) * \operatorname{sqrt}(c*x + 1) * \operatorname{sqrt}(-c*x + 1) + 3) / ((b^2 * c^4 * x^4 - 2 * b^2 * c^2 * x^2 + b^2) * x^4 * \log(x) + ((b^2 * c^4 * \log(c) - a * b * c^4) * x^4 - 2 * (b^2 * c^2 * \log(c) - a * b * c^2) * x^2 + b^2 * \log(c) - a * b) * x^4 - (b^2 * x^4 * \log(x) + (b^2 * \log(c) - a * b) * x^4) * (c*x + 1) * (c*x - 1) - 2 * ((b^2 * c^2 * x^2 - b^2) * x^4 * \log(x) + ((b^2 * c^2 * \log(c) - a * b * c^2) * x^2 - b^2 * \log(c) + a * b) * x^4) * \operatorname{sqrt}(c*x + 1) * \operatorname{sqrt}(-c*x + 1) + ((c*x + 1) * (c*x - 1) * b^2 * x^4 + 2 * (b^2 * c^2 * x^2 - b^2) * \operatorname{sqrt}(c*x + 1) * \operatorname{sqrt}(-c*x + 1) * x^4 - (b^2 * c^4 * x^4 - 2 * b^2 * c^2 * x^2 + b^2) * x^4) * \log(\operatorname{sqrt}(c*x + 1) * \operatorname{sqrt}(-c*x + 1) + 1)), x)$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left(\frac{1}{b^2 x^4 \operatorname{ar} \operatorname{sech}(cx)^2 + 2 a b x^4 \operatorname{ar} \operatorname{sech}(cx) + a^2 x^4}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^4/(a+b*arcsech(c*x))^2,x, algorithm="fricas")`

[Out] `integral(1/(b^2*x^4*arcsech(c*x)^2 + 2*a*b*x^4*arcsech(c*x) + a^2*x^4), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 (a + b \operatorname{arsech}(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**4/(a+b*arsech(c*x))**2,x)`

[Out] `Integral(1/(x**4*(a + b*arsech(c*x))**2), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{arsech}(cx) + a)^2 x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^4/(a+b*arcsech(c*x))^2,x, algorithm="giac")`

[Out] `integrate(1/((b*arcsech(c*x) + a)^2*x^4), x)`

$$3.63 \quad \int \frac{x}{\left(a + b \operatorname{sech}^{-1}(cx)\right)^3} dx$$

Optimal. Leaf size=14

$$\text{Unintegrable}\left(\frac{x}{\left(a + b \operatorname{sech}^{-1}(cx)\right)^3}, x\right)$$

[Out] Unintegrable[x/(a + b*ArcSech[c*x])^3, x]

Rubi [A] time = 0.0158324, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{x}{\left(a + b \operatorname{sech}^{-1}(cx)\right)^3} dx$$

Verification is Not applicable to the result.

[In] Int[x/(a + b*ArcSech[c*x])^3, x]

[Out] Defer[Int][x/(a + b*ArcSech[c*x])^3, x]

Rubi steps

$$\int \frac{x}{\left(a + b \operatorname{sech}^{-1}(cx)\right)^3} dx = \int \frac{x}{\left(a + b \operatorname{sech}^{-1}(cx)\right)^3} dx$$

Mathematica [A] time = 5.79114, size = 0, normalized size = 0.

$$\int \frac{x}{\left(a + b \operatorname{sech}^{-1}(cx)\right)^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[x/(a + b*ArcSech[c*x])^3, x]

[Out] Integrate[x/(a + b*ArcSech[c*x])^3, x]

Maple [A] time = 0.67, size = 0, normalized size = 0.

$$\int \frac{x}{\left(a + b \operatorname{arcsech}(cx)\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+b*arcsech(c*x))^3, x)

[Out] $\int (x/(a+b*\operatorname{arcsech}(c*x))^3, x)$

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a+b*arcsech(c*x))^3,x, algorithm="maxima")`

[Out]
$$-1/2*((2*(2*b*c^4*x^5 - 3*b*c^2*x^3 + b*x)*x*\log(x) + (4*(b*c^4*\log(c) - a*c^4)*x^5 - (b*c^2*(6*\log(c) + 1) - 6*a*c^2)*x^3 + (b*(2*\log(c) + 1) - 2*a)*x)*x)*(c*x + 1)^{(3/2)}*(-c*x + 1)^{(3/2)} + (3*(b*c^6*x^7 - 5*b*c^4*x^5 + 6*b*c^2*x^3 - 2*b*x)*x*\log(x) + (3*(b*c^6*\log(c) - a*c^6)*x^7 - (b*c^4*(15*\log(c) + 2) - 15*a*c^4)*x^5 + (b*c^2*(18*\log(c) + 5) - 18*a*c^2)*x^3 - 3*(b*(2*\log(c) + 1) - 2*a)*x)*x)*(c*x + 1)*(c*x - 1) - 2*(b*c^6*x^7 - 3*b*c^4*x^5 + 3*b*c^2*x^3 - b*x)*x*\log(x) - ((5*b*c^6*x^7 - 17*b*c^4*x^5 + 18*b*c^2*x^3 - 6*b*x)*x*\log(x) + ((b*c^6*(5*\log(c) + 1) - 5*a*c^6)*x^7 - (b*c^4*(17*\log(c) + 5) - 17*a*c^4)*x^5 + (b*c^2*(18*\log(c) + 7) - 18*a*c^2)*x^3 - 3*(b*(2*\log(c) + 1) - 2*a)*x)*x)*\sqrt{c*x + 1}*\sqrt{-c*x + 1} - ((b*c^6*(2*\log(c) + 1) - 2*a*c^6)*x^7 - 3*(b*c^4*(2*\log(c) + 1) - 2*a*c^4)*x^5 + 3*(b*c^2*(2*\log(c) + 1) - 2*a*c^2)*x^3 - (b*(2*\log(c) + 1) - 2*a)*x)*x - (2*(2*b*c^4*x^5 - 3*b*c^2*x^3 + b*x)*(c*x + 1)^{(3/2)}*(-c*x + 1)^{(3/2)}*x + 3*(b*c^6*x^7 - 5*b*c^4*x^5 + 6*b*c^2*x^3 - 2*b*x)*(c*x + 1)*(c*x - 1)*x - (5*b*c^6*x^7 - 17*b*c^4*x^5 + 18*b*c^2*x^3 - 6*b*x)*\sqrt{c*x + 1}*\sqrt{-c*x + 1}*x - 2*(b*c^6*x^7 - 3*b*c^4*x^5 + 3*b*c^2*x^3 - b*x)*x)*\log(\sqrt{c*x + 1}*\sqrt{-c*x + 1} + 1))/((b^4*c^6*\log(c)^2 - 2*a*b^3*c^6*\log(c) + a^2*b^2*c^6)*x^6 - b^4*\log(c)^2 - 3*(b^4*c^4*\log(c)^2 - 2*a*b^3*c^4*\log(c) + a^2*b^2*c^4)*x^4 + 2*a*b^3*\log(c) - (b^4*\log(c)^2 + b^4*\log(x)^2 - 2*a*b^3*\log(c) + a^2*b^2 + 2*(b^4*\log(c) - a*b^3)*\log(x))*(c*x + 1)^{(3/2)}*(-c*x + 1)^{(3/2)} - a^2*b^2 + 3*(b^4*\log(c)^2 - 2*a*b^3*\log(c) + a^2*b^2 - (b^4*c^2*\log(c)^2 - 2*a*b^3*c^2*\log(c) + a^2*b^2*c^2)*x^2 - (b^4*c^2*x^2 - b^4)*\log(x)^2 + 2*(b^4*\log(c) - a*b^3 - (b^4*c^2*\log(c) - a*b^3*c^2)*x^2)*\log(x))*(c*x + 1)*(c*x - 1) + 3*(b^4*c^2*\log(c)^2 - 2*a*b^3*c^2*\log(c) + a^2*b^2*c^2)*x^2 + (b^4*c^6*x^6 - 3*b^4*c^4*x^4 + 3*b^4*c^2*x^2 - (c*x + 1)^{(3/2)}*(-c*x + 1)^{(3/2)}*b^4 - b^4 - 3*(b^4*c^2*x^2 - b^4)*(c*x + 1)*(c*x - 1) - 3*(b^4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4)*\sqrt{c*x + 1}*\sqrt{-c*x + 1})*\log(\sqrt{c*x + 1}*\sqrt{-c*x + 1} + 1)^2 + (b^4*c^6*x^6 - 3*b^4*c^4*x^4 + 3*b^4*c^2*x^2 - b^4)*\log(x)^2 - 3*(b^4*\log(c)^2 + (b^4*c^4*\log(c)^2 - 2*a*b^3*c^4*\log(c) + a^2*b^2*c^4)*x^4 - 2*a*b^3*\log(c) + a^2*b^2 - 2*(b^4*c^2*\log(c)^2 - 2*a*b^3*c^2*\log(c) + a^2*b^2*c^2)*x^2 + (b^4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4)*\log(x)^2 + 2*((b^4*c^4*\log(c) - a*b^3*c^4)*x^4 + b^4*\log(c) - a*b^3 - 2*(b^4*c^2*\log(c) - a*b^3*c^2)*x^2)*\log(x))*\sqrt{c*x + 1}*\sqrt{-c*x + 1} - 2*((b^4*c^6*\log(c) - a*b^3*c^6)*x^6 - 3*(b^4*c^4*\log(c) - a*b^3*c^4)*x^4 - b^4*\log(c) - (b^4*\log(c) + b^4*\log(x) - a*b^3)*(c*x + 1)^{(3/2)}*(-c*x + 1)^{(3/2)} + a*b^3 + 3*(b^4*\log(c) - a*b^3 - (b^4*c^2*\log(c) - a*b^3*c^2)*x^2 - (b^4*c^2*x^2 - b^4)*\log(x))*(c*x + 1)*(c*x - 1) + 3*(b^4*c^2*\log(c) - a*b^3*c^2)*x^2 - 3*((b^4*c^4*\log(c) - a*b^3*c^4)*x^4 + b^4*\log(c) - a*b^3 - 2*(b^4*c^2*\log(c) - a*b^3*c^2)*x^2 + (b^4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4)*\log(x))*\sqrt{c*x + 1}*\sqrt{-c*x + 1} + (b^4*c^6*x^6 - 3*b^4*c^4*x^4 + 3*b^4*c^2*x^2 - b^4)*\log(x))*\log(\sqrt{c*x + 1}*\sqrt{-c*x + 1} + 1) + 2*((b^4*c^6*\log(c) - a*b^3*c^6)*x^6 - 3*(b^4*c^4*\log(c) - a*b^3*c^4)*x^4 - b^4*\log(c) + a*b^3 + 3*(b^4*c^2*\log(c) - a*b^3*c^2)*x^2)*\log(x)) + \int (-1/2*(4*(6*c^4*x^4 - 6*c^2*x^2 + 1)*(c*x + 1)^2*(c*x - 1)^2*x - (33*c^6*x^6 - 108*c^4*x^4 + 88*c^2*x^2 - 16)*(c*x + 1)^{(3/2)}*(-c*x + 1)^{(3/2)}*x - 12*(c^8*x^8 - 7*c^6*x^6 + 14*c^4*x^4 - 10*c^2*x^2 + 2)*(c*x + 1)*(c*x - 1)*x + (15*c^8*x^8 - 67*c^6*x^6 + 108*c^4*x^4 - 72*c^2*x^2 + 16)*\sqrt{c*x + 1}*\sqrt{-c*x + 1})*x + 4*(c^8*x^8 - 4*c^6*x^6 + 6*c^4*x^4$$

- 4*c^2*x^2 + 1)*x)/((b^3*c^8*log(c) - a*b^2*c^8)*x^8 - 4*(b^3*c^6*log(c) - a*b^2*c^6)*x^6 + (b^3*log(c) + b^3*log(x) - a*b^2)*(c*x + 1)^2*(c*x - 1)^2 + 6*(b^3*c^4*log(c) - a*b^2*c^4)*x^4 + 4*(b^3*log(c) - a*b^2 - (b^3*c^2*log(c) - a*b^2*c^2)*x^2 - (b^3*c^2*x^2 - b^3)*log(x))*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) + b^3*log(c) - 6*((b^3*c^4*log(c) - a*b^2*c^4)*x^4 + b^3*log(c) - a*b^2 - 2*(b^3*c^2*log(c) - a*b^2*c^2)*x^2 + (b^3*c^4*x^4 - 2*b^3*c^2*x^2 + b^3)*log(x))*(c*x + 1)*(c*x - 1) - a*b^2 - 4*(b^3*c^2*log(c) - a*b^2*c^2)*x^2 - 4*((b^3*c^6*log(c) - a*b^2*c^6)*x^6 - 3*(b^3*c^4*log(c) - a*b^2*c^4)*x^4 - b^3*log(c) + a*b^2 + 3*(b^3*c^2*log(c) - a*b^2*c^2)*x^2 + (b^3*c^6*x^6 - 3*b^3*c^4*x^4 + 3*b^3*c^2*x^2 - b^3)*log(x))*sqrt(c*x + 1)*sqrt(-c*x + 1) - (b^3*c^8*x^8 - 4*b^3*c^6*x^6 + 6*b^3*c^4*x^4 + (c*x + 1)^2*(c*x - 1)^2*b^3 - 4*b^3*c^2*x^2 - 4*(b^3*c^2*x^2 - b^3)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) - 6*(b^3*c^4*x^4 - 2*b^3*c^2*x^2 + b^3)*(c*x + 1)*(c*x - 1) + b^3 - 4*(b^3*c^6*x^6 - 3*b^3*c^4*x^4 + 3*b^3*c^2*x^2 - b^3)*sqrt(c*x + 1)*sqrt(-c*x + 1))*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) + (b^3*c^8*x^8 - 4*b^3*c^6*x^6 + 6*b^3*c^4*x^4 - 4*b^3*c^2*x^2 + b^3)*log(x)), x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{b^3 \operatorname{arsech}(cx)^3 + 3ab^2 \operatorname{arsech}(cx)^2 + 3a^2b \operatorname{arsech}(cx) + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*arcsech(c*x))^3,x, algorithm="fricas")

[Out] integral(x/(b^3*arcsech(c*x)^3 + 3*a*b^2*arcsech(c*x)^2 + 3*a^2*b*arcsech(c*x) + a^3), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(a + b \operatorname{asech}(cx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*asech(c*x))**3,x)

[Out] Integral(x/(a + b*asech(c*x))**3, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(b \operatorname{arsech}(cx) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*arcsech(c*x))^3,x, algorithm="giac")

[Out] integrate(x/(b*arcsech(c*x) + a)^3, x)

$$3.64 \quad \int \frac{1}{\left(a + b \operatorname{sech}^{-1}(cx)\right)^3} dx$$

Optimal. Leaf size=12

$$\text{Unintegrable}\left(\frac{1}{\left(a + b \operatorname{sech}^{-1}(cx)\right)^3}, x\right)$$

[Out] Unintegrable[(a + b*ArcSech[c*x])^(-3), x]

Rubi [A] time = 0.0059659, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{\left(a + b \operatorname{sech}^{-1}(cx)\right)^3} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*ArcSech[c*x])^(-3), x]

[Out] Defer[Int] [(a + b*ArcSech[c*x])^(-3), x]

Rubi steps

$$\int \frac{1}{\left(a + b \operatorname{sech}^{-1}(cx)\right)^3} dx = \int \frac{1}{\left(a + b \operatorname{sech}^{-1}(cx)\right)^3} dx$$

Mathematica [A] time = 3.58711, size = 0, normalized size = 0.

$$\int \frac{1}{\left(a + b \operatorname{sech}^{-1}(cx)\right)^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*ArcSech[c*x])^(-3), x]

[Out] Integrate[(a + b*ArcSech[c*x])^(-3), x]

Maple [A] time = 0.589, size = 0, normalized size = 0.

$$\int (a + b \operatorname{arcsech}(cx))^{-3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*arcsech(c*x))^3, x)

[Out] $\int (1/(a+b*\operatorname{arcsech}(c*x))^3, x)$

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*arcsech(c*x))^3,x, algorithm="maxima")`

[Out]
$$\begin{aligned} & 1/2*((b*c^6*(\log(c) + 1) - a*c^6)*x^7 - 3*(b*c^4*(\log(c) + 1) - a*c^4)*x^5 \\ & - (3*(b*c^4*\log(c) - a*c^4)*x^5 - (b*c^2*(4*\log(c) + 1) - 4*a*c^2)*x^3 + (b \\ & *(\log(c) + 1) - a)*x + (3*b*c^4*x^5 - 4*b*c^2*x^3 + b*x)*\log(x))*(c*x + 1)^{3/2} \\ & *(-c*x + 1)^{3/2} + 3*(b*c^2*(\log(c) + 1) - a*c^2)*x^3 - (2*(b*c^6*\log(c) - a*c^6) \\ & *x^7 - 2*(b*c^4*(5*\log(c) + 1) - 5*a*c^4)*x^5 + (b*c^2*(11*\log(c) + 5) - 11*a*c^2) \\ & *x^3 - 3*(b*(\log(c) + 1) - a)*x + (2*b*c^6*x^7 - 10*b*c^4*x^5 + 11*b*c^2*x^3 - 3*b*x) \\ & *\log(x))*(c*x + 1)*(c*x - 1) + ((b*c^6*(3*\log(c) + 1) - 3*a*c^6)*x^7 - 5*(b*c^4*(2*\log(c) + 1) - 2*a*c^4) \\ & *x^5 + (b*c^2*(10*\log(c) + 7) - 10*a*c^2)*x^3 - 3*(b*(\log(c) + 1) - a)*x + (3*b*c^6*x^7 - 1 \\ & 0*b*c^4*x^5 + 10*b*c^2*x^3 - 3*b*x)*\log(x))*\sqrt{c*x + 1}*\sqrt{-c*x + 1} - (b*(\log(c) + 1) - a) \\ & *x - (b*c^6*x^7 - 3*b*c^4*x^5 + 3*b*c^2*x^3 - (3*b*c^4*x^5 - 4*b*c^2*x^3 + b*x)*(c*x + 1)^{3/2} \\ & *(-c*x + 1)^{3/2} - (2*b*c^6*x^7 - 10*b*c^4*x^5 + 11*b*c^2*x^3 - 3*b*x)*(c*x + 1)*(c*x - 1) + (3*b*c^6*x^7 - 1 \\ & 0*b*c^4*x^5 + 10*b*c^2*x^3 - 3*b*x)*\sqrt{c*x + 1}*\sqrt{-c*x + 1} - b*x)*\log(\sqrt{c*x + 1}*\sqrt{-c*x + 1} + 1) \\ & + (b*c^6*x^7 - 3*b*c^4*x^5 + 3*b*c^2*x^3 - b*x)*\log(x))/((b^4*c^6*\log(c)^2 - 2*a*b^3*c^6*\log(c) + a^2*b^2*c^6)*x^6 \\ & - b^4*\log(c)^2 - 3*(b^4*c^4*\log(c)^2 - 2*a*b^3*c^4*\log(c) + a^2*b^2*c^4)*x^4 + 2*a*b^3*\log(c) - (b^4*\log(c)^2 + b^4*\log(x)^2 - 2*a*b^3*\log(c) + a^2*b^2 \\ & + 2*(b^4*\log(c) - a*b^3)*\log(x))*(c*x + 1)^{3/2}*(-c*x + 1)^{3/2} - a^2*b^2 + 3*(b^4*\log(c)^2 - 2*a*b^3*\log(c) + a^2*b^2 - (b^4*c^2*\log(c)^2 - 2*a*b^3*c^2*\log(c) + a^2*b^2*c^2)*x^2 - (b^4*c^2*x^2 - b^4)*\log(x)^2 + 2*(b^4*\log(c) - a*b^3 - (b^4*c^2*\log(c) - a*b^3*c^2)*x^2)*\log(x))*(c*x + 1)*(c*x - 1) + 3*(b^4*c^2*\log(c)^2 - 2*a*b^3*c^2*\log(c) + a^2*b^2*c^2)*x^2 + (b^4*c^6*x^6 - 3*b^4*c^4*x^4 + 3*b^4*c^2*x^2 - (c*x + 1)^{3/2}*(-c*x + 1)^{3/2}*b^4 - b^4 - 3*(b^4*c^2*x^2 - b^4)*(c*x + 1)*(c*x - 1) - 3*(b^4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4))*\sqrt{c*x + 1}*\sqrt{-c*x + 1})*\log(\sqrt{c*x + 1}*\sqrt{-c*x + 1} + 1)^2 + (b^4*c^6*x^6 - 3*b^4*c^4*x^4 + 3*b^4*c^2*x^2 - b^4)*\log(x)^2 - 3*(b^4*\log(c)^2 + (b^4*c^4*\log(c)^2 - 2*a*b^3*c^4*\log(c) + a^2*b^2*c^4)*x^4 - 2*a*b^3*\log(c) + a^2*b^2 - 2*(b^4*c^2*\log(c)^2 - 2*a*b^3*c^2*\log(c) + a^2*b^2*c^2)*x^2 + (b^4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4)*\log(x)^2 + 2*((b^4*c^4*\log(c) - a*b^3*c^4)*x^4 + b^4*\log(c) - a*b^3 - 2*(b^4*c^2*\log(c) - a*b^3*c^2)*x^2)*\log(x))*\sqrt{c*x + 1}*\sqrt{-c*x + 1} - 2*((b^4*c^6*\log(c) - a*b^3*c^6)*x^6 - 3*(b^4*c^4*\log(c) - a*b^3*c^4)*x^4 - b^4*\log(c) - (b^4*\log(c) + b^4*\log(x) - a*b^3)*(c*x + 1)^{3/2}*(-c*x + 1)^{3/2} + a*b^3 + 3*(b^4*\log(c) - a*b^3 - (b^4*c^2*\log(c) - a*b^3*c^2)*x^2 - (b^4*c^2*x^2 - b^4)*\log(x))*(c*x + 1)*(c*x - 1) + 3*(b^4*c^2*\log(c) - a*b^3*c^2)*x^2 - 3*((b^4*c^4*\log(c) - a*b^3*c^4)*x^4 + b^4*\log(c) - a*b^3 - 2*(b^4*c^2*\log(c) - a*b^3*c^2)*x^2 + (b^4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4)*\log(x))*\sqrt{c*x + 1}*\sqrt{-c*x + 1} + (b^4*c^6*x^6 - 3*b^4*c^4*x^4 + 3*b^4*c^2*x^2 - b^4)*\log(x))*\log(\sqrt{c*x + 1}*\sqrt{-c*x + 1} + 1) + 2*((b^4*c^6*\log(c) - a*b^3*c^6)*x^6 - 3*(b^4*c^4*\log(c) - a*b^3*c^4)*x^4 - b^4*\log(c) + a*b^3 + 3*(b^4*c^2*\log(c) - a*b^3*c^2)*x^2)*\log(x)) + \int (-1/2*(c^8*x^8 - 4*c^6*x^6 + 6*c^4*x^4 + (15*c^4*x^4 - 12*c^2*x^2 + 1)*(c*x + 1)^2*(c*x - 1)^2 - (18*c^6*x^6 - 57*c^4*x^4 + 40*c^2*x^2 - 4)*(c*x + 1)^{3/2}*(-c*x + 1)^{3/2} - 4*c^2*x^2 - 3*(2*c^8*x^8 - 13*c^6*x^6 + 25*c^4*x^4 - 16*c^2*x^2 + 2)*(c*x + 1)*(c*x - 1) + (6*c^8*x^8 - 25*c^6*x^6 + 39*c^4*x^4 - 24*c^2*x^2 + 4))*\sqrt{c*x + 1}*\sqrt{-c*x + 1} + 1)/((b^3*c^8*\log(c) - a*b^2*c^8)*x^8 - 4*(b^3*c^6*\log(c) - a*b^2*c^6) \end{aligned}$$

$6)x^6 + (b^3 \log(c) + b^3 \log(x) - a b^2)(c x + 1)^2 (c x - 1)^2 + 6(b^3 c^4 \log(c) - a b^2 c^4)x^4 + 4(b^3 \log(c) - a b^2 - (b^3 c^2 \log(c) - a b^2 c^2)x^2 - (b^3 c^2 x^2 - b^3) \log(x))(c x + 1)^{3/2} (-c x + 1)^{3/2} + b^3 \log(c) - 6((b^3 c^4 \log(c) - a b^2 c^4)x^4 + b^3 \log(c) - a b^2 - 2(b^3 c^2 \log(c) - a b^2 c^2)x^2 + (b^3 c^4 x^4 - 2 b^3 c^2 x^2 + b^3) \log(x))(c x + 1)(c x - 1) - a b^2 - 4(b^3 c^2 \log(c) - a b^2 c^2)x^2 - 4((b^3 c^6 \log(c) - a b^2 c^6)x^6 - 3(b^3 c^4 \log(c) - a b^2 c^4)x^4 - b^3 \log(c) + a b^2 + 3(b^3 c^2 \log(c) - a b^2 c^2)x^2 + (b^3 c^6 x^6 - 3 b^3 c^4 x^4 + 3 b^3 c^2 x^2 - b^3) \log(x)) \sqrt{c x + 1} \sqrt{-c x + 1} - (b^3 c^8 x^8 - 4 b^3 c^6 x^6 + 6 b^3 c^4 x^4 + (c x + 1)^2 (c x - 1)^2 b^3 - 4 b^3 c^2 x^2 - 4(b^3 c^2 x^2 - b^3)(c x + 1)^{3/2} (-c x + 1)^{3/2} - 6(b^3 c^4 x^4 - 2 b^3 c^2 x^2 + b^3)(c x + 1)(c x - 1) + b^3 - 4(b^3 c^6 x^6 - 3 b^3 c^4 x^4 + 3 b^3 c^2 x^2 - b^3) \sqrt{c x + 1} \sqrt{-c x + 1}) \log(\sqrt{c x + 1} \sqrt{-c x + 1} + 1) + (b^3 c^8 x^8 - 4 b^3 c^6 x^6 + 6 b^3 c^4 x^4 - 4 b^3 c^2 x^2 + b^3) \log(x), x$

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b^3 \operatorname{arsech}(cx)^3 + 3 a b^2 \operatorname{arsech}(cx)^2 + 3 a^2 b \operatorname{arsech}(cx) + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*arcsech(c*x))^3,x, algorithm="fricas")

[Out] integral(1/(b^3*arcsech(c*x)^3 + 3*a*b^2*arcsech(c*x)^2 + 3*a^2*b*arcsech(c*x) + a^3), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + b \operatorname{asech}(cx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*asech(c*x))**3,x)

[Out] Integral((a + b*asech(c*x))**(-3), x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{arsech}(cx) + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*arcsech(c*x))^3,x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)^(-3), x)

$$3.65 \quad \int \frac{1}{x(a+b\operatorname{sech}^{-1}(cx))^3} dx$$

Optimal. Leaf size=16

$$\text{Unintegrable}\left(\frac{1}{x(a+b\operatorname{sech}^{-1}(cx))^3}, x\right)$$

[Out] Unintegrable[1/(x*(a + b*ArcSech[c*x])^3), x]

Rubi [A] time = 0.0248769, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{1}{x(a+b\operatorname{sech}^{-1}(cx))^3} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x*(a + b*ArcSech[c*x])^3), x]

[Out] Defer[Int][1/(x*(a + b*ArcSech[c*x])^3), x]

Rubi steps

$$\int \frac{1}{x(a+b\operatorname{sech}^{-1}(cx))^3} dx = \int \frac{1}{x(a+b\operatorname{sech}^{-1}(cx))^3} dx$$

Mathematica [A] time = 2.49891, size = 0, normalized size = 0.

$$\int \frac{1}{x(a+b\operatorname{sech}^{-1}(cx))^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x*(a + b*ArcSech[c*x])^3), x]

[Out] Integrate[1/(x*(a + b*ArcSech[c*x])^3), x]

Maple [A] time = 0.276, size = 0, normalized size = 0.

$$\int \frac{1}{x(a+b\operatorname{arcsech}(cx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a+b*arcsech(c*x))^3, x)

[Out] $\int (1/x/(a+b*\operatorname{arcsech}(c*x))^3, x)$

Maxima [A] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a+b*arcsech(c*x))^3,x, algorithm="maxima")`

[Out]
$$-1/2*(b*c^6*x^7 - 3*b*c^4*x^5 + 3*b*c^2*x^3 - (2*(b*c^4*\log(c) - a*c^4)*x^5 - (b*c^2*(2*\log(c) + 1) - 2*a*c^2)*x^3 + b*x + 2*(b*c^4*x^5 - b*c^2*x^3)*\log(x))*(c*x + 1)^{(3/2)}*(-c*x + 1)^{(3/2)} - ((b*c^6*\log(c) - a*c^6)*x^7 - (b*c^4*(5*\log(c) + 2) - 5*a*c^4)*x^5 + (b*c^2*(4*\log(c) + 5) - 4*a*c^2)*x^3 - 3*b*x + (b*c^6*x^7 - 5*b*c^4*x^5 + 4*b*c^2*x^3)*\log(x))*(c*x + 1)*(c*x - 1) + ((b*c^6*(\log(c) + 1) - a*c^6)*x^7 - (b*c^4*(3*\log(c) + 5) - 3*a*c^4)*x^5 + (b*c^2*(2*\log(c) + 7) - 2*a*c^2)*x^3 - 3*b*x + (b*c^6*x^7 - 3*b*c^4*x^5 + 2*b*c^2*x^3)*\log(x))*\sqrt{c*x + 1}*\sqrt{-c*x + 1} - b*x + (2*(b*c^4*x^5 - b*c^2*x^3)*(c*x + 1)^{(3/2)}*(-c*x + 1)^{(3/2)} + (b*c^6*x^7 - 5*b*c^4*x^5 + 4*b*c^2*x^3)*(c*x + 1)*(c*x - 1) - (b*c^6*x^7 - 3*b*c^4*x^5 + 2*b*c^2*x^3)*\sqrt{c*x + 1}*\sqrt{-c*x + 1})*\log(\sqrt{c*x + 1}*\sqrt{-c*x + 1} + 1))/((b^4*x*\log(x)^2 + 2*(b^4*\log(c) - a*b^3)*x*\log(x) + (b^4*\log(c)^2 - 2*a*b^3*\log(c) + a^2*b^2)*x)*(c*x + 1)^{(3/2)}*(-c*x + 1)^{(3/2)} - (b^4*c^6*x^6 - 3*b^4*c^4*x^4 + 3*b^4*c^2*x^2 - b^4)*x*\log(x)^2 + 3*((b^4*c^2*x^2 - b^4)*x*\log(x)^2 - 2*(b^4*\log(c) - a*b^3 - (b^4*c^2*\log(c) - a*b^3*c^2)*x^2)*x*\log(x) - (b^4*\log(c)^2 - 2*a*b^3*\log(c) + a^2*b^2 - (b^4*c^2*\log(c)^2 - 2*a*b^3*c^2*\log(c) + a^2*b^2*c^2)*x^2)*x)*(c*x + 1)*(c*x - 1) + ((c*x + 1)^{(3/2)}*(-c*x + 1)^{(3/2)}*b^4*x + 3*(b^4*c^2*x^2 - b^4)*(c*x + 1)*(c*x - 1)*x + 3*(b^4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4)*\sqrt{c*x + 1}*\sqrt{-c*x + 1}*x - (b^4*c^6*x^6 - 3*b^4*c^4*x^4 + 3*b^4*c^2*x^2 - b^4)*x)*\log(\sqrt{c*x + 1}*\sqrt{-c*x + 1} + 1)^2 - 2*((b^4*c^6*\log(c) - a*b^3*c^6)*x^6 - 3*(b^4*c^4*\log(c) - a*b^3*c^4)*x^4 - b^4*\log(c) + a*b^3 + 3*(b^4*c^2*\log(c) - a*b^3*c^2)*x^2)*x*\log(x) + 3*((b^4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4)*x*\log(x)^2 + 2*((b^4*c^4*\log(c) - a*b^3*c^4)*x^4 + b^4*\log(c) - a*b^3 - 2*(b^4*c^2*\log(c) - a*b^3*c^2)*x^2)*x*\log(x) + (b^4*\log(c)^2 + (b^4*c^4*\log(c)^2 - 2*a*b^3*c^4*\log(c) + a^2*b^2*c^4)*x^4 - 2*a*b^3*\log(c) + a^2*b^2 - 2*(b^4*c^2*\log(c)^2 - 2*a*b^3*c^2*\log(c) + a^2*b^2*c^2)*x^2)*x)*\sqrt{c*x + 1}*\sqrt{-c*x + 1} - ((b^4*c^6*\log(c)^2 - 2*a*b^3*c^6*\log(c) + a^2*b^2*c^6)*x^6 - b^4*\log(c)^2 - 3*(b^4*c^4*\log(c)^2 - 2*a*b^3*c^4*\log(c) + a^2*b^2*c^4)*x^4 + 2*a*b^3*\log(c) - a^2*b^2 + 3*(b^4*c^2*\log(c)^2 - 2*a*b^3*c^2*\log(c) + a^2*b^2*c^2)*x^2)*x - 2*((b^4*x*\log(x) + (b^4*\log(c) - a*b^3)*x)*(c*x + 1)^{(3/2)}*(-c*x + 1)^{(3/2)} + 3*((b^4*c^2*x^2 - b^4)*x*\log(x) - (b^4*\log(c) - a*b^3 - (b^4*c^2*\log(c) - a*b^3*c^2)*x^2)*x)*(c*x + 1)*(c*x - 1) - (b^4*c^6*x^6 - 3*b^4*c^4*x^4 + 3*b^4*c^2*x^2 - b^4)*x*\log(x) + 3*((b^4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4)*x*\log(x) + ((b^4*c^4*\log(c) - a*b^3*c^4)*x^4 + b^4*\log(c) - a*b^3 - 2*(b^4*c^2*\log(c) - a*b^3*c^2)*x^2)*x)*\sqrt{c*x + 1}*\sqrt{-c*x + 1} - ((b^4*c^6*\log(c) - a*b^3*c^6)*x^6 - 3*(b^4*c^4*\log(c) - a*b^3*c^4)*x^4 - b^4*\log(c) + a*b^3 + 3*(b^4*c^2*\log(c) - a*b^3*c^2)*x^2)*x)*\log(\sqrt{c*x + 1}*\sqrt{-c*x + 1} + 1)) + integrate(-1/2*(4*(2*c^4*x^4 - c^2*x^2)*(c*x + 1)^2*(c*x - 1)^2 - (7*c^6*x^6 - 22*c^4*x^4 + 12*c^2*x^2)*(c*x + 1)^{(3/2)}*(-c*x + 1)^{(3/2)} - 2*(c^8*x^8 - 5*c^6*x^6 + 10*c^4*x^4 - 6*c^2*x^2)*(c*x + 1)*(c*x - 1) + (c^8*x^8 - 3*c^6*x^6 + 6*c^4*x^4 - 4*c^2*x^2)*\sqrt{c*x + 1}*\sqrt{-c*x + 1}))/((b^3*x*\log(x) + (b^3*\log(c) - a*b^2)*x)*(c*x + 1)^2*(c*x - 1)^2 - 4*((b^3*c^2*x^2 - b^3)*x*\log(x) - (b^3*\log(c) - a*b^2 - (b^3*c^2*\log(c) - a*b^2*c^2)*x^2)*x)*(c*x + 1)^{(3/2)}*(-c*x + 1)^{(3/2)} - 6*((b^3*c^4*x^4 - 2*b^3*c^2*x^2 + b^3)*x*\log(x) + ((b^3*c^4*\log(c) - a*b^2*c^4)*x^4 + b^3*\log(c) - a*b^2 - 2*(b^3*c^2*\log(c) - a*b^2*c^2)*x^2)*x)*(c*x + 1)*(c*x - 1) + (b^3*c^8*x^8 - 4*b^3*c^6*x^6 + 6*b^3$$

```
*c^4*x^4 - 4*b^3*c^2*x^2 + b^3)*x*log(x) - 4*((b^3*c^6*x^6 - 3*b^3*c^4*x^4
+ 3*b^3*c^2*x^2 - b^3)*x*log(x) + ((b^3*c^6*log(c) - a*b^2*c^6)*x^6 - 3*(b^
3*c^4*log(c) - a*b^2*c^4)*x^4 - b^3*log(c) + a*b^2 + 3*(b^3*c^2*log(c) - a*
b^2*c^2)*x^2)*x)*sqrt(c*x + 1)*sqrt(-c*x + 1) + ((b^3*c^8*log(c) - a*b^2*c^
8)*x^8 - 4*(b^3*c^6*log(c) - a*b^2*c^6)*x^6 + 6*(b^3*c^4*log(c) - a*b^2*c^4
)*x^4 + b^3*log(c) - a*b^2 - 4*(b^3*c^2*log(c) - a*b^2*c^2)*x^2)*x - ((c*x
+ 1)^2*(c*x - 1)^2*b^3*x - 4*(b^3*c^2*x^2 - b^3)*(c*x + 1)^(3/2)*(-c*x + 1)
^(3/2)*x - 6*(b^3*c^4*x^4 - 2*b^3*c^2*x^2 + b^3)*(c*x + 1)*(c*x - 1)*x - 4*
(b^3*c^6*x^6 - 3*b^3*c^4*x^4 + 3*b^3*c^2*x^2 - b^3)*sqrt(c*x + 1)*sqrt(-c*x
+ 1)*x + (b^3*c^8*x^8 - 4*b^3*c^6*x^6 + 6*b^3*c^4*x^4 - 4*b^3*c^2*x^2 + b^
3)*x)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b^3x \operatorname{arsech}(cx)^3 + 3ab^2x \operatorname{arsech}(cx)^2 + 3a^2bx \operatorname{arsech}(cx) + a^3x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arcsech(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(1/(b^3*x*arcsech(c*x)^3 + 3*a*b^2*x*arcsech(c*x)^2 + 3*a^2*b*x*arc
sech(c*x) + a^3*x), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x(a + b \operatorname{asech}(cx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*asech(c*x))**3,x)
```

```
[Out] Integral(1/(x*(a + b*asech(c*x))**3), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{arsech}(cx) + a)^3 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arcsech(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate(1/((b*arcsech(c*x) + a)^3*x), x)
```

$$3.66 \quad \int \frac{1}{x^2 \left(a + b \operatorname{sech}^{-1}(cx) \right)^3} dx$$

Optimal. Leaf size=114

$$\frac{c \sinh\left(\frac{a}{b}\right) \operatorname{Chi}\left(\frac{a}{b} + \operatorname{sech}^{-1}(cx)\right)}{2b^3} - \frac{c \cosh\left(\frac{a}{b}\right) \operatorname{Shi}\left(\frac{a}{b} + \operatorname{sech}^{-1}(cx)\right)}{2b^3} + \frac{1}{2b^2 x \left(a + b \operatorname{sech}^{-1}(cx) \right)} + \frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{2bx \left(a + b \operatorname{sech}^{-1}(cx) \right)}$$

[Out] (Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(2*b*x*(a + b*ArcSech[c*x])^2) + 1/(2*b^2*x*(a + b*ArcSech[c*x])) + (c*CoshIntegral[a/b + ArcSech[c*x]]*Sinh[a/b])/ (2*b^3) - (c*Cosh[a/b]*SinhIntegral[a/b + ArcSech[c*x]])/(2*b^3)

Rubi [A] time = 0.175679, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {6285, 3297, 3303, 3298, 3301}

$$\frac{c \sinh\left(\frac{a}{b}\right) \operatorname{Chi}\left(\frac{a}{b} + \operatorname{sech}^{-1}(cx)\right)}{2b^3} - \frac{c \cosh\left(\frac{a}{b}\right) \operatorname{Shi}\left(\frac{a}{b} + \operatorname{sech}^{-1}(cx)\right)}{2b^3} + \frac{1}{2b^2 x \left(a + b \operatorname{sech}^{-1}(cx) \right)} + \frac{\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{2bx \left(a + b \operatorname{sech}^{-1}(cx) \right)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(a + b*ArcSech[c*x])^3), x]

[Out] (Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(2*b*x*(a + b*ArcSech[c*x])^2) + 1/(2*b^2*x*(a + b*ArcSech[c*x])) + (c*CoshIntegral[a/b + ArcSech[c*x]]*Sinh[a/b])/ (2*b^3) - (c*Cosh[a/b]*SinhIntegral[a/b + ArcSech[c*x]])/(2*b^3)

Rule 6285

Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))^n_.*(x_)^m_.), x_Symbol] :> -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

Rule 3297

Int[((c_.) + (d_.)*(x_.))^m_.*sin[(e_.) + (f_.)*(x_.)], x_Symbol] :> Simp[(c + d*x)^(m + 1)*Sin[e + f*x]/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3303

Int[sin[(e_.) + (f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 3298

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_.)]/((c_.) + (d_.)*(x_.)), x_Symbol] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x^2 (a + b \operatorname{sech}^{-1}(cx))^3} dx &= - \left(c \operatorname{Subst} \left(\int \frac{\sinh(x)}{(a + bx)^3} dx, x, \operatorname{sech}^{-1}(cx) \right) \right) \\ &= \frac{\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{2bx (a + b \operatorname{sech}^{-1}(cx))^2} - \frac{c \operatorname{Subst} \left(\int \frac{\cosh(x)}{(a+bx)^2} dx, x, \operatorname{sech}^{-1}(cx) \right)}{2b} \\ &= \frac{\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{2bx (a + b \operatorname{sech}^{-1}(cx))^2} + \frac{1}{2b^2 x (a + b \operatorname{sech}^{-1}(cx))} - \frac{c \operatorname{Subst} \left(\int \frac{\sinh(x)}{a+bx} dx, x, \operatorname{sech}^{-1}(cx) \right)}{2b^2} \\ &= \frac{\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{2bx (a + b \operatorname{sech}^{-1}(cx))^2} + \frac{1}{2b^2 x (a + b \operatorname{sech}^{-1}(cx))} - \frac{(c \cosh(\frac{a}{b})) \operatorname{Subst} \left(\int \frac{\sinh(\frac{a}{b}+x)}{a+bx} dx, x, \operatorname{sech}^{-1}(cx) \right)}{2b^2} \\ &= \frac{\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{2bx (a + b \operatorname{sech}^{-1}(cx))^2} + \frac{1}{2b^2 x (a + b \operatorname{sech}^{-1}(cx))} + \frac{c \operatorname{Chi}(\frac{a}{b} + \operatorname{sech}^{-1}(cx)) \sinh(\frac{a}{b})}{2b^3} - \frac{c \operatorname{Shi}(\frac{a}{b} + \operatorname{sech}^{-1}(cx))}{2b^3} \end{aligned}$$

Mathematica [A] time = 0.287881, size = 103, normalized size = 0.9

$$\frac{b^2 \sqrt{\frac{1-cx}{cx+1}}(cx+1)}{x(a+b \operatorname{sech}^{-1}(cx))^2} + c \left(\sinh\left(\frac{a}{b}\right) \operatorname{Chi}\left(\frac{a}{b} + \operatorname{sech}^{-1}(cx)\right) - \cosh\left(\frac{a}{b}\right) \operatorname{Shi}\left(\frac{a}{b} + \operatorname{sech}^{-1}(cx)\right) \right) + \frac{b}{ax + b \operatorname{sech}^{-1}(cx)}$$

$$2b^3$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*ArcSech[c*x])^3), x]
```

```
[Out] ((b^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(x*(a + b*ArcSech[c*x])^2) + b/(a*x + b*x*ArcSech[c*x]) + c*(CoshIntegral[a/b + ArcSech[c*x]]*Sinh[a/b] - Cosh[a/b]*SinhIntegral[a/b + ArcSech[c*x]]))/(2*b^3)
```

Maple [B] time = 0.231, size = 244, normalized size = 2.1

$$c \left(- \frac{b \operatorname{arcsech}(cx) + a - b}{4 c x b^2 \left((\operatorname{arcsech}(cx))^2 b^2 + 2 \operatorname{arcsech}(cx) a b + a^2 \right)} \left(\sqrt{\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} cx - 1 \right) - \frac{1}{4 b^3} e^{\frac{a}{b}} \operatorname{Ei} \left(1, \frac{a}{b} + \operatorname{arcsech}(cx) \right) \right) + \frac{b}{ax + b \operatorname{sech}^{-1}(cx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*arcsech(c*x))^3, x)
```

```
[Out] c*(-1/4*((-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*c*x-1)*(b*arcsech(c*x)+a-b)/c/x/b^2/(arcsech(c*x)^2*b^2+2*arcsech(c*x)*a*b+a^2)-1/4/b^3*exp(a/b)*Ei(1, a/b+arcsech(c*x))+1/4/b*((-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*c*x+1)/c/x/(a+b*arcsech(c*x))^2+1/4/b^2*((-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*
```


$$c*x+1)/c/x/(a+b*\text{arcsech}(c*x))+1/4/b^3*\exp(-a/b)*\text{Ei}(1,-\text{arcsech}(c*x)-a/b))$$

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b*arcsech(c*x))^3,x, algorithm="maxima")

[Out]
$$\begin{aligned} & -1/2*((b*c^6*(\log(c) - 1) - a*c^6)*x^7 - 3*(b*c^4*(\log(c) - 1) - a*c^4)*x^5 \\ & - (b*c^2*x^3 - (b*c^4*\log(c) - a*c^4)*x^5 + (b*(\log(c) - 1) - a)*x - (b*c^4*x^5 - b*x)*\log(x))*(c*x + 1)^{(3/2)}*(-c*x + 1)^{(3/2)} + 3*(b*c^2*(\log(c) - \\ & 1) - a*c^2)*x^3 - (2*b*c^4*x^5 + (b*c^2*(3*\log(c) - 5) - 3*a*c^2)*x^3 - 3*(\\ & b*(\log(c) - 1) - a)*x + 3*(b*c^2*x^3 - b*x)*\log(x))*(c*x + 1)*(c*x - 1) + (\\ & (b*c^6*(\log(c) - 1) - a*c^6)*x^7 - (b*c^4*(4*\log(c) - 5) - 4*a*c^4)*x^5 + (\\ & b*c^2*(6*\log(c) - 7) - 6*a*c^2)*x^3 - 3*(b*(\log(c) - 1) - a)*x + (b*c^6*x^7 \\ & - 4*b*c^4*x^5 + 6*b*c^2*x^3 - 3*b*x)*\log(x))*\text{sqrt}(c*x + 1)*\text{sqrt}(-c*x + 1) \\ & - (b*(\log(c) - 1) - a)*x - (b*c^6*x^7 - 3*b*c^4*x^5 + 3*b*c^2*x^3 + (b*c^4*x^5 - b*x)* \\ & (c*x + 1)^{(3/2)}*(-c*x + 1)^{(3/2)} - 3*(b*c^2*x^3 - b*x)*(c*x + 1) \\ & *(c*x - 1) + (b*c^6*x^7 - 4*b*c^4*x^5 + 6*b*c^2*x^3 - 3*b*x)*\text{sqrt}(c*x + 1)* \\ & \text{sqrt}(-c*x + 1) - b*x)*\log(\text{sqrt}(c*x + 1)*\text{sqrt}(-c*x + 1) + 1) + (b*c^6*x^7 - \\ & 3*b*c^4*x^5 + 3*b*c^2*x^3 - b*x)*\log(x))/((b^4*c^6*x^6 - 3*b^4*c^4*x^4 + 3* \\ & b^4*c^2*x^2 - b^4)*x^2*\log(x)^2 - (b^4*x^2*\log(x)^2 + 2*(b^4*\log(c) - a*b^3) \\ &)*x^2*\log(x) + (b^4*\log(c)^2 - 2*a*b^3*\log(c) + a^2*b^2)*x^2)*(c*x + 1)^{(3/2)} \\ & *(-c*x + 1)^{(3/2)} + 2*((b^4*c^6*\log(c) - a*b^3*c^6)*x^6 - 3*(b^4*c^4*\log(c) - \\ & a*b^3*c^4)*x^4 - b^4*\log(c) + a*b^3 + 3*(b^4*c^2*\log(c) - a*b^3*c^2)*x^2) \\ & *x^2*\log(x) - 3*((b^4*c^2*x^2 - b^4)*x^2*\log(x)^2 - 2*(b^4*\log(c) - a*b^3 - \\ & (b^4*c^2*\log(c) - a*b^3*c^2)*x^2)*x^2*\log(x) - (b^4*\log(c)^2 - 2*a*b^3* \\ & \log(c) + a^2*b^2 - (b^4*c^2*\log(c)^2 - 2*a*b^3*c^2*\log(c) + a^2*b^2*c^2)*x^2) \\ & *(c*x + 1)*(c*x - 1) + ((b^4*c^6*\log(c)^2 - 2*a*b^3*c^6*\log(c) + a^2 \\ & *b^2*c^6)*x^6 - b^4*\log(c)^2 - 3*(b^4*c^4*\log(c)^2 - 2*a*b^3*c^4*\log(c) + a^2 \\ & *b^2*c^4)*x^4 + 2*a*b^3*\log(c) - a^2*b^2 + 3*(b^4*c^2*\log(c)^2 - 2*a*b^3* \\ & c^2*\log(c) + a^2*b^2*c^2)*x^2)*x^2 - ((c*x + 1)^{(3/2)}*(-c*x + 1)^{(3/2)}*b^4*x^2 \\ & + 3*(b^4*c^2*x^2 - b^4)*(c*x + 1)*(c*x - 1)*x^2 + 3*(b^4*c^4*x^4 - 2*b^4 \\ & *c^2*x^2 + b^4)*\text{sqrt}(c*x + 1)*\text{sqrt}(-c*x + 1)*x^2 - (b^4*c^6*x^6 - 3*b^4*c^4 \\ & *x^4 + 3*b^4*c^2*x^2 - b^4)*x^2)*\log(\text{sqrt}(c*x + 1)*\text{sqrt}(-c*x + 1) + 1)^2 - \\ & 3*((b^4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4)*x^2*\log(x)^2 + 2*((b^4*c^4*\log(c) - \\ & a*b^3*c^4)*x^4 + b^4*\log(c) - a*b^3 - 2*(b^4*c^2*\log(c) - a*b^3*c^2)*x^2)* \\ & x^2*\log(x) + (b^4*\log(c)^2 + (b^4*c^4*\log(c)^2 - 2*a*b^3*c^4*\log(c) + a^2*b^2 \\ & *c^4)*x^4 - 2*a*b^3*\log(c) + a^2*b^2 - 2*(b^4*c^2*\log(c)^2 - 2*a*b^3*c^2* \\ & \log(c) + a^2*b^2*c^2)*x^2)*x^2)*\text{sqrt}(c*x + 1)*\text{sqrt}(-c*x + 1) + 2*((b^4*x^2* \\ & \log(x) + (b^4*\log(c) - a*b^3)*x^2)*(c*x + 1)^{(3/2)}*(-c*x + 1)^{(3/2)} - (b^4*c^6*x^6 - \\ & 3*b^4*c^4*x^4 + 3*b^4*c^2*x^2 - b^4)*x^2*\log(x) + 3*((b^4*c^2*x^2 - b^4)*x^2* \\ & \log(x) - (b^4*\log(c) - a*b^3 - (b^4*c^2*\log(c) - a*b^3*c^2)*x^2) \\ &)*x^2)*(c*x + 1)*(c*x - 1) - ((b^4*c^6*\log(c) - a*b^3*c^6)*x^6 - 3*(b^4*c^4 \\ & *\log(c) - a*b^3*c^4)*x^4 - b^4*\log(c) + a*b^3 + 3*(b^4*c^2*\log(c) - a*b^3*c^2) \\ & *x^2)*x^2 + 3*((b^4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4)*x^2*\log(x) + ((b^4*c^4*\log(c) - \\ & a*b^3*c^4)*x^4 + b^4*\log(c) - a*b^3 - 2*(b^4*c^2*\log(c) - a*b^3* \\ & c^2)*x^2)*x^2)*\text{sqrt}(c*x + 1)*\text{sqrt}(-c*x + 1))*\log(\text{sqrt}(c*x + 1)*\text{sqrt}(-c*x + \\ & 1) + 1) + \text{integrate}(-1/2*(c^8*x^8 - 4*c^6*x^6 + 6*c^4*x^4 + (3*c^4*x^4 + 1) \\ &)*(c*x + 1)^2*(c*x - 1)^2 + (3*c^4*x^4 - 4*c^2*x^2 + 4)*(c*x + 1)^{(3/2)}*(-c \\ & *x + 1)^{(3/2)} - 4*c^2*x^2 - 3*(c^6*x^6 + c^4*x^4 - 4*c^2*x^2 + 2)*(c*x + 1) \\ & *(c*x - 1) - (c^6*x^6 - 9*c^4*x^4 + 12*c^2*x^2 - 4)*\text{sqrt}(c*x + 1)*\text{sqrt}(-c*x \\ & + 1) + 1)/((b^3*x^2*\log(x) + (b^3*\log(c) - a*b^2)*x^2)*(c*x + 1)^2*(c*x - \\ & 1)^2 - 4*((b^3*c^2*x^2 - b^3)*x^2*\log(x) - (b^3*\log(c) - a*b^2 - (b^3*c^2* \\ & \log(c) - a*b^2*c^2)*x^2)*x^2)*(c*x + 1)^{(3/2)}*(-c*x + 1)^{(3/2)} + (b^3*c^8*x^8 \end{aligned}$$

$8 - 4b^3c^6x^6 + 6b^3c^4x^4 - 4b^3c^2x^2 + b^3)x^2\log(x) - 6((b^3c^4x^4 - 2b^3c^2x^2 + b^3)x^2\log(x) + ((b^3c^4\log(c) - ab^2c^4)x^4 + b^3\log(c) - ab^2 - 2(b^3c^2\log(c) - ab^2c^2)x^2)(cx + 1)(cx - 1) + ((b^3c^8\log(c) - ab^2c^8)x^8 - 4(b^3c^6\log(c) - ab^2c^6)x^6 + 6(b^3c^4\log(c) - ab^2c^4)x^4 + b^3\log(c) - ab^2 - 4(b^3c^2\log(c) - ab^2c^2)x^2)x^2 - 4((b^3c^6x^6 - 3b^3c^4x^4 + 3b^3c^2x^2 - b^3)x^2\log(x) + ((b^3c^6\log(c) - ab^2c^6)x^6 - 3(b^3c^4\log(c) - ab^2c^4)x^4 - b^3\log(c) + ab^2 + 3(b^3c^2\log(c) - ab^2c^2)x^2)x^2)\sqrt{cx + 1}\sqrt{-cx + 1} - ((cx + 1)^2(cx - 1)^2b^3x^2 - 4(b^3c^2x^2 - b^3)(cx + 1)^{(3/2)}(-cx + 1)^{(3/2)}x^2 - 6(b^3c^4x^4 - 2b^3c^2x^2 + b^3)(cx + 1)(cx - 1)x^2 - 4(b^3c^6x^6 - 3b^3c^4x^4 + 3b^3c^2x^2 - b^3)\sqrt{cx + 1}\sqrt{-cx + 1}x^2 + (b^3c^8x^8 - 4b^3c^6x^6 + 6b^3c^4x^4 - 4b^3c^2x^2 + b^3)x^2)\log(\sqrt{cx + 1}\sqrt{-cx + 1} + 1)), x$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b^3x^2 \operatorname{arsech}(cx)^3 + 3ab^2x^2 \operatorname{arsech}(cx)^2 + 3a^2bx^2 \operatorname{arsech}(cx) + a^3x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b*arcsech(c*x))^3,x, algorithm="fricas")

[Out] integral(1/(b^3*x^2*arcsech(c*x)^3 + 3*a*b^2*x^2*arcsech(c*x)^2 + 3*a^2*b*x^2*arcsech(c*x) + a^3*x^2), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (a + b \operatorname{asech}(cx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(a+b*asech(c*x))**3,x)

[Out] Integral(1/(x**2*(a + b*asech(c*x))**3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{arsech}(cx) + a)^3 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b*arcsech(c*x))^3,x, algorithm="giac")

[Out] integrate(1/((b*arcsech(c*x) + a)^3*x^2), x)

$$3.67 \quad \int \frac{1}{x^3 (a + b \operatorname{sech}^{-1}(cx))^3} dx$$

Optimal. Leaf size=112

$$\frac{c^2 \sinh\left(\frac{2a}{b}\right) \operatorname{Chi}\left(\frac{2a}{b} + 2 \operatorname{sech}^{-1}(cx)\right)}{b^3} - \frac{c^2 \cosh\left(\frac{2a}{b}\right) \operatorname{Shi}\left(\frac{2a}{b} + 2 \operatorname{sech}^{-1}(cx)\right)}{b^3} + \frac{c^2 \cosh\left(2 \operatorname{sech}^{-1}(cx)\right)}{2b^2 (a + b \operatorname{sech}^{-1}(cx))} + \frac{c^2 \sinh\left(2 \operatorname{sech}^{-1}(cx)\right)}{4b (a + b \operatorname{sech}^{-1}(cx))}$$

[Out] (c^2*Cosh[2*ArcSech[c*x]])/(2*b^2*(a + b*ArcSech[c*x])) + (c^2*CoshIntegral[(2*a)/b + 2*ArcSech[c*x]]*Sinh[(2*a)/b])/b^3 + (c^2*Sinh[2*ArcSech[c*x]])/(4*b*(a + b*ArcSech[c*x])^2) - (c^2*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSech[c*x]])/b^3

Rubi [A] time = 0.206464, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6285, 5448, 12, 3297, 3303, 3298, 3301}

$$\frac{c^2 \sinh\left(\frac{2a}{b}\right) \operatorname{Chi}\left(\frac{2a}{b} + 2 \operatorname{sech}^{-1}(cx)\right)}{b^3} - \frac{c^2 \cosh\left(\frac{2a}{b}\right) \operatorname{Shi}\left(\frac{2a}{b} + 2 \operatorname{sech}^{-1}(cx)\right)}{b^3} + \frac{c^2 \cosh\left(2 \operatorname{sech}^{-1}(cx)\right)}{2b^2 (a + b \operatorname{sech}^{-1}(cx))} + \frac{c^2 \sinh\left(2 \operatorname{sech}^{-1}(cx)\right)}{4b (a + b \operatorname{sech}^{-1}(cx))}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(a + b*ArcSech[c*x])^3),x]

[Out] (c^2*Cosh[2*ArcSech[c*x]])/(2*b^2*(a + b*ArcSech[c*x])) + (c^2*CoshIntegral[(2*a)/b + 2*ArcSech[c*x]]*Sinh[(2*a)/b])/b^3 + (c^2*Sinh[2*ArcSech[c*x]])/(4*b*(a + b*ArcSech[c*x])^2) - (c^2*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcSech[c*x]])/b^3

Rule 6285

Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))^n*(x_)^m, x_Symbol] := -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

Rule 5448

Int[Cosh[(a_.) + (b_.)*(x_.)]^p*((c_.) + (d_.)*(x_.))^m*Sinh[(a_.) + (b_.)*(x_.)]^n, x_Symbol] := Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a + b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 3297

Int[((c_.) + (d_.)*(x_.))^m*sin[(e_.) + (f_.)*(x_.)], x_Symbol] := Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

]

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3 (a + b \operatorname{sech}^{-1}(cx))^3} dx &= - \left(c^2 \operatorname{Subst} \left(\int \frac{\cosh(x) \sinh(x)}{(a + bx)^3} dx, x, \operatorname{sech}^{-1}(cx) \right) \right) \\
&= - \left(c^2 \operatorname{Subst} \left(\int \frac{\sinh(2x)}{2(a + bx)^3} dx, x, \operatorname{sech}^{-1}(cx) \right) \right) \\
&= - \left(\frac{1}{2} c^2 \operatorname{Subst} \left(\int \frac{\sinh(2x)}{(a + bx)^3} dx, x, \operatorname{sech}^{-1}(cx) \right) \right) \\
&= \frac{c^2 \sinh(2 \operatorname{sech}^{-1}(cx))}{4b (a + b \operatorname{sech}^{-1}(cx))^2} - \frac{c^2 \operatorname{Subst} \left(\int \frac{\cosh(2x)}{(a + bx)^2} dx, x, \operatorname{sech}^{-1}(cx) \right)}{2b} \\
&= \frac{c^2 \cosh(2 \operatorname{sech}^{-1}(cx))}{2b^2 (a + b \operatorname{sech}^{-1}(cx))} + \frac{c^2 \sinh(2 \operatorname{sech}^{-1}(cx))}{4b (a + b \operatorname{sech}^{-1}(cx))^2} - \frac{c^2 \operatorname{Subst} \left(\int \frac{\sinh(2x)}{a + bx} dx, x, \operatorname{sech}^{-1}(cx) \right)}{b^2} \\
&= \frac{c^2 \cosh(2 \operatorname{sech}^{-1}(cx))}{2b^2 (a + b \operatorname{sech}^{-1}(cx))} + \frac{c^2 \sinh(2 \operatorname{sech}^{-1}(cx))}{4b (a + b \operatorname{sech}^{-1}(cx))^2} - \frac{\left(c^2 \cosh\left(\frac{2a}{b}\right) \right) \operatorname{Subst} \left(\int \frac{\sinh\left(\frac{2a}{b} + 2x\right)}{a + bx} dx \right)}{b^2} \\
&= \frac{c^2 \cosh(2 \operatorname{sech}^{-1}(cx))}{2b^2 (a + b \operatorname{sech}^{-1}(cx))} + \frac{c^2 \operatorname{Chi}\left(\frac{2a}{b} + 2 \operatorname{sech}^{-1}(cx)\right) \sinh\left(\frac{2a}{b}\right)}{b^3} + \frac{c^2 \sinh(2 \operatorname{sech}^{-1}(cx))}{4b (a + b \operatorname{sech}^{-1}(cx))^2} -
\end{aligned}$$

Mathematica [A] time = 0.404743, size = 122, normalized size = 1.09

$$\frac{b^2 \sqrt{\frac{1-cx}{cx+1}} (cx+1)}{x^2 (a + b \operatorname{sech}^{-1}(cx))^2} + 2c^2 \left(\sinh\left(\frac{2a}{b}\right) \operatorname{Chi}\left(2\left(\frac{a}{b} + \operatorname{sech}^{-1}(cx)\right)\right) - \cosh\left(\frac{2a}{b}\right) \operatorname{Shi}\left(2\left(\frac{a}{b} + \operatorname{sech}^{-1}(cx)\right)\right) \right) + \frac{b(2-c^2x^2)}{x^2 (a + b \operatorname{sech}^{-1}(cx))}$$

$$2b^3$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*ArcSech[c*x])^3),x]
```

```
[Out] ((b^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(x^2*(a + b*ArcSech[c*x])^2) + (
b*(2 - c^2*x^2))/(x^2*(a + b*ArcSech[c*x])) + 2*c^2*(CoshIntegral[2*(a/b +
ArcSech[c*x]])*Sinh[(2*a)/b] - Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcSech[
c*x])]))/(2*b^3)
```

Maple [B] time = 0.293, size = 277, normalized size = 2.5

$$c^2 \left(\frac{2 \operatorname{arcsech}(cx) + 2a - b}{8b^2c^2x^2 \left((\operatorname{arcsech}(cx))^2 b^2 + 2 \operatorname{arcsech}(cx) ab + a^2 \right)} \left(2 \sqrt{\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} cx + c^2x^2 - 2 \right) - \frac{1}{2b^3} e^{2\frac{a}{b}} \operatorname{Ei} \left(1, 2\frac{a}{b} + \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b*arcsech(c*x))^3,x)
```

```
[Out] c^2*(-1/8*(2*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*c*x+c^2*x^2-2)*(2*b*a
rcsech(c*x)+2*a-b)/c^2/x^2/b^2/(arcsech(c*x)^2*b^2+2*arcsech(c*x)*a*b+a^2)-
1/2/b^3*exp(2*a/b)*Ei(1,2*a/b+2*arcsech(c*x))-1/8/b*(c^2*x^2-2-2*(-(c*x-1)/
c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*c*x)/c^2/x^2/(a+b*arcsech(c*x))^2-1/4/b^2*(c
^2*x^2-2-2*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*c*x)/c^2/x^2/(a+b*arcse
ch(c*x))+1/2/b^3*exp(-2*a/b)*Ei(1,-2*arcsech(c*x)-2*a/b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*arcsech(c*x))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((b*c^6*(2*log(c) - 1) - 2*a*c^6)*x^7 - 3*(b*c^4*(2*log(c) - 1) - 2*a*
c^4)*x^5 + ((b*c^2*(2*log(c) - 1) - 2*a*c^2)*x^3 - (b*(2*log(c) - 1) - 2*a)
*x + 2*(b*c^2*x^3 - b*x)*log(x))*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) + 3*(b*c^
2*(2*log(c) - 1) - 2*a*c^2)*x^3 - ((b*c^6*log(c) - a*c^6)*x^7 - (b*c^4*(5*log
(c) - 2) - 5*a*c^4)*x^5 + 5*(b*c^2*(2*log(c) - 1) - 2*a*c^2)*x^3 - 3*(b*(
2*log(c) - 1) - 2*a)*x + (b*c^6*x^7 - 5*b*c^4*x^5 + 10*b*c^2*x^3 - 6*b*x)*log
(x))*(c*x + 1)*(c*x - 1) + ((b*c^6*(3*log(c) - 1) - 3*a*c^6)*x^7 - (b*c^4
*(11*log(c) - 5) - 11*a*c^4)*x^5 + 7*(b*c^2*(2*log(c) - 1) - 2*a*c^2)*x^3 -
3*(b*(2*log(c) - 1) - 2*a)*x + (3*b*c^6*x^7 - 11*b*c^4*x^5 + 14*b*c^2*x^3
- 6*b*x)*log(x))*sqrt(c*x + 1)*sqrt(-c*x + 1) - (b*(2*log(c) - 1) - 2*a)*x
- (2*b*c^6*x^7 - 6*b*c^4*x^5 + 6*b*c^2*x^3 + 2*(b*c^2*x^3 - b*x)*(c*x + 1)^(
3/2)*(-c*x + 1)^(3/2) - (b*c^6*x^7 - 5*b*c^4*x^5 + 10*b*c^2*x^3 - 6*b*x)*(
c*x + 1)*(c*x - 1) + (3*b*c^6*x^7 - 11*b*c^4*x^5 + 14*b*c^2*x^3 - 6*b*x)*sq
rt(c*x + 1)*sqrt(-c*x + 1) - 2*b*x)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) +
2*(b*c^6*x^7 - 3*b*c^4*x^5 + 3*b*c^2*x^3 - b*x)*log(x))/((b^4*c^6*x^6 - 3*
b^4*c^4*x^4 + 3*b^4*c^2*x^2 - b^4)*x^3*log(x)^2 + 2*((b^4*c^6*log(c) - a*b^
3*c^6)*x^6 - 3*(b^4*c^4*log(c) - a*b^3*c^4)*x^4 - b^4*log(c) + a*b^3 + 3*(b
^4*c^2*log(c) - a*b^3*c^2)*x^2)*x^3*log(x) - (b^4*x^3*log(x)^2 + 2*(b^4*log
(c) - a*b^3)*x^3*log(x) + (b^4*log(c))^2 - 2*a*b^3*log(c) + a^2*b^2)*x^3*(c
*x + 1)^(3/2)*(-c*x + 1)^(3/2) + ((b^4*c^6*log(c))^2 - 2*a*b^3*c^6*log(c) +
a^2*b^2*c^6)*x^6 - b^4*log(c)^2 - 3*(b^4*c^4*log(c))^2 - 2*a*b^3*c^4*log(c)
+ a^2*b^2*c^4)*x^4 + 2*a*b^3*log(c) - a^2*b^2 + 3*(b^4*c^2*log(c))^2 - 2*a*b
^3*c^2*log(c) + a^2*b^2*c^2)*x^2)*x^3 - 3*((b^4*c^2*x^2 - b^4)*x^3*log(x)^2
- 2*(b^4*log(c) - a*b^3 - (b^4*c^2*log(c) - a*b^3*c^2)*x^2)*x^3*log(x) - (
b^4*log(c))^2 - 2*a*b^3*log(c) + a^2*b^2 - (b^4*c^2*log(c))^2 - 2*a*b^3*c^2*
```

```

og(c) + a^2*b^2*c^2)*x^2)*x^3)*(c*x + 1)*(c*x - 1) - ((c*x + 1)^(3/2)*(-c*x
+ 1)^(3/2)*b^4*x^3 + 3*(b^4*c^2*x^2 - b^4)*(c*x + 1)*(c*x - 1)*x^3 + 3*(b^
4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^3 - (b^4*c^
6*x^6 - 3*b^4*c^4*x^4 + 3*b^4*c^2*x^2 - b^4)*x^3)*log(sqrt(c*x + 1)*sqrt(-c
*x + 1) + 1)^2 - 3*((b^4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4)*x^3*log(x)^2 + 2*((
b^4*c^4*log(c) - a*b^3*c^4)*x^4 + b^4*log(c) - a*b^3 - 2*(b^4*c^2*log(c) -
a*b^3*c^2)*x^2)*x^3*log(x) + (b^4*log(c)^2 + (b^4*c^4*log(c))^2 - 2*a*b^3*c^
4*log(c) + a^2*b^2*c^4)*x^4 - 2*a*b^3*log(c) + a^2*b^2 - 2*(b^4*c^2*log(c))^
2 - 2*a*b^3*c^2*log(c) + a^2*b^2*c^2)*x^2)*x^3)*sqrt(c*x + 1)*sqrt(-c*x + 1
) - 2*((b^4*c^6*x^6 - 3*b^4*c^4*x^4 + 3*b^4*c^2*x^2 - b^4)*x^3*log(x) - (b^
4*x^3*log(x) + (b^4*log(c) - a*b^3)*x^3)*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) +
((b^4*c^6*log(c) - a*b^3*c^6)*x^6 - 3*(b^4*c^4*log(c) - a*b^3*c^4)*x^4 - b
^4*log(c) + a*b^3 + 3*(b^4*c^2*log(c) - a*b^3*c^2)*x^2)*x^3 - 3*((b^4*c^2*x
^2 - b^4)*x^3*log(x) - (b^4*log(c) - a*b^3 - (b^4*c^2*log(c) - a*b^3*c^2)*x
^2)*x^3)*(c*x + 1)*(c*x - 1) - 3*((b^4*c^4*x^4 - 2*b^4*c^2*x^2 + b^4)*x^3*1
og(x) + ((b^4*c^4*log(c) - a*b^3*c^4)*x^4 + b^4*log(c) - a*b^3 - 2*(b^4*c^2
*log(c) - a*b^3*c^2)*x^2)*x^3)*sqrt(c*x + 1)*sqrt(-c*x + 1))*log(sqrt(c*x +
1)*sqrt(-c*x + 1) + 1)) + integrate(-1/2*(4*c^8*x^8 - 16*c^6*x^6 + 24*c^4*x
^4 + 4*(c*x + 1)^2*(c*x - 1)^2 + (3*c^6*x^6 - 16*c^2*x^2 + 16)*(c*x + 1)^(
3/2)*(-c*x + 1)^(3/2) - 16*c^2*x^2 - 24*(c^4*x^4 - 2*c^2*x^2 + 1)*(c*x + 1)
*(c*x - 1) + (3*c^8*x^8 - 19*c^6*x^6 + 48*c^4*x^4 - 48*c^2*x^2 + 16)*sqrt(c
*x + 1)*sqrt(-c*x + 1) + 4)/((b^3*x^3*log(x) + (b^3*log(c) - a*b^2)*x^3)*(c
*x + 1)^2*(c*x - 1)^2 + (b^3*c^8*x^8 - 4*b^3*c^6*x^6 + 6*b^3*c^4*x^4 - 4*b^
3*c^2*x^2 + b^3)*x^3*log(x) - 4*((b^3*c^2*x^2 - b^3)*x^3*log(x) - (b^3*log(
c) - a*b^2 - (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*x^3)*(c*x + 1)^(3/2)*(-c*x +
1)^(3/2) + ((b^3*c^8*log(c) - a*b^2*c^8)*x^8 - 4*(b^3*c^6*log(c) - a*b^2*c
^6)*x^6 + 6*(b^3*c^4*log(c) - a*b^2*c^4)*x^4 + b^3*log(c) - a*b^2 - 4*(b^3*
c^2*log(c) - a*b^2*c^2)*x^2)*x^3 - 6*((b^3*c^4*x^4 - 2*b^3*c^2*x^2 + b^3)*x
^3*log(x) + ((b^3*c^4*log(c) - a*b^2*c^4)*x^4 + b^3*log(c) - a*b^2 - 2*(b^3
*c^2*log(c) - a*b^2*c^2)*x^2)*x^3)*(c*x + 1)*(c*x - 1) - 4*((b^3*c^6*x^6 -
3*b^3*c^4*x^4 + 3*b^3*c^2*x^2 - b^3)*x^3*log(x) + ((b^3*c^6*log(c) - a*b^2*
c^6)*x^6 - 3*(b^3*c^4*log(c) - a*b^2*c^4)*x^4 - b^3*log(c) + a*b^2 + 3*(b^3
*c^2*log(c) - a*b^2*c^2)*x^2)*x^3)*sqrt(c*x + 1)*sqrt(-c*x + 1) - ((c*x + 1
)^2*(c*x - 1)^2*b^3*x^3 - 4*(b^3*c^2*x^2 - b^3)*(c*x + 1)^(3/2)*(-c*x + 1)^(
3/2)*x^3 - 6*(b^3*c^4*x^4 - 2*b^3*c^2*x^2 + b^3)*(c*x + 1)*(c*x - 1)*x^3 -
4*(b^3*c^6*x^6 - 3*b^3*c^4*x^4 + 3*b^3*c^2*x^2 - b^3)*sqrt(c*x + 1)*sqrt(-
c*x + 1)*x^3 + (b^3*c^8*x^8 - 4*b^3*c^6*x^6 + 6*b^3*c^4*x^4 - 4*b^3*c^2*x^2
+ b^3)*x^3)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)), x)

```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b^3x^3 \operatorname{arsech}(cx)^3 + 3ab^2x^3 \operatorname{arsech}(cx)^2 + 3a^2bx^3 \operatorname{arsech}(cx) + a^3x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b*arcsech(c*x))^3,x, algorithm="fricas")

[Out] integral(1/(b^3*x^3*arcsech(c*x)^3 + 3*a*b^2*x^3*arcsech(c*x)^2 + 3*a^2*b*x^3*arcsech(c*x) + a^3*x^3), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 (a + b \operatorname{asech}(cx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(a+b*asech(c*x))**3,x)

[Out] Integral(1/(x**3*(a + b*asech(c*x))**3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{ar} \operatorname{sech}(cx) + a)^3 x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b*arcsech(c*x))^3,x, algorithm="giac")

[Out] integrate(1/((b*arcsech(c*x) + a)^3*x^3), x)

$$3.68 \quad \int \frac{1}{x^4 (a + b \operatorname{sech}^{-1}(cx))^3} dx$$

Optimal. Leaf size=240

$$\frac{c^3 \sinh\left(\frac{a}{b}\right) \operatorname{Chi}\left(\frac{a}{b} + \operatorname{sech}^{-1}(cx)\right)}{8b^3} + \frac{9c^3 \sinh\left(\frac{3a}{b}\right) \operatorname{Chi}\left(\frac{3a}{b} + 3\operatorname{sech}^{-1}(cx)\right)}{8b^3} - \frac{c^3 \cosh\left(\frac{a}{b}\right) \operatorname{Shi}\left(\frac{a}{b} + \operatorname{sech}^{-1}(cx)\right)}{8b^3} - \frac{9c^3 \cosh\left(\frac{3a}{b}\right) \operatorname{Shi}\left(\frac{3a}{b} + 3\operatorname{sech}^{-1}(cx)\right)}{8b^3}$$

[Out] (c^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(8*b*x*(a + b*ArcSech[c*x])^2) + c^2/(8*b^2*x*(a + b*ArcSech[c*x])) + (3*c^3*Cosh[3*ArcSech[c*x]])/(8*b^2*(a + b*ArcSech[c*x])) + (c^3*CoshIntegral[a/b + ArcSech[c*x]]*Sinh[a/b])/(8*b^3) + (9*c^3*CoshIntegral[(3*a)/b + 3*ArcSech[c*x]]*Sinh[(3*a)/b])/(8*b^3) + (c^3*Sinh[3*ArcSech[c*x]])/(8*b*(a + b*ArcSech[c*x])^2) - (c^3*Cosh[a/b]*SinhIntegral[a/b + ArcSech[c*x]])/(8*b^3) - (9*c^3*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSech[c*x]])/(8*b^3)

Rubi [A] time = 0.371114, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6285, 5448, 3297, 3303, 3298, 3301}

$$\frac{c^3 \sinh\left(\frac{a}{b}\right) \operatorname{Chi}\left(\frac{a}{b} + \operatorname{sech}^{-1}(cx)\right)}{8b^3} + \frac{9c^3 \sinh\left(\frac{3a}{b}\right) \operatorname{Chi}\left(\frac{3a}{b} + 3\operatorname{sech}^{-1}(cx)\right)}{8b^3} - \frac{c^3 \cosh\left(\frac{a}{b}\right) \operatorname{Shi}\left(\frac{a}{b} + \operatorname{sech}^{-1}(cx)\right)}{8b^3} - \frac{9c^3 \cosh\left(\frac{3a}{b}\right) \operatorname{Shi}\left(\frac{3a}{b} + 3\operatorname{sech}^{-1}(cx)\right)}{8b^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(a + b*ArcSech[c*x])^3),x]

[Out] (c^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(8*b*x*(a + b*ArcSech[c*x])^2) + c^2/(8*b^2*x*(a + b*ArcSech[c*x])) + (3*c^3*Cosh[3*ArcSech[c*x]])/(8*b^2*(a + b*ArcSech[c*x])) + (c^3*CoshIntegral[a/b + ArcSech[c*x]]*Sinh[a/b])/(8*b^3) + (9*c^3*CoshIntegral[(3*a)/b + 3*ArcSech[c*x]]*Sinh[(3*a)/b])/(8*b^3) + (c^3*Sinh[3*ArcSech[c*x]])/(8*b*(a + b*ArcSech[c*x])^2) - (c^3*Cosh[a/b]*SinhIntegral[a/b + ArcSech[c*x]])/(8*b^3) - (9*c^3*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcSech[c*x]])/(8*b^3)

Rule 6285

Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))^ (n_.)*(x_.)^(m_.), x_Symbol] :> -Dist[(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Sech[x]^(m + 1)*Tanh[x], x], x, ArcSech[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (GtQ[n, 0] || LtQ[m, -1])

Rule 5448

Int[Cosh[(a_.) + (b_.)*(x_.)]^(p_.)*((c_.) + (d_.)*(x_.))^(m_.)*Sinh[(a_.) + (b_.)*(x_.)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a + b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3297

Int[((c_.) + (d_.)*(x_.))^(m_.)*sin[(e_.) + (f_.)*(x_.)], x_Symbol] :> Simp[((c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

]

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^4 (a + b \operatorname{sech}^{-1}(cx))^3} dx &= - \left(c^3 \operatorname{Subst} \left(\int \frac{\cosh^2(x) \sinh(x)}{(a + bx)^3} dx, x, \operatorname{sech}^{-1}(cx) \right) \right) \\
&= - \left(c^3 \operatorname{Subst} \left(\int \left(\frac{\sinh(x)}{4(a + bx)^3} + \frac{\sinh(3x)}{4(a + bx)^3} \right) dx, x, \operatorname{sech}^{-1}(cx) \right) \right) \\
&= - \left(\frac{1}{4} c^3 \operatorname{Subst} \left(\int \frac{\sinh(x)}{(a + bx)^3} dx, x, \operatorname{sech}^{-1}(cx) \right) \right) - \frac{1}{4} c^3 \operatorname{Subst} \left(\int \frac{\sinh(3x)}{(a + bx)^3} dx, x, \operatorname{sech}^{-1}(cx) \right) \\
&= \frac{c^2 \sqrt{\frac{1-cx}{1+cx}} (1+cx)}{8bx (a + b \operatorname{sech}^{-1}(cx))^2} + \frac{c^3 \sinh(3 \operatorname{sech}^{-1}(cx))}{8b (a + b \operatorname{sech}^{-1}(cx))^2} - \frac{c^3 \operatorname{Subst} \left(\int \frac{\cosh(x)}{(a+bx)^2} dx, x, \operatorname{sech}^{-1}(cx) \right)}{8b} \\
&= \frac{c^2 \sqrt{\frac{1-cx}{1+cx}} (1+cx)}{8bx (a + b \operatorname{sech}^{-1}(cx))^2} + \frac{c^2}{8b^2 x (a + b \operatorname{sech}^{-1}(cx))} + \frac{3c^3 \cosh(3 \operatorname{sech}^{-1}(cx))}{8b^2 (a + b \operatorname{sech}^{-1}(cx))} + \frac{c^3 \sinh(3 \operatorname{sech}^{-1}(cx))}{8b (a + b \operatorname{sech}^{-1}(cx))} \\
&= \frac{c^2 \sqrt{\frac{1-cx}{1+cx}} (1+cx)}{8bx (a + b \operatorname{sech}^{-1}(cx))^2} + \frac{c^2}{8b^2 x (a + b \operatorname{sech}^{-1}(cx))} + \frac{3c^3 \cosh(3 \operatorname{sech}^{-1}(cx))}{8b^2 (a + b \operatorname{sech}^{-1}(cx))} + \frac{c^3 \sinh(3 \operatorname{sech}^{-1}(cx))}{8b (a + b \operatorname{sech}^{-1}(cx))} \\
&= \frac{c^2 \sqrt{\frac{1-cx}{1+cx}} (1+cx)}{8bx (a + b \operatorname{sech}^{-1}(cx))^2} + \frac{c^2}{8b^2 x (a + b \operatorname{sech}^{-1}(cx))} + \frac{3c^3 \cosh(3 \operatorname{sech}^{-1}(cx))}{8b^2 (a + b \operatorname{sech}^{-1}(cx))} + \frac{c^3 \operatorname{Chi} \left(\frac{a}{b} + \operatorname{sech}^{-1}(cx) \right)}{8b (a + b \operatorname{sech}^{-1}(cx))}
\end{aligned}$$

Mathematica [A] time = 0.554671, size = 204, normalized size = 0.85

$$\frac{4b^2 \sqrt{\frac{1-cx}{cx+1}} (cx+1)}{x^3 (a + b \operatorname{sech}^{-1}(cx))^2} - 8c^3 \left(\sinh \left(\frac{a}{b} \right) \operatorname{Chi} \left(\frac{a}{b} + \operatorname{sech}^{-1}(cx) \right) - \cosh \left(\frac{a}{b} \right) \operatorname{Shi} \left(\frac{a}{b} + \operatorname{sech}^{-1}(cx) \right) \right) + 9c^3 \left(\sinh \left(\frac{a}{b} \right) \operatorname{Chi} \left(\frac{a}{b} + \operatorname{sech}^{-1}(cx) \right) + \cosh \left(\frac{a}{b} \right) \operatorname{Shi} \left(\frac{a}{b} + \operatorname{sech}^{-1}(cx) \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(x^4*(a + b*ArcSech[c*x])^3), x]

```
[Out] ((4*b^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(x^3*(a + b*ArcSech[c*x])^2) +
(4*b*(3 - 2*c^2*x^2))/(x^3*(a + b*ArcSech[c*x])) - 8*c^3*(CoshIntegral[a/b
+ ArcSech[c*x]]*Sinh[a/b] - Cosh[a/b]*SinhIntegral[a/b + ArcSech[c*x]]) +
9*c^3*(CoshIntegral[a/b + ArcSech[c*x]]*Sinh[a/b] + CoshIntegral[3*(a/b + A
rcSech[c*x]])*Sinh[(3*a)/b] - Cosh[a/b]*SinhIntegral[a/b + ArcSech[c*x]] -
Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcSech[c*x])]))/(8*b^3)
```

Maple [B] time = 0.329, size = 628, normalized size = 2.6

$$c^3 \left(\frac{3 \operatorname{arcsech}(cx) + 3a - b}{16c^3x^3b^2 \left((\operatorname{arcsech}(cx))^2 b^2 + 2 \operatorname{arcsech}(cx) ab + a^2 \right)} \left(\sqrt{\frac{cx+1}{cx}} \sqrt{\frac{cx-1}{cx}} c^3 x^3 - 4 \sqrt{\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} cx - 3c^2 x^2 + \dots \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a+b*arcsech(c*x))^3,x)
```

```
[Out] c^3*(1/16*(((c*x+1)/c/x)^(1/2)*(-(c*x-1)/c/x)^(1/2)*c^3*x^3-4*(-(c*x-1)/c/x)
)^(1/2)*((c*x+1)/c/x)^(1/2)*c*x-3*c^2*x^2+4)*(3*b*arcsech(c*x)+3*a-b)/c^3/x
^3/b^2/(arcsech(c*x)^2*b^2+2*arcsech(c*x)*a*b+a^2)-9/16/b^3*exp(3*a/b)*Ei(1
,3*a/b+3*arcsech(c*x))-1/16*((-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*c*x-1
)*(b*arcsech(c*x)+a-b)/c/x/b^2/(arcsech(c*x)^2*b^2+2*arcsech(c*x)*a*b+a^2)-
1/16/b^3*exp(a/b)*Ei(1,a/b+arcsech(c*x))+1/16/b*((-(c*x-1)/c/x)^(1/2)*((c*x
+1)/c/x)^(1/2)*c*x+1)/c/x/(a+b*arcsech(c*x))^2+1/16/b^2*((-(c*x-1)/c/x)^(1/
2)*((c*x+1)/c/x)^(1/2)*c*x+1)/c/x/(a+b*arcsech(c*x))+1/16/b^3*exp(-a/b)*Ei(
1,-arcsech(c*x)-a/b)-1/16/b*((c*x+1)/c/x)^(1/2)*(-(c*x-1)/c/x)^(1/2)*c^3*x
^3-4*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*c*x+3*c^2*x^2-4)/c^3/x^3/(a+b
*arcsech(c*x))^2-3/16/b^2*((c*x+1)/c/x)^(1/2)*(-(c*x-1)/c/x)^(1/2)*c^3*x^3
-4*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*c*x+3*c^2*x^2-4)/c^3/x^3/(a+b*a
rcsech(c*x))+9/16/b^3*exp(-3*a/b)*Ei(1,-3*arcsech(c*x)-3*a/b)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*arcsech(c*x))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((b*c^6*(3*log(c) - 1) - 3*a*c^6)*x^7 - 3*(b*c^4*(3*log(c) - 1) - 3*a*
c^4)*x^5 - ((b*c^4*log(c) - a*c^4)*x^5 - (b*c^2*(4*log(c) - 1) - 4*a*c^2)*x
^3 + (b*(3*log(c) - 1) - 3*a)*x + (b*c^4*x^5 - 4*b*c^2*x^3 + 3*b*x)*log(x))
*(c*x + 1)^(3/2)*(-c*x + 1)^(3/2) + 3*(b*c^2*(3*log(c) - 1) - 3*a*c^2)*x^3
- (2*(b*c^6*log(c) - a*c^6)*x^7 - 2*(b*c^4*(5*log(c) - 1) - 5*a*c^4)*x^5 +
(b*c^2*(17*log(c) - 5) - 17*a*c^2)*x^3 - 3*(b*(3*log(c) - 1) - 3*a)*x + (2*
b*c^6*x^7 - 10*b*c^4*x^5 + 17*b*c^2*x^3 - 9*b*x)*log(x))*(c*x + 1)*(c*x - 1
) + ((b*c^6*(5*log(c) - 1) - 5*a*c^6)*x^7 - (b*c^4*(18*log(c) - 5) - 18*a*c
^4)*x^5 + (b*c^2*(22*log(c) - 7) - 22*a*c^2)*x^3 - 3*(b*(3*log(c) - 1) - 3*
a)*x + (5*b*c^6*x^7 - 18*b*c^4*x^5 + 22*b*c^2*x^3 - 9*b*x)*log(x))*sqrt(c*x
+ 1)*sqrt(-c*x + 1) - (b*(3*log(c) - 1) - 3*a)*x - (3*b*c^6*x^7 - 9*b*c^4*
x^5 + 9*b*c^2*x^3 - (b*c^4*x^5 - 4*b*c^2*x^3 + 3*b*x)*(c*x + 1)^(3/2)*(-c*x
+ 1)^(3/2) - (2*b*c^6*x^7 - 10*b*c^4*x^5 + 17*b*c^2*x^3 - 9*b*x)*(c*x + 1)
*(c*x - 1) + (5*b*c^6*x^7 - 18*b*c^4*x^5 + 22*b*c^2*x^3 - 9*b*x)*sqrt(c*x +
1)*sqrt(-c*x + 1) - 3*b*x)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) + 3*(b*c^
6*x^7 - 3*b*c^4*x^5 + 3*b*c^2*x^3 - b*x)*log(x))/((b^4*c^6*x^6 - 3*b^4*c^4*
```

$$\begin{aligned}
& x^4 + 3b^4c^2x^2 - b^4)x^4\log(x)^2 + 2*((b^4c^6\log(c) - ab^3c^6)*x \\
& ^6 - 3*(b^4c^4\log(c) - ab^3c^4)*x^4 - b^4\log(c) + ab^3 + 3*(b^4c^2\log(c) - ab^3c^2)*x^2)*x^4\log(x) + ((b^4c^6\log(c)^2 - 2*ab^3c^6\log(c) \\
&) + a^2b^2c^6)*x^6 - b^4\log(c)^2 - 3*(b^4c^4\log(c)^2 - 2*ab^3c^4\log(c) + a^2b^2c^4)*x^4 + 2*ab^3\log(c) - a^2b^2 + 3*(b^4c^2\log(c)^2 - 2 \\
& *ab^3c^2\log(c) + a^2b^2c^2)*x^2)*x^4 - (b^4x^4\log(x)^2 + 2*(b^4\log(c) - ab^3)*x^4\log(x) + (b^4\log(c)^2 - 2*ab^3\log(c) + a^2b^2)*x^4)*(c*x \\
& + 1)^{(3/2)}*(-c*x + 1)^{(3/2)} - 3*((b^4c^2x^2 - b^4)*x^4\log(x)^2 - 2*(b^4\log(c) - ab^3 - (b^4c^2\log(c) - ab^3c^2)*x^2)*x^4\log(x) - (b^4\log(c) \\
& ^2 - 2*ab^3\log(c) + a^2b^2 - (b^4c^2\log(c)^2 - 2*ab^3c^2\log(c) + a^2b^2c^2)*x^2)*x^4)*(c*x + 1)*(c*x - 1) - ((c*x + 1)^{(3/2)}*(-c*x + 1)^{(3 \\
& /2)}*b^4*x^4 + 3*(b^4c^2*x^2 - b^4)*(c*x + 1)*(c*x - 1)*x^4 + 3*(b^4c^4*x^4 - 2*b^4c^2*x^2 + b^4)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^4 - (b^4c^6*x^6 - \\
& 3*b^4c^4*x^4 + 3*b^4c^2*x^2 - b^4)*x^4)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)^2 - 3*((b^4c^4*x^4 - 2*b^4c^2*x^2 + b^4)*x^4\log(x)^2 + 2*((b^4c^4\log(c) - ab^3c^4)*x^4 + b^4\log(c) - ab^3 - 2*(b^4c^2\log(c) - ab^3c^2)*x^2)*x^4\log(x) + (b^4\log(c)^2 + (b^4c^4\log(c)^2 - 2*ab^3c^4\log(c) + a^2b^2c^4)*x^4 - 2*ab^3\log(c) + a^2b^2 - 2*(b^4c^2\log(c)^2 - 2*ab^3c^2\log(c) + a^2b^2c^2)*x^2)*x^4)*sqrt(c*x + 1)*sqrt(-c*x + 1) - 2*((b^4c^6*x^6 - 3*b^4c^4*x^4 + 3*b^4c^2*x^2 - b^4)*x^4\log(x) + ((b^4c^6\log(c) - ab^3c^6)*x^6 - 3*(b^4c^4\log(c) - ab^3c^4)*x^4 - b^4\log(c) + ab^3 + 3*(b^4c^2\log(c) - ab^3c^2)*x^2)*x^4 - (b^4x^4\log(x) + (b^4\log(c) - ab^3)*x^4)*(c*x + 1)^{(3/2)}*(-c*x + 1)^{(3/2)} - 3*((b^4c^2*x^2 - b^4)*x^4\log(x) - (b^4\log(c) - ab^3 - (b^4c^2\log(c) - ab^3c^2)*x^2)*x^4)*(c*x + 1)*(c*x - 1) - 3*((b^4c^4*x^4 - 2*b^4c^2*x^2 + b^4)*x^4\log(x) + ((b^4c^4\log(c) - ab^3c^4)*x^4 + b^4\log(c) - ab^3 - 2*(b^4c^2\log(c) - ab^3c^2)*x^2)*x^4)*sqrt(c*x + 1)*sqrt(-c*x + 1))*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)) - integrate(1/2*(9*c^8*x^8 - 36*c^6*x^6 + 54*c^4*x^4 - (c^4*x^4 + 4*c^2*x^2 - 9)*(c*x + 1)^2*(c*x - 1)^2 + (2*c^6*x^6 + 13*c^4*x^4 - 48*c^2*x^2 + 36)*(c*x + 1)^{(3/2)}*(-c*x + 1)^{(3/2)} - 36*c^2*x^2 - (2*c^8*x^8 - 19*c^6*x^6 + 83*c^4*x^4 - 120*c^2*x^2 + 54)*(c*x + 1)*(c*x - 1) + (10*c^8*x^8 - 57*c^6*x^6 + 123*c^4*x^4 - 112*c^2*x^2 + 36)*sqrt(c*x + 1)*sqrt(-c*x + 1) + 9)/((b^3*c^8*x^8 - 4*b^3*c^6*x^6 + 6*b^3*c^4*x^4 - 4*b^3*c^2*x^2 + b^3)*x^4\log(x) + (b^3*x^4\log(x) + (b^3\log(c) - ab^2)*x^4)*(c*x + 1)^2*(c*x - 1)^2 + ((b^3*c^8\log(c) - ab^2*c^8)*x^8 - 4*(b^3*c^6\log(c) - ab^2*c^6)*x^6 + 6*(b^3*c^4\log(c) - ab^2*c^4)*x^4 + b^3\log(c) - ab^2 - 4*(b^3*c^2\log(c) - ab^2*c^2)*x^2)*x^4 - 4*((b^3*c^2*x^2 - b^3)*x^4\log(x) - (b^3\log(c) - ab^2 - (b^3*c^2\log(c) - ab^2*c^2)*x^2)*x^4)*(c*x + 1)^{(3/2)}*(-c*x + 1)^{(3/2)} - 6*((b^3*c^4*x^4 - 2*b^3*c^2*x^2 + b^3)*x^4\log(x) + ((b^3*c^4\log(c) - ab^2*c^4)*x^4 + b^3\log(c) - ab^2 - 2*(b^3*c^2\log(c) - ab^2*c^2)*x^2)*x^4)*(c*x + 1)*(c*x - 1) - 4*((b^3*c^6*x^6 - 3*b^3*c^4*x^4 + 3*b^3*c^2*x^2 - b^3)*x^4\log(x) + ((b^3*c^6\log(c) - ab^2*c^6)*x^6 - 3*(b^3*c^4\log(c) - ab^2*c^4)*x^4 - b^3\log(c) + ab^2 + 3*(b^3*c^2\log(c) - ab^2*c^2)*x^2)*x^4)*sqrt(c*x + 1)*sqrt(-c*x + 1) - ((c*x + 1)^2*(c*x - 1)^2*b^3*x^4 - 4*(b^3*c^2*x^2 - b^3)*(c*x + 1)^{(3/2)}*(-c*x + 1)^{(3/2)}*x^4 - 6*(b^3*c^4*x^4 - 2*b^3*c^2*x^2 + b^3)*(c*x + 1)*(c*x - 1)*x^4 - 4*(b^3*c^6*x^6 - 3*b^3*c^4*x^4 + 3*b^3*c^2*x^2 - b^3)*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^4 + (b^3*c^8*x^8 - 4*b^3*c^6*x^6 + 6*b^3*c^4*x^4 - 4*b^3*c^2*x^2 + b^3)*x^4)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)), x)
\end{aligned}$$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{b^3x^4 \operatorname{arsech}(cx)^3 + 3ab^2x^4 \operatorname{arsech}(cx)^2 + 3a^2bx^4 \operatorname{arsech}(cx) + a^3x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a+b*arcsech(c*x))^3,x, algorithm="fricas")

[Out] `integral(1/(b^3*x^4*arcsech(c*x)^3 + 3*a*b^2*x^4*arcsech(c*x)^2 + 3*a^2*b*x^4*arcsech(c*x) + a^3*x^4), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 (a + b \operatorname{arsech}(cx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**4/(a+b*arsech(c*x))**3,x)`

[Out] `Integral(1/(x**4*(a + b*arsech(c*x))**3), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \operatorname{arsech}(cx) + a)^3 x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^4/(a+b*arcsech(c*x))^3,x, algorithm="giac")`

[Out] `integrate(1/((b*arcsech(c*x) + a)^3*x^4), x)`

$$3.69 \quad \int (dx)^m \left(a + b \operatorname{sech}^{-1}(cx) \right)^3 dx$$

Optimal. Leaf size=18

$$\text{Unintegrable} \left((dx)^m \left(a + b \operatorname{sech}^{-1}(cx) \right)^3, x \right)$$

[Out] Unintegrable[(d*x)^m*(a + b*ArcSech[c*x])^3, x]

Rubi [A] time = 0.0237076, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (dx)^m \left(a + b \operatorname{sech}^{-1}(cx) \right)^3 dx$$

Verification is Not applicable to the result.

[In] Int[(d*x)^m*(a + b*ArcSech[c*x])^3,x]

[Out] Defer[Int] [(d*x)^m*(a + b*ArcSech[c*x])^3, x]

Rubi steps

$$\int (dx)^m \left(a + b \operatorname{sech}^{-1}(cx) \right)^3 dx = \int (dx)^m \left(a + b \operatorname{sech}^{-1}(cx) \right)^3 dx$$

Mathematica [A] time = 5.07665, size = 0, normalized size = 0.

$$\int (dx)^m \left(a + b \operatorname{sech}^{-1}(cx) \right)^3 dx$$

Verification is Not applicable to the result.

[In] Integrate[(d*x)^m*(a + b*ArcSech[c*x])^3,x]

[Out] Integrate[(d*x)^m*(a + b*ArcSech[c*x])^3, x]

Maple [A] time = 1.97, size = 0, normalized size = 0.

$$\int (dx)^m \left(a + b \operatorname{arcsech}(cx) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x)^m*(a+b*arcsech(c*x))^3,x)

[Out] int((d*x)^m*(a+b*arcsech(c*x))^3,x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(a+b*arcsech(c*x))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^3 \operatorname{arsech}(cx)^3 + 3ab^2 \operatorname{arsech}(cx)^2 + 3a^2b \operatorname{arsech}(cx) + a^3\right)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(a+b*arcsech(c*x))^3,x, algorithm="fricas")

[Out] integral((b^3*arcsech(c*x)^3 + 3*a*b^2*arcsech(c*x)^2 + 3*a^2*b*arcsech(c*x) + a^3)*(d*x)^m, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m (a + b \operatorname{asech}(cx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)**m*(a+b*asech(c*x))**3,x)

[Out] Integral((d*x)**m*(a + b*asech(c*x))**3, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arsech}(cx) + a)^3 (dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(a+b*arcsech(c*x))^3,x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)^3*(d*x)^m, x)

$$3.70 \quad \int (dx)^m \left(a + b \operatorname{sech}^{-1}(cx) \right)^2 dx$$

Optimal. Leaf size=18

$$\text{Unintegrable} \left((dx)^m \left(a + b \operatorname{sech}^{-1}(cx) \right)^2, x \right)$$

[Out] Unintegrable[(d*x)^m*(a + b*ArcSech[c*x])^2, x]

Rubi [A] time = 0.0255439, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (dx)^m \left(a + b \operatorname{sech}^{-1}(cx) \right)^2 dx$$

Verification is Not applicable to the result.

[In] Int[(d*x)^m*(a + b*ArcSech[c*x])^2,x]

[Out] Defer[Int] [(d*x)^m*(a + b*ArcSech[c*x])^2, x]

Rubi steps

$$\int (dx)^m \left(a + b \operatorname{sech}^{-1}(cx) \right)^2 dx = \int (dx)^m \left(a + b \operatorname{sech}^{-1}(cx) \right)^2 dx$$

Mathematica [A] time = 3.33555, size = 0, normalized size = 0.

$$\int (dx)^m \left(a + b \operatorname{sech}^{-1}(cx) \right)^2 dx$$

Verification is Not applicable to the result.

[In] Integrate[(d*x)^m*(a + b*ArcSech[c*x])^2,x]

[Out] Integrate[(d*x)^m*(a + b*ArcSech[c*x])^2, x]

Maple [A] time = 1.681, size = 0, normalized size = 0.

$$\int (dx)^m \left(a + b \operatorname{arcsech}(cx) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x)^m*(a+b*arcsech(c*x))^2,x)

[Out] int((d*x)^m*(a+b*arcsech(c*x))^2,x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(a+b*arcsech(c*x))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2 \operatorname{arsech}(cx)^2 + 2ab \operatorname{arsech}(cx) + a^2\right)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(a+b*arcsech(c*x))^2,x, algorithm="fricas")

[Out] integral((b^2*arcsech(c*x)^2 + 2*a*b*arcsech(c*x) + a^2)*(d*x)^m, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m (a + b \operatorname{asech}(cx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)**m*(a+b*asech(c*x))**2,x)

[Out] Integral((d*x)**m*(a + b*asech(c*x))**2, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arsech}(cx) + a)^2 (dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(a+b*arcsech(c*x))^2,x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)^2*(d*x)^m, x)

3.71 $\int (dx)^m \left(a + b \operatorname{sech}^{-1}(cx) \right) dx$

Optimal. Leaf size=87

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(dx)^{m+1}\operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+1}{2}, \frac{m+3}{2}, c^2x^2\right)}{d(m+1)^2} + \frac{(dx)^{m+1}\left(a + b\operatorname{sech}^{-1}(cx)\right)}{d(m+1)}$$

[Out] $((d*x)^{(1+m)}*(a + b*\operatorname{ArcSech}[c*x]))/(d*(1+m)) + (b*(d*x)^{(1+m)}*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{Hypergeometric2F1}[1/2, (1+m)/2, (3+m)/2, c^2*x^2])/(d*(1+m)^2)$

Rubi [A] time = 0.0371767, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {6283, 125, 364}

$$\frac{(dx)^{m+1}\left(a + b\operatorname{sech}^{-1}(cx)\right)}{d(m+1)} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(dx)^{m+1} {}_2F_1\left(\frac{1}{2}, \frac{m+1}{2}; \frac{m+3}{2}; c^2x^2\right)}{d(m+1)^2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(d*x)^m*(a + b*\operatorname{ArcSech}[c*x]), x]$

[Out] $((d*x)^{(1+m)}*(a + b*\operatorname{ArcSech}[c*x]))/(d*(1+m)) + (b*(d*x)^{(1+m)}*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{Hypergeometric2F1}[1/2, (1+m)/2, (3+m)/2, c^2*x^2])/(d*(1+m)^2)$

Rule 6283

$\operatorname{Int}[(a + \operatorname{ArcSech}[c*x])*(b*x)^m*(d*x)^m, x_Symbol] \rightarrow \operatorname{Simp}[(d*x)^{m+1}*(a + b*\operatorname{ArcSech}[c*x])/(d*(m+1)), x] + \operatorname{Dist}[(b*\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[1/(1 + c*x)])/(m+1), \operatorname{Int}[(d*x)^m/(\operatorname{Sqrt}[1 - c*x]*\operatorname{Sqrt}[1 + c*x]), x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, m, x\} \ \&\& \ \operatorname{NeQ}[m, -1]$

Rule 125

$\operatorname{Int}[(f*x)^p*(a + b*x)^m*(c + d*x)^n, x_Symbol] \rightarrow \operatorname{Int}[(a*c + b*d*x^2)^m*(f*x)^p, x] /;$ $\operatorname{FreeQ}\{a, b, c, d, f, m, n, p, x\} \ \&\& \ \operatorname{EqQ}[b*c + a*d, 0] \ \&\& \ \operatorname{EqQ}[m - n, 0] \ \&\& \ \operatorname{GtQ}[a, 0] \ \&\& \ \operatorname{GtQ}[c, 0]$

Rule 364

$\operatorname{Int}[(c*x)^m*(a + b*x)^n*(d*x)^p, x_Symbol] \rightarrow \operatorname{Simp}[(a^p*(c*x)^{m+1}*\operatorname{Hypergeometric2F1}[-p, (m+1)/n, (m+1)/n + 1, -(b*x^n)/a])/(c*(m+1)), x] /;$ $\operatorname{FreeQ}\{a, b, c, m, n, p, x\} \ \&\& \ !\operatorname{IGtQ}[p, 0] \ \&\& \ (\operatorname{ILtQ}[p, 0] \ || \ \operatorname{GtQ}[a, 0])$

Rubi steps

$$\begin{aligned} \int (dx)^m (a + b \operatorname{sech}^{-1}(cx)) dx &= \frac{(dx)^{1+m} (a + b \operatorname{sech}^{-1}(cx))}{d(1+m)} + \frac{\left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx}\right) \int \frac{(dx)^m}{\sqrt{1-cx}\sqrt{1+cx}} dx}{1+m} \\ &= \frac{(dx)^{1+m} (a + b \operatorname{sech}^{-1}(cx))}{d(1+m)} + \frac{\left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx}\right) \int \frac{(dx)^m}{\sqrt{1-c^2x^2}} dx}{1+m} \\ &= \frac{(dx)^{1+m} (a + b \operatorname{sech}^{-1}(cx))}{d(1+m)} + \frac{b(dx)^{1+m} \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} {}_2F_1\left(\frac{1}{2}, \frac{1+m}{2}; \frac{3+m}{2}; c^2x^2\right)}{d(1+m)^2} \end{aligned}$$

Mathematica [A] time = 0.147044, size = 97, normalized size = 1.11

$$\frac{x(dx)^m \left((m+1)(cx-1)(a + b \operatorname{sech}^{-1}(cx)) - b \sqrt{\frac{1-cx}{cx+1}} \sqrt{1-c^2x^2} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+1}{2}, \frac{m+3}{2}, c^2x^2\right) \right)}{(m+1)^2(cx-1)}$$

Antiderivative was successfully verified.

[In] Integrate[(d*x)^m*(a + b*ArcSech[c*x]),x]

[Out] (x*(d*x)^m*((1+m)*(-1+c*x)*(a + b*ArcSech[c*x]) - b*Sqrt[(1-c*x)/(1+c*x)]*Sqrt[1-c^2*x^2]*Hypergeometric2F1[1/2, (1+m)/2, (3+m)/2, c^2*x^2]))/((1+m)^2*(-1+c*x))

Maple [F] time = 1.601, size = 0, normalized size = 0.

$$\int (dx)^m (a + b \operatorname{arcsech}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x)^m*(a+b*arcsech(c*x)),x)

[Out] int((d*x)^m*(a+b*arcsech(c*x)),x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m*(a+b*arcsech(c*x)),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left((b \operatorname{ar} \operatorname{sech}(cx) + a)(dx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] integral((b*arcsech(c*x) + a)*(d*x)^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (dx)^m (a + b \operatorname{asech}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*asech(c*x)),x)
```

```
[Out] Integral((d*x)**m*(a + b*asech(c*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arsech}(cx) + a) (dx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*(d*x)^m, x)
```

$$3.72 \quad \int \frac{(dx)^m}{a+b\operatorname{sech}^{-1}(cx)} dx$$

Optimal. Leaf size=18

$$\text{Unintegrable}\left(\frac{(dx)^m}{a+b\operatorname{sech}^{-1}(cx)}, x\right)$$

[Out] Unintegrable[(d*x)^m/(a + b*ArcSech[c*x]), x]

Rubi [A] time = 0.0279905, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(dx)^m}{a+b\operatorname{sech}^{-1}(cx)} dx$$

Verification is Not applicable to the result.

[In] Int[(d*x)^m/(a + b*ArcSech[c*x]), x]

[Out] Defer[Int] [(d*x)^m/(a + b*ArcSech[c*x]), x]

Rubi steps

$$\int \frac{(dx)^m}{a+b\operatorname{sech}^{-1}(cx)} dx = \int \frac{(dx)^m}{a+b\operatorname{sech}^{-1}(cx)} dx$$

Mathematica [A] time = 0.39426, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a+b\operatorname{sech}^{-1}(cx)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(d*x)^m/(a + b*ArcSech[c*x]), x]

[Out] Integrate[(d*x)^m/(a + b*ArcSech[c*x]), x]

Maple [A] time = 1.291, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a+b\operatorname{arcsech}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x)^m/(a+b*arcsech(c*x)), x)

[Out] int((d*x)^m/(a+b*arcsech(c*x)), x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b \operatorname{arsech}(cx) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m/(a+b*arcsech(c*x)),x, algorithm="maxima")

[Out] integrate((d*x)^m/(b*arcsech(c*x) + a), x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(dx)^m}{b \operatorname{arsech}(cx) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m/(a+b*arcsech(c*x)),x, algorithm="fricas")

[Out] integral((d*x)^m/(b*arcsech(c*x) + a), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{a + b \operatorname{asech}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)**m/(a+b*asech(c*x)),x)

[Out] Integral((d*x)**m/(a + b*asech(c*x)), x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{b \operatorname{arsech}(cx) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x)^m/(a+b*arcsech(c*x)),x, algorithm="giac")

[Out] integrate((d*x)^m/(b*arcsech(c*x) + a), x)

$$3.73 \quad \int \frac{(dx)^m}{\left(a + b \operatorname{sech}^{-1}(cx)\right)^2} dx$$

Optimal. Leaf size=18

$$\text{Unintegrable} \left(\frac{(dx)^m}{\left(a + b \operatorname{sech}^{-1}(cx)\right)^2}, x \right)$$

[Out] Unintegrable[(d*x)^m/(a + b*ArcSech[c*x])^2, x]

Rubi [A] time = 0.0273039, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(dx)^m}{\left(a + b \operatorname{sech}^{-1}(cx)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(d*x)^m/(a + b*ArcSech[c*x])^2, x]

[Out] Defer[Int] [(d*x)^m/(a + b*ArcSech[c*x])^2, x]

Rubi steps

$$\int \frac{(dx)^m}{\left(a + b \operatorname{sech}^{-1}(cx)\right)^2} dx = \int \frac{(dx)^m}{\left(a + b \operatorname{sech}^{-1}(cx)\right)^2} dx$$

Mathematica [A] time = 0.805672, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(a + b \operatorname{sech}^{-1}(cx)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(d*x)^m/(a + b*ArcSech[c*x])^2, x]

[Out] Integrate[(d*x)^m/(a + b*ArcSech[c*x])^2, x]

Maple [A] time = 1.267, size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{\left(a + b \operatorname{arcsech}(cx)\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x)^m/(a+b*arcsech(c*x))^2, x)

[Out] $\int ((d*x)^m / (a+b*\operatorname{arcsech}(c*x))^2, x)$

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\frac{(c^2 d^m x^3 - d^m x) \sqrt{cx+1} \sqrt{-cx+1} x^m + (c^2 d^m x^3 - d^m x) x^n}{(b^2 c^2 \log(c) - abc^2) x^2 - b^2 \log(c) - (b^2 \log(c) + b^2 \log(x) - ab) \sqrt{cx+1} \sqrt{-cx+1} + ab - (b^2 c^2 x^2 - \sqrt{cx+1} \sqrt{-cx+1})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{integrate}((d*x)^m / (a+b*\operatorname{arcsech}(c*x))^2, x, \text{algorithm}="maxima")$

[Out] $-\left(\frac{c^2 d^m x^3 - d^m x}{b^2 c^2 \log(c) - a b c^2}\right) \sqrt{c x + 1} \sqrt{-c x + 1} x^m + \frac{c^2 d^m x^3 - d^m x}{b^2 c^2 \log(c) - a b c^2} x^m$
 $-\frac{(b^2 c^2 \log(c) - a b c^2) x^2 - b^2 \log(c) - (b^2 \log(c) + b^2 \log(x) - a b) \sqrt{c x + 1} \sqrt{-c x + 1} + a b - (b^2 c^2 x^2 - \sqrt{c x + 1} \sqrt{-c x + 1})}{(b^2 c^2 \log(c) - a b c^2) x^2 - b^2 \log(c) - (b^2 \log(c) + b^2 \log(x) - a b) \sqrt{c x + 1} \sqrt{-c x + 1} + a b - (b^2 c^2 x^2 - \sqrt{c x + 1} \sqrt{-c x + 1})} + \frac{(b^2 c^2 x^2 - b^2) \log(\sqrt{c x + 1} \sqrt{-c x + 1} + 1) + (b^2 c^2 x^2 - b^2) \log(x)}{(b^2 c^2 \log(c) - a b c^2) x^2 - b^2 \log(c) - (b^2 \log(c) + b^2 \log(x) - a b) \sqrt{c x + 1} \sqrt{-c x + 1} + a b - (b^2 c^2 x^2 - \sqrt{c x + 1} \sqrt{-c x + 1})} + \operatorname{integrate}\left(\frac{(c^2 d^m (m+3) x^2 - d^m (m+1)) (c x + 1) (c x - 1) x^m + (c^4 d^m (m+2) x^4 - c^2 d^m (3m+5) x^2 + 2 d^m (m+1)) \sqrt{c x + 1} \sqrt{-c x + 1} x^m + (c^4 d^m (m+1) x^4 - 2 c^2 d^m (m+1) x^2 + d^m (m+1)) x^m}{(b^2 c^4 \log(c) - a b c^4) x^4 - (b^2 \log(c) + b^2 \log(x) - a b) (c x + 1) (c x - 1) - 2 (b^2 c^2 \log(c) - a b c^2) x^2 + b^2 \log(c) - 2 ((b^2 c^2 \log(c) - a b c^2) x^2 - b^2 \log(c) + a b + (b^2 c^2 x^2 - b^2) \log(x)) \sqrt{c x + 1} \sqrt{-c x + 1} - a b - (b^2 c^4 x^4 - 2 b^2 c^2 x^2 - (c x + 1) (c x - 1) b^2 - 2 (b^2 c^2 x^2 - b^2) \sqrt{c x + 1} \sqrt{-c x + 1} + b^2) \log(\sqrt{c x + 1} \sqrt{-c x + 1} + 1) + (b^2 c^4 x^4 - 2 b^2 c^2 x^2 + b^2) \log(x)}, x\right)$

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(dx)^m}{b^2 \operatorname{arsech}(cx)^2 + 2 ab \operatorname{arsech}(cx) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{integrate}((d*x)^m / (a+b*\operatorname{arcsech}(c*x))^2, x, \text{algorithm}="fricas")$

[Out] $\operatorname{integral}((d*x)^m / (b^2*\operatorname{arcsech}(c*x)^2 + 2*a*b*\operatorname{arcsech}(c*x) + a^2), x)$

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(a + b \operatorname{asech}(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{integrate}((d*x)**m / (a+b*\operatorname{asech}(c*x))**2, x)$

[Out] $\operatorname{Integral}((d*x)**m / (a + b*\operatorname{asech}(c*x))**2, x)$

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx)^m}{(b \operatorname{ar} \operatorname{sech}(cx) + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arcsech(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arcsech(c*x) + a)^2, x)
```


3.74 $\int (d + ex)^3 (a + b \operatorname{sech}^{-1}(cx)) dx$

Optimal. Leaf size=264

$$\frac{(d + ex)^4 (a + b \operatorname{sech}^{-1}(cx))}{4e} - \frac{be \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \sqrt{1-c^2x^2} (9c^2d^2 + e^2)}{6c^4} + \frac{bd \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} (2c^2d^2 + e^2) \sin^{-1}(cx)}{2c^3} - \frac{bd^2 \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \sqrt{1-c^2x^2} \operatorname{sech}^{-1}(cx)}{2c^3}$$

```
[Out] -(b*e*(9*c^2*d^2 + e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2
])/ (6*c^4) - (b*d*e^2*x*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2
])/ (2*c^2) - (b*e^3*x^2*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2
])/ (12*c^2) + ((d + e*x)^4*(a + b*ArcSech[c*x]))/(4*e) + (b*d*(2*c^2*d^2 +
e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x])/ (2*c^3) - (b*d^4*Sqrt[
(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[1 - c^2*x^2]])/(4*e)
```

Rubi [A] time = 0.362204, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {6288, 1809, 844, 216, 266, 63, 208}

$$\frac{(d + ex)^4 (a + b \operatorname{sech}^{-1}(cx))}{4e} - \frac{be \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \sqrt{1-c^2x^2} (9c^2d^2 + e^2)}{6c^4} + \frac{bd \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} (2c^2d^2 + e^2) \sin^{-1}(cx)}{2c^3} - \frac{bd^2 \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \sqrt{1-c^2x^2} \operatorname{sech}^{-1}(cx)}{2c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*ArcSech[c*x]), x]
```

```
[Out] -(b*e*(9*c^2*d^2 + e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2
])/ (6*c^4) - (b*d*e^2*x*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2
])/ (2*c^2) - (b*e^3*x^2*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2
])/ (12*c^2) + ((d + e*x)^4*(a + b*ArcSech[c*x]))/(4*e) + (b*d*(2*c^2*d^2 +
e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x])/ (2*c^3) - (b*d^4*Sqrt[
(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[1 - c^2*x^2]])/(4*e)
```

Rule 6288

```
Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))*((d_.) + (e_.)*(x_.))^(m_.), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(a + b*ArcSech[c*x]))/(e*(m + 1)), x] + Dist[
(b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)])/(e*(m + 1)), Int[(d + e*x)^(m + 1)/(x*S
qrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 1809

```
Int[(Pq_.)*((c_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.)^2)^(p_.), x_Symbol]
:> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && (!IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_.))^(m_.)*((f_.) + (g_.)*(x_.))*((a_.) + (c_.)*(x_.)^2)^(p
_.), x_Symbol]
:> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 216

$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] := \text{Simp}[\text{ArcSin}[(\text{Rt}[-b, 2]*x)/\text{Sqrt}[a]]/\text{Rt}[-b, 2], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{GtQ}[a, 0] \ \&\& \ \text{NegQ}[b]$

Rule 266

$\text{Int}[(x_)^{(m_)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] := \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p}, x], x, x^n], x] /; \text{FreeQ}[\{a, b, m, n, p\}, x] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rule 63

$\text{Int}[(a_ + (b_)*(x_))^{(m_)}*((c_ + (d_)*(x_))^{(n_)}), x_Symbol] := \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n}, x], x, (a + b*x)^{(1/p)}], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 208

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] := \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$

Rubi steps

$$\begin{aligned} \int (d + ex)^3 (a + b \operatorname{sech}^{-1}(cx)) dx &= \frac{(d + ex)^4 (a + b \operatorname{sech}^{-1}(cx))}{4e} + \frac{\left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx}\right) \int \frac{(d+ex)^4}{x \sqrt{1-c^2x^2}} dx}{4e} \\ &= -\frac{be^3x^2 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{12c^2} + \frac{(d + ex)^4 (a + b \operatorname{sech}^{-1}(cx))}{4e} - \frac{\left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx}\right) \int \frac{(d+ex)^4}{x \sqrt{1-c^2x^2}} dx}{4e} \\ &= -\frac{bde^2x \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{2c^2} - \frac{be^3x^2 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{12c^2} + \frac{(d + ex)^4 (a + b \operatorname{sech}^{-1}(cx))}{4e} \\ &= -\frac{be(9c^2d^2 + e^2) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{6c^4} - \frac{bde^2x \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{2c^2} - \frac{be^3x^2 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{12c^2} \\ &= -\frac{be(9c^2d^2 + e^2) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{6c^4} - \frac{bde^2x \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{2c^2} - \frac{be^3x^2 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{12c^2} \\ &= -\frac{be(9c^2d^2 + e^2) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{6c^4} - \frac{bde^2x \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{2c^2} - \frac{be^3x^2 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{12c^2} \\ &= -\frac{be(9c^2d^2 + e^2) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{6c^4} - \frac{bde^2x \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{2c^2} - \frac{be^3x^2 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{12c^2} \\ &= -\frac{be(9c^2d^2 + e^2) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{6c^4} - \frac{bde^2x \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{2c^2} - \frac{be^3x^2 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{12c^2} \end{aligned}$$

Mathematica [C] time = 0.396662, size = 190, normalized size = 0.72

$$\frac{1}{4} \left(6ad^2ex^2 + 4ad^3x + 4ade^2x^3 + ae^3x^4 - \frac{be \sqrt{\frac{1-cx}{cx+1}} (cx+1) (c^2 (18d^2 + 6dex + e^2x^2) + 2e^2)}{3c^4} + \frac{2ibd (2c^2d^2 + e^2) \log \left(2\sqrt{\frac{1-cx}{cx+1}} \right)}{c^3} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)^3*(a + b*ArcSech[c*x]), x]

[Out] $(4*a*d^3*x + 6*a*d^2*e*x^2 + 4*a*d*e^2*x^3 + a*e^3*x^4 - (b*e*\sqrt{(1 - c*x)/(1 + c*x)}*(1 + c*x)*(2*e^2 + c^2*(18*d^2 + 6*d*e*x + e^2*x^2)))/(3*c^4) + b*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3)*\text{ArcSech}[c*x] + ((2*I)*b*d*(2*c^2*d^2 + e^2)*\text{Log}[(-2*I)*c*x + 2*\sqrt{(1 - c*x)/(1 + c*x)}*(1 + c*x)]) / c^3) / 4$

Maple [A] time = 0.196, size = 283, normalized size = 1.1

$$\frac{1}{c} \left(\frac{(cxe + cd)^4 a}{4c^3e} + \frac{b}{c^3} \left(\frac{e^3 \text{arcsech}(cx) c^4 x^4}{4} + e^2 \text{arcsech}(cx) c^4 x^3 d + \frac{3e \text{arcsech}(cx) c^4 x^2 d^2}{2} + \text{arcsech}(cx) c^4 x d^3 + \dots \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^3*(a+b*arcsech(c*x)), x)

[Out] $1/c*(1/4*(c*e*x+c*d)^4*a/c^3/e+b/c^3*(1/4*e^3*\text{arcsech}(c*x)*c^4*x^4+e^2*\text{arcsech}(c*x)*c^4*x^3*d+3/2*e*\text{arcsech}(c*x)*c^4*x^2*d^2+\text{arcsech}(c*x)*c^4*x*d^3+1/4/e*\text{arcsech}(c*x)*c^4*d^4+1/12/e*(-(c*x-1)/c/x)^{(1/2)}*c*x*((c*x+1)/c/x)^{(1/2)}*(-3*c^4*d^4*\text{arctanh}(1/(-c^2*x^2+1)^{(1/2)})+12*c^3*d^3*e*\text{arcsin}(c*x)-c^2*x^2*e^4*(-c^2*x^2+1)^{(1/2)}-6*c^2*d*e^3*x*(-c^2*x^2+1)^{(1/2)}-18*c^2*d^2*e^2*(-c^2*x^2+1)^{(1/2)}+6*c*d*e^3*\text{arcsin}(c*x)-2*e^4*(-c^2*x^2+1)^{(1/2)})/(-c^2*x^2+1)^{(1/2))}$

Maxima [A] time = 1.51568, size = 298, normalized size = 1.13

$$\frac{1}{4} a e^3 x^4 + a d e^2 x^3 + \frac{3}{2} a d^2 e x^2 + \frac{3}{2} \left(x^2 \text{arsech}(cx) - \frac{x \sqrt{\frac{1}{c^2 x^2} - 1}}{c} \right) b d^2 e + \frac{1}{2} \left(2 x^3 \text{arsech}(cx) - \frac{\frac{\sqrt{\frac{1}{c^2 x^2} - 1}}{c^2 \left(\frac{1}{c^2 x^2} - 1 \right) + c^2} + \frac{\text{arctan}\left(\sqrt{\frac{1}{c^2 x^2} - 1}\right)}{c^2}}{c} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^3*(a+b*arcsech(c*x)), x, algorithm="maxima")

[Out] $1/4*a*e^3*x^4 + a*d*e^2*x^3 + 3/2*a*d^2*e*x^2 + 3/2*(x^2*\text{arcsech}(c*x) - x*\sqrt{(1/(c^2*x^2) - 1)/c}*b*d^2*e + 1/2*(2*x^3*\text{arcsech}(c*x) - (\sqrt{(1/(c^2*x^2) - 1)/(c^2*(1/(c^2*x^2) - 1) + c^2)} + \text{arctan}(\sqrt{(1/(c^2*x^2) - 1)/c^2})/c)*b*d*e^2 + 1/12*(3*x^4*\text{arcsech}(c*x) + (c^2*x^3*(1/(c^2*x^2) - 1)^{(3/2)} - 3*x*\sqrt{(1/(c^2*x^2) - 1)/c^3}*b*e^3 + a*d^3*x + (c*x*\text{arcsech}(c*x) - \text{arctan}(\sqrt{(1/(c^2*x^2) - 1)})))*b*d^3/c$

Fricas [B] time = 2.41672, size = 764, normalized size = 2.89

$$3 a c^3 e^3 x^4 + 12 a c^3 d e^2 x^3 + 18 a c^3 d^2 e x^2 + 12 a c^3 d^3 x - 12 (2 b c^2 d^3 + b d e^2) \text{arctan} \left(\frac{c x \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2} - 1}}{c x} \right) - 3 (4 b c^3 d^3 + 6 b c^3 d^2 e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*c^3*e^3*x^4 + 12*a*c^3*d*e^2*x^3 + 18*a*c^3*d^2*e*x^2 + 12*a*c^3*d^3*x - 12*(2*b*c^2*d^3 + b*d*e^2)*arctan((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c*x)) - 3*(4*b*c^3*d^3 + 6*b*c^3*d^2*e + 4*b*c^3*d*e^2 + b*c^3*e^3)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + 3*(b*c^3*e^3*x^4 + 4*b*c^3*d*e^2*x^3 + 6*b*c^3*d^2*e*x^2 + 4*b*c^3*d^3*x - 4*b*c^3*d^3 - 6*b*c^3*d^2*e - 4*b*c^3*d*e^2 - b*c^3*e^3)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (b*c^2*e^3*x^3 + 6*b*c^2*d*e^2*x^2 + 2*(9*b*c^2*d^2*e + b*e^3)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/c^3
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{asech}(cx))(d + ex)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(a+b*asech(c*x)),x)
```

```
[Out] Integral((a + b*asech(c*x))*(d + e*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^3 (b \operatorname{arsech}(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(b*arcsech(c*x) + a), x)
```

3.75 $\int (d + ex)^2 (a + b \operatorname{sech}^{-1}(cx)) dx$

Optimal. Leaf size=201

$$\frac{(d + ex)^3 (a + b \operatorname{sech}^{-1}(cx))}{3e} + \frac{b \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} (6c^2 d^2 + e^2) \sin^{-1}(cx)}{6c^3} - \frac{bd^3 \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \tanh^{-1}(\sqrt{1 - c^2 x^2})}{3e} - \frac{bde}{3e}$$

```
[Out] -((b*d*e*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/c^2) - (b*e^
2*x*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(6*c^2) + ((d + e
*x)^3*(a + b*ArcSech[c*x]))/(3*e) + (b*(6*c^2*d^2 + e^2)*Sqrt[(1 + c*x)^(-1
)])*Sqrt[1 + c*x]*ArcSin[c*x])/(6*c^3) - (b*d^3*Sqrt[(1 + c*x)^(-1)]*Sqrt[1
+ c*x]*ArcTanh[Sqrt[1 - c^2*x^2]])/(3*e)
```

Rubi [A] time = 0.217373, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {6288, 1809, 844, 216, 266, 63, 208}

$$\frac{(d + ex)^3 (a + b \operatorname{sech}^{-1}(cx))}{3e} + \frac{b \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} (6c^2 d^2 + e^2) \sin^{-1}(cx)}{6c^3} - \frac{bd^3 \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \tanh^{-1}(\sqrt{1 - c^2 x^2})}{3e} - \frac{bde}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + b*ArcSech[c*x]), x]
```

```
[Out] -((b*d*e*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/c^2) - (b*e^
2*x*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(6*c^2) + ((d + e
*x)^3*(a + b*ArcSech[c*x]))/(3*e) + (b*(6*c^2*d^2 + e^2)*Sqrt[(1 + c*x)^(-1
)])*Sqrt[1 + c*x]*ArcSin[c*x])/(6*c^3) - (b*d^3*Sqrt[(1 + c*x)^(-1)]*Sqrt[1
+ c*x]*ArcTanh[Sqrt[1 - c^2*x^2]])/(3*e)
```

Rule 6288

```
Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))*((d_.) + (e_.)*(x_.))^(m_.), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(a + b*ArcSech[c*x]))/(e*(m + 1)), x] + Dist[
(b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)])/(e*(m + 1)), Int[(d + e*x)^(m + 1)/(x*S
qrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.)^2)^(p_), x_Symbol]
:> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && (!IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_.))^(m_.)*((f_.) + (g_.)*(x_.))*((a_.) + (c_.)*(x_.)^2)^(p
_.), x_Symbol]
:> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned} \int (d + ex)^2 (a + b \operatorname{sech}^{-1}(cx)) dx &= \frac{(d + ex)^3 (a + b \operatorname{sech}^{-1}(cx))}{3e} + \frac{\left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx}\right) \int \frac{(d+ex)^3}{x \sqrt{1-c^2x^2}} dx}{3e} \\ &= -\frac{be^2x \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{6c^2} + \frac{(d + ex)^3 (a + b \operatorname{sech}^{-1}(cx))}{3e} - \frac{\left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx}\right) \int \frac{(d+ex)^3}{x \sqrt{1-c^2x^2}} dx}{3e} \\ &= -\frac{bde \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{c^2} - \frac{be^2x \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{6c^2} + \frac{(d + ex)^3 (a + b \operatorname{sech}^{-1}(cx))}{3e} \\ &= -\frac{bde \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{c^2} - \frac{be^2x \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{6c^2} + \frac{(d + ex)^3 (a + b \operatorname{sech}^{-1}(cx))}{3e} \\ &= -\frac{bde \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{c^2} - \frac{be^2x \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{6c^2} + \frac{(d + ex)^3 (a + b \operatorname{sech}^{-1}(cx))}{3e} \\ &= -\frac{bde \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{c^2} - \frac{be^2x \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{6c^2} + \frac{(d + ex)^3 (a + b \operatorname{sech}^{-1}(cx))}{3e} \\ &= -\frac{bde \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{c^2} - \frac{be^2x \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{6c^2} + \frac{(d + ex)^3 (a + b \operatorname{sech}^{-1}(cx))}{3e} \end{aligned}$$

Mathematica [C] time = 0.214008, size = 147, normalized size = 0.73

$$\frac{2ac^3x(3d^2 + 3dex + e^2x^2) + 2bc^3x \operatorname{sech}^{-1}(cx)(3d^2 + 3dex + e^2x^2) + ib(6c^2d^2 + e^2) \log\left(2\sqrt{\frac{1-cx}{cx+1}}(cx+1) - 2icx\right) - bce\sqrt{\frac{1-cx}{cx+1}}}{6c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + b*ArcSech[c*x]),x]
```

```
[Out] -(b*c*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(6*d + e*x)) + 2*a*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2) + 2*b*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2)*ArcSech[c*x]
```

$$+ I*b*(6*c^2*d^2 + e^2)*Log[(-2*I)*c*x + 2*sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)]/(6*c^3)$$

Maple [A] time = 0.213, size = 215, normalized size = 1.1

$$\frac{1}{c} \left(\frac{(cxe + cd)^3 a}{3ec^2} + \frac{b}{c^2} \left(\frac{e^2 \operatorname{arcsech}(cx) c^3 x^3}{3} + e \operatorname{arcsech}(cx) c^3 x^2 d + \operatorname{arcsech}(cx) c^3 x d^2 + \frac{\operatorname{arcsech}(cx) c^3 d^3}{3e} + \frac{cx}{6e} \sqrt{-\dots} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)^2*(a+b*arcsech(c*x)),x)

[Out] $\frac{1}{c} \left(\frac{1}{3} (cex + cd)^3 \frac{a}{c^2} + \frac{b}{c^2} \left(\frac{1}{3} e^2 \operatorname{arcsech}(cx) c^3 x^3 + e \operatorname{arcsech}(cx) c^3 x^2 d + \operatorname{arcsech}(cx) c^3 x d^2 + \frac{\operatorname{arcsech}(cx) c^3 d^3}{3e} + \frac{cx}{6e} \sqrt{-\dots} \right) \right)$

Maxima [A] time = 1.50683, size = 205, normalized size = 1.02

$$\frac{1}{3} ae^2 x^3 + adex^2 + \left(x^2 \operatorname{arseth}(cx) - \frac{x \sqrt{\frac{1}{c^2 x^2} - 1}}{c} \right) bde + \frac{1}{6} \left(2x^3 \operatorname{arseth}(cx) - \frac{\frac{\sqrt{\frac{1}{c^2 x^2} - 1}}{c^2} + \frac{\arctan\left(\sqrt{\frac{1}{c^2 x^2} - 1}\right)}{c^2}}{c} \right) be^2 + ad^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2*(a+b*arcsech(c*x)),x, algorithm="maxima")

[Out] $\frac{1}{3} a e^2 x^3 + a d e x^2 + (x^2 \operatorname{arseth}(cx) - x \sqrt{(1/(c^2 x^2) - 1)}/c) * b d e + \frac{1}{6} (2 x^3 \operatorname{arseth}(cx) - (\sqrt{(1/(c^2 x^2) - 1)}/(c^2 (1/(c^2 x^2) - 1) + c^2) + \arctan(\sqrt{(1/(c^2 x^2) - 1)}/c^2)}/c) * b e^2 + a d^2 x + (c x \operatorname{arseth}(cx) - \arctan(\sqrt{(1/(c^2 x^2) - 1)}/c^2)) * b d^2 / c$

Fricas [B] time = 2.12997, size = 601, normalized size = 2.99

$$2ac^3e^2x^3 + 6ac^3dex^2 + 6ac^3d^2x - 2(6bc^2d^2 + be^2) \arctan\left(\frac{cx \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2}}}{cx}\right) - 2(3bc^3d^2 + 3bc^3de + bc^3e^2) \log\left(\frac{cx \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2}}}{cx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2*(a+b*arcsech(c*x)),x, algorithm="fricas")

[Out] $\frac{1}{6} (2 a c^3 e^2 x^3 + 6 a c^3 d e x^2 + 6 a c^3 d^2 x - 2 (6 b c^2 d^2 + b e^2) \arctan((c x \sqrt{-(c^2 x^2 - 1)/(c^2 x^2)} - 1)/(c x)) - 2 (3 b c^3 d^2 + 3 b c^3 d e + b c^3 e^2) \log((c x \sqrt{-(c^2 x^2 - 1)/(c^2 x^2)} - 1)/x) + 2 (b c^3 e^2 x^3 + 3 b c^3 d e x^2 + 3 b c^3 d^2 x - 3 b c^3 d^2 - 3 b c^3 d e - b c^3 e^2) \log((c x \sqrt{-(c^2 x^2 - 1)/(c^2 x^2)} + 1)/(c x)) -$

$$(b*c^2*e^2*x^2 + 6*b*c^2*d*e*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/c^3$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{asech}(cx))(d + ex)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)**2*(a+b*asech(c*x)),x)

[Out] Integral((a + b*asech(c*x))*(d + e*x)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^2(b \operatorname{arsech}(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^2*(a+b*arcsech(c*x)),x, algorithm="giac")

[Out] integrate((e*x + d)^2*(b*arcsech(c*x) + a), x)

3.76 $\int (d + ex) (a + b \operatorname{sech}^{-1}(cx)) dx$

Optimal. Leaf size=142

$$\frac{(d + ex)^2 (a + b \operatorname{sech}^{-1}(cx))}{2e} - \frac{bd^2 \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \tanh^{-1}(\sqrt{1-c^2x^2})}{2e} - \frac{be \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \sqrt{1-c^2x^2}}{2c^2} + \frac{bd \sqrt{\frac{1}{cx+1}} \sqrt{cx+1}}{c}$$

[Out] $-(b * e * \operatorname{Sqrt}[(1 + c * x)^{-1}] * \operatorname{Sqrt}[1 + c * x] * \operatorname{Sqrt}[1 - c^2 * x^2]) / (2 * c^2) + ((d + e * x)^2 * (a + b * \operatorname{ArcSech}[c * x])) / (2 * e) + (b * d * \operatorname{Sqrt}[(1 + c * x)^{-1}] * \operatorname{Sqrt}[1 + c * x] * \operatorname{ArcSin}[c * x]) / c - (b * d^2 * \operatorname{Sqrt}[(1 + c * x)^{-1}] * \operatorname{Sqrt}[1 + c * x] * \operatorname{ArcTanh}[\operatorname{Sqrt}[1 - c^2 * x^2]]) / (2 * e)$

Rubi [A] time = 0.117831, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6288, 1809, 844, 216, 266, 63, 208}

$$\frac{(d + ex)^2 (a + b \operatorname{sech}^{-1}(cx))}{2e} - \frac{bd^2 \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \tanh^{-1}(\sqrt{1-c^2x^2})}{2e} - \frac{be \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \sqrt{1-c^2x^2}}{2c^2} + \frac{bd \sqrt{\frac{1}{cx+1}} \sqrt{cx+1}}{c}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(d + e * x) * (a + b * \operatorname{ArcSech}[c * x]), x]$

[Out] $-(b * e * \operatorname{Sqrt}[(1 + c * x)^{-1}] * \operatorname{Sqrt}[1 + c * x] * \operatorname{Sqrt}[1 - c^2 * x^2]) / (2 * c^2) + ((d + e * x)^2 * (a + b * \operatorname{ArcSech}[c * x])) / (2 * e) + (b * d * \operatorname{Sqrt}[(1 + c * x)^{-1}] * \operatorname{Sqrt}[1 + c * x] * \operatorname{ArcSin}[c * x]) / c - (b * d^2 * \operatorname{Sqrt}[(1 + c * x)^{-1}] * \operatorname{Sqrt}[1 + c * x] * \operatorname{ArcTanh}[\operatorname{Sqrt}[1 - c^2 * x^2]]) / (2 * e)$

Rule 6288

$\operatorname{Int}[(a + \operatorname{ArcSech}[c * x]) * (d + e * x)^m, x] \rightarrow \operatorname{Simp}[(d + e * x)^{m+1} * (a + b * \operatorname{ArcSech}[c * x]) / (e * (m + 1)), x] + \operatorname{Dist}[b * \operatorname{Sqrt}[1 + c * x] * \operatorname{Sqrt}[1 / (1 + c * x)] / (e * (m + 1)), \operatorname{Int}[(d + e * x)^{m+1} / (x * \operatorname{Sqrt}[1 - c^2 * x^2]), x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, e, m\}, x \ \&\& \ \operatorname{NeQ}[m, -1]$

Rule 1809

$\operatorname{Int}[(Pq) * (c * x)^m * (a + b * x^2)^p, x] \rightarrow \operatorname{With}\{q = \operatorname{Expon}[Pq, x], f = \operatorname{Coeff}[Pq, x, \operatorname{Expon}[Pq, x]]\}, \operatorname{Simp}[(f * (c * x)^{m+q-1} * (a + b * x^2)^{p+1}) / (b * c^{q-1} * (m + q + 2 * p + 1)), x] + \operatorname{Dist}[1 / (b * (m + q + 2 * p + 1)), \operatorname{Int}[(c * x)^m * (a + b * x^2)^p * \operatorname{ExpandToSum}[b * (m + q + 2 * p + 1) * Pq - b * f * (m + q + 2 * p + 1) * x^q - a * f * (m + q - 1) * x^{q-2}], x], x] /;$ $\operatorname{GtQ}[q, 1] \ \&\& \ \operatorname{NeQ}[m + q + 2 * p + 1, 0] /;$ $\operatorname{FreeQ}\{a, b, c, m, p\}, x \ \&\& \ \operatorname{PolyQ}[Pq, x] \ \&\& \ (!\operatorname{IGtQ}[m, 0] \ || \ \operatorname{IGtQ}[p + 1/2, -1])$

Rule 844

$\operatorname{Int}[(d + e * x)^m * (f + g * x^2)^p, x] \rightarrow \operatorname{Dist}[g / e, \operatorname{Int}[(d + e * x)^{m+1} * (a + c * x^2)^p, x], x] + \operatorname{Dist}[(e * f - d * g) / e, \operatorname{Int}[(d + e * x)^m * (a + c * x^2)^p, x], x] /;$ $\operatorname{FreeQ}\{a, c, d, e, f, g, m, p\}, x \ \&\& \ \operatorname{NeQ}[c * d^2 + a * e^2, 0] \ \&\& \ !\operatorname{IGtQ}[m, 0]$

Rule 216

`Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]`

Rule 266

`Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]`

Rule 63

`Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

Rule 208

`Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

Rubi steps

$$\begin{aligned}
 \int (d + ex)(a + b \operatorname{sech}^{-1}(cx)) dx &= \frac{(d + ex)^2 (a + b \operatorname{sech}^{-1}(cx))}{2e} + \frac{\left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx}\right) \int \frac{(d+ex)^2}{x \sqrt{1-c^2x^2}} dx}{2e} \\
 &= -\frac{be \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{2c^2} + \frac{(d + ex)^2 (a + b \operatorname{sech}^{-1}(cx))}{2e} - \frac{\left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx}\right) \int \frac{-c^2}{x} dx}{2c^2e} \\
 &= -\frac{be \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{2c^2} + \frac{(d + ex)^2 (a + b \operatorname{sech}^{-1}(cx))}{2e} + \left(bd \sqrt{\frac{1}{1+cx}} \sqrt{1+cx}\right) \int \frac{1}{x} dx \\
 &= -\frac{be \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{2c^2} + \frac{(d + ex)^2 (a + b \operatorname{sech}^{-1}(cx))}{2e} + \frac{bd \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sin^{-1}(cx)}{c} \\
 &= -\frac{be \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{2c^2} + \frac{(d + ex)^2 (a + b \operatorname{sech}^{-1}(cx))}{2e} + \frac{bd \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sin^{-1}(cx)}{c} \\
 &= -\frac{be \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{2c^2} + \frac{(d + ex)^2 (a + b \operatorname{sech}^{-1}(cx))}{2e} + \frac{bd \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sin^{-1}(cx)}{c}
 \end{aligned}$$

Mathematica [A] time = 0.34379, size = 122, normalized size = 0.86

$$adx + \frac{1}{2} aex^2 - \frac{bd \sqrt{\frac{1-cx}{cx+1}} \sqrt{1-c^2x^2} \sin^{-1}(cx)}{c(cx-1)} + be \left(-\frac{1}{2c^2} - \frac{x}{2c} \right) \sqrt{\frac{1-cx}{cx+1}} + bdx \operatorname{sech}^{-1}(cx) + \frac{1}{2} bex^2 \operatorname{sech}^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x)*(a + b*ArcSech[c*x]),x]

[Out] a*d*x + (a*e*x^2)/2 + b*e*(-1/(2*c^2) - x/(2*c))*Sqrt[(1 - c*x)/(1 + c*x)] + b*d*x*ArcSech[c*x] + (b*e*x^2*ArcSech[c*x])/2 - (b*d*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*ArcSin[c*x])/(c*(-1 + c*x))

Maple [A] time = 0.193, size = 125, normalized size = 0.9

$$\frac{1}{c} \left(\frac{a}{c} \left(\frac{c^2 x^2 e}{2} + c^2 dx \right) + \frac{b}{c} \left(\frac{\operatorname{arcsech}(cx) c^2 x^2 e}{2} + \operatorname{arcsech}(cx) c^2 x d + \frac{cx}{2} \sqrt{\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} \left(2cd \arcsin(cx) - e\sqrt{-c^2 x^2} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d)*(a+b*arcsech(c*x)),x)

[Out] 1/c*(a/c*(1/2*c^2*x^2*e+c^2*d*x)+b/c*(1/2*arcsech(c*x)*c^2*x^2*e+arcsech(c*x)*c^2*x*d+1/2*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)*(2*c*d*arcsin(c*x)-e*(-c^2*x^2+1)^(1/2)))/(-c^2*x^2+1)^(1/2))

Maxima [A] time = 0.989862, size = 95, normalized size = 0.67

$$\frac{1}{2} a e x^2 + \frac{1}{2} \left(x^2 \operatorname{arsech}(cx) - \frac{x \sqrt{\frac{1}{c^2 x^2} - 1}}{c} \right) b e + a d x + \frac{\left(c x \operatorname{arsech}(cx) - \arctan\left(\sqrt{\frac{1}{c^2 x^2} - 1}\right) \right) b d}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)*(a+b*arcsech(c*x)),x, algorithm="maxima")

[Out] 1/2*a*e*x^2 + 1/2*(x^2*arcsech(c*x) - x*sqrt(1/(c^2*x^2) - 1)/c)*b*e + a*d*x + (c*x*arcsech(c*x) - arctan(sqrt(1/(c^2*x^2) - 1)))*b*d/c

Fricas [B] time = 1.85227, size = 401, normalized size = 2.82

$$\frac{a c e x^2 + 2 a c d x - b e x \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2}} - 4 b d \arctan\left(\frac{c x \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2}}}{c x}\right) - (2 b c d + b c e) \log\left(\frac{c x \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2}}}{x}\right) + (b c e x^2 + 2 b c d x - 2 b c d)}{2 c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)*(a+b*arcsech(c*x)),x, algorithm="fricas")

[Out] 1/2*(a*c*e*x^2 + 2*a*c*d*x - b*e*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 4*b*d*arctan((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c*x)) - (2*b*c*d + b*c*e)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + (b*c*e*x^2 + 2*b*c*d*x - 2*b*c*d - b*c*e)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/c

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{asech}(cx))(d + ex) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)*(a+b*asech(c*x)),x)

[Out] Integral((a + b*asech(c*x))*(d + e*x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex + d)(b \operatorname{arsech}(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)*(a+b*arcsech(c*x)),x, algorithm="giac")

[Out] integrate((e*x + d)*(b*arcsech(c*x) + a), x)

3.77 $\int (a + b \operatorname{sech}^{-1}(cx)) dx$

Optimal. Leaf size=40

$$ax + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sin^{-1}(cx)}{c} + bx\operatorname{sech}^{-1}(cx)$$

[Out] a*x + b*x*ArcSech[c*x] + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x])/c

Rubi [A] time = 0.0151617, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {6277, 216}

$$ax + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sin^{-1}(cx)}{c} + bx\operatorname{sech}^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Int[a + b*ArcSech[c*x], x]

[Out] a*x + b*x*ArcSech[c*x] + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x])/c

Rule 6277

Int[ArcSech[(c_.)*(x_)], x_Symbol] := Simp[x*ArcSech[c*x], x] + Dist[Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[1/Sqrt[1 - c^2*x^2], x], x] /; FreeQ[c, x]

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rubi steps

$$\begin{aligned} \int (a + b \operatorname{sech}^{-1}(cx)) dx &= ax + b \int \operatorname{sech}^{-1}(cx) dx \\ &= ax + bx \operatorname{sech}^{-1}(cx) + \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{1}{\sqrt{1-c^2x^2}} dx \\ &= ax + bx \operatorname{sech}^{-1}(cx) + \frac{b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sin^{-1}(cx)}{c} \end{aligned}$$

Mathematica [A] time = 0.0874106, size = 60, normalized size = 1.5

$$ax - \frac{b\sqrt{\frac{1-cx}{cx+1}}\sqrt{1-c^2x^2}\sin^{-1}(cx)}{c(cx-1)} + bx\operatorname{sech}^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[a + b*ArcSech[c*x], x]

[Out] $a*x + b*x*\text{ArcSech}[c*x] - (b*\text{Sqrt}[(1 - c*x)/(1 + c*x)]*\text{Sqrt}[1 - c^2*x^2]*\text{ArcSin}[c*x])/(c*(-1 + c*x))$

Maple [A] time = 0.165, size = 42, normalized size = 1.1

$$ax + bx\text{arcsech}(cx) - \frac{b}{c} \arctan\left(\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(a+b*arcsech(c*x),x)`

[Out] $a*x + b*x*\text{arcsech}(c*x) - b/c*\arctan((-1 + 1/c/x)^{(1/2)}*(1 + 1/c/x)^{(1/2)})$

Maxima [A] time = 0.970528, size = 42, normalized size = 1.05

$$ax + \frac{\left(cx \operatorname{ar} \operatorname{sech}(cx) - \arctan\left(\sqrt{\frac{1}{c^2 x^2} - 1}\right)\right) b}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a+b*arcsech(c*x),x, algorithm="maxima")`

[Out] $a*x + (c*x*\text{arcsech}(c*x) - \arctan(\text{sqrt}(1/(c^2*x^2) - 1)))*b/c$

Fricas [B] time = 1.69404, size = 262, normalized size = 6.55

$$\frac{acx - bc \log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{x}\right) - 2b \arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right) + (bcx - bc) \log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a+b*arcsech(c*x),x, algorithm="fricas")`

[Out] $(a*c*x - b*c*\log((c*x*\text{sqrt}(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) - 2*b*\arctan((c*x*\text{sqrt}(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c*x)) + (b*c*x - b*c)*\log((c*x*\text{sqrt}(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/c$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{asech}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a+b*asech(c*x),x)`

[Out] Integral(a + b*asech(c*x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int b \operatorname{arsech}(cx) + a dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b*arcsech(c*x),x, algorithm="giac")

[Out] integrate(b*arcsech(c*x) + a, x)

$$3.78 \quad \int \frac{a+b\operatorname{sech}^{-1}(cx)}{d+ex} dx$$

Optimal. Leaf size=229

$$\frac{b\operatorname{PolyLog}\left(2, -\frac{(e-\sqrt{e^2-c^2d^2})e^{-\operatorname{sech}^{-1}(cx)}}{cd}\right)}{e} - \frac{b\operatorname{PolyLog}\left(2, -\frac{(\sqrt{e^2-c^2d^2}+e)e^{-\operatorname{sech}^{-1}(cx)}}{cd}\right)}{e} + \frac{b\operatorname{PolyLog}\left(2, -e^{-2\operatorname{sech}^{-1}(cx)}\right)}{2e} + \frac{(a+bs)}{e}$$

[Out] -(((a + b*ArcSech[c*x])*Log[1 + E^(-2*ArcSech[c*x])])/e) + ((a + b*ArcSech[c*x])*Log[1 + (e - Sqrt[-(c^2*d^2) + e^2])/(c*d*E^ArcSech[c*x])])/e + ((a + b*ArcSech[c*x])*Log[1 + (e + Sqrt[-(c^2*d^2) + e^2])/(c*d*E^ArcSech[c*x])])/e + (b*PolyLog[2, -E^(-2*ArcSech[c*x])]/(2*e) - (b*PolyLog[2, -((e - Sqrt[-(c^2*d^2) + e^2])/(c*d*E^ArcSech[c*x])]))/e - (b*PolyLog[2, -((e + Sqrt[-(c^2*d^2) + e^2])/(c*d*E^ArcSech[c*x])]))/e

Rubi [A] time = 0.933496, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6287, 2518}

$$\frac{b\operatorname{PolyLog}\left(2, -\frac{(e-\sqrt{e^2-c^2d^2})e^{-\operatorname{sech}^{-1}(cx)}}{cd}\right)}{e} - \frac{b\operatorname{PolyLog}\left(2, -\frac{(\sqrt{e^2-c^2d^2}+e)e^{-\operatorname{sech}^{-1}(cx)}}{cd}\right)}{e} + \frac{b\operatorname{PolyLog}\left(2, -e^{-2\operatorname{sech}^{-1}(cx)}\right)}{2e} + \frac{(a+bs)}{e}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcSech[c*x])/(d + e*x), x]

[Out] -(((a + b*ArcSech[c*x])*Log[1 + E^(-2*ArcSech[c*x])])/e) + ((a + b*ArcSech[c*x])*Log[1 + (e - Sqrt[-(c^2*d^2) + e^2])/(c*d*E^ArcSech[c*x])])/e + ((a + b*ArcSech[c*x])*Log[1 + (e + Sqrt[-(c^2*d^2) + e^2])/(c*d*E^ArcSech[c*x])])/e + (b*PolyLog[2, -E^(-2*ArcSech[c*x])]/(2*e) - (b*PolyLog[2, -((e - Sqrt[-(c^2*d^2) + e^2])/(c*d*E^ArcSech[c*x])]))/e - (b*PolyLog[2, -((e + Sqrt[-(c^2*d^2) + e^2])/(c*d*E^ArcSech[c*x])]))/e

Rule 6287

Int[((a_.) + ArcSech[(c_.)*(x_)])*(b_.))/((d_.) + (e_.)*(x_)), x_Symbol] :> Simp[((a + b*ArcSech[c*x])*Log[1 + (e - Sqrt[-(c^2*d^2) + e^2])/(c*d*E^ArcSech[c*x])])/e, x] + (Dist[b/e, Int[(Sqrt[(1 - c*x)/(1 + c*x)]*Log[1 + (e - Sqrt[-(c^2*d^2) + e^2])/(c*d*E^ArcSech[c*x])])/(x*(1 - c*x)), x], x] + Dist[b/e, Int[(Sqrt[(1 - c*x)/(1 + c*x)]*Log[1 + (e + Sqrt[-(c^2*d^2) + e^2])/(c*d*E^ArcSech[c*x])])/(x*(1 - c*x)), x], x] - Dist[b/e, Int[(Sqrt[(1 - c*x)/(1 + c*x)]*Log[1 + 1/E^(2*ArcSech[c*x])])/(x*(1 - c*x)), x], x] + Simp[((a + b*ArcSech[c*x])*Log[1 + (e + Sqrt[-(c^2*d^2) + e^2])/(c*d*E^ArcSech[c*x])])/e, x] - Simp[((a + b*ArcSech[c*x])*Log[1 + 1/E^(2*ArcSech[c*x])])/e, x] /; FreeQ[{a, b, c, d, e}, x]

Rule 2518

Int[Log[v_]*(u_), x_Symbol] :> With[{w = DerivativeDivides[v, u*(1 - v), x]}, Simp[w*PolyLog[2, 1 - v], x] /; !FalseQ[w]]

Rubi steps

$$\int \frac{a + b \operatorname{sech}^{-1}(cx)}{d + ex} dx = -\frac{(a + b \operatorname{sech}^{-1}(cx)) \log\left(1 + e^{-2 \operatorname{sech}^{-1}(cx)}\right)}{e} + \frac{(a + b \operatorname{sech}^{-1}(cx)) \log\left(1 + \frac{(e - \sqrt{-c^2 d^2 + e^2}) e^{-\operatorname{sech}^{-1}(cx)}}{cd}\right)}{e}$$

$$= -\frac{(a + b \operatorname{sech}^{-1}(cx)) \log\left(1 + e^{-2 \operatorname{sech}^{-1}(cx)}\right)}{e} + \frac{(a + b \operatorname{sech}^{-1}(cx)) \log\left(1 + \frac{(e - \sqrt{-c^2 d^2 + e^2}) e^{-\operatorname{sech}^{-1}(cx)}}{cd}\right)}{e}$$

Mathematica [C] time = 0.522587, size = 393, normalized size = 1.72

$$\frac{a \log(d + ex)}{e} + \frac{b \left(\operatorname{PolyLog}\left(2, -e^{-2 \operatorname{sech}^{-1}(cx)}\right) - 2 \operatorname{PolyLog}\left(2, \frac{(\sqrt{e^2 - c^2 d^2} - e) e^{-\operatorname{sech}^{-1}(cx)}}{cd}\right) + \operatorname{PolyLog}\left(2, -\frac{(\sqrt{e^2 - c^2 d^2} + e) e^{-\operatorname{sech}^{-1}(cx)}}{cd}\right) \right)}{e}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*ArcSech[c*x])/(d + e*x), x]

[Out] (a*Log[d + e*x])/e + (b*(PolyLog[2, -E^(-2*ArcSech[c*x])] - 2*((-4*I)*ArcSin[Sqrt[1 + e/(c*d)]]/Sqrt[2]]*ArcTanh[((-c*d) + e)*Tanh[ArcSech[c*x]/2]]/Sqrt[-(c^2*d^2) + e^2] + ArcSech[c*x]*Log[1 + E^(-2*ArcSech[c*x])] - ArcSech[c*x]*Log[1 + (e - Sqrt[-(c^2*d^2) + e^2])/(c*d*E^ArcSech[c*x])] + (2*I)*ArcSin[Sqrt[1 + e/(c*d)]]/Sqrt[2]]*Log[1 + (e - Sqrt[-(c^2*d^2) + e^2])/(c*d*E^ArcSech[c*x])] - ArcSech[c*x]*Log[1 + (e + Sqrt[-(c^2*d^2) + e^2])/(c*d*E^ArcSech[c*x])] - (2*I)*ArcSin[Sqrt[1 + e/(c*d)]]/Sqrt[2]]*Log[1 + (e + Sqrt[-(c^2*d^2) + e^2])/(c*d*E^ArcSech[c*x])] + PolyLog[2, (-e + Sqrt[-(c^2*d^2) + e^2])/(c*d*E^ArcSech[c*x])] + PolyLog[2, -((e + Sqrt[-(c^2*d^2) + e^2])/(c*d*E^ArcSech[c*x]))])))/(2*e)

Maple [C] time = 0.271, size = 514, normalized size = 2.2

$$\frac{a \ln(cxe + cd)}{e} + \frac{b \operatorname{arcsech}(cx)}{e} \ln\left(\left(-cd\left(\frac{1}{cx} + \sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}\right) + \sqrt{-c^2 d^2 + e^2} - e\right)\left(-e + \sqrt{-c^2 d^2 + e^2}\right)^{-1}\right) + \frac{ba}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))/(e*x+d), x)

[Out] a*ln(c*e*x+c*d)/e+b/e*arcsech(c*x)*ln((-c*d*(1/c/x+(-1+1/c/x)^(1/2))*(1+1/c/x)^(1/2))+(-c^2*d^2+e^2)^(1/2)-e)/(-e+(-c^2*d^2+e^2)^(1/2))+b/e*arcsech(c*x)*ln((c*d*(1/c/x+(-1+1/c/x)^(1/2))*(1+1/c/x)^(1/2))+(-c^2*d^2+e^2)^(1/2)+e)/(e+(-c^2*d^2+e^2)^(1/2))+b/e*dilog((c*d*(1/c/x+(-1+1/c/x)^(1/2))*(1+1/c/x)^(1/2))+(-c^2*d^2+e^2)^(1/2)+e)/(e+(-c^2*d^2+e^2)^(1/2))+b/e*dilog((-c*d*(1/c/x+(-1+1/c/x)^(1/2))*(1+1/c/x)^(1/2))+(-c^2*d^2+e^2)^(1/2)-e)/(-e+(-c^2*d^2+e^2)^(1/2))-b/e*arcsech(c*x)*ln(1+I*(1/c/x+(-1+1/c/x)^(1/2))*(1+1/c/x)^(1/2))-b/e*arcsech(c*x)*ln(1-I*(1/c/x+(-1+1/c/x)^(1/2))*(1+1/c/x)^(1/2))-b/e*dilog(1+I*(1/c/x+(-1+1/c/x)^(1/2))*(1+1/c/x)^(1/2))-b/e*dilog(1-I*(1/c/x+(-1+1/c/x)^(1/2))*(1+1/c/x)^(1/2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b \int \frac{\log\left(\sqrt{\frac{1}{cx} + 1}\sqrt{\frac{1}{cx} - 1} + \frac{1}{cx}\right)}{ex + d} dx + \frac{a \log(ex + d)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/(e*x+d),x, algorithm="maxima")

[Out] b*integrate(log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e*x + d), x) + a*log(e*x + d)/e

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b \operatorname{arsech}(cx) + a}{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/(e*x+d),x, algorithm="fricas")

[Out] integral((b*arcsech(c*x) + a)/(e*x + d), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{asech}(cx)}{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asech(c*x))/(e*x+d),x)

[Out] Integral((a + b*asech(c*x))/(d + e*x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsech}(cx) + a}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/(e*x+d),x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)/(e*x + d), x)

$$3.79 \quad \int \frac{a+b\operatorname{sech}^{-1}(cx)}{(d+ex)^2} dx$$

Optimal. Leaf size=147

$$-\frac{a+b\operatorname{sech}^{-1}(cx)}{e(d+ex)} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tan^{-1}\left(\frac{c^2dx+e}{\sqrt{1-c^2x^2}\sqrt{c^2d^2-e^2}}\right)}{d\sqrt{c^2d^2-e^2}} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{de}$$

[Out] $-\left(\frac{a+b\operatorname{ArcSech}[c*x]}{e*(d+e*x)}\right) + \left(\frac{b*\operatorname{Sqrt}[(1+c*x)^{-1}]*\operatorname{Sqrt}[1+c*x]*\operatorname{ArcTan}[(e+c^2*d*x)/(\operatorname{Sqrt}[c^2*d^2-e^2]*\operatorname{Sqrt}[1-c^2*x^2])]}{d*\operatorname{Sqrt}[c^2*d^2-e^2]} + \frac{b*\operatorname{Sqrt}[(1+c*x)^{-1}]*\operatorname{Sqrt}[1+c*x]*\operatorname{ArcTanh}[\operatorname{Sqrt}[1-c^2*x^2]]}{d*e}\right)$

Rubi [A] time = 0.119007, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {6288, 961, 266, 63, 208, 725, 204}

$$-\frac{a+b\operatorname{sech}^{-1}(cx)}{e(d+ex)} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tan^{-1}\left(\frac{c^2dx+e}{\sqrt{1-c^2x^2}\sqrt{c^2d^2-e^2}}\right)}{d\sqrt{c^2d^2-e^2}} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{de}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a+b*\operatorname{ArcSech}[c*x])/(d+e*x)^2,x]$

[Out] $-\left(\frac{a+b\operatorname{ArcSech}[c*x]}{e*(d+e*x)}\right) + \left(\frac{b*\operatorname{Sqrt}[(1+c*x)^{-1}]*\operatorname{Sqrt}[1+c*x]*\operatorname{ArcTan}[(e+c^2*d*x)/(\operatorname{Sqrt}[c^2*d^2-e^2]*\operatorname{Sqrt}[1-c^2*x^2])]}{d*\operatorname{Sqrt}[c^2*d^2-e^2]} + \frac{b*\operatorname{Sqrt}[(1+c*x)^{-1}]*\operatorname{Sqrt}[1+c*x]*\operatorname{ArcTanh}[\operatorname{Sqrt}[1-c^2*x^2]]}{d*e}\right)$

Rule 6288

$\operatorname{Int}[(a + \operatorname{ArcSech}[c*x])*(d + e*x)^m, x] \rightarrow \operatorname{Simp}[(d + e*x)^{m+1}*(a + b*\operatorname{ArcSech}[c*x])/(e*(m+1)), x] + \operatorname{Dist}[b*\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[1/(1 + c*x)]/(e*(m+1)), \operatorname{Int}[(d + e*x)^{m+1}/(x*\operatorname{Sqrt}[1 - c^2*x^2]), x], x] /;$ FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]

Rule 961

$\operatorname{Int}[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /;$ FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) && !(IGtQ[m, 0] || IGtQ[n, 0])

Rule 266

$\operatorname{Int}[x^m*(a + b*x^n)^p, x] \rightarrow \operatorname{Dist}[1/n, \operatorname{Subst}[\operatorname{Int}[x^{(Simplify[(m+1)/n]-1)}*(a + b*x)^p, x], x, x^n], x] /;$ FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m+1)/n]]

Rule 63

$\operatorname{Int}[(a + b*x)^m*(c + d*x)^n, x] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{p*(m+1)-1}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x] /;$ FreeQ[{a, b, c, d}, x] && NeQ

$[b*c - a*d, 0] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 208

$\text{Int}[(a_) + (b_.)*(x_)^2)^{-1}, x_Symbol] \ :> \ \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] \ /; \ \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$

Rule 725

$\text{Int}[1/(((d_) + (e_.)*(x_))*\text{Sqrt}[(a_) + (c_.)*(x_)^2]), x_Symbol] \ :> \ -\text{Subst}[\text{Int}[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/\text{Sqrt}[a + c*x^2]] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x]$

Rule 204

$\text{Int}[(a_) + (b_.)*(x_)^2)^{-1}, x_Symbol] \ :> \ -\text{Simp}[\text{ArcTan}[(\text{Rt}[-b, 2]*x)/\text{Rt}[-a, 2]]/(\text{Rt}[-a, 2]*\text{Rt}[-b, 2]), x] \ /; \ \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned} \int \frac{a + b \operatorname{sech}^{-1}(cx)}{(d + ex)^2} dx &= -\frac{a + b \operatorname{sech}^{-1}(cx)}{e(d + ex)} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{1}{x(d+ex)\sqrt{1-c^2x^2}} dx}{e} \\ &= -\frac{a + b \operatorname{sech}^{-1}(cx)}{e(d + ex)} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \left(\frac{1}{dx\sqrt{1-c^2x^2}} - \frac{e}{d(d+ex)\sqrt{1-c^2x^2}}\right) dx}{e} \\ &= -\frac{a + b \operatorname{sech}^{-1}(cx)}{e(d + ex)} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{1}{(d+ex)\sqrt{1-c^2x^2}} dx}{d} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{1}{x\sqrt{1-c^2x^2}} dx}{de} \\ &= -\frac{a + b \operatorname{sech}^{-1}(cx)}{e(d + ex)} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \operatorname{Subst}\left(\int \frac{1}{-c^2d^2+e^2-x^2} dx, x, \frac{e+c^2dx}{\sqrt{1-c^2x^2}}\right)}{d} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{1}{x\sqrt{1-c^2x^2}} dx}{c^2de} \\ &= -\frac{a + b \operatorname{sech}^{-1}(cx)}{e(d + ex)} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx} \tan^{-1}\left(\frac{e+c^2dx}{\sqrt{c^2d^2-e^2}\sqrt{1-c^2x^2}}\right)}{d\sqrt{c^2d^2-e^2}} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-c^2x^2}} dx, x, \frac{e+c^2dx}{\sqrt{1-c^2x^2}}\right)}{c^2de} \\ &= -\frac{a + b \operatorname{sech}^{-1}(cx)}{e(d + ex)} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx} \tan^{-1}\left(\frac{e+c^2dx}{\sqrt{c^2d^2-e^2}\sqrt{1-c^2x^2}}\right)}{d\sqrt{c^2d^2-e^2}} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx} \tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{de} \end{aligned}$$

Mathematica [A] time = 0.225787, size = 222, normalized size = 1.51

$$-\frac{a}{e(d+ex)} + \frac{b \log(d+ex)}{d\sqrt{e^2-c^2d^2}} - \frac{b \log\left(cx\sqrt{\frac{1-cx}{cx+1}}\sqrt{e^2-c^2d^2} + \sqrt{\frac{1-cx}{cx+1}}\sqrt{e^2-c^2d^2} + c^2dx + e\right)}{d\sqrt{e^2-c^2d^2}} + \frac{b \log\left(cx\sqrt{\frac{1-cx}{cx+1}} + \sqrt{\frac{1-cx}{cx+1}} + 1\right)}{de}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcSech[c*x])/(d + e*x)^2,x]

[Out] $-(a/(e*(d + e*x))) - (b*ArcSech[c*x])/(e*(d + e*x)) - (b*Log[x])/(d*e) + (b*Log[d + e*x])/(d*Sqrt[-(c^2*d^2) + e^2]) + (b*Log[1 + Sqrt[(1 - c*x)/(1 + c*x)]] + c*x*Sqrt[(1 - c*x)/(1 + c*x)])/(d*e) - (b*Log[e + c^2*d*x + Sqrt[-(c^2*d^2) + e^2]]*Sqrt[(1 - c*x)/(1 + c*x)] + c*Sqrt[-(c^2*d^2) + e^2]*x*Sqr$

$t[(1 - cx)/(1 + cx)]/(d\sqrt{-(c^2d^2) + e^2})$

Maple [A] time = 0.29, size = 231, normalized size = 1.6

$$-\frac{ac}{(cxe + cd)e} - \frac{b\operatorname{arcsech}(cx)}{(cxe + cd)e} + \frac{xbc}{de} \sqrt{\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} \operatorname{Artanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right) \frac{1}{\sqrt{-c^2x^2+1}} - \frac{xbc}{de} \sqrt{\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))/(e*x+d)^2,x)

[Out] $-c*a/(c*e*x+c*d)/e - c*b/(c*e*x+c*d)/e * \operatorname{arcsech}(c*x) + c*b/e * (-c*x-1)/c/x^{1/2} * x * ((c*x+1)/c/x)^{1/2} / d / (-c^2*x^2+1)^{1/2} * \operatorname{arctanh}(1/(-c^2*x^2+1)^{1/2}) - c*b/e * (-c*x-1)/c/x^{1/2} * x * ((c*x+1)/c/x)^{1/2} / (-c^2*d^2-e^2)/e^2)^{1/2} / d / (-c^2*x^2+1)^{1/2} * \ln(2 * ((-c^2*d^2-e^2)/e^2)^{1/2} * (-c^2*x^2+1)^{1/2} * e + c^2*d*x + e) / (c*e*x+c*d)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\left(c^2 \int \frac{x^2}{c^2 d^2 x^2 + (c^2 d^2 x^2 - d^2 + (c^2 d e x^2 - d e) x) \sqrt{c x + 1} \sqrt{-c x + 1} - d^2 + (c^2 d e x^2 - d e) x} dx + \frac{x \log(\sqrt{c x + 1} \sqrt{-c x + 1})}{d e x} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/(e*x+d)^2,x, algorithm="maxima")

[Out] $(c^2 * \operatorname{integrate}(x^2 / (c^2 * d^2 * x^2 + (c^2 * d^2 * x^2 - d^2 + (c^2 * d * e * x^2 - d * e) * x) * \sqrt{c * x + 1} * \sqrt{-c * x + 1} - d^2 + (c^2 * d * e * x^2 - d * e) * x), x) + (x * \log(\sqrt{c * x + 1} * \sqrt{-c * x + 1}) + 1) - x * \log(c) - x * \log(x)) / (d * e * x + d^2) - \operatorname{integrate}(1 / (c^2 * d^2 * x^2 - d^2 + (c^2 * d * e * x^2 - d * e) * x), x) * b - a / (e^2 * x + d * e)$

Fricas [B] time = 1.8242, size = 1162, normalized size = 7.9

$$\frac{ac^2d^3 - ade^2 + \sqrt{-c^2d^2 + e^2}(be^2x + bde) \log\left(\frac{c^2dex - (c^3d^2 - ce^2)x \sqrt{\frac{c^2x^2-1}{c^2x^2}} + e^2 - \sqrt{-c^2d^2 + e^2}(c^2dx + cex \sqrt{\frac{c^2x^2-1}{c^2x^2}} + e)}{ex+d}\right) + (bc^2d^3 - bde^2)}{c^2d^4e - d^2e^3 + (c^2d^3e^2 - de^4)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/(e*x+d)^2,x, algorithm="fricas")

[Out] $[-(a*c^2*d^3 - a*d*e^2 + \sqrt{-c^2*d^2 + e^2} * (b*e^2*x + b*d*e) * \log((c^2*d*e*x - (c^3*d^2 - c*e^2) * x * \sqrt{-(c^2*x^2 - 1)/(c^2*x^2)}) + e^2 - \sqrt{-c^2*d^2 + e^2} * (c^2*d*x + c*e*x * \sqrt{-(c^2*x^2 - 1)/(c^2*x^2)}) + e)) / (e*x + d) + (b*c^2*d^3 - b*d*e^2 + (b*c^2*d^2*e - b*e^3) * x) * \log((c*x * \sqrt{-(c^2*x^2 - 1)/(c^2*x^2)}) - 1) / x + (b*c^2*d^3 - b*d*e^2) * \log((c*x * \sqrt{-(c^2*x^2 - 1)/(c^2*x^2)}) - 1) / x]$

```
)/(c^2*x^2) + 1)/(c*x)))/(c^2*d^4*e - d^2*e^3 + (c^2*d^3*e^2 - d*e^4)*x),
-(a*c^2*d^3 - a*d*e^2 - 2*sqrt(c^2*d^2 - e^2)*(b*e^2*x + b*d*e)*arctan(-(sqrt(c^2*d^2 - e^2)*c*d*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - sqrt(c^2*d^2 - e^2)*(e*x + d))/((c^2*d^2 - e^2)*x)) + (b*c^2*d^3 - b*d*e^2 + (b*c^2*d^2*e - b*e^3)*x)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + (b*c^2*d^3 - b*d*e^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/(c^2*d^4*e - d^2*e^3 + (c^2*d^3*e^2 - d*e^4)*x]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{asech}(cx)}{(d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))/(e*x+d)**2,x)
```

```
[Out] Integral((a + b*asech(c*x))/(d + e*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsech}(cx) + a}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/(e*x + d)^2, x)
```

$$3.80 \quad \int \frac{a+b\operatorname{sech}^{-1}(cx)}{(d+ex)^3} dx$$

Optimal. Leaf size=306

$$\frac{a+b\operatorname{sech}^{-1}(cx)}{2e(d+ex)^2} + \frac{be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{2d(c^2d^2-e^2)(d+ex)} + \frac{bc^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tan^{-1}\left(\frac{c^2dx+e}{\sqrt{1-c^2x^2}\sqrt{c^2d^2-e^2}}\right)}{2(c^2d^2-e^2)^{3/2}} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tan^{-1}}{2d^2\sqrt{c^2d^2}}$$

[Out] (b*e*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(2*d*(c^2*d^2 - e^2)*(d + e*x)) - (a + b*ArcSech[c*x])/(2*e*(d + e*x)^2) + (b*c^2*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTan[(e + c^2*d*x)/(Sqrt[c^2*d^2 - e^2]*Sqrt[1 - c^2*x^2])])/(2*(c^2*d^2 - e^2)^(3/2)) + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTan[(e + c^2*d*x)/(Sqrt[c^2*d^2 - e^2]*Sqrt[1 - c^2*x^2])])/(2*d^2*Sqrt[c^2*d^2 - e^2]) + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[1 - c^2*x^2]])/(2*d^2*e)

Rubi [A] time = 0.193247, antiderivative size = 306, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {6288, 961, 266, 63, 208, 731, 725, 204}

$$\frac{a+b\operatorname{sech}^{-1}(cx)}{2e(d+ex)^2} + \frac{be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{2d(c^2d^2-e^2)(d+ex)} + \frac{bc^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tan^{-1}\left(\frac{c^2dx+e}{\sqrt{1-c^2x^2}\sqrt{c^2d^2-e^2}}\right)}{2(c^2d^2-e^2)^{3/2}} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tan^{-1}}{2d^2\sqrt{c^2d^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcSech[c*x])/(d + e*x)^3, x]

[Out] (b*e*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(2*d*(c^2*d^2 - e^2)*(d + e*x)) - (a + b*ArcSech[c*x])/(2*e*(d + e*x)^2) + (b*c^2*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTan[(e + c^2*d*x)/(Sqrt[c^2*d^2 - e^2]*Sqrt[1 - c^2*x^2])])/(2*(c^2*d^2 - e^2)^(3/2)) + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTan[(e + c^2*d*x)/(Sqrt[c^2*d^2 - e^2]*Sqrt[1 - c^2*x^2])])/(2*d^2*Sqrt[c^2*d^2 - e^2]) + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[1 - c^2*x^2]])/(2*d^2*e)

Rule 6288

Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))*((d_.) + (e_.)*(x_.))^(m_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcSech[c*x]))/(e*(m + 1)), x] + Dist[(b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)])/(e*(m + 1)), Int[(d + e*x)^(m + 1)/(x*Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]

Rule 961

Int[((d_.) + (e_.)*(x_.))^(m_.)*((f_.) + (g_.)*(x_.))^(n_.)*((a_.) + (c_.)*(x_.)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) && !(IGtQ[m, 0] || IGtQ[n, 0])

Rule 266

Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b

, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 731

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]

Rule 725

Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{a + b \operatorname{sech}^{-1}(cx)}{(d + ex)^3} dx &= -\frac{a + b \operatorname{sech}^{-1}(cx)}{2e(d + ex)^2} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{1}{x(d+ex)^2\sqrt{1-c^2x^2}} dx}{2e} \\
 &= -\frac{a + b \operatorname{sech}^{-1}(cx)}{2e(d + ex)^2} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \left(\frac{1}{d^2x\sqrt{1-c^2x^2}} - \frac{e}{d(d+ex)^2\sqrt{1-c^2x^2}} - \frac{e}{d^2(d+ex)\sqrt{1-c^2x^2}}\right) dx}{2e} \\
 &= -\frac{a + b \operatorname{sech}^{-1}(cx)}{2e(d + ex)^2} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{1}{(d+ex)\sqrt{1-c^2x^2}} dx}{2d^2} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{1}{(d+ex)^2\sqrt{1-c^2x^2}} dx}{2d} \\
 &= \frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{2d(c^2d^2 - e^2)(d + ex)} - \frac{a + b \operatorname{sech}^{-1}(cx)}{2e(d + ex)^2} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \operatorname{Subst}\left(\int \frac{1}{-c^2d^2 + e^2 - x^2} dx, x, \frac{e + c^2dx}{\sqrt{c^2d^2 - e^2}\sqrt{1-c^2x^2}}\right)}{2d^2} \\
 &= \frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{2d(c^2d^2 - e^2)(d + ex)} - \frac{a + b \operatorname{sech}^{-1}(cx)}{2e(d + ex)^2} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx} \tan^{-1}\left(\frac{e + c^2dx}{\sqrt{c^2d^2 - e^2}\sqrt{1-c^2x^2}}\right)}{2d^2\sqrt{c^2d^2 - e^2}} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \operatorname{Subst}\left(\int \frac{1}{-c^2d^2 + e^2 - x^2} dx, x, \frac{e + c^2dx}{\sqrt{c^2d^2 - e^2}\sqrt{1-c^2x^2}}\right)}{2d} \\
 &= \frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{2d(c^2d^2 - e^2)(d + ex)} - \frac{a + b \operatorname{sech}^{-1}(cx)}{2e(d + ex)^2} + \frac{bc^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx} \tan^{-1}\left(\frac{e + c^2dx}{\sqrt{c^2d^2 - e^2}\sqrt{1-c^2x^2}}\right)}{2(c^2d^2 - e^2)^{3/2}} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}}{2d}
 \end{aligned}$$

Mathematica [C] time = 0.629015, size = 342, normalized size = 1.12

$$\frac{1}{2} \left[\frac{a}{e(d+ex)^2} - \frac{ib(2c^2d^2 - e^2) \log \left(\frac{4d^2e\sqrt{c^2d^2 - e^2} \left(cx\sqrt{\frac{1-cx}{cx+1}}\sqrt{c^2d^2 - e^2} + \sqrt{\frac{1-cx}{cx+1}}\sqrt{c^2d^2 - e^2} + ic^2dx + ie \right)}{b(2c^2d^2 - e^2)(d+ex)} \right)}{d^2(cd - e)(cd + e)\sqrt{c^2d^2 - e^2}} \right] + \frac{b \log \left(cx\sqrt{\frac{1-cx}{cx+1}} + \sqrt{\frac{1-cx}{cx+1}} \right)}{d^2e}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcSech[c*x])/(d + e*x)^3, x]

[Out]
$$\begin{aligned} & -(a/(e*(d + e*x)^2)) + (b*\text{Sqrt}[(1 - c*x)/(1 + c*x)]*(e + c*e*x))/(d*(c*d - \\ & e)*(c*d + e)*(d + e*x)) - (b*\text{ArcSech}[c*x])/(e*(d + e*x)^2) - (b*\text{Log}[x])/(d \\ & ^2*e) + (b*\text{Log}[1 + \text{Sqrt}[(1 - c*x)/(1 + c*x)] + c*x*\text{Sqrt}[(1 - c*x)/(1 + c*x) \\ &]])/(d^2*e) - (I*b*(2*c^2*d^2 - e^2)*\text{Log}[(4*d^2*e*\text{Sqrt}[c^2*d^2 - e^2]*(I*e \\ & + I*c^2*d*x + \text{Sqrt}[c^2*d^2 - e^2]*\text{Sqrt}[(1 - c*x)/(1 + c*x)] + c*\text{Sqrt}[c^2*d^2 \\ & ^2 - e^2]*x*\text{Sqrt}[(1 - c*x)/(1 + c*x)])])/(b*(2*c^2*d^2 - e^2)*(d + e*x)))/(d \\ & ^2*(c*d - e)*(c*d + e)*\text{Sqrt}[c^2*d^2 - e^2])/2 \end{aligned}$$

Maple [B] time = 0.274, size = 1090, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))/(e*x+d)^3, x)

[Out]
$$\begin{aligned} & -1/2*c^2*a/(c*e*x+c*d)^2/e - 1/2*c^2*b/(c*e*x+c*d)^2/e*\text{arcsech}(c*x) + 1/2*c^4*b \\ & *(-c*x-1)/c/x)^{(1/2)}*x^2*((c*x+1)/c/x)^{(1/2)}/(-c^2*x^2+1)^{(1/2)}/(c*d+e)/(c \\ & *d-e)/(c*e*x+c*d)*\text{arctanh}(1/(-c^2*x^2+1)^{(1/2)}) + 1/2*c^4*b/e*(-c*x-1)/c/x)^{(1/2)} \\ & *x*((c*x+1)/c/x)^{(1/2)}/(-c^2*x^2+1)^{(1/2)}*d/(c*d+e)/(c*d-e)/(c*e*x+c*d) \\ &)*\text{arctanh}(1/(-c^2*x^2+1)^{(1/2)}) - c^4*b*(-c*x-1)/c/x)^{(1/2)}*x^2*((c*x+1)/c/x \\ &)^{(1/2)}/(-c^2*x^2+1)^{(1/2)}/(c*d+e)/(c*d-e)/(-c^2*d^2-e^2)/e^2)^{(1/2)}/(c*e \\ & x+c*d)*\ln(2*((-c^2*d^2-e^2)/e^2)^{(1/2)}*(-c^2*x^2+1)^{(1/2)}*e+c^2*d*x+e)/(c* \\ & e*x+c*d)) - c^4*b/e*(-c*x-1)/c/x)^{(1/2)}*x*((c*x+1)/c/x)^{(1/2)}/(-c^2*x^2+1)^{(1/2)} \\ & *d/(c*d+e)/(c*d-e)/(-c^2*d^2-e^2)/e^2)^{(1/2)}/(c*e*x+c*d)*\ln(2*((-c^2* \\ & d^2-e^2)/e^2)^{(1/2)}*(-c^2*x^2+1)^{(1/2)}*e+c^2*d*x+e)/(c*e*x+c*d)) - 1/2*c^2*b* \\ & e^2*(-c*x-1)/c/x)^{(1/2)}*x^2*((c*x+1)/c/x)^{(1/2)}/(-c^2*x^2+1)^{(1/2)}/d^2/(c* \\ & d+e)/(c*d-e)/(c*e*x+c*d)*\text{arctanh}(1/(-c^2*x^2+1)^{(1/2)}) - 1/2*c^2*b*e*(-c*x-1) \\ &)/c/x)^{(1/2)}*x*((c*x+1)/c/x)^{(1/2)}/(-c^2*x^2+1)^{(1/2)}/d/(c*d+e)/(c*d-e)/(c* \\ & e*x+c*d)*\text{arctanh}(1/(-c^2*x^2+1)^{(1/2)}) + 1/2*c^2*b*e*(-c*x-1)/c/x)^{(1/2)}*x* \\ & ((c*x+1)/c/x)^{(1/2)}/d/(c*d+e)/(c*d-e)/(c*e*x+c*d) + 1/2*c^2*b*e^2*(-c*x-1)/c/ \\ & x)^{(1/2)}*x^2*((c*x+1)/c/x)^{(1/2)}/(-c^2*x^2+1)^{(1/2)}/d^2/(c*d+e)/(c*d-e)/(- \\ & c^2*d^2-e^2)/e^2)^{(1/2)}/(c*e*x+c*d)*\ln(2*((-c^2*d^2-e^2)/e^2)^{(1/2)}*(-c^2* \\ & x^2+1)^{(1/2)}*e+c^2*d*x+e)/(c*e*x+c*d)) + 1/2*c^2*b*e*(-c*x-1)/c/x)^{(1/2)}*x* \\ & ((c*x+1)/c/x)^{(1/2)}/(-c^2*x^2+1)^{(1/2)}/d/(c*d+e)/(c*d-e)/(-c^2*d^2-e^2)/e^2 \\ &)^{(1/2)}/(c*e*x+c*d)*\ln(2*((-c^2*d^2-e^2)/e^2)^{(1/2)}*(-c^2*x^2+1)^{(1/2)}*e+c \\ & ^2*d*x+e)/(c*e*x+c*d)) \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/(e*x+d)^3,x, algorithm="maxima")

[Out] Exception raised: RuntimeError

Fricas [B] time = 2.68898, size = 2423, normalized size = 7.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/(e*x+d)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/2*(a*c^4*d^6 - (2*a + b)*c^2*d^4*e^2 + (a + b)*d^2*e^4 - (b*c^2*d^2*e^4 - b*e^6)*x^2 + (2*b*c^2*d^4*e - b*d^2*e^3 + (2*b*c^2*d^2*e^3 - b*e^5)*x^2 \\ & + 2*(2*b*c^2*d^3*e^2 - b*d*e^4)*x)*\sqrt{-c^2*d^2 + e^2}*\log((c^2*d*e*x - (c^3*d^2 - c*e^2)*x*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)} + e^2 - \sqrt{-c^2*d^2 + e^2} \\ & 2)*(c^2*d*x + c*e*x*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)} + e))/(e*x + d) - 2*(b*c^2*d^3*e^3 - b*d*e^5)*x + (b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^2*e^4 + (b*c^4*d^4*e^2 - 2*b*c^2*d^2*e^4 + b*e^6)*x^2 + 2*(b*c^4*d^5*e - 2*b*c^2*d^3*e^3 + b*d*e^5)*x)*\log((c*x*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)} - 1)/x) + (b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^2*e^4)*\log((c*x*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)} + 1)/(c*x)) - ((b*c^3*d^3*e^3 - b*c*d*e^5)*x^2 + (b*c^3*d^4*e^2 - b*c*d^2*e^4)*x)*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)))/(c^4*d^8*e - 2*c^2*d^6*e^3 + d^4*e^5 + (c^4*d^6*e^3 - 2*c^2*d^4*e^5 + d^2*e^7)*x^2 + 2*(c^4*d^7*e^2 - 2*c^2*d^5*e^4 + d^3*e^6)*x), -1/2*(a*c^4*d^6 - (2*a + b)*c^2*d^4*e^2 + (a + b)*d^2*e^4 - (b*c^2*d^2*e^4 - b*e^6)*x^2 - 2*(2*b*c^2*d^4*e - b*d^2*e^3 + (2*b*c^2*d^2*e^3 - b*e^5)*x^2 + 2*(2*b*c^2*d^3*e^2 - b*d*e^4)*x)*\sqrt{c^2*d^2 - e^2}*\arctan(-(\sqrt{c^2*d^2 - e^2}*c*d*x*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)} - \sqrt{c^2*d^2 - e^2})*(e*x + d))/((c^2*d^2 - e^2)*x)) - 2*(b*c^2*d^3*e^3 - b*d*e^5)*x + (b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^2*e^4 + (b*c^4*d^4*e^2 - 2*b*c^2*d^2*e^4 + b*e^6)*x^2 + 2*(b*c^4*d^5*e - 2*b*c^2*d^3*e^3 + b*d*e^5)*x)*\log((c*x*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)} - 1)/x) + (b*c^4*d^6 - 2*b*c^2*d^4*e^2 + b*d^2*e^4)*\log((c*x*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)} + 1)/(c*x)) - ((b*c^3*d^3*e^3 - b*c*d*e^5)*x^2 + (b*c^3*d^4*e^2 - b*c*d^2*e^4)*x)*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)))/(c^4*d^8*e - 2*c^2*d^6*e^3 + d^4*e^5 + (c^4*d^6*e^3 - 2*c^2*d^4*e^5 + d^2*e^7)*x^2 + 2*(c^4*d^7*e^2 - 2*c^2*d^5*e^4 + d^3*e^6)*x)] \end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{asech}(cx)}{(d + ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asech(c*x))/(e*x+d)**3,x)

[Out] Integral((a + b*asech(c*x))/(d + e*x)**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsech}(cx) + a}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/(e*x + d)^3, x)
```

3.81 $\int (d + ex)^{3/2} (a + b \operatorname{sech}^{-1}(cx)) dx$

Optimal. Leaf size=343

$$\frac{4b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(2c^2d^2 + e^2)\sqrt{\frac{c(d+ex)}{cd+e}}\operatorname{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right), \frac{2e}{cd+e}\right)}{15c^3\sqrt{d+ex}} + \frac{2(d+ex)^{5/2}(a + b\operatorname{sech}^{-1}(cx))}{5e} - \frac{4be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-cx}}{15c^2}$$

[Out] $(-4*b*e*\sqrt{(1 + c*x)^{-1}}*\sqrt{1 + c*x}*\sqrt{d + e*x}*\sqrt{1 - c^2*x^2})/(15*c^2) + (2*(d + e*x)^{(5/2)}*(a + b*\operatorname{ArcSech}[c*x]))/(5*e) - (28*b*d*\sqrt{(1 + c*x)^{-1}}*\sqrt{1 + c*x}*\sqrt{d + e*x}*\operatorname{EllipticE}[\operatorname{ArcSin}[\sqrt{1 - c*x}/\sqrt{2}], (2*e)/(c*d + e)])/(15*c*\sqrt{(c*(d + e*x))/(c*d + e)}) - (4*b*(2*c^2*d^2 + e^2)*\sqrt{(1 + c*x)^{-1}}*\sqrt{1 + c*x}*\sqrt{(c*(d + e*x))/(c*d + e)}*\operatorname{EllipticF}[\operatorname{ArcSin}[\sqrt{1 - c*x}/\sqrt{2}], (2*e)/(c*d + e)])/(15*c^3*\sqrt{d + e*x}) - (4*b*d^3*\sqrt{(1 + c*x)^{-1}}*\sqrt{1 + c*x}*\sqrt{(c*(d + e*x))/(c*d + e)}*\operatorname{EllipticPi}[2, \operatorname{ArcSin}[\sqrt{1 - c*x}/\sqrt{2}], (2*e)/(c*d + e)])/(5*e*\sqrt{d + e*x})$

Rubi [A] time = 0.621292, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6288, 958, 719, 419, 932, 168, 538, 537, 844, 424, 931, 1584}

$$\frac{2(d+ex)^{5/2}(a + b\operatorname{sech}^{-1}(cx))}{5e} - \frac{4b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(2c^2d^2 + e^2)\sqrt{\frac{c(d+ex)}{cd+e}}F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right) \middle| \frac{2e}{cd+e}\right)}{15c^3\sqrt{d+ex}} - \frac{4be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-cx}}{15c^2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(d + e*x)^{(3/2)}*(a + b*\operatorname{ArcSech}[c*x]), x]$

[Out] $(-4*b*e*\sqrt{(1 + c*x)^{-1}}*\sqrt{1 + c*x}*\sqrt{d + e*x}*\sqrt{1 - c^2*x^2})/(15*c^2) + (2*(d + e*x)^{(5/2)}*(a + b*\operatorname{ArcSech}[c*x]))/(5*e) - (28*b*d*\sqrt{(1 + c*x)^{-1}}*\sqrt{1 + c*x}*\sqrt{d + e*x}*\operatorname{EllipticE}[\operatorname{ArcSin}[\sqrt{1 - c*x}/\sqrt{2}], (2*e)/(c*d + e)])/(15*c*\sqrt{(c*(d + e*x))/(c*d + e)}) - (4*b*(2*c^2*d^2 + e^2)*\sqrt{(1 + c*x)^{-1}}*\sqrt{1 + c*x}*\sqrt{(c*(d + e*x))/(c*d + e)}*\operatorname{EllipticF}[\operatorname{ArcSin}[\sqrt{1 - c*x}/\sqrt{2}], (2*e)/(c*d + e)])/(15*c^3*\sqrt{d + e*x}) - (4*b*d^3*\sqrt{(1 + c*x)^{-1}}*\sqrt{1 + c*x}*\sqrt{(c*(d + e*x))/(c*d + e)}*\operatorname{EllipticPi}[2, \operatorname{ArcSin}[\sqrt{1 - c*x}/\sqrt{2}], (2*e)/(c*d + e)])/(5*e*\sqrt{d + e*x})$

Rule 6288

$\operatorname{Int}[(a + \operatorname{ArcSech}[c*x])*(b*x)^m, x] \rightarrow \operatorname{Simp}[(d + e*x)^{m+1}*(a + b*\operatorname{ArcSech}[c*x])/(e*(m+1)), x] + \operatorname{Dist}[b*\sqrt{1 + c*x}*\sqrt{1/(1 + c*x)}]/(e*(m+1)), \operatorname{Int}[(d + e*x)^{m+1}/(x*\sqrt{1 - c^2*x^2}), x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, m\}, x \ \&\& \operatorname{NeQ}[m, -1]$

Rule 958

$\operatorname{Int}[(f + g*x)^n/((d + e*x)*\sqrt{a + c*x^2}), x] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[1/(\sqrt{f + g*x}*\sqrt{a + c*x^2}), (f + g*x)^{n+1/2}/(d + e*x), x], x] /; \operatorname{FreeQ}\{a, c, d, e, f, g\}, x \ \&\& \operatorname{NeQ}[e*f - d*g, 0] \ \&\& \operatorname{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \operatorname{IntegerQ}[n + 1/2]$

Rule 719

Int[((d_) + (e_)*(x_)^m)/Sqrt[(a_) + (c_)*(x_)^2], x_Symbol] := Dist[(2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]

Rule 419

Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := Simp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])

Rule 932

Int[1/(((d_) + (e_)*(x_))*Sqrt[(f_) + (g_)*(x_)]*Sqrt[(a_) + (c_)*(x_)^2]), x_Symbol] := With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]

Rule 168

Int[1/(((a_) + (b_)*(x_))*Sqrt[(c_) + (d_)*(x_)]*Sqrt[(e_) + (f_)*(x_)]*Sqrt[(g_) + (h_)*(x_)]), x_Symbol] := Dist[-2, Subst[Int[1/(Simp[b*c - a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g - c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]

Rule 538

Int[1/(((a_) + (b_)*(x_)^2)*Sqrt[(c_) + (d_)*(x_)^2]*Sqrt[(e_) + (f_)*(x_)^2]), x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a + b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[c, 0]

Rule 537

Int[1/(((a_) + (b_)*(x_)^2)*Sqrt[(c_) + (d_)*(x_)^2]*Sqrt[(e_) + (f_)*(x_)^2]), x_Symbol] := Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !(!GtQ[f/e, 0] && SimplerSqrtQ[-(f/e), -(d/c)])

Rule 844

Int[((d_) + (e_)*(x_)^m)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^p, x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]

Rule 424

Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] := Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rule 931

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (c_.)*(x_)^2]), x_Symbol] := Simp[(2*e^2*(d + e*x)^(m - 2)*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(c*g*(2*m - 1)), x] - Dist[1/(c*g*(2*m - 1)), Int[((d + e*x)^(m - 3))*Simp[a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^3*g*(2*m - 1) + e*(e*(a*e*g*(2*m - 3)) + c*d*(2*e*f - 3*d*g*(2*m - 1)))*x + 2*e^2*(c*e*f - 3*c*d*g)*(m - 1)*x^2, x)]/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GeQ[m, 2]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned} \int (d + ex)^{3/2} (a + b \operatorname{sech}^{-1}(cx)) dx &= \frac{2(d + ex)^{5/2} (a + b \operatorname{sech}^{-1}(cx))}{5e} + \frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{(d+ex)^{5/2}}{x\sqrt{1-c^2x^2}} dx}{5e} \\ &= \frac{2(d + ex)^{5/2} (a + b \operatorname{sech}^{-1}(cx))}{5e} + \frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \left(\frac{3d^2e}{\sqrt{d+ex}\sqrt{1-c^2x^2}} + \frac{d^3}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}\right) dx}{5e} \\ &= \frac{2(d + ex)^{5/2} (a + b \operatorname{sech}^{-1}(cx))}{5e} + \frac{1}{5} \left(6bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}} dx \\ &= -\frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}\sqrt{1-c^2x^2}}{15c^2} + \frac{2(d + ex)^{5/2} (a + b \operatorname{sech}^{-1}(cx))}{5e} + \frac{1}{5} \left(6bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}} dx \\ &= -\frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}\sqrt{1-c^2x^2}}{15c^2} + \frac{2(d + ex)^{5/2} (a + b \operatorname{sech}^{-1}(cx))}{5e} - \frac{12bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}}{5e} \int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}} dx \\ &= -\frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}\sqrt{1-c^2x^2}}{15c^2} + \frac{2(d + ex)^{5/2} (a + b \operatorname{sech}^{-1}(cx))}{5e} - \frac{12bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}}{5e} \int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}} dx \\ &= -\frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}\sqrt{1-c^2x^2}}{15c^2} + \frac{2(d + ex)^{5/2} (a + b \operatorname{sech}^{-1}(cx))}{5e} - \frac{12bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}}{5e} \int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}} dx \\ &= -\frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}\sqrt{1-c^2x^2}}{15c^2} + \frac{2(d + ex)^{5/2} (a + b \operatorname{sech}^{-1}(cx))}{5e} - \frac{28bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}}{5e} \int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}} dx \end{aligned}$$

Mathematica [C] time = 9.87643, size = 2653, normalized size = 7.73

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(d + e*x)^(3/2)*(a + b*ArcSech[c*x]),x]

[Out] Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[d + e*x]*((-4*b*e)/(15*c^2) - (4*b*e*x)/(15*c)) + Sqrt[d + e*x]*((2*a*d^2)/(5*e) + (4*a*d*x)/5 + (2*a*e*x^2)/5) + (2*b*(d + e*x)^(5/2)*ArcSech[c*x])/(5*e) - (4*b*(7*c*d*e*Sqrt[(1 - c*x)/(1 + c*x)])*(e - (e*(1 - c*x))/(1 + c*x) + c*d*(1 + (1 - c*x)/(1 + c*x))) + ((7*I)*c^2*d^2*e*(c*d + e)*Sqrt[1 + (1 - c*x)/(1 + c*x)]*Sqrt[(e - (e*(1 - c*x))/(1 + c*x) + c*d*(1 + (1 - c*x)/(1 + c*x)))]/(c*d + e)]*(EllipticE[I*ArcSinh[Sqrt[(1 - c*x)/(1 + c*x)]], (c*d - e)/(c*d + e)] - EllipticF[I*ArcSinh[Sqrt[(1 - c*x)/(1 + c*x)]], (c*d - e)/(c*d + e)]))/((c*d - e) - ((7*I)*c*d*e^2*(c*d + e)*Sqrt[1 + (1 - c*x)/(1 + c*x)]*Sqrt[(e - (e*(1 - c*x))/(1 + c*x) + c*d*(1 + (1 - c*x)/(1 + c*x)))]/(c*d + e)]*(EllipticE[I*ArcSinh[Sqrt[(1 - c*x)/(1 + c*x)]], (c*d - e)/(c*d + e)] - EllipticF[I*ArcSinh[Sqrt[(1 - c*x)/(1 + c*x)]], (c*d - e)/(c*d + e)]))/((c*d - e) + (3*I)*c^3*d^3*Sqrt[1 + (1 - c*x)/(1 + c*x)]*Sqrt[(e - (e*(1 - c*x))/(1 + c*x) + c*d*(1 + (1 - c*x)/(1 + c*x)))]/(c*d + e)]*EllipticF[I*ArcSinh[Sqrt[(1 - c*x)/(1 + c*x)]], (c*d - e)/(c*d + e)] - (2*I)*c^2*d^2*e*Sqrt[1 + (1 - c*x)/(1 + c*x)]*Sqrt[(e - (e*(1 - c*x))/(1 + c*x) + c*d*(1 + (1 - c*x)/(1 + c*x)))]/(c*d + e)]*EllipticF[I*ArcSinh[Sqrt[(1 - c*x)/(1 + c*x)]], (c*d - e)/(c*d + e)] - I*e^3*Sqrt[1 + (1 - c*x)/(1 + c*x)]*Sqrt[(e - (e*(1 - c*x))/(1 + c*x) + c*d*(1 + (1 - c*x)/(1 + c*x)))]/(c*d + e)]*EllipticF[I*ArcSinh[Sqrt[(1 - c*x)/(1 + c*x)]], (c*d - e)/(c*d + e)] + ((3 + 3*I)*c^3*d^3*(-I + Sqrt[(1 - c*x)/(1 + c*x)])*(I + Sqrt[(1 - c*x)/(1 + c*x)])*Sqrt[((-I)*(Sqrt[-(c*d) - e]*Sqrt[c*d - e] + c*d*Sqrt[(1 - c*x)/(1 + c*x)] - e*Sqrt[(1 - c*x)/(1 + c*x)])]/(((-I)*c*d + Sqrt[-(c*d) - e]*Sqrt[c*d - e] + I*e)*(-I + Sqrt[(1 - c*x)/(1 + c*x)])))*Sqrt[((-I)*(Sqrt[-(c*d) - e]*Sqrt[c*d - e] - c*d*Sqrt[(1 - c*x)/(1 + c*x)] + e*Sqrt[(1 - c*x)/(1 + c*x)])]/((I*c*d + Sqrt[-(c*d) - e]*Sqrt[c*d - e] - I*e)*(-I + Sqrt[(1 - c*x)/(1 + c*x)])))*EllipticF[ArcSin[Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)])]/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]]], (Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])^2/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])^2] - (1 - I)*EllipticPi[(I*Sqrt[-(c*d) - e] - Sqrt[c*d - e])/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])], ArcSin[Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)])]/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]]], (Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])^2/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])^2))/Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)])]/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)])))] + ((3 + 3*I)*c^3*d^3*(1 + I*Sqrt[(1 - c*x)/(1 + c*x)])*(I + Sqrt[(1 - c*x)/(1 + c*x)])*Sqrt[((-I)*(Sqrt[-(c*d) - e]*Sqrt[c*d - e] + c*d*Sqrt[(1 - c*x)/(1 + c*x)] - e*Sqrt[(1 - c*x)/(1 + c*x)])]/(((-I)*c*d + Sqrt[-(c*d) - e]*Sqrt[c*d - e] + I*e)*(-I + Sqrt[(1 - c*x)/(1 + c*x)])))*Sqrt[((-I)*(Sqrt[-(c*d) - e]*Sqrt[c*d - e] - c*d*Sqrt[(1 - c*x)/(1 + c*x)] + e*Sqrt[(1 - c*x)/(1 + c*x)])]/((I*c*d + Sqrt[-(c*d) - e]*Sqrt[c*d - e] - I*e)*(-I + Sqrt[(1 - c*x)/(1 + c*x)])))*EllipticF[ArcSin[Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)])]/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]]], (Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])^2/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])^2] - (1 + I)*EllipticPi[((-I)*Sqrt[-(c*d) - e] + Sqrt[c*d - e])/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])], ArcSin[Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)])]/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]]], (Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])^2/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])^2))/Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)])]/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)])))]/(15*c^3*e*(1 + (1 - c*x)/(1 + c*x))*Sqrt[(c*d + e + (c*d*(1 - c*x))/(1 + c*x) - (e*(1 - c*x))/(1 + c*x))/(c + (c*(1 - c*x))/(1 + c*x)))]

Maple [B] time = 0.395, size = 830, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((e*x+d)^{(3/2)}*(a+b*\text{arcsech}(c*x)),x)$

[Out] $2/e*(1/5*(e*x+d)^{(5/2)}*a+b*(1/5*(e*x+d)^{(5/2)}*\text{arcsech}(c*x)-2/15/c*e^2*(-((e*x+d)*c-c*d-e)/c/x/e)^{(1/2)}*x*((e*x+d)*c-c*d+e)/c/x/e)^{(1/2)}*((c/(c*d+e))^{(1/2)}*(e*x+d)^{(5/2)}*c^2+9*(-((e*x+d)*c-c*d-e)/(c*d+e))^{(1/2)}*(-((e*x+d)*c-c*d+e)/(c*d-e))^{(1/2)}*\text{EllipticF}((e*x+d)^{(1/2)}*(c/(c*d+e))^{(1/2)},((c*d+e)/(c*d-e))^{(1/2)})*c^2*d^2-7*(-((e*x+d)*c-c*d-e)/(c*d+e))^{(1/2)}*(-((e*x+d)*c-c*d+e)/(c*d-e))^{(1/2)}*\text{EllipticE}((e*x+d)^{(1/2)}*(c/(c*d+e))^{(1/2)},((c*d+e)/(c*d-e))^{(1/2)})*c^2*d^2-3*(-((e*x+d)*c-c*d-e)/(c*d+e))^{(1/2)}*(-((e*x+d)*c-c*d+e)/(c*d-e))^{(1/2)}*\text{EllipticPi}((e*x+d)^{(1/2)}*(c/(c*d+e))^{(1/2)},1/c*(c*d+e)/d,(c/(c*d-e))^{(1/2)}/(c/(c*d+e))^{(1/2)})*c^2*d^2-2*(c/(c*d+e))^{(1/2)}*(e*x+d)^{(3/2)}*c^2*d-7*(-((e*x+d)*c-c*d-e)/(c*d+e))^{(1/2)}*(-((e*x+d)*c-c*d+e)/(c*d-e))^{(1/2)}*\text{EllipticF}((e*x+d)^{(1/2)}*(c/(c*d+e))^{(1/2)},((c*d+e)/(c*d-e))^{(1/2)})*c*d*e+7*(-((e*x+d)*c-c*d-e)/(c*d+e))^{(1/2)}*(-((e*x+d)*c-c*d+e)/(c*d-e))^{(1/2)}*\text{EllipticE}((e*x+d)^{(1/2)}*(c/(c*d+e))^{(1/2)},((c*d+e)/(c*d-e))^{(1/2)})*c*d*e+(c/(c*d+e))^{(1/2)}*(e*x+d)^{(1/2)}*c^2*d^2+(-((e*x+d)*c-c*d-e)/(c*d+e))^{(1/2)}*(-((e*x+d)*c-c*d+e)/(c*d-e))^{(1/2)}*\text{EllipticF}((e*x+d)^{(1/2)}*(c/(c*d+e))^{(1/2)},((c*d+e)/(c*d-e))^{(1/2)})*e^2-(c/(c*d+e))^{(1/2)}*(e*x+d)^{(1/2)}*e^2)/(c/(c*d+e))^{(1/2)}/((e*x+d)^2*c^2-2*(e*x+d)*c^2*d+c^2*d^2-e^2))$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((e*x+d)^{(3/2)}*(a+b*\text{arcsech}(c*x)),x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((e*x+d)^{(3/2)}*(a+b*\text{arcsech}(c*x)),x, \text{algorithm}="fricas")$

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)**(3/2)*(a+b*asech(c*x)),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^{\frac{3}{2}}(b \operatorname{arsech}(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="giac")

[Out] integrate((e*x + d)^(3/2)*(b*arcsech(c*x) + a), x)

3.82 $\int \sqrt{d+ex} \left(a + b \operatorname{sech}^{-1}(cx) \right) dx$

Optimal. Leaf size=279

$$\frac{4bd\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{c(d+ex)}{cd+e}}\operatorname{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{3c\sqrt{d+ex}} + \frac{2(d+ex)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{3e} - \frac{4bd^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{c(d+ex)}{cd+e}}}{3e\sqrt{d+ex}}$$

```
[Out] (2*(d + e*x)^(3/2)*(a + b*ArcSech[c*x]))/(3*e) - (4*b*Sqrt[(1 + c*x)^(-1)]*
Sqrt[1 + c*x]*Sqrt[d + e*x]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/
(c*d + e)])/(3*c*Sqrt[(c*(d + e*x))/(c*d + e)]) - (4*b*d*Sqrt[(1 + c*x)^(-1)]*
Sqrt[1 + c*x]*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticF[ArcSin[Sqrt[1 - c*
x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c*Sqrt[d + e*x]) - (4*b*d^2*Sqrt[(1 + c*x)
]^(-1)]*Sqrt[1 + c*x]*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticPi[2, ArcSin[Sq
rt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*e*Sqrt[d + e*x])
```

Rubi [A] time = 0.372203, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6288, 958, 719, 419, 932, 168, 538, 537, 844, 424}

$$\frac{2(d+ex)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{3e} - \frac{4bd^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{c(d+ex)}{cd+e}}\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\middle|\frac{2e}{cd+e}\right)}{3e\sqrt{d+ex}} - \frac{4bd\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{c(d+ex)}{cd+e}}F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\middle|\frac{2e}{cd+e}\right)}{3c\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*(a + b*ArcSech[c*x]),x]
```

```
[Out] (2*(d + e*x)^(3/2)*(a + b*ArcSech[c*x]))/(3*e) - (4*b*Sqrt[(1 + c*x)^(-1)]*
Sqrt[1 + c*x]*Sqrt[d + e*x]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/
(c*d + e)])/(3*c*Sqrt[(c*(d + e*x))/(c*d + e)]) - (4*b*d*Sqrt[(1 + c*x)^(-1)]*
Sqrt[1 + c*x]*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticF[ArcSin[Sqrt[1 - c*
x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*c*Sqrt[d + e*x]) - (4*b*d^2*Sqrt[(1 + c*x)
]^(-1)]*Sqrt[1 + c*x]*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticPi[2, ArcSin[Sq
rt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*e*Sqrt[d + e*x])
```

Rule 6288

```
Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))*((d_.) + (e_.)*(x_.))^(m_.), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(a + b*ArcSech[c*x]))/(e*(m + 1)), x] + Dist[
(b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)])/(e*(m + 1)), Int[(d + e*x)^(m + 1)/(x*S
qrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 958

```
Int[((f_.) + (g_.)*(x_.))^(n_/)((d_.) + (e_.)*(x_.))*Sqrt[(a_.) + (c_.)*(x_.)^
2]), x_Symbol]
:> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
+ g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 719

```
Int[((d_.) + (e_.)*(x_.))^(m_)/Sqrt[(a_.) + (c_.)*(x_.)^2], x_Symbol]
:> Dist[(2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
```

a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]

Rule 419

Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := Simp[1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])

Rule 932

Int[1/(((d_) + (e_)*(x_))*Sqrt[(f_) + (g_)*(x_)]*Sqrt[(a_) + (c_)*(x_)^2]), x_Symbol] := With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]

Rule 168

Int[1/(((a_) + (b_)*(x_))*Sqrt[(c_) + (d_)*(x_)]*Sqrt[(e_) + (f_)*(x_)]*Sqrt[(g_) + (h_)*(x_)]), x_Symbol] := Dist[-2, Subst[Int[1/(Simp[b*c - a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g - c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]

Rule 538

Int[1/(((a_) + (b_)*(x_)^2)*Sqrt[(c_) + (d_)*(x_)^2]*Sqrt[(e_) + (f_)*(x_)^2]), x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a + b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[c, 0]

Rule 537

Int[1/(((a_) + (b_)*(x_)^2)*Sqrt[(c_) + (d_)*(x_)^2]*Sqrt[(e_) + (f_)*(x_)^2]), x_Symbol] := Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x], (c*f)/(d*e)]/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !(!GtQ[f/e, 0] && SimplerSqrtQ[-(f/e), -(d/c)])

Rule 844

Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]

Rule 424

Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] := Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rubi steps

$$\begin{aligned}
\int \sqrt{d+ex}(a+b\operatorname{sech}^{-1}(cx))dx &= \frac{2(d+ex)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{3e} + \frac{(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx})\int \frac{(d+ex)^{3/2}}{x\sqrt{1-c^2x^2}}dx}{3e} \\
&= \frac{2(d+ex)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{3e} + \frac{(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx})\int \left(\frac{2de}{\sqrt{d+ex}\sqrt{1-c^2x^2}} + \frac{d^2}{x\sqrt{d+ex}\sqrt{1-c^2x^2}}\right)dx}{3e} \\
&= \frac{2(d+ex)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{3e} + \frac{1}{3}\left(4bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx + \dots \\
&= \frac{2(d+ex)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{3e} + \frac{1}{3}\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{\sqrt{d+ex}}{\sqrt{1-c^2x^2}}dx - \frac{1}{3}\left(2bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}}dx + \dots \\
&= \frac{2(d+ex)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{3e} - \frac{8bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\middle|\frac{2e}{cd+e}\right)}{3c\sqrt{d+ex}} + \dots \\
&= \frac{2(d+ex)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{3e} - \frac{4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\middle|\frac{2e}{cd+e}\right)}{3c\sqrt{\frac{c(d+ex)}{cd+e}}} + \dots \\
&= \frac{2(d+ex)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{3e} - \frac{4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\middle|\frac{2e}{cd+e}\right)}{3c\sqrt{\frac{c(d+ex)}{cd+e}}} + \dots
\end{aligned}$$

Mathematica [C] time = 12.9828, size = 2938, normalized size = 10.53

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[d + e*x]*(a + b*ArcSech[c*x]), x]

[Out] ((2*a*d)/(3*e) + (2*a*x)/3)*Sqrt[d + e*x] + (2*b*(d + e*x)^(3/2)*ArcSech[c*x])/((3*e) + (4*b*(-((e*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[c*(1 + (1 - c*x)/(1 + c*x))])*Sqrt[c + (c*(1 - c*x))/(1 + c*x])*Sqrt[(c*d + e + (c*d*(1 - c*x))/(1 + c*x) - (e*(1 - c*x))/(1 + c*x))/(c + (c*(1 - c*x))/(1 + c*x))])/(c*(1 + (1 - c*x)/(1 + c*x)))) + (Sqrt[c*(1 + (1 - c*x)/(1 + c*x))]*Sqrt[c + (c*(1 - c*x)/(1 + c*x))])*Sqrt[c*(1 + (1 - c*x)/(1 + c*x))*(c*d + e + (c*d*(1 - c*x))/(1 + c*x) - (e*(1 - c*x))/(1 + c*x))])*Sqrt[(c*d + e + (c*d*(1 - c*x))/(1 + c*x) - (e*(1 - c*x))/(1 + c*x))/(c + (c*(1 - c*x))/(1 + c*x))]*((I*c*d*(-(c*d) - e)*e*Sqrt[1 + (1 - c*x)/(1 + c*x)]*Sqrt[1 - ((c*d - e)*(1 - c*x))/((-c*d) - e)*(1 + c*x)])*(EllipticE[I*ArcSinh[Sqrt[(1 - c*x)/(1 + c*x)]], -((c*d - e)/(-c*d) - e)] - EllipticF[I*ArcSinh[Sqrt[(1 - c*x)/(1 + c*x)]], -((c*d - e)/(-c*d) - e)))/((c*d - e)*Sqrt[c*(1 + (1 - c*x)/(1 + c*x))*(c*d + e + ((c*d - e)*(1 - c*x))/(1 + c*x))]) - (I*(-(c*d) - e)*e^2*Sqrt[1 + (1 - c*x)/(1 + c*x)]*Sqrt[1 - ((c*d - e)*(1 - c*x))/((-c*d) - e)*(1 + c*x)])*(EllipticE[I*ArcSinh[Sqrt[(1 - c*x)/(1 + c*x)]], -((c*d - e)/(-c*d) - e)] - EllipticF[I*ArcSinh[Sqrt[(1 - c*x)/(1 + c*x)]], -((c*d - e)/(-c*d) - e)))/((c*d - e)*Sqrt[c*(1 + (1 - c*x)/(1 + c*x))*(c*d + e + ((c*d - e)*(1 - c*x))/(1 + c*x))]) - (I*c^2*d^2*Sqrt[1 + (1 - c*x)/(1 + c*x)]*Sqrt[1 - ((c*d - e)*(1 - c*x))/((-c*d) - e)*(1 + c*x)])*EllipticF[I*ArcSinh[Sqr


```
pticE((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/2))*e)*(-(e*x+d)
)*c-c*d+e)/(c*d-e))^(1/2)*(-(e*x+d)*c-c*d-e)/(c*d+e))^(1/2)/(c/(c*d+e))^(1
/2)/((e*x+d)^2*c^2-2*(e*x+d)*c^2*d+c^2*d^2-e^2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(a+b*arcsech(c*x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{ex+d}(b \operatorname{ar}\operatorname{sech}(cx) + a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(a+b*arcsech(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*(b*arcsech(c*x) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{asech}(cx)) \sqrt{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)*(a+b*asech(c*x)),x)
```

```
[Out] Integral((a + b*asech(c*x))*sqrt(d + e*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex+d}(b \operatorname{ar}\operatorname{sech}(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(1/2)*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*(b*arcsech(c*x) + a), x)
```

$$3.83 \quad \int \frac{a+b\operatorname{sech}^{-1}(cx)}{\sqrt{d+ex}} dx$$

Optimal. Leaf size=187

$$\frac{4b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{c(d+ex)}{cd+e}}\operatorname{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right), \frac{2e}{cd+e}\right)}{c\sqrt{d+ex}} + \frac{2\sqrt{d+ex}(a+b\operatorname{sech}^{-1}(cx))}{e} - \frac{4bd\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{c(d+ex)}{cd+e}}}{e\sqrt{d+ex}}$$

```
[Out] (2*Sqrt[d + e*x]*(a + b*ArcSech[c*x]))/e - (4*b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c*Sqrt[d + e*x]) - (4*b*d*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(e*Sqrt[d + e*x])
```

Rubi [A] time = 0.246547, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {6288, 944, 719, 419, 932, 168, 538, 537}

$$\frac{2\sqrt{d+ex}(a+b\operatorname{sech}^{-1}(cx))}{e} - \frac{4b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{c(d+ex)}{cd+e}}F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\middle|\frac{2e}{cd+e}\right)}{c\sqrt{d+ex}} - \frac{4bd\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{c(d+ex)}{cd+e}}\Pi\left(2; \sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\middle|\frac{2e}{cd+e}\right)}{e\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])/Sqrt[d + e*x], x]
```

```
[Out] (2*Sqrt[d + e*x]*(a + b*ArcSech[c*x]))/e - (4*b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(c*Sqrt[d + e*x]) - (4*b*d*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(e*Sqrt[d + e*x])
```

Rule 6288

```
Int[((a_.) + ArcSech[(c_.)*(x_)])*(b_.)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(a + b*ArcSech[c*x]))/(e*(m + 1)), x] + Dist[
(b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)])/(e*(m + 1)), Int[(d + e*x)^(m + 1)/(x*S
qrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 944

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol]
:> Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] + Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 719

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol]
:> Dist[(2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x], Sqrt[(1 - Rt[-(c/a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 932

```
Int[1/(((d_) + (e_)*(x_))*Sqrt[(f_) + (g_)*(x_)]*Sqrt[(a_) + (c_)*(x_)
^2]), x_Symbol] := With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_) + (b_)*(x_))*Sqrt[(c_) + (d_)*(x_)]*Sqrt[(e_) + (f_)*(x_)
]*Sqrt[(g_) + (h_)*(x_)]), x_Symbol] := Dist[-2, Subst[Int[1/(Simp[b*c -
a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_)*(x_)^2)*Sqrt[(c_) + (d_)*(x_)^2]*Sqrt[(e_) + (f_)*(x_)
^2]), x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_)*(x_)^2)*Sqrt[(c_) + (d_)*(x_)^2]*Sqrt[(e_) + (f_)*(x_)
^2]), x_Symbol] := Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)]/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\begin{aligned}
\int \frac{a + b \operatorname{sech}^{-1}(cx)}{\sqrt{d+ex}} dx &= \frac{2\sqrt{d+ex}(a + b \operatorname{sech}^{-1}(cx))}{e} + \frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{\sqrt{d+ex}}{x\sqrt{1-c^2x^2}} dx}{e} \\
&= \frac{2\sqrt{d+ex}(a + b \operatorname{sech}^{-1}(cx))}{e} + \left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{1}{\sqrt{d+ex}\sqrt{1-c^2x^2}} dx + \frac{\left(2bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)}{e} \\
&= \frac{2\sqrt{d+ex}(a + b \operatorname{sech}^{-1}(cx))}{e} + \frac{\left(2bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex}} dx}{e} - \frac{\left(4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)}{e} \\
&= \frac{2\sqrt{d+ex}(a + b \operatorname{sech}^{-1}(cx))}{e} - \frac{4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}} F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right) \middle| \frac{2e}{cd+e}\right)}{c\sqrt{d+ex}} - \frac{\left(4bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)}{e} \\
&= \frac{2\sqrt{d+ex}(a + b \operatorname{sech}^{-1}(cx))}{e} - \frac{4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}} F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right) \middle| \frac{2e}{cd+e}\right)}{c\sqrt{d+ex}} - \frac{\left(4bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)}{e} \\
&= \frac{2\sqrt{d+ex}(a + b \operatorname{sech}^{-1}(cx))}{e} - \frac{4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}} F\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right) \middle| \frac{2e}{cd+e}\right)}{c\sqrt{d+ex}} - \frac{4bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}}{e}
\end{aligned}$$

Mathematica [C] time = 10.7838, size = 1707, normalized size = 9.13

$$\frac{2\sqrt{d+ex}a}{e} + \frac{2b\sqrt{d+ex}\operatorname{sech}^{-1}(cx)}{e} - \frac{4ib\sqrt{\frac{cd+\frac{c(1-cx)d}{cx+1}+e-\frac{e(1-cx)}{cx+1}}{\frac{(1-cx)c}{cx+1}+c}} \left(2cd \sqrt{\frac{i\left(c\sqrt{\frac{1-cx}{cx+1}}d+\sqrt{-cd-e}\sqrt{cd-e}-e\sqrt{\frac{1-cx}{cx+1}}\right)}{(-icd+ie+\sqrt{-cd-e}\sqrt{cd-e})\left(\sqrt{\frac{1-cx}{cx+1}}-i\right)}} \sqrt{\frac{i\left(-c\sqrt{\frac{1-cx}{cx+1}}d+\sqrt{-cd-e}\sqrt{cd-e}-e\sqrt{\frac{1-cx}{cx+1}}\right)}{(icd-ie+\sqrt{-cd-e}\sqrt{cd-e})}} \right)}{e}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*ArcSech[c*x])/Sqrt[d + e*x], x]

[Out] (2*a*Sqrt[d + e*x])/e + (2*b*Sqrt[d + e*x]*ArcSech[c*x])/e - ((4*I)*b*Sqrt[(c*d + e + (c*d*(1 - c*x))/(1 + c*x) - (e*(1 - c*x))/(1 + c*x))/(c + (c*(1 - c*x))/(1 + c*x))]*(2*c*d*Sqrt[(-I)*(Sqrt[-(c*d) - e]*Sqrt[c*d - e] + c*d*Sqrt[(1 - c*x)/(1 + c*x)] - e*Sqrt[(1 - c*x)/(1 + c*x)])]/((-I)*c*d + Sqrt[-(c*d) - e]*Sqrt[c*d - e] + I*e)*(-I + Sqrt[(1 - c*x)/(1 + c*x)])]*Sqrt[(-I)*(Sqrt[-(c*d) - e]*Sqrt[c*d - e] - c*d*Sqrt[(1 - c*x)/(1 + c*x)] + e*Sqrt[(1 - c*x)/(1 + c*x)])]/((I*c*d + Sqrt[-(c*d) - e]*Sqrt[c*d - e] - I*e)*(-I + Sqrt[(1 - c*x)/(1 + c*x)])]*(1 + (1 - c*x)/(1 + c*x))*EllipticF[ArcSin[Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)])]/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))], (Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])^2/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])^2 + (c*d - e)*Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)])]/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*Sqrt[1 + (1 - c*x)/(1 + c*x)]*Sqrt[(e - (e*(1 - c*x))/(1 + c*x) + c*d*(1 + (1 - c*x)/(1 + c*x)))/(c*d + e)]*EllipticF[I*ArcSinh[Sqrt[(1 - c*x)/(1 + c*x)]], (c*d - e)/(c*d + e)] + (2*I)*c*d*Sqrt[(-I)*(Sqrt[-(c*d) - e]*Sqrt[c*d - e] + c*d*Sqrt[(1 - c*x)/(1 + c*x)] - e*Sqrt[(1 - c*x)/(1 + c*x)])]/((-I)*c*d + Sqrt[-(c*d) - e]*Sqrt[c*d - e] + I*e)*(-I + Sqrt[(1 - c*x)/(1 + c*x)])]

$$\begin{aligned} & t[(1 - c*x)/(1 + c*x)] * \text{Sqrt}[((-I) * (\text{Sqrt}[-(c*d) - e] * \text{Sqrt}[c*d - e] - c*d * \\ & \text{Sqrt}[(1 - c*x)/(1 + c*x)] + e * \text{Sqrt}[(1 - c*x)/(1 + c*x]))] / ((I * c*d + \text{Sqrt}[-(c*d) - e] * \\ & \text{Sqrt}[c*d - e] - I * e) * (-I + \text{Sqrt}[(1 - c*x)/(1 + c*x)])) * (1 + (1 - c*x)/(1 + c*x)) * \\ & (\text{EllipticPi}[(I * \text{Sqrt}[-(c*d) - e] - \text{Sqrt}[c*d - e]) / (\text{Sqrt}[-(c*d) - e] - I * \text{Sqrt}[c*d - e]), \\ & \text{ArcSin}[\text{Sqrt}[(\text{Sqrt}[-(c*d) - e] - I * \text{Sqrt}[c*d - e]) * (I + \text{Sqrt}[(1 - c*x)/(1 + c*x]))] / \\ & ((\text{Sqrt}[-(c*d) - e] + I * \text{Sqrt}[c*d - e]) * (-I + \text{Sqrt}[(1 - c*x)/(1 + c*x]))]), \\ & (\text{Sqrt}[-(c*d) - e] + I * \text{Sqrt}[c*d - e])^2 / (\text{Sqrt}[-(c*d) - e] - I * \text{Sqrt}[c*d - e])^2] - \\ & \text{EllipticPi}[((-I) * \text{Sqrt}[-(c*d) - e] + \text{Sqrt}[c*d - e]) / (\text{Sqrt}[-(c*d) - e] - I * \text{Sqrt}[c*d - e]), \\ & \text{ArcSin}[\text{Sqrt}[(\text{Sqrt}[-(c*d) - e] - I * \text{Sqrt}[c*d - e]) * (I + \text{Sqrt}[(1 - c*x)/(1 + c*x]))] / \\ & ((\text{Sqrt}[-(c*d) - e] + I * \text{Sqrt}[c*d - e]) * (-I + \text{Sqrt}[(1 - c*x)/(1 + c*x]))]), \\ & (\text{Sqrt}[-(c*d) - e] + I * \text{Sqrt}[c*d - e])^2 / (\text{Sqrt}[-(c*d) - e] - I * \text{Sqrt}[c*d - e])^2])) / (e * \\ & \text{Sqrt}[(\text{Sqrt}[-(c*d) - e] - I * \text{Sqrt}[c*d - e]) * (I + \text{Sqrt}[(1 - c*x)/(1 + c*x]))] / \\ & ((\text{Sqrt}[-(c*d) - e] + I * \text{Sqrt}[c*d - e]) * (-I + \text{Sqrt}[(1 - c*x)/(1 + c*x])])) * (e - \\ & (e * (1 - c*x)) / (1 + c*x) + c*d * (1 + (1 - c*x) / (1 + c*x)))) \end{aligned}$$

Maple [A] time = 0.275, size = 288, normalized size = 1.5

$$2 \frac{1}{e} \left(a \sqrt{ex+d} + b \left(\sqrt{ex+d} \operatorname{arcsech}(cx) - 2 \frac{ce^2x}{(ex+d)^2 c^2 - 2(ex+d)c^2d + c^2d^2 - e^2} \sqrt{\frac{(ex+d)c - cd - e}{cxe}} \sqrt{\frac{(ex+d)c - cd - e}{cxe}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))/(e*x+d)^(1/2),x)

[Out] 2/e*(a*(e*x+d)^(1/2)+b*((e*x+d)^(1/2)*arcsech(c*x)-2*c*e^2*(-((e*x+d)*c-c*d-e)/c/x/e)^(1/2)*x*((e*x+d)*c-c*d+e)/c/x/e)^(1/2)*(EllipticF((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),((c*d+e)/(c*d-e))^(1/2))-EllipticPi((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),1/c*(c*d+e)/d,(c/(c*d-e))^(1/2)/(c/(c*d+e))^(1/2)))*(-((e*x+d)*c-c*d+e)/(c*d-e))^(1/2)*(-((e*x+d)*c-c*d-e)/(c*d+e))^(1/2)/(c/(c*d+e))^(1/2))/((e*x+d)^2*c^2-2*(e*x+d)*c^2*d+c^2*d^2-e^2))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{asech}(cx)}{\sqrt{d + ex}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asech(c*x))/(e*x+d)**(1/2),x)

[Out] Integral((a + b*asech(c*x))/sqrt(d + e*x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsech}(cx) + a}{\sqrt{ex + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/(e*x+d)^(1/2),x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)/sqrt(e*x + d), x)

$$3.84 \quad \int \frac{a+b\operatorname{sech}^{-1}(cx)}{(d+ex)^{3/2}} dx$$

Optimal. Leaf size=105

$$\frac{4b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{c(d+ex)}{cd+e}}\Pi\left(2; \sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\middle|\frac{2e}{cd+e}\right)}{e\sqrt{d+ex}} - \frac{2(a+b\operatorname{sech}^{-1}(cx))}{e\sqrt{d+ex}}$$

[Out] (-2*(a + b*ArcSech[c*x]))/(e*Sqrt[d + e*x]) + (4*b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(e*Sqrt[d + e*x])

Rubi [A] time = 0.177725, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6288, 932, 168, 538, 537}

$$\frac{4b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{c(d+ex)}{cd+e}}\Pi\left(2; \sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\middle|\frac{2e}{cd+e}\right)}{e\sqrt{d+ex}} - \frac{2(a+b\operatorname{sech}^{-1}(cx))}{e\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcSech[c*x])/(d + e*x)^(3/2),x]

[Out] (-2*(a + b*ArcSech[c*x]))/(e*Sqrt[d + e*x]) + (4*b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(e*Sqrt[d + e*x])

Rule 6288

Int[((a_.) + ArcSech[(c_.)*(x_)])*(b_.))*((d_.) + (e_.)*(x_)^(m_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*ArcSech[c*x]))/(e*(m + 1)), x] + Dist[(b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)])/(e*(m + 1)), Int[(d + e*x)^(m + 1)/(x*Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]

Rule 932

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (c_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]

Rule 168

Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] := Dist[-2, Subst[Int[1/(Simp[b*c - a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g - c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]

Rule 538

Int[1/(((a_.) + (b_.)*(x_)^2)*Sqrt[(c_.) + (d_.)*(x_)^2]*Sqrt[(e_.) + (f_.)*(x_)^2]), x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a + b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[c, 0]

Rule 537

```
Int[1/(((a_) + (b_)*(x_)^2)*Sqrt[(c_) + (d_)*(x_)^2]*Sqrt[(e_) + (f_)*(x_)^2]), x_Symbol] :> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x], (c*f)/(d*e)]]/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && SimplerSqrtQ[-(f/e), -(d/c)])]
```

Rubi steps

$$\begin{aligned} \int \frac{a + b \operatorname{sech}^{-1}(cx)}{(d + ex)^{3/2}} dx &= -\frac{2(a + b \operatorname{sech}^{-1}(cx))}{e\sqrt{d + ex}} - \frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{1}{x\sqrt{d+ex}\sqrt{1-c^2x^2}} dx}{e} \\ &= -\frac{2(a + b \operatorname{sech}^{-1}(cx))}{e\sqrt{d + ex}} - \frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex}} dx}{e} \\ &= -\frac{2(a + b \operatorname{sech}^{-1}(cx))}{e\sqrt{d + ex}} + \frac{\left(4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{d+\frac{e}{c}-\frac{ex^2}{c}} dx, x, \sqrt{1-cx}\right)}{e} \\ &= -\frac{2(a + b \operatorname{sech}^{-1}(cx))}{e\sqrt{d + ex}} + \frac{\left(4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\right) \operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\sqrt{1-\frac{ex^2}{c(d+\frac{e}{c})}} dx, x, \sqrt{1-cx}\right)}{e\sqrt{d + ex}} \\ &= -\frac{2(a + b \operatorname{sech}^{-1}(cx))}{e\sqrt{d + ex}} + \frac{4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}} \Pi\left(2; \sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right) \Big|_{\frac{2e}{cd+e}}\right)}{e\sqrt{d + ex}} \end{aligned}$$

Mathematica [C] time = 10.3983, size = 1675, normalized size = 15.95

$$\frac{2a}{e\sqrt{d + ex}} - \frac{2b \operatorname{sech}^{-1}(cx)}{e\sqrt{d + ex}} + \frac{4ib \left(2 \sqrt{\frac{i\left(c\sqrt{\frac{1-cx}{cx+1}}d + \sqrt{-cd-e}\sqrt{cd-e} - e\sqrt{\frac{1-cx}{cx+1}}\right)}{(-icd+ie + \sqrt{-cd-e}\sqrt{cd-e})\left(\sqrt{\frac{1-cx}{cx+1}} - i\right)}} \sqrt{\frac{i\left(-c\sqrt{\frac{1-cx}{cx+1}}d + \sqrt{-cd-e}\sqrt{cd-e} + e\sqrt{\frac{1-cx}{cx+1}}\right)}{(icd-ie + \sqrt{-cd-e}\sqrt{cd-e})\left(\sqrt{\frac{1-cx}{cx+1}} - i\right)}} \left(\frac{1-cx}{cx+1} + 1\right) \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1-cx}}{\sqrt{2}}\right], \frac{2e}{cd+e}\right] \right)}{e\sqrt{d + ex}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*ArcSech[c*x])/(d + e*x)^(3/2), x]

[Out] (-2*a)/(e*Sqrt[d + e*x]) - (2*b*ArcSech[c*x])/(e*Sqrt[d + e*x]) + ((4*I)*b*(2*Sqrt[((-I)*(Sqrt[-(c*d) - e])*Sqrt[c*d - e] + c*d*Sqrt[(1 - c*x)/(1 + c*x]) - e*Sqrt[(1 - c*x)/(1 + c*x])])]/(((-I)*c*d + Sqrt[-(c*d) - e]*Sqrt[c*d - e] + I*e)*(-I + Sqrt[(1 - c*x)/(1 + c*x)])))*Sqrt[((-I)*(Sqrt[-(c*d) - e])*Sqrt[c*d - e] - c*d*Sqrt[(1 - c*x)/(1 + c*x]) + e*Sqrt[(1 - c*x)/(1 + c*x])])]/((I*c*d + Sqrt[-(c*d) - e]*Sqrt[c*d - e] - I*e)*(-I + Sqrt[(1 - c*x)/(1 + c*x)])))* (1 + (1 - c*x)/(1 + c*x))*EllipticF[ArcSin[Sqrt[(-(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x])])]/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]], (Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])^2/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])^2 + Sqrt[(-(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x])])]/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)])))*Sqrt[1 + (1 - c*x)/(1 + c*x)]]*Sqrt[(e - (e*(1 - c*x))/(1 + c*x) + c*d*(1 + (1 - c*x)/(1 + c*x)))]

$$c*x)))/(c*d + e)]*EllipticF[I*ArcSinh[Sqrt[(1 - c*x)/(1 + c*x)]], (c*d - e)/(c*d + e)] + (2*I)*Sqrt[((-I)*(Sqrt[-(c*d) - e]*Sqrt[c*d - e] + c*d*Sqrt[(1 - c*x)/(1 + c*x)] - e*Sqrt[(1 - c*x)/(1 + c*x)])))/(((-I)*c*d + Sqrt[-(c*d) - e]*Sqrt[c*d - e] + I*e)*(-I + Sqrt[(1 - c*x)/(1 + c*x)]))]*Sqrt[((-I)*(Sqrt[-(c*d) - e]*Sqrt[c*d - e] - c*d*Sqrt[(1 - c*x)/(1 + c*x)] + e*Sqrt[(1 - c*x)/(1 + c*x)])))/((I*c*d + Sqrt[-(c*d) - e]*Sqrt[c*d - e] - I*e)*(-I + Sqrt[(1 - c*x)/(1 + c*x)])))]*(1 + (1 - c*x)/(1 + c*x))*EllipticPi[(I*Sqrt[-(c*d) - e] - Sqrt[c*d - e])/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e]), ArcSin[Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)])))/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)])))]], (Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])^2/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])^2 - EllipticPi[((-I)*Sqrt[-(c*d) - e] + Sqrt[c*d - e])/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e]), ArcSin[Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)])))/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)])))]], (Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])^2/(Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])^2)))/(e*Sqrt[((Sqrt[-(c*d) - e] - I*Sqrt[c*d - e])*(I + Sqrt[(1 - c*x)/(1 + c*x)])))/((Sqrt[-(c*d) - e] + I*Sqrt[c*d - e])*(-I + Sqrt[(1 - c*x)/(1 + c*x)])))]*(1 + (1 - c*x)/(1 + c*x))*Sqrt[(c*d + e + (c*d*(1 - c*x))/(1 + c*x) - (e*(1 - c*x))/(1 + c*x))/(c + (c*(1 - c*x))/(1 + c*x)))]$$

Maple [B] time = 0.271, size = 253, normalized size = 2.4

$$2 \frac{1}{e} \left(-\frac{a}{\sqrt{ex+d}} + b \left(-\frac{\operatorname{arcsech}(cx)}{\sqrt{ex+d}} - 2 \frac{ce^2x}{d((ex+d)^2c^2 - 2(ex+d)c^2d + c^2d^2 - e^2)} \sqrt{\frac{(ex+d)c - cd - e}{cxe}} \sqrt{\frac{(ex+d)c - c}{cxe}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))/(e*x+d)^(3/2),x)

[Out] 2/e*(-a/(e*x+d)^(1/2)+b*(-1/(e*x+d)^(1/2)*arcsech(c*x)-2*c*e^2*(-((e*x+d)*c-c*d-e)/c/x/e)^(1/2)*x*(((e*x+d)*c-c*d+e)/c/x/e)^(1/2)*EllipticPi((e*x+d)^(1/2)*(c/(c*d+e))^(1/2),1/c*(c*d+e)/d,(c/(c*d-e))^(1/2)/(c/(c*d+e))^(1/2))*(-(e*x+d)*c-c*d+e)/(c*d-e)^(1/2)*(-(e*x+d)*c-c*d-e)/(c*d+e)^(1/2)/d/(c/(c*d+e))^(1/2)/((e*x+d)^2*c^2-2*(e*x+d)*c^2*d+c^2*d^2-e^2)))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/(e*x+d)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{ex+d}(b \operatorname{ar} \operatorname{sech}(cx) + a)}{e^2x^2 + 2dex + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*(b*arcsech(c*x) + a)/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{arsech}(cx)}{(d + ex)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arsech(c*x))/(e*x+d)**(3/2),x)
```

```
[Out] Integral((a + b*arsech(c*x))/(d + e*x)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsech}(cx) + a}{(ex + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/(e*x + d)^(3/2), x)
```

$$3.85 \quad \int \frac{a+b\operatorname{sech}^{-1}(cx)}{(d+ex)^{5/2}} dx$$

Optimal. Leaf size=278

$$-\frac{2(a+b\operatorname{sech}^{-1}(cx))}{3e(d+ex)^{3/2}} + \frac{4be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{3d(c^2d^2-e^2)\sqrt{d+ex}} - \frac{4bc\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\middle|\frac{2e}{cd+e}\right)}{3d(c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}} + \frac{4b\sqrt{\frac{1}{cx+1}}\sqrt{cx}}{3e(d+ex)^{3/2}}$$

```
[Out] (4*b*e*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(3*d*(c^2*d^2 - e^2)*Sqrt[d + e*x]) - (2*(a + b*ArcSech[c*x]))/(3*e*(d + e*x)^(3/2)) - (4*b*c*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[d + e*x]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*d*(c^2*d^2 - e^2)*Sqrt[(c*(d + e*x))/(c*d + e)]) + (4*b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*d*e*Sqrt[d + e*x])
```

Rubi [A] time = 0.31253, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {6288, 958, 745, 21, 719, 424, 932, 168, 538, 537}

$$-\frac{2(a+b\operatorname{sech}^{-1}(cx))}{3e(d+ex)^{3/2}} + \frac{4be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{3d(c^2d^2-e^2)\sqrt{d+ex}} - \frac{4bc\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\middle|\frac{2e}{cd+e}\right)}{3d(c^2d^2-e^2)\sqrt{\frac{c(d+ex)}{cd+e}}} + \frac{4b\sqrt{\frac{1}{cx+1}}\sqrt{cx}}{3e(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])/(d + e*x)^(5/2), x]
```

```
[Out] (4*b*e*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(3*d*(c^2*d^2 - e^2)*Sqrt[d + e*x]) - (2*(a + b*ArcSech[c*x]))/(3*e*(d + e*x)^(3/2)) - (4*b*c*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[d + e*x]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*d*(c^2*d^2 - e^2)*Sqrt[(c*(d + e*x))/(c*d + e)]) + (4*b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(3*d*e*Sqrt[d + e*x])
```

Rule 6288

```
Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))*((d_.) + (e_.)*(x_.))^(m_.), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(a + b*ArcSech[c*x]))/(e*(m + 1)), x] + Dist[
(b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)])/(e*(m + 1)), Int[(d + e*x)^(m + 1)/(x*Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rule 958

```
Int[((f_.) + (g_.)*(x_.))^(n_.)/(((d_.) + (e_.)*(x_.))*Sqrt[(a_.) + (c_.)*(x_.)^2]), x_Symbol]
:> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 745

```
Int[((d_.) + (e_.)*(x_.))^(m_.)*((a_.) + (c_.)*(x_.)^2)^(p_.), x_Symbol]
:> Simp[(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
```



```
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :=>
Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
&& EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x,
a + b*x])
```

Rule 719

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :=> Dist[
(2*a*Rt[-(c/a), 2]*(d + e*x)^m*Sqrt[1 + (c*x^2)/a])/(c*Sqrt[a + c*x^2]*((c*(
d + e*x))/(c*d - a*e*Rt[-(c/a), 2]))^m), Subst[Int[(1 + (2*a*e*Rt[-(c/a), 2
]*x^2)/(c*d - a*e*Rt[-(c/a), 2]))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-(c/
a), 2]*x)/2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ
[m^2, 1/4]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :=> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :=> With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_)
]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :=> Dist[-2, Subst[Int[1/(Simp[b*c -
a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :=> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :=> Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e)])/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !(GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\begin{aligned}
\int \frac{a + b \operatorname{sech}^{-1}(cx)}{(d + ex)^{5/2}} dx &= -\frac{2(a + b \operatorname{sech}^{-1}(cx))}{3e(d + ex)^{3/2}} - \frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{1}{x(d+ex)^{3/2}\sqrt{1-c^2x^2}} dx}{3e} \\
&= -\frac{2(a + b \operatorname{sech}^{-1}(cx))}{3e(d + ex)^{3/2}} - \frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \left(-\frac{e}{d(d+ex)^{3/2}\sqrt{1-c^2x^2}} + \frac{1}{dx\sqrt{d+ex}\sqrt{1-c^2x^2}}\right) dx}{3e} \\
&= -\frac{2(a + b \operatorname{sech}^{-1}(cx))}{3e(d + ex)^{3/2}} + \frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{1}{(d+ex)^{3/2}\sqrt{1-c^2x^2}} dx}{3d} - \frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{1}{x\sqrt{d+ex}} dx}{3de} \\
&= \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d(c^2d^2 - e^2)\sqrt{d+ex}} - \frac{2(a + b \operatorname{sech}^{-1}(cx))}{3e(d + ex)^{3/2}} - \frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex}} dx}{3de} \\
&= \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d(c^2d^2 - e^2)\sqrt{d+ex}} - \frac{2(a + b \operatorname{sech}^{-1}(cx))}{3e(d + ex)^{3/2}} + \frac{\left(4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}} dx\right)}{3de} \\
&= \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d(c^2d^2 - e^2)\sqrt{d+ex}} - \frac{2(a + b \operatorname{sech}^{-1}(cx))}{3e(d + ex)^{3/2}} + \frac{\left(4b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{\frac{c(d+ex)}{cd+e}}\right) \operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}} dx\right)}{3de\sqrt{d+ex}} \\
&= \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d(c^2d^2 - e^2)\sqrt{d+ex}} - \frac{2(a + b \operatorname{sech}^{-1}(cx))}{3e(d + ex)^{3/2}} - \frac{4bc\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{d+ex}E\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\right)}{3d(c^2d^2 - e^2)\sqrt{\frac{c(d+ex)}{cd+e}}}
\end{aligned}$$

Mathematica [C] time = 12.6615, size = 4527, normalized size = 16.28

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*ArcSech[c*x])/(d + e*x)^(5/2), x]

[Out] $(-2*a)/(3*e*(d + e*x)^{(3/2)}) + \operatorname{Sqrt}[(1 - c*x)/(1 + c*x)]*\operatorname{Sqrt}[d + e*x]*((4*b*c)/(3*d*(c^2*d^2 - e^2)) - (4*b)/(3*d*(c*d + e)*(d + e*x))) - (2*b*\operatorname{ArcSech}[c*x])/(3*e*(d + e*x)^{(3/2)}) - (4*b*((e*\operatorname{Sqrt}[(1 - c*x)/(1 + c*x)]*\operatorname{Sqrt}[c*(1 + (1 - c*x)/(1 + c*x))]*(c*d + e + (c*d*(1 - c*x))/(1 + c*x) - (e*(1 - c*x))/(1 + c*x)))/((1 + (1 - c*x)/(1 + c*x))*\operatorname{Sqrt}[c + (c*(1 - c*x))/(1 + c*x)])*\operatorname{Sqrt}[(c*d + e + (c*d*(1 - c*x))/(1 + c*x) - (e*(1 - c*x))/(1 + c*x))]/(c + (c*(1 - c*x))/(1 + c*x))) - ((c*d - e)*\operatorname{Sqrt}[c*(1 + (1 - c*x)/(1 + c*x))]*\operatorname{Sqrt}[c*(1 + (1 - c*x)/(1 + c*x))*(c*d + e + (c*d*(1 - c*x))/(1 + c*x) - (e*(1 - c*x))/(1 + c*x))]*(I*(-(c*d) - e)*e*\operatorname{Sqrt}[1 + (1 - c*x)/(1 + c*x)]*\operatorname{Sqrt}[1 - ((c*d - e)*(1 - c*x))/((-c*d) - e)*(1 + c*x)])*(\operatorname{EllipticE}[I*\operatorname{ArcSinh}[\operatorname{Sqrt}[(1 - c*x)/(1 + c*x)]], -((c*d - e)/(-(c*d) - e))] - \operatorname{EllipticF}[I*\operatorname{ArcSinh}[\operatorname{Sqrt}[(1 - c*x)/(1 + c*x)]], -((c*d - e)/(-(c*d) - e)))]/(c*d - e)*\operatorname{Sqrt}[c*(1 + (1 - c*x)/(1 + c*x))*(c*d + e + ((c*d - e)*(1 - c*x))/(1 + c*x))]) + (I*c*d*\operatorname{Sqrt}[1 + (1 - c*x)/(1 + c*x)]*\operatorname{Sqrt}[1 - ((c*d - e)*(1 - c*x))/((-c*d) - e)*(1 + c*x)])*\operatorname{EllipticF}[I*\operatorname{ArcSinh}[\operatorname{Sqrt}[(1 - c*x)/(1 + c*x)]], -((c*d - e)/(-(c*d) - e))]/\operatorname{Sqrt}[c*(1 + (1 - c*x)/(1 + c*x))*(c*d + e + ((c*d - e)*(1 - c*x))/(1 + c*x))]) + (I*e*\operatorname{Sqrt}[1 + (1 - c*x)/(1 + c*x)]*\operatorname{Sqrt}[1 - ((c*d - e)*(1 - c*x))/((-c*d) - e)*(1 + c*x)])*\operatorname{EllipticF}[I*\operatorname{ArcSinh}[\operatorname{Sqrt}[(1 - c*x)/(1 + c*x)]], -((c*d - e)/(-(c*d) - e))]/\operatorname{Sqrt}[c*(1 + (1 - c*x)/(1 + c*x))*(c*d + e + ((c*d - e)*(1 - c*x))/(1 + c*x))]) - (I*c*d*(I + \operatorname{Sqrt}[-(c*d) -$

$$\begin{aligned}
& e]/\text{Sqrt}[c*d - e]*(-I + \text{Sqrt}[(1 - c*x)/(1 + c*x)])^2*\text{Sqrt}[(\text{Sqrt}[-(c*d) - e] \\
& - I*\text{Sqrt}[c*d - e]*(I + \text{Sqrt}[(1 - c*x)/(1 + c*x)]))/(\text{Sqrt}[-(c*d) - e] + \\
& I*\text{Sqrt}[c*d - e]*(-I + \text{Sqrt}[(1 - c*x)/(1 + c*x)]))] * \text{Sqrt}[(I*(-\text{Sqrt}[-(c*d) \\
& - e]/\text{Sqrt}[c*d - e]) + \text{Sqrt}[(1 - c*x)/(1 + c*x)])]/((I + \text{Sqrt}[-(c*d) - e]/\text{Sqrt}[c*d \\
& - e])*(-I + \text{Sqrt}[(1 - c*x)/(1 + c*x)]))] * \text{Sqrt}[(I*(\text{Sqrt}[-(c*d) - e]/\text{S} \\
& \text{qrt}[c*d - e] + \text{Sqrt}[(1 - c*x)/(1 + c*x)]))/((I - \text{Sqrt}[-(c*d) - e]/\text{Sqrt}[c*d \\
& - e])*(-I + \text{Sqrt}[(1 - c*x)/(1 + c*x)]))] * ((1 + I)*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[(\text{S} \\
& \text{qrt}[-(c*d) - e] - I*\text{Sqrt}[c*d - e])*(I + \text{Sqrt}[(1 - c*x)/(1 + c*x]))]/((\text{Sqrt}[- \\
& -(c*d) - e] + I*\text{Sqrt}[c*d - e])*(-I + \text{Sqrt}[(1 - c*x)/(1 + c*x)]))], (\text{Sqrt}[- \\
& -(c*d) - e] + I*\text{Sqrt}[c*d - e])^2/(\text{Sqrt}[-(c*d) - e] - I*\text{Sqrt}[c*d - e])^2] - (\\
& 2*I)*\text{EllipticPi}[((-I)*(I + \text{Sqrt}[-(c*d) - e]/\text{Sqrt}[c*d - e]))/(-I + \text{Sqrt}[-(c* \\
& d) - e]/\text{Sqrt}[c*d - e]), \text{ArcSin}[\text{Sqrt}[(\text{Sqrt}[-(c*d) - e] - I*\text{Sqrt}[c*d - e])*(\\
& I + \text{Sqrt}[(1 - c*x)/(1 + c*x]))]/((\text{Sqrt}[-(c*d) - e] + I*\text{Sqrt}[c*d - e])*(-I + \\
& \text{Sqrt}[(1 - c*x)/(1 + c*x)]))]], (\text{Sqrt}[-(c*d) - e] + I*\text{Sqrt}[c*d - e])^2/(\text{Sqr} \\
& \text{t}[-(c*d) - e] - I*\text{Sqrt}[c*d - e])^2)]/((I - \text{Sqrt}[-(c*d) - e]/\text{Sqrt}[c*d - e]) \\
& * \text{Sqrt}[c*(1 + (1 - c*x)/(1 + c*x))*(c*d + e + ((c*d - e)*(1 - c*x))/(1 + c*x \\
&))] - (I*(I + \text{Sqrt}[-(c*d) - e]/\text{Sqrt}[c*d - e])*e*(-I + \text{Sqrt}[(1 - c*x)/(1 + \\
& c*x]))^2*\text{Sqrt}[(\text{Sqrt}[-(c*d) - e] - I*\text{Sqrt}[c*d - e])*(I + \text{Sqrt}[(1 - c*x)/(1 \\
& + c*x]))]/((\text{Sqrt}[-(c*d) - e] + I*\text{Sqrt}[c*d - e])*(-I + \text{Sqrt}[(1 - c*x)/(1 + c \\
& *x)]))] * \text{Sqrt}[(I*(-\text{Sqrt}[-(c*d) - e]/\text{Sqrt}[c*d - e]) + \text{Sqrt}[(1 - c*x)/(1 + c* \\
& x)])]/((I + \text{Sqrt}[-(c*d) - e]/\text{Sqrt}[c*d - e])*(-I + \text{Sqrt}[(1 - c*x)/(1 + c*x)] \\
&)) * \text{Sqrt}[(I*(\text{Sqrt}[-(c*d) - e]/\text{Sqrt}[c*d - e] + \text{Sqrt}[(1 - c*x)/(1 + c*x]))]/((\\
& (I - \text{Sqrt}[-(c*d) - e]/\text{Sqrt}[c*d - e])*(-I + \text{Sqrt}[(1 - c*x)/(1 + c*x)]))] * ((1 \\
& + I)*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[(\text{Sqrt}[-(c*d) - e] - I*\text{Sqrt}[c*d - e])*(I + \text{Sqrt} \\
& [(1 - c*x)/(1 + c*x]))]/((\text{Sqrt}[-(c*d) - e] + I*\text{Sqrt}[c*d - e])*(-I + \text{Sqrt}[(1 \\
& - c*x)/(1 + c*x)]))], (\text{Sqrt}[-(c*d) - e] + I*\text{Sqrt}[c*d - e])^2/(\text{Sqrt}[-(c*d) \\
& - e] - I*\text{Sqrt}[c*d - e])^2] - (2*I)*\text{EllipticPi}[((-I)*(I + \text{Sqrt}[-(c*d) - e]/ \\
& \text{Sqrt}[c*d - e]))/(-I + \text{Sqrt}[-(c*d) - e]/\text{Sqrt}[c*d - e]), \text{ArcSin}[\text{Sqrt}[(\text{Sqrt}[- \\
& -(c*d) - e] - I*\text{Sqrt}[c*d - e])*(I + \text{Sqrt}[(1 - c*x)/(1 + c*x]))]/((\text{Sqrt}[-(c*d) \\
&) - e] + I*\text{Sqrt}[c*d - e])*(-I + \text{Sqrt}[(1 - c*x)/(1 + c*x)]))], (\text{Sqrt}[-(c*d) \\
& - e] + I*\text{Sqrt}[c*d - e])^2/(\text{Sqrt}[-(c*d) - e] - I*\text{Sqrt}[c*d - e])^2)]/((I - \\
& \text{Sqrt}[-(c*d) - e]/\text{Sqrt}[c*d - e]) * \text{Sqrt}[c*(1 + (1 - c*x)/(1 + c*x))*(c*d + e + \\
& ((c*d - e)*(1 - c*x))/(1 + c*x))] + (I*c*d*(I + \text{Sqrt}[-(c*d) - e]/\text{Sqrt}[c*d \\
& - e])*(-I + \text{Sqrt}[(1 - c*x)/(1 + c*x)])^2*\text{Sqrt}[(\text{Sqrt}[-(c*d) - e] - I*\text{Sqrt}[\\
& c*d - e])*(I + \text{Sqrt}[(1 - c*x)/(1 + c*x]))]/((\text{Sqrt}[-(c*d) - e] + I*\text{Sqrt}[c*d \\
& - e])*(-I + \text{Sqrt}[(1 - c*x)/(1 + c*x)]))] * \text{Sqrt}[(I*(-\text{Sqrt}[-(c*d) - e]/\text{Sqrt}[c \\
& *d - e]) + \text{Sqrt}[(1 - c*x)/(1 + c*x)])]/((I + \text{Sqrt}[-(c*d) - e]/\text{Sqrt}[c*d - e] \\
&)*(-I + \text{Sqrt}[(1 - c*x)/(1 + c*x)]))] * \text{Sqrt}[(I*(\text{Sqrt}[-(c*d) - e]/\text{Sqrt}[c*d - e] \\
& + \text{Sqrt}[(1 - c*x)/(1 + c*x]))]/((I - \text{Sqrt}[-(c*d) - e]/\text{Sqrt}[c*d - e])*(-I + \\
& \text{Sqrt}[(1 - c*x)/(1 + c*x)]))] * ((-1 + I)*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[(\text{Sqrt}[-(c*d) \\
& - e] - I*\text{Sqrt}[c*d - e])*(I + \text{Sqrt}[(1 - c*x)/(1 + c*x]))]/((\text{Sqrt}[-(c*d) - e] \\
& + I*\text{Sqrt}[c*d - e])*(-I + \text{Sqrt}[(1 - c*x)/(1 + c*x)]))], (\text{Sqrt}[-(c*d) - e] \\
& + I*\text{Sqrt}[c*d - e])^2/(\text{Sqrt}[-(c*d) - e] - I*\text{Sqrt}[c*d - e])^2] - (2*I)*\text{Ellip} \\
& \text{ticPi}[(I*(I + \text{Sqrt}[-(c*d) - e]/\text{Sqrt}[c*d - e]))/(-I + \text{Sqrt}[-(c*d) - e]/\text{Sqrt}[\\
& c*d - e]), \text{ArcSin}[\text{Sqrt}[(\text{Sqrt}[-(c*d) - e] - I*\text{Sqrt}[c*d - e])*(I + \text{Sqrt}[(1 - \\
& c*x)/(1 + c*x]))]/((\text{Sqrt}[-(c*d) - e] + I*\text{Sqrt}[c*d - e])*(-I + \text{Sqrt}[(1 - c* \\
& x)/(1 + c*x)]))], (\text{Sqrt}[-(c*d) - e] + I*\text{Sqrt}[c*d - e])^2/(\text{Sqrt}[-(c*d) - e] \\
& - I*\text{Sqrt}[c*d - e])^2)]/((I - \text{Sqrt}[-(c*d) - e]/\text{Sqrt}[c*d - e]) * \text{Sqrt}[c*(1 + \\
& (1 - c*x)/(1 + c*x))*(c*d + e + ((c*d - e)*(1 - c*x))/(1 + c*x))] + (I*(I \\
& + \text{Sqrt}[-(c*d) - e]/\text{Sqrt}[c*d - e])*e*(-I + \text{Sqrt}[(1 - c*x)/(1 + c*x)])^2*\text{Sqrt} \\
& [(\text{Sqrt}[-(c*d) - e] - I*\text{Sqrt}[c*d - e])*(I + \text{Sqrt}[(1 - c*x)/(1 + c*x]))]/((\text{S} \\
& \text{qrt}[-(c*d) - e] + I*\text{Sqrt}[c*d - e])*(-I + \text{Sqrt}[(1 - c*x)/(1 + c*x)]))] * \text{Sqrt} [\\
& (I*(-\text{Sqrt}[-(c*d) - e]/\text{Sqrt}[c*d - e]) + \text{Sqrt}[(1 - c*x)/(1 + c*x)])]/((I + \text{S} \\
& \text{qrt}[-(c*d) - e]/\text{Sqrt}[c*d - e])*(-I + \text{Sqrt}[(1 - c*x)/(1 + c*x)]))] * \text{Sqrt}[(I*(\\
& \text{Sqrt}[-(c*d) - e]/\text{Sqrt}[c*d - e] + \text{Sqrt}[(1 - c*x)/(1 + c*x]))]/((I - \text{Sqrt}[-(c \\
& *d) - e]/\text{Sqrt}[c*d - e])*(-I + \text{Sqrt}[(1 - c*x)/(1 + c*x)]))] * ((-1 + I)*\text{Ellipt} \\
& \text{icF}[\text{ArcSin}[\text{Sqrt}[(\text{Sqrt}[-(c*d) - e] - I*\text{Sqrt}[c*d - e])*(I + \text{Sqrt}[(1 - c*x)/(\\
& 1 + c*x]))]/((\text{Sqrt}[-(c*d) - e] + I*\text{Sqrt}[c*d - e])*(-I + \text{Sqrt}[(1 - c*x)/(1 + \\
& c*x)]))], (\text{Sqrt}[-(c*d) - e] + I*\text{Sqrt}[c*d - e])^2/(\text{Sqrt}[-(c*d) - e] - I*\text{Sqrt}
\end{aligned}$$

$$\begin{aligned} & \text{rt}[c*d - e]^2] - (2*I)*\text{EllipticPi}[(I*(I + \text{Sqrt}[-(c*d) - e]/\text{Sqrt}[c*d - e])) \\ & /(-I + \text{Sqrt}[-(c*d) - e]/\text{Sqrt}[c*d - e]), \text{ArcSin}[\text{Sqrt}[(\text{Sqrt}[-(c*d) - e] - I* \\ & \text{Sqrt}[c*d - e])*(I + \text{Sqrt}[(1 - c*x)/(1 + c*x]))]/((\text{Sqrt}[-(c*d) - e] + I*\text{Sqrt} \\ & [c*d - e])*(-I + \text{Sqrt}[(1 - c*x)/(1 + c*x)]))], (\text{Sqrt}[-(c*d) - e] + I*\text{Sqrt} \\ & [c*d - e])^2/(\text{Sqrt}[-(c*d) - e] - I*\text{Sqrt}[c*d - e])^2)/((I - \text{Sqrt}[-(c*d) - e] \\ &]/\text{Sqrt}[c*d - e])* \text{Sqrt}[c*(1 + (1 - c*x)/(1 + c*x))*(c*d + e + ((c*d - e)*(1 \\ & - c*x))/(1 + c*x)))]/((1 + (1 - c*x)/(1 + c*x))* \text{Sqrt}[c + (c*(1 - c*x))/(1 \\ & + c*x)]*\text{Sqrt}[(c*d + e + (c*d*(1 - c*x))/(1 + c*x) - (e*(1 - c*x))/(1 + c*x) \\ &))/(c + (c*(1 - c*x))/(1 + c*x)))]/((3*d*e*(c^2*d^2 - e^2)) \end{aligned}$$

Maple [B] time = 0.335, size = 902, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arcsech(c*x))/(e*x+d)^(5/2),x)`

[Out]
$$\begin{aligned} & 2/e*(-1/3*a/(e*x+d)^{(3/2)}+b*(-1/3/(e*x+d)^{(3/2)}*\text{arcsech}(c*x)+2/3*c*e^2*(-((\\ & e*x+d)*c-c*d-e)/c/x/e)^{(1/2)}*x*((e*x+d)*c-c*d+e)/c/x/e)^{(1/2)}*((c/(c*d+e)) \\ & ^{(1/2)}*(e*x+d)^2*c^2*d-(-(e*x+d)*c-c*d-e)/(c*d+e))^{(1/2)}*(-((e*x+d)*c-c*d+ \\ & e)/(c*d-e))^{(1/2)}*\text{EllipticF}((e*x+d)^{(1/2)}*(c/(c*d+e))^{(1/2)},((c*d+e)/(c*d-e) \\ &))^{(1/2)}*(e*x+d)^{(1/2)}*c^2*d^2+c^2*(-((e*x+d)*c-c*d-e)/(c*d+e))^{(1/2)}*(-((\\ & e*x+d)*c-c*d+e)/(c*d-e))^{(1/2)}*\text{EllipticE}((e*x+d)^{(1/2)}*(c/(c*d+e))^{(1/2)},((\\ & c*d+e)/(c*d-e))^{(1/2)})*d^2*(e*x+d)^{(1/2)}-(-(e*x+d)*c-c*d-e)/(c*d+e))^{(1/2)} \\ & *(-(e*x+d)*c-c*d+e)/(c*d-e))^{(1/2)}*\text{EllipticPi}((e*x+d)^{(1/2)}*(c/(c*d+e))^{(1 \\ & /2)},1/c*(c*d+e)/d,(c/(c*d-e))^{(1/2)}/(c/(c*d+e))^{(1/2)})*(e*x+d)^{(1/2)}*c^2*d^ \\ & 2-2*(c/(c*d+e))^{(1/2)}*(e*x+d)*c^2*d^2+(-(e*x+d)*c-c*d-e)/(c*d+e))^{(1/2)}*(- \\ & ((e*x+d)*c-c*d+e)/(c*d-e))^{(1/2)}*\text{EllipticF}((e*x+d)^{(1/2)}*(c/(c*d+e))^{(1/2)}, \\ & ((c*d+e)/(c*d-e))^{(1/2)})*(e*x+d)^{(1/2)}*c*d*e-(-(e*x+d)*c-c*d-e)/(c*d+e))^{(\\ & 1/2)}*(-((e*x+d)*c-c*d+e)/(c*d-e))^{(1/2)}*\text{EllipticE}((e*x+d)^{(1/2)}*(c/(c*d+e)) \\ & ^{(1/2)},((c*d+e)/(c*d-e))^{(1/2)})*(e*x+d)^{(1/2)}*c*d*e+(c/(c*d+e))^{(1/2)}*c^2*d \\ & ^3+(-(e*x+d)*c-c*d-e)/(c*d+e))^{(1/2)}*(-((e*x+d)*c-c*d+e)/(c*d-e))^{(1/2)}*\text{El \\ & lipticPi}((e*x+d)^{(1/2)}*(c/(c*d+e))^{(1/2)},1/c*(c*d+e)/d,(c/(c*d-e))^{(1/2)}/(c \\ & /(c*d+e))^{(1/2)})*(e*x+d)^{(1/2)}*e^2-(c/(c*d+e))^{(1/2)}*d*e^2)/(c/(c*d+e))^{(1/ \\ & 2)}/(e*x+d)^{(1/2)}/d^2/(c*d+e)/(c*d-e)/((e*x+d)^2*c^2-2*(e*x+d)*c^2*d+c^2*d^2 \\ & -e^2)) \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsech(c*x))/(e*x+d)^(5/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{ex+d}(b \operatorname{ar} \operatorname{sech}(cx) + a)}{e^3 x^3 + 3 d e^2 x^2 + 3 d^2 e x + d^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x + d)*(b*arcsech(c*x) + a)/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*
e*x + d^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*asech(c*x))/(e*x+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsech}(cx) + a}{(ex + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/(e*x + d)^(5/2), x)
```

$$3.86 \quad \int \frac{a+b\operatorname{sech}^{-1}(cx)}{(d+ex)^{7/2}} dx$$

Optimal. Leaf size=609

$$\frac{4bc\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{c(d+ex)}{cd+e}}\operatorname{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right),\frac{2e}{cd+e}\right)}{15d(c^2d^2-e^2)\sqrt{d+ex}} - \frac{2(a+b\operatorname{sech}^{-1}(cx))}{5e(d+ex)^{5/2}} + \frac{16bc^2e\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{15(c^2d^2-e^2)^2\sqrt{d+ex}} + \dots$$

[Out] (4*b*e*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(15*d*(c^2*d^2 - e^2)*(d + e*x)^(3/2)) + (16*b*c^2*e*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(15*(c^2*d^2 - e^2)^2*Sqrt[d + e*x]) + (4*b*e*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(5*d^2*(c^2*d^2 - e^2)*Sqrt[d + e*x]) - (2*(a + b*ArcSech[c*x]))/(5*e*(d + e*x)^(5/2)) - (16*b*c^3*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[d + e*x]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*(c^2*d^2 - e^2)^2*Sqrt[(c*(d + e*x))/(c*d + e)]) - (4*b*c*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[d + e*x]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(5*d^2*(c^2*d^2 - e^2)*Sqrt[(c*(d + e*x))/(c*d + e)]) + (4*b*c*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*d*(c^2*d^2 - e^2)*Sqrt[d + e*x]) + (4*b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(5*d^2*e*Sqrt[d + e*x])

Rubi [A] time = 0.594117, antiderivative size = 609, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$, Rules used = {6288, 958, 745, 835, 844, 719, 424, 419, 21, 932, 168, 538, 537}

$$-\frac{2(a+b\operatorname{sech}^{-1}(cx))}{5e(d+ex)^{5/2}} + \frac{16bc^2e\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{15(c^2d^2-e^2)^2\sqrt{d+ex}} + \frac{4be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{5d^2(c^2d^2-e^2)\sqrt{d+ex}} + \frac{4be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{15d(c^2d^2-e^2)(d+ex)^{3/2}} + \dots$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcSech[c*x])/(d + e*x)^(7/2), x]

[Out] (4*b*e*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(15*d*(c^2*d^2 - e^2)*(d + e*x)^(3/2)) + (16*b*c^2*e*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(15*(c^2*d^2 - e^2)^2*Sqrt[d + e*x]) + (4*b*e*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(5*d^2*(c^2*d^2 - e^2)*Sqrt[d + e*x]) - (2*(a + b*ArcSech[c*x]))/(5*e*(d + e*x)^(5/2)) - (16*b*c^3*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[d + e*x]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*(c^2*d^2 - e^2)^2*Sqrt[(c*(d + e*x))/(c*d + e)]) - (4*b*c*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[d + e*x]*EllipticE[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(5*d^2*(c^2*d^2 - e^2)*Sqrt[(c*(d + e*x))/(c*d + e)]) + (4*b*c*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticF[ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(15*d*(c^2*d^2 - e^2)*Sqrt[d + e*x]) + (4*b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[(c*(d + e*x))/(c*d + e)]*EllipticPi[2, ArcSin[Sqrt[1 - c*x]/Sqrt[2]], (2*e)/(c*d + e)])/(5*d^2*e*Sqrt[d + e*x])

Rule 6288

Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))*((d_.) + (e_.)*(x_.))^(m_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*ArcSech[c*x]))/(e*(m + 1)), x] + Dist[(b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)])/(e*(m + 1)), Int[(d + e*x)^(m + 1)/(x*S

$\text{qrt}[1 - c^2x^2], x], x] /; \text{FreeQ}\{a, b, c, d, e, m\}, x\} \&\& \text{NeQ}[m, -1]$

Rule 958

$\text{Int}[(f_.) + (g_.)*(x_.)^{(n_.)}/((d_.) + (e_.)*(x_.)*\text{Sqrt}[(a_.) + (c_.)*(x_.)^2]), x_Symbol] :> \text{Int}[\text{ExpandIntegrand}[1/(\text{Sqrt}[f + gx]*\text{Sqrt}[a + cx^2]), (f + gx)^{(n + 1/2)}/(d + ex), x], x] /; \text{FreeQ}\{a, c, d, e, f, g\}, x\} \&\& \text{NeQ}[e*f - d*g, 0] \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{IntegerQ}[n + 1/2]$

Rule 745

$\text{Int}[(d_.) + (e_.)*(x_.)^{(m_.)}*((a_.) + (c_.)*(x_.)^2)^{(p_.)}, x_Symbol] :> \text{Simp}[(e*(d + ex)^{(m + 1)}*(a + cx^2)^{(p + 1)})/((m + 1)*(c*d^2 + a*e^2)), x] + \text{Dist}[c/((m + 1)*(c*d^2 + a*e^2)), \text{Int}[(d + ex)^{(m + 1)}*\text{Simp}[d*(m + 1) - e*(m + 2*p + 3)*x, x]*(a + cx^2)^p, x], x] /; \text{FreeQ}\{a, c, d, e, m, p\}, x\} \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{NeQ}[m, -1] \&\& ((\text{LtQ}[m, -1] \&\& \text{IntQuadraticQ}[a, 0, c, d, e, m, p, x]) \|\ (\text{SumSimplerQ}[m, 1] \&\& \text{IntegerQ}[p]) \|\ \text{ILtQ}[\text{Simplify}[m + 2*p + 3], 0])$

Rule 835

$\text{Int}[(d_.) + (e_.)*(x_.)^{(m_.)}*((f_.) + (g_.)*(x_.)*((a_.) + (c_.)*(x_.)^2)^{(p_.)}), x_Symbol] :> \text{Simp}[(e*f - d*g)*(d + ex)^{(m + 1)}*(a + cx^2)^{(p + 1)})/((m + 1)*(c*d^2 + a*e^2)), x] + \text{Dist}[1/((m + 1)*(c*d^2 + a*e^2)), \text{Int}[(d + ex)^{(m + 1)}*(a + cx^2)^p*\text{Simp}[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m + 2*p + 3)*x, x], x], x] /; \text{FreeQ}\{a, c, d, e, f, g, p\}, x\} \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{LtQ}[m, -1] \&\& (\text{IntegerQ}[m] \|\ \text{IntegerQ}[p] \|\ \text{IntegersQ}[2*m, 2*p])$

Rule 844

$\text{Int}[(d_.) + (e_.)*(x_.)^{(m_.)}*((f_.) + (g_.)*(x_.)*((a_.) + (c_.)*(x_.)^2)^{(p_.)}), x_Symbol] :> \text{Dist}[g/e, \text{Int}[(d + ex)^{(m + 1)}*(a + cx^2)^p, x], x] + \text{Dist}[(e*f - d*g)/e, \text{Int}[(d + ex)^m*(a + cx^2)^p, x], x] /; \text{FreeQ}\{a, c, d, e, f, g, m, p\}, x\} \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& !\text{IGtQ}[m, 0]$

Rule 719

$\text{Int}[(d_.) + (e_.)*(x_.)^{(m_.)}/\text{Sqrt}[(a_.) + (c_.)*(x_.)^2], x_Symbol] :> \text{Dist}[(2*a*\text{Rt}[-(c/a), 2]*(d + ex)^m*\text{Sqrt}[1 + (cx^2)/a])/(c*\text{Sqrt}[a + cx^2]*((c*(d + ex))/(c*d - a*e*\text{Rt}[-(c/a), 2]))^m), \text{Subst}[\text{Int}[(1 + (2*a*e*\text{Rt}[-(c/a), 2]*x^2)/(c*d - a*e*\text{Rt}[-(c/a), 2]))^m/\text{Sqrt}[1 - x^2], x], x, \text{Sqrt}[(1 - \text{Rt}[-(c/a), 2]*x)/2]], x] /; \text{FreeQ}\{a, c, d, e\}, x\} \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{EqQ}[m^2, 1/4]$

Rule 424

$\text{Int}[\text{Sqrt}[(a_.) + (b_.)*(x_.)^2]/\text{Sqrt}[(c_.) + (d_.)*(x_.)^2], x_Symbol] :> \text{Simp}[(\text{Sqrt}[a]*\text{EllipticE}[\text{ArcSin}[\text{Rt}[-(d/c), 2]*x], (b*c)/(a*d)])/(\text{Sqrt}[c]*\text{Rt}[-(d/c), 2]), x] /; \text{FreeQ}\{a, b, c, d\}, x\} \&\& \text{NegQ}[d/c] \&\& \text{GtQ}[c, 0] \&\& \text{GtQ}[a, 0]$

Rule 419

$\text{Int}[1/(\text{Sqrt}[(a_.) + (b_.)*(x_.)^2]*\text{Sqrt}[(c_.) + (d_.)*(x_.)^2]), x_Symbol] :> \text{Simp}[(1*\text{EllipticF}[\text{ArcSin}[\text{Rt}[-(d/c), 2]*x], (b*c)/(a*d)])/(\text{Sqrt}[a]*\text{Sqrt}[c]*\text{Rt}[-(d/c), 2]), x] /; \text{FreeQ}\{a, b, c, d\}, x\} \&\& \text{NegQ}[d/c] \&\& \text{GtQ}[c, 0] \&\& \text{GtQ}[a, 0] \&\& !(\text{NegQ}[b/a] \&\& \text{SimplerSqrtQ}[-(b/a), -(d/c)])$

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :=
  Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
  && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x,
  a + b*x])
```

Rule 932

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] := With[{q = Rt[-(c/a), 2]}, Dist[1/Sqrt[a], Int[1/((d + e*
x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x] /; FreeQ[{a, c, d, e
, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 168

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] := Dist[-2, Subst[Int[1/(Simp[b*c -
a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + (f*x^2)/d, x]]*Sqrt[Simp[(d*g -
c*h)/d + (h*x^2)/d, x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 538

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 537

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] := Simp[(1*EllipticPi[(b*c)/(a*d), ArcSin[Rt[-(d/c), 2]*x
], (c*f)/(d*e))]/(a*Sqrt[c]*Sqrt[e]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d
, e, f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0]
&& SimplerSqrtQ[-(f/e), -(d/c)])
```

Rubi steps

$$\begin{aligned}
\int \frac{a + b \operatorname{sech}^{-1}(cx)}{(d + ex)^{7/2}} dx &= -\frac{2(a + b \operatorname{sech}^{-1}(cx))}{5e(d + ex)^{5/2}} - \frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{1}{x(d+ex)^{5/2}\sqrt{1-c^2x^2}} dx}{5e} \\
&= -\frac{2(a + b \operatorname{sech}^{-1}(cx))}{5e(d + ex)^{5/2}} - \frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \left(-\frac{e}{d(d+ex)^{5/2}\sqrt{1-c^2x^2}} - \frac{e}{d^2(d+ex)^{3/2}\sqrt{1-c^2x^2}} + \frac{e}{d^2x\sqrt{d+ex}}\right) dx}{5e} \\
&= -\frac{2(a + b \operatorname{sech}^{-1}(cx))}{5e(d + ex)^{5/2}} + \frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{1}{(d+ex)^{3/2}\sqrt{1-c^2x^2}} dx}{5d^2} + \frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{e}{d^2x\sqrt{d+ex}} dx}{5d} \\
&= \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{15d(c^2d^2 - e^2)(d + ex)^{3/2}} + \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{5d^2(c^2d^2 - e^2)\sqrt{d + ex}} - \frac{2(a + b \operatorname{sech}^{-1}(cx))}{5e(d + ex)^{5/2}} - \frac{\left(2b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{e}{d^2x\sqrt{d+ex}} dx}{5d} \\
&= \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{15d(c^2d^2 - e^2)(d + ex)^{3/2}} + \frac{16bc^2e\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{15(c^2d^2 - e^2)^2\sqrt{d + ex}} + \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{5d^2(c^2d^2 - e^2)\sqrt{d + ex}} \\
&= \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{15d(c^2d^2 - e^2)(d + ex)^{3/2}} + \frac{16bc^2e\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{15(c^2d^2 - e^2)^2\sqrt{d + ex}} + \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{5d^2(c^2d^2 - e^2)\sqrt{d + ex}} \\
&= \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{15d(c^2d^2 - e^2)(d + ex)^{3/2}} + \frac{16bc^2e\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{15(c^2d^2 - e^2)^2\sqrt{d + ex}} + \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{5d^2(c^2d^2 - e^2)\sqrt{d + ex}} \\
&= \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{15d(c^2d^2 - e^2)(d + ex)^{3/2}} + \frac{16bc^2e\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{15(c^2d^2 - e^2)^2\sqrt{d + ex}} + \frac{4be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{5d^2(c^2d^2 - e^2)\sqrt{d + ex}}
\end{aligned}$$

Mathematica [C] time = 12.9241, size = 8675, normalized size = 14.24

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*ArcSech[c*x])/(d + e*x)^(7/2), x]

[Out] Result too large to show

Maple [B] time = 0.352, size = 1632, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))/(e*x+d)^(7/2), x)

[Out] $2/e*(-1/5*a/(e*x+d)^{(5/2)}+b*(-1/5/(e*x+d)^{(5/2)}*\operatorname{arcsech}(c*x)+2/15*c*e^2*(-(e*x+d)*c-c*d-e)/c/x/e)^{(1/2)}*x*((e*x+d)*c-c*d+e)/c/x/e)^{(1/2)}*(7*(c/(c*d+e))^{(1/2)}+2/15*c/e)^{(1/2)}$

$$\begin{aligned}
& e)^{(1/2)} * (e*x+d)^3 * c^4 * d^3 - 6 * (-((e*x+d)*c-c*d-e)/(c*d+e))^{(1/2)} * (-((e*x+d) \\
& *c-c*d+e)/(c*d-e))^{(1/2)} * \text{EllipticF}((e*x+d)^{(1/2)} * (c/(c*d+e))^{(1/2)}, ((c*d+e) \\
& / (c*d-e))^{(1/2)}) * (e*x+d)^{(3/2)} * c^4 * d^4 + 7 * (-((e*x+d)*c-c*d-e)/(c*d+e))^{(1/2)} \\
& * (-((e*x+d)*c-c*d+e)/(c*d-e))^{(1/2)} * \text{EllipticE}((e*x+d)^{(1/2)} * (c/(c*d+e))^{(1/2)} \\
& / ((c*d+e)/(c*d-e))^{(1/2)}) * (e*x+d)^{(3/2)} * c^4 * d^4 - 3 * (-((e*x+d)*c-c*d-e)/(c* \\
& d+e))^{(1/2)} * (-((e*x+d)*c-c*d+e)/(c*d-e))^{(1/2)} * \text{EllipticPi}((e*x+d)^{(1/2)} * (c/ \\
& (c*d+e))^{(1/2)}, 1/c * (c*d+e)/d, (c/(c*d-e))^{(1/2)} / (c/(c*d+e))^{(1/2)}) * (e*x+d)^{(3/2)} * c^4 * d^4 - 13 * (c/(c*d+e))^{(1/2)} * (e*x+d)^2 * c^4 * d^4 + 7 * (-((e*x+d)*c-c*d-e)/(c*d+e))^{(1/2)} * (-((e*x+d)*c-c*d+e)/(c*d-e))^{(1/2)} * \text{EllipticF}((e*x+d)^{(1/2)} * (c/(c*d+e))^{(1/2)}, ((c*d+e)/(c*d-e))^{(1/2)}) * (e*x+d)^{(3/2)} * c^3 * d^3 * e - 7 * (-((e*x+d)*c-c*d-e)/(c*d+e))^{(1/2)} * (-((e*x+d)*c-c*d+e)/(c*d-e))^{(1/2)} * \text{EllipticE}((e*x+d)^{(1/2)} * (c/(c*d+e))^{(1/2)}, ((c*d+e)/(c*d-e))^{(1/2)}) * (e*x+d)^{(3/2)} * c^3 * d^3 * e - 3 * (c/(c*d+e))^{(1/2)} * (e*x+d)^3 * c^2 * d * e^2 + 5 * (c/(c*d+e))^{(1/2)} * (e*x+d) * c^4 * d^5 + 2 * (-((e*x+d)*c-c*d-e)/(c*d+e))^{(1/2)} * (-((e*x+d)*c-c*d+e)/(c*d-e))^{(1/2)} * \text{EllipticF}((e*x+d)^{(1/2)} * (c/(c*d+e))^{(1/2)}, ((c*d+e)/(c*d-e))^{(1/2)}) * (e*x+d)^{(3/2)} * c^2 * d^2 * e^2 - 3 * (-((e*x+d)*c-c*d-e)/(c*d+e))^{(1/2)} * (-((e*x+d)*c-c*d+e)/(c*d-e))^{(1/2)} * \text{EllipticE}((e*x+d)^{(1/2)} * (c/(c*d+e))^{(1/2)}, ((c*d+e)/(c*d-e))^{(1/2)}) * (e*x+d)^{(3/2)} * c^2 * d^2 * e^2 + 6 * (-((e*x+d)*c-c*d-e)/(c*d+e))^{(1/2)} * (-((e*x+d)*c-c*d+e)/(c*d-e))^{(1/2)} * \text{EllipticPi}((e*x+d)^{(1/2)} * (c/(c*d+e))^{(1/2)}, 1/c * (c*d+e)/d, (c/(c*d-e))^{(1/2)} / (c/(c*d+e))^{(1/2)}) * (e*x+d)^{(3/2)} * c^2 * d^2 * e^2 + 5 * (c/(c*d+e))^{(1/2)} * (e*x+d)^2 * c^2 * d^2 * e^2 + (c/(c*d+e))^{(1/2)} * c^4 * d^6 - 3 * (-((e*x+d)*c-c*d-e)/(c*d+e))^{(1/2)} * (-((e*x+d)*c-c*d+e)/(c*d-e))^{(1/2)} * \text{EllipticF}((e*x+d)^{(1/2)} * (c/(c*d+e))^{(1/2)}, ((c*d+e)/(c*d-e))^{(1/2)}) * (e*x+d)^{(3/2)} * c * d * e^3 + 3 * (-((e*x+d)*c-c*d-e)/(c*d+e))^{(1/2)} * (-((e*x+d)*c-c*d+e)/(c*d-e))^{(1/2)} * \text{EllipticE}((e*x+d)^{(1/2)} * (c/(c*d+e))^{(1/2)}, ((c*d+e)/(c*d-e))^{(1/2)}) * (e*x+d)^{(3/2)} * c * d * e^3 - 8 * (c/(c*d+e))^{(1/2)} * (e*x+d) * c^2 * d^3 * e^2 - 3 * (-((e*x+d)*c-c*d-e)/(c*d+e))^{(1/2)} * (-((e*x+d)*c-c*d+e)/(c*d-e))^{(1/2)} * \text{EllipticPi}((e*x+d)^{(1/2)} * (c/(c*d+e))^{(1/2)}, 1/c * (c*d+e)/d, (c/(c*d-e))^{(1/2)} / (c/(c*d+e))^{(1/2)}) * (e*x+d)^{(3/2)} * e^4 - 2 * (c/(c*d+e))^{(1/2)} * c^2 * d^4 * e^2 + 3 * (c/(c*d+e))^{(1/2)} * (e*x+d) * d * e^4 + (c/(c*d+e))^{(1/2)} * d^2 * e^4) / ((e*x+d)^2 * c^2 - 2 * (e*x+d) * c^2 * d + c^2 * d^2 - e^2) / (c*d-e) / (c*d+e) / d^3 / (e*x+d)^{(3/2)} / (c/(c*d+e))^{(1/2)} / (c^2 * d^2 - e^2)))
\end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/(e*x+d)^(7/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/(e*x+d)^(7/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asech(c*x))/(e*x+d)**(7/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsech}(cx) + a}{(ex + d)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/(e*x+d)^(7/2),x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)/(e*x + d)^(7/2), x)

3.87 $\int (d + ex)^m (a + b \operatorname{sech}^{-1}(cx)) dx$

Optimal. Leaf size=86

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\operatorname{Unintegrable}\left(\frac{(d+ex)^{m+1}}{x\sqrt{1-c^2x^2}}, x\right)}{e(m+1)} + \frac{(d+ex)^{m+1}(a+b\operatorname{sech}^{-1}(cx))}{e(m+1)}$$

[Out] $((d + e*x)^{(1 + m)*(a + b*\operatorname{ArcSech}[c*x]))/(e*(1 + m)) + (b*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{Unintegrable}[(d + e*x)^{(1 + m)}/(x*\operatorname{Sqrt}[1 - c^2*x^2]), x])/(e*(1 + m))$

Rubi [A] time = 0.0481412, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (d + ex)^m (a + b \operatorname{sech}^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] $\operatorname{Int}[(d + e*x)^m*(a + b*\operatorname{ArcSech}[c*x]), x]$

[Out] $((d + e*x)^{(1 + m)*(a + b*\operatorname{ArcSech}[c*x]))/(e*(1 + m)) + (b*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{Defer}[\operatorname{Int}[(d + e*x)^{(1 + m)}/(x*\operatorname{Sqrt}[1 - c^2*x^2]), x])/(e*(1 + m))$

Rubi steps

$$\int (d + ex)^m (a + b \operatorname{sech}^{-1}(cx)) dx = \frac{(d + ex)^{1+m} (a + b \operatorname{sech}^{-1}(cx))}{e(1 + m)} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{(d+ex)^{1+m}}{x\sqrt{1-c^2x^2}} dx}{e(1 + m)}$$

Mathematica [A] time = 1.83619, size = 0, normalized size = 0.

$$\int (d + ex)^m (a + b \operatorname{sech}^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] $\operatorname{Integrate}[(d + e*x)^m*(a + b*\operatorname{ArcSech}[c*x]), x]$

[Out] $\operatorname{Integrate}[(d + e*x)^m*(a + b*\operatorname{ArcSech}[c*x]), x]$

Maple [A] time = 1.454, size = 0, normalized size = 0.

$$\int (ex + d)^m (a + b \operatorname{arcsech}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{int}((e*x+d)^m*(a+b*\operatorname{arcsech}(c*x)), x)$

[Out] `int((e*x+d)^m*(a+b*arcsech(c*x)),x)`

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^m*(a+b*arcsech(c*x)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}((b \operatorname{arsech}(cx) + a)(ex + d)^m, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^m*(a+b*arcsech(c*x)),x, algorithm="fricas")`

[Out] `integral((b*arcsech(c*x) + a)*(e*x + d)^m, x)`

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{asech}(cx))(d + ex)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**m*(a+b*asech(c*x)),x)`

[Out] `Integral((a + b*asech(c*x))*(d + e*x)**m, x)`

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b \operatorname{arsech}(cx) + a)(ex + d)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^m*(a+b*arcsech(c*x)),x, algorithm="giac")`

[Out] `integrate((b*arcsech(c*x) + a)*(e*x + d)^m, x)`

3.88 $\int x^4 (d + ex^2) (a + b \operatorname{sech}^{-1}(cx)) dx$

Optimal. Leaf size=229

$$\frac{1}{5}dx^5 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{7}ex^7 (a + b \operatorname{sech}^{-1}(cx)) - \frac{bx^3 \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \sqrt{1-c^2x^2} (42c^2d + 25e)}{840c^4} - \frac{bx \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \sqrt{1-c^2x^2}}{560c^6}$$

```
[Out] -(b*(42*c^2*d + 25*e)*x*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2
])/ (560*c^6) - (b*(42*c^2*d + 25*e)*x^3*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*
Sqrt[1 - c^2*x^2])/ (840*c^4) - (b*e*x^5*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*
Sqrt[1 - c^2*x^2])/ (42*c^2) + (d*x^5*(a + b*ArcSech[c*x]))/5 + (e*x^7*(a +
b*ArcSech[c*x]))/7 + (b*(42*c^2*d + 25*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x
]*ArcSin[c*x])/ (560*c^7)
```

Rubi [A] time = 0.127182, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {14, 6301, 12, 459, 321, 216}

$$\frac{1}{5}dx^5 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{7}ex^7 (a + b \operatorname{sech}^{-1}(cx)) - \frac{bx^3 \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \sqrt{1-c^2x^2} (42c^2d + 25e)}{840c^4} - \frac{bx \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \sqrt{1-c^2x^2}}{560c^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] -(b*(42*c^2*d + 25*e)*x*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2
])/ (560*c^6) - (b*(42*c^2*d + 25*e)*x^3*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*
Sqrt[1 - c^2*x^2])/ (840*c^4) - (b*e*x^5*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*
Sqrt[1 - c^2*x^2])/ (42*c^2) + (d*x^5*(a + b*ArcSech[c*x]))/5 + (e*x^7*(a +
b*ArcSech[c*x]))/7 + (b*(42*c^2*d + 25*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x
]*ArcSin[c*x])/ (560*c^7)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_
+ (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)])*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p +
3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_.)*(v_)] /; FreeQ[b, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned} \int x^4 (d + ex^2) (a + b \operatorname{sech}^{-1}(cx)) dx &= \frac{1}{5} dx^5 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{7} ex^7 (a + b \operatorname{sech}^{-1}(cx)) + \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int x^4 dx \\ &= \frac{1}{5} dx^5 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{7} ex^7 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{35} \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int x^4 dx \\ &= -\frac{bex^5 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{42c^2} + \frac{1}{5} dx^5 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{7} ex^7 (a + b \operatorname{sech}^{-1}(cx)) \\ &= -\frac{b(42c^2d + 25e)x^3 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{840c^4} - \frac{bex^5 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{42c^2} \\ &= -\frac{b(42c^2d + 25e)x \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{560c^6} - \frac{b(42c^2d + 25e)x^3 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{840c^4} \\ &= -\frac{b(42c^2d + 25e)x \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{560c^6} - \frac{b(42c^2d + 25e)x^3 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{840c^4} \end{aligned}$$

Mathematica [C] time = 0.317618, size = 162, normalized size = 0.71

$$\frac{48ac^7x^5(7d + 5ex^2) - bcx\sqrt{\frac{1-cx}{cx+1}}(cx+1)(c^4(84dx^2 + 40ex^4) + 2c^2(63d + 25ex^2) + 75e) + 48bc^7x^5\operatorname{sech}^{-1}(cx)(7d + 5ex^2)}{1680c^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x^2)*(a + b*ArcSech[c*x]), x]
```

```
[Out] (48*a*c^7*x^5*(7*d + 5*e*x^2) - b*c*x*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(75*e + 2*c^2*(63*d + 25*e*x^2) + c^4*(84*d*x^2 + 40*e*x^4)) + 48*b*c^7*x^5*(7*d + 5*e*x^2)*ArcSech[c*x] + (3*I)*b*(42*c^2*d + 25*e)*Log[(-2*I)*c*x + 2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)]/(1680*c^7)
```

Maple [A] time = 0.185, size = 224, normalized size = 1.

$$\frac{1}{c^5} \left(\frac{a}{c^2} \left(\frac{ec^7x^7}{7} + \frac{c^7x^5d}{5} \right) + \frac{b}{c^2} \left(\frac{\operatorname{arcsech}(cx)ec^7x^7}{7} + \frac{\operatorname{arcsech}(cx)c^7x^5d}{5} + \frac{cx}{1680} \sqrt{\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} \left(-40c^5x^5e\sqrt{-c^2x^2+1} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(e*x^2+d)*(a+b*arcsech(c*x)),x)

[Out] $\frac{1}{c^5} \left(\frac{a}{c^2} \left(\frac{1}{7} e c^7 x^7 + \frac{1}{5} c^7 x^5 d \right) + \frac{b}{c^2} \left(\frac{1}{7} \operatorname{arcsech}(c x) e c^7 x^7 + \frac{1}{5} \operatorname{arcsech}(c x) c^7 x^5 d + \frac{1}{1680} \left(-\frac{c x - 1}{c x} \right)^{\frac{1}{2}} \frac{c x + 1}{c x} \left(-40 c^5 x^5 e \sqrt{-c^2 x^2 + 1} \right) \right) \right)$

Maxima [A] time = 1.47971, size = 329, normalized size = 1.44

$$\frac{1}{7} a e x^7 + \frac{1}{5} a d x^5 + \frac{1}{40} \left(8 x^5 \operatorname{arseth}(c x) - \frac{3 \left(\frac{1}{c^2 x^2} - 1 \right)^{\frac{3}{2}} + 5 \sqrt{\frac{1}{c^2 x^2} - 1}}{c^4 \left(\frac{1}{c^2 x^2} - 1 \right)^2 + 2 c^4 \left(\frac{1}{c^2 x^2} - 1 \right) + c^4} + \frac{3 \operatorname{arctan}\left(\sqrt{\frac{1}{c^2 x^2} - 1}\right)}{c^4} \right) b d + \frac{1}{336} \left(48 x^7 \operatorname{arseth}(c x) - \frac{15 \left(\frac{1}{c^2 x^2} - 1 \right)^{\frac{5}{2}} + 40 \left(\frac{1}{c^2 x^2} - 1 \right)^{\frac{3}{2}} + 33 \sqrt{\frac{1}{c^2 x^2} - 1}}{c^6 \left(\frac{1}{c^2 x^2} - 1 \right)^3 + 3 c^6 \left(\frac{1}{c^2 x^2} - 1 \right) + c^6} + 15 \operatorname{arctan}\left(\sqrt{\frac{1}{c^2 x^2} - 1}\right) \right) c b e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="maxima")

[Out] $\frac{1}{7} a e x^7 + \frac{1}{5} a d x^5 + \frac{1}{40} \left(8 x^5 \operatorname{arseth}(c x) - \left(\frac{3 \left(\frac{1}{c^2 x^2} - 1 \right)^{\frac{3}{2}} + 5 \sqrt{\frac{1}{c^2 x^2} - 1}}{c^4 \left(\frac{1}{c^2 x^2} - 1 \right)^2 + 2 c^4 \left(\frac{1}{c^2 x^2} - 1 \right) + c^4} + 3 \operatorname{arctan}\left(\sqrt{\frac{1}{c^2 x^2} - 1}\right) / c \right) b d + \frac{1}{336} \left(48 x^7 \operatorname{arseth}(c x) - \left(\frac{15 \left(\frac{1}{c^2 x^2} - 1 \right)^{\frac{5}{2}} + 40 \left(\frac{1}{c^2 x^2} - 1 \right)^{\frac{3}{2}} + 33 \sqrt{\frac{1}{c^2 x^2} - 1}}{c^6 \left(\frac{1}{c^2 x^2} - 1 \right)^3 + 3 c^6 \left(\frac{1}{c^2 x^2} - 1 \right) + c^6} + 15 \operatorname{arctan}\left(\sqrt{\frac{1}{c^2 x^2} - 1}\right) / c \right) c b e \right)$

Fricas [A] time = 2.98418, size = 590, normalized size = 2.58

$$240 a c^7 e x^7 + 336 a c^7 d x^5 - 6 (42 b c^2 d + 25 b e) \operatorname{arctan}\left(\frac{c x \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2}}}{c x}\right) - 48 (7 b c^7 d + 5 b c^7 e) \log\left(\frac{c x \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2}}}{x}\right) + 48 (5 b c^7 d + 7 b c^7 e) \log\left(\frac{c x \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2}}}{c x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="fricas")

[Out] $\frac{1}{1680} \left(240 a c^7 e x^7 + 336 a c^7 d x^5 - 6 (42 b c^2 d + 25 b e) \operatorname{arctan}\left(\frac{c x \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2}}}{c x}\right) - 48 (7 b c^7 d + 5 b c^7 e) \log\left(\frac{c x \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2}}}{x}\right) + 48 (5 b c^7 d + 7 b c^7 e) \log\left(\frac{c x \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2}}}{c x}\right) - (40 b c^6 e x^6 + 2 (42 b c^6 d + 25 b c^4 e) x^4 + 3 (42 b c^6 d + 25 b c^4 e) x^2 + 3 (42 b c^6 d + 25 b c^4 e) \right) \right)$

$$c^4*d + 25*b*c^2*e)*x^2)*\text{sqrt}(-(c^2*x^2 - 1)/(c^2*x^2)))/c^7$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^4 (a + b \operatorname{asech}(cx)) (d + ex^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(e*x**2+d)*(a+b*asech(c*x)),x)

[Out] Integral(x**4*(a + b*asech(c*x))*(d + e*x**2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + d)(b \operatorname{arsech}(cx) + a)x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="giac")

[Out] integrate((e*x^2 + d)*(b*arcsech(c*x) + a)*x^4, x)

3.89 $\int x^2 (d + ex^2) (a + b \operatorname{sech}^{-1}(cx)) dx$

Optimal. Leaf size=174

$$\frac{1}{3} dx^3 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{5} ex^5 (a + b \operatorname{sech}^{-1}(cx)) - \frac{bx \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \sqrt{1-c^2x^2} (20c^2d + 9e)}{120c^4} + \frac{b \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} (20c^2d + 9e)}{120c^5}$$

```
[Out] -(b*(20*c^2*d + 9*e)*x*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(120*c^4) - (b*e*x^3*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(20*c^2) + (d*x^3*(a + b*ArcSech[c*x]))/3 + (e*x^5*(a + b*ArcSech[c*x]))/5 + (b*(20*c^2*d + 9*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x])/(120*c^5)
```

Rubi [A] time = 0.102986, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {14, 6301, 12, 459, 321, 216}

$$\frac{1}{3} dx^3 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{5} ex^5 (a + b \operatorname{sech}^{-1}(cx)) - \frac{bx \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \sqrt{1-c^2x^2} (20c^2d + 9e)}{120c^4} + \frac{b \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} (20c^2d + 9e)}{120c^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] -(b*(20*c^2*d + 9*e)*x*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(120*c^4) - (b*e*x^3*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(20*c^2) + (d*x^3*(a + b*ArcSech[c*x]))/3 + (e*x^5*(a + b*ArcSech[c*x]))/5 + (b*(20*c^2*d + 9*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x])/(120*c^5)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)])*(b_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_.)*(v_)] /; FreeQ[b, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p + 1) + 1)), x]
```

+ 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]

Rule 321

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 216

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rubi steps

$$\begin{aligned} \int x^2 (d + ex^2) (a + b \operatorname{sech}^{-1}(cx)) dx &= \frac{1}{3} dx^3 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{5} ex^5 (a + b \operatorname{sech}^{-1}(cx)) + \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int -x \\ &= \frac{1}{3} dx^3 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{5} ex^5 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{15} \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \\ &= -\frac{bex^3 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{20c^2} + \frac{1}{3} dx^3 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{5} ex^5 (a + b \operatorname{sech}^{-1}(cx)) \\ &= -\frac{b(20c^2d + 9e)x \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{120c^4} - \frac{bex^3 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{20c^2} + \frac{1}{3} dx^3 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{5} ex^5 (a + b \operatorname{sech}^{-1}(cx)) \\ &= -\frac{b(20c^2d + 9e)x \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{120c^4} - \frac{bex^3 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{20c^2} + \frac{1}{3} dx^3 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{5} ex^5 (a + b \operatorname{sech}^{-1}(cx)) \end{aligned}$$

Mathematica [C] time = 0.208183, size = 144, normalized size = 0.83

$$\frac{8ac^5x^3(5d + 3ex^2) - bcx\sqrt{\frac{1-cx}{cx+1}}(cx+1)(c^2(20d + 6ex^2) + 9e) + 8bc^5x^3\operatorname{sech}^{-1}(cx)(5d + 3ex^2) + ib(20c^2d + 9e)\log\left(\frac{cx+1}{1-cx}\right)}{120c^5}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(d + e*x^2)*(a + b*ArcSech[c*x]),x]

[Out] (8*a*c^5*x^3*(5*d + 3*e*x^2) - b*c*x*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(9*e + c^2*(20*d + 6*e*x^2)) + 8*b*c^5*x^3*(5*d + 3*e*x^2)*ArcSech[c*x] + I*b*(20*c^2*d + 9*e)*Log[(-2*I)*c*x + 2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)]/(120*c^5)

Maple [A] time = 0.185, size = 182, normalized size = 1.1

$$\frac{1}{c^3} \left(\frac{a}{c^2} \left(\frac{c^5 x^5 e}{5} + \frac{c^5 x^3 d}{3} \right) + \frac{b}{c^2} \left(\frac{\operatorname{arcsech}(cx) c^5 x^5 e}{5} + \frac{\operatorname{arcsech}(cx) c^5 x^3 d}{3} - \frac{cx}{120} \sqrt{\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} \left(6ec^3x^3\sqrt{-c^2x^2+1} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(e*x^2+d)*(a+b*arcsech(c*x)),x)`

[Out] $\frac{1}{c^3} \left(\frac{a}{c^2} \left(\frac{1}{5} c^5 x^5 e + \frac{1}{3} c^5 x^3 d \right) + b \left(\frac{1}{5} \operatorname{arcsech}(c x) c^5 x^5 e + \frac{1}{3} \operatorname{arcsech}(c x) c^5 x^3 d - \frac{1}{120} \left(-\frac{c x - 1}{c x} \right)^{\frac{1}{2}} c x \left(\frac{c x + 1}{c x} \right)^{\frac{1}{2}} \left(6 e c^3 x^3 \left(-c^2 x^2 + 1 \right)^{\frac{1}{2}} + 20 \left(-c^2 x^2 + 1 \right)^{\frac{1}{2}} c^3 x d - 20 \arcsin(c x) c^2 d + 9 e c x \left(-c^2 x^2 + 1 \right)^{\frac{1}{2}} - 9 e \arcsin(c x) \right) / \left(-c^2 x^2 + 1 \right)^{\frac{1}{2}} \right) \right)$

Maxima [A] time = 1.50551, size = 246, normalized size = 1.41

$$\frac{1}{5} a e x^5 + \frac{1}{3} a d x^3 + \frac{1}{6} \left(2 x^3 \operatorname{arosech}(c x) - \frac{\frac{\sqrt{\frac{1}{c^2 x^2} - 1}}{c^2 \left(\frac{1}{c^2 x^2} - 1 \right) + c^2} + \frac{\arctan\left(\sqrt{\frac{1}{c^2 x^2} - 1}\right)}{c^2}}{c} \right) b d + \frac{1}{40} \left(8 x^5 \operatorname{arosech}(c x) - \frac{3 \left(\frac{1}{c^2 x^2} - 1 \right)^{\frac{3}{2}} + 5 \sqrt{\frac{1}{c^2 x^2} - 1}}{c^4 \left(\frac{1}{c^2 x^2} - 1 \right)^2 + 2 c^4 \left(\frac{1}{c^2 x^2} - 1 \right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="maxima")`

[Out] $\frac{1}{5} a e x^5 + \frac{1}{3} a d x^3 + \frac{1}{6} \left(2 x^3 \operatorname{arosech}(c x) - \left(\sqrt{\frac{1}{c^2 x^2} - 1} - 1 \right) / \left(c^2 \left(\frac{1}{c^2 x^2} - 1 \right) + c^2 \right) + \arctan\left(\sqrt{\frac{1}{c^2 x^2} - 1}\right) / c \right) b d + \frac{1}{40} \left(8 x^5 \operatorname{arosech}(c x) - \left(\left(\frac{1}{c^2 x^2} - 1 \right)^{\frac{3}{2}} + 5 \sqrt{\frac{1}{c^2 x^2} - 1} \right) / \left(c^4 \left(\frac{1}{c^2 x^2} - 1 \right)^2 + 2 c^4 \left(\frac{1}{c^2 x^2} - 1 \right) + c^4 \right) + 3 \arctan\left(\sqrt{\frac{1}{c^2 x^2} - 1}\right) / c \right) b e$

Fricas [B] time = 2.73193, size = 531, normalized size = 3.05

$$\frac{24 a c^5 e x^5 + 40 a c^5 d x^3 - 2 \left(20 b c^2 d + 9 b e \right) \arctan\left(\frac{c x \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2} - 1}}{c x}\right) - 8 \left(5 b c^5 d + 3 b c^5 e \right) \log\left(\frac{c x \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2} - 1}}{x}\right) + 8 \left(3 b c^5 e x^5 + 5 b c^5 d x^3 - 5 b c^5 d - 3 b c^5 e \right) \log\left(\frac{c x \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2} - 1}}{c x}\right) - \left(6 b c^4 e x^4 + \left(20 b c^4 d + 9 b c^2 e \right) x^2 \right) \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2}}}{120 c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="fricas")`

[Out] $\frac{1}{120} \left(24 a c^5 e x^5 + 40 a c^5 d x^3 - 2 \left(20 b c^2 d + 9 b e \right) \arctan\left(\frac{c x \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2} - 1}}{c x}\right) - 8 \left(5 b c^5 d + 3 b c^5 e \right) \log\left(\frac{c x \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2} - 1}}{x}\right) + 8 \left(3 b c^5 e x^5 + 5 b c^5 d x^3 - 5 b c^5 d - 3 b c^5 e \right) \log\left(\frac{c x \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2} - 1}}{c x}\right) - \left(6 b c^4 e x^4 + \left(20 b c^4 d + 9 b c^2 e \right) x^2 \right) \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2}} \right) / c^5$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 (a + b \operatorname{asech}(c x)) (d + e x^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)*(a+b*asech(c*x)),x)
```

```
[Out] Integral(x**2*(a + b*asech(c*x))*(d + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + d)(b \operatorname{ar} \operatorname{sech}(cx) + a)x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arcsech(c*x) + a)*x^2, x)
```

3.90 $\int (d + ex^2) (a + b \operatorname{sech}^{-1}(cx)) dx$

Optimal. Leaf size=112

$$dx (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{3} ex^3 (a + b \operatorname{sech}^{-1}(cx)) + \frac{b \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} (6c^2d + e) \sin^{-1}(cx)}{6c^3} - \frac{bex \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \sqrt{1-c^2x^2}}{6c^2}$$

[Out] $-(bex\sqrt{1+c^2x^2})\sqrt{1+c^2x^2}/(6c^2) + dx$
 $*(a + b\operatorname{ArcSech}[c*x]) + (ex^3*(a + b\operatorname{ArcSech}[c*x]))/3 + (b*(6c^2*d + e)*S$
 $\operatorname{qrt}[(1 + c*x)^{-1}]\sqrt{1 + c*x}*\operatorname{ArcSin}[c*x])/(6c^3)$

Rubi [A] time = 0.0508121, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {6291, 12, 388, 216}

$$dx (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{3} ex^3 (a + b \operatorname{sech}^{-1}(cx)) + \frac{b \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} (6c^2d + e) \sin^{-1}(cx)}{6c^3} - \frac{bex \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \sqrt{1-c^2x^2}}{6c^2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(d + ex^2)*(a + b\operatorname{ArcSech}[c*x]), x]$

[Out] $-(bex\sqrt{1+c^2x^2})\sqrt{1+c^2x^2}/(6c^2) + dx$
 $*(a + b\operatorname{ArcSech}[c*x]) + (ex^3*(a + b\operatorname{ArcSech}[c*x]))/3 + (b*(6c^2*d + e)*S$
 $\operatorname{qrt}[(1 + c*x)^{-1}]\sqrt{1 + c*x}*\operatorname{ArcSin}[c*x])/(6c^3)$

Rule 6291

$\operatorname{Int}[(a_.) + \operatorname{ArcSech}[(c_.)*(x_)]*(b_.)]*((d_.) + (e_.)*(x_)^2)^{(p_.)}, x_Symbol] :> \operatorname{With}[\{u = \operatorname{IntHide}[(d + ex^2)^p, x]\}, \operatorname{Dist}[a + b\operatorname{ArcSech}[c*x], u, x] + \operatorname{Dist}[b\sqrt{1 + c*x}*\sqrt{1/(1 + c*x)}], \operatorname{Int}[\operatorname{SimplifyIntegrand}[u/(x*\sqrt{1 - c*x}*\sqrt{1 + c*x}), x], x], x] /; \operatorname{FreeQ}[\{a, b, c, d, e\}, x] \&\& (\operatorname{IGtQ}[p, 0] \|\operatorname{ILtQ}[p + 1/2, 0])$

Rule 12

$\operatorname{Int}[(a_)*(u_), x_Symbol] :> \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \&\& !\operatorname{MatchQ}[u, (b_)*(v_)] /; \operatorname{FreeQ}[b, x]$

Rule 388

$\operatorname{Int}[(a_.) + (b_.)*(x_)^{(n_)}]^{(p_)}*((c_.) + (d_.)*(x_)^{(n_)}), x_Symbol] :> \operatorname{Simp}[(d*x*(a + b*x^n)^{(p+1)})/(b*(n*(p+1) + 1)), x] - \operatorname{Dist}[(a*d - b*c*(n*(p+1) + 1))/(b*(n*(p+1) + 1)), \operatorname{Int}[(a + b*x^n)^p, x], x] /; \operatorname{FreeQ}[\{a, b, c, d, n\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{NeQ}[n*(p+1) + 1, 0]$

Rule 216

$\operatorname{Int}[1/\sqrt{(a_.) + (b_.)*(x_)^2}, x_Symbol] :> \operatorname{Simp}[\operatorname{ArcSin}[(\operatorname{Rt}[-b, 2]*x)/\sqrt{a}]]/\operatorname{Rt}[-b, 2], x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{GtQ}[a, 0] \&\& \operatorname{NegQ}[b]$

Rubi steps

$$\begin{aligned}
\int (d + ex^2)(a + b\operatorname{sech}^{-1}(cx)) dx &= dx(a + b\operatorname{sech}^{-1}(cx)) + \frac{1}{3}ex^3(a + b\operatorname{sech}^{-1}(cx)) + \left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{3d +}{3\sqrt{1-}} \\
&= dx(a + b\operatorname{sech}^{-1}(cx)) + \frac{1}{3}ex^3(a + b\operatorname{sech}^{-1}(cx)) + \frac{1}{3}\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{3d -}{\sqrt{1-}} \\
&= -\frac{bex\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^2} + dx(a + b\operatorname{sech}^{-1}(cx)) + \frac{1}{3}ex^3(a + b\operatorname{sech}^{-1}(cx)) \\
&= -\frac{bex\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{6c^2} + dx(a + b\operatorname{sech}^{-1}(cx)) + \frac{1}{3}ex^3(a + b\operatorname{sech}^{-1}(cx))
\end{aligned}$$

Mathematica [C] time = 0.356175, size = 169, normalized size = 1.51

$$adx + \frac{1}{3}aex^3 - \frac{bd\sqrt{\frac{1-cx}{cx+1}}\sqrt{1-c^2x^2}\sin^{-1}(cx)}{c(cx-1)} + be\sqrt{\frac{1-cx}{cx+1}}\left(-\frac{x}{6c^2} - \frac{x^2}{6c}\right) + \frac{ibe\log\left(2\sqrt{\frac{1-cx}{cx+1}}(cx+1) - 2icx\right)}{6c^3} + bdx\operatorname{sech}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)*(a + b*ArcSech[c*x]), x]

[Out] a*d*x + (a*e*x^3)/3 + b*e*Sqrt[(1 - c*x)/(1 + c*x)]*(-x/(6*c^2) - x^2/(6*c)) + b*d*x*ArcSech[c*x] + (b*e*x^3*ArcSech[c*x])/3 - (b*d*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*ArcSin[c*x])/(c*(-1 + c*x)) + ((I/6)*b*e*Log[(-2*I)*c*x + 2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)])/c^3

Maple [A] time = 0.175, size = 135, normalized size = 1.2

$$\frac{1}{c}\left(\frac{a}{c^2}\left(\frac{ec^3x^3}{3} + xc^3d\right) + \frac{b}{c^2}\left(\frac{\operatorname{arcsech}(cx)ec^3x^3}{3} + \operatorname{arcsech}(cx)c^3dx + \frac{cx}{6}\sqrt{\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\left(6\arcsin(cx)c^2d - ecx\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)*(a+b*arcsech(c*x)), x)

[Out] 1/c*(a/c^2*(1/3*e*c^3*x^3+x*c^3*d)+b/c^2*(1/3*arcsech(c*x)*e*c^3*x^3+arcsech(c*x)*c^3*d*x+1/6*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)*(6*arcsin(c*x)*c^2*d-e*c*x*(-c^2*x^2+1)^(1/2)+e*arcsin(c*x))/(-c^2*x^2+1)^(1/2)))

Maxima [A] time = 1.49131, size = 144, normalized size = 1.29

$$\frac{1}{3}aex^3 + \frac{1}{6}\left(2x^3\operatorname{arsech}(cx) - \frac{\frac{\sqrt{\frac{1}{c^2x^2}-1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^2}}{c}\right)be + adx + \frac{\left(cx\operatorname{arsech}(cx) - \arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)\right)bd}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(a+b*arcsech(c*x)), x, algorithm="maxima")

[Out] $\frac{1}{3}ae^x x^3 + \frac{1}{6}(2x^3 \operatorname{arcsech}(cx) - (\sqrt{1/(c^2 x^2) - 1})/(c^2(1/(c^2 x^2) - 1) + c^2) + \arctan(\sqrt{1/(c^2 x^2) - 1}))/c^2)/c * b * e + a * d * x + (c * x * \operatorname{arcsech}(cx) - \arctan(\sqrt{1/(c^2 x^2) - 1})) * b * d / c$

Fricas [B] time = 2.5259, size = 460, normalized size = 4.11

$$\frac{2ac^3ex^3 - bc^2ex^2\sqrt{-\frac{c^2x^2-1}{c^2x^2}} + 6ac^3dx - 2(6bc^2d + be)\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right) - 2(3bc^3d + bc^3e)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{x}\right) + 2(b^2c^3d + bc^3e)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right)}{6c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="fricas")

[Out] $\frac{1}{6}(2a*c^3*e*x^3 - b*c^2*e*x^2*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)}) + 6*a*c^3*d*x - 2*(6*b*c^2*d + b*e)*\arctan((c*x*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)}) - 1)/(c*x)) - 2*(3*b*c^3*d + b*c^3*e)*\log((c*x*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)}) - 1)/x) + 2*(b*c^3*e*x^3 + 3*b*c^3*d*x - 3*b*c^3*d - b*c^3*e)*\log((c*x*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)}) + 1)/(c*x))/c^3$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{asech}(cx))(d + ex^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)*(a+b*asech(c*x)),x)

[Out] Integral((a + b*asech(c*x))*(d + e*x**2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + d)(b \operatorname{arsech}(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="giac")

[Out] integrate((e*x^2 + d)*(b*arcsech(c*x) + a), x)

$$3.91 \quad \int \frac{(d+ex^2)(a+b\operatorname{sech}^{-1}(cx))}{x^2} dx$$

Optimal. Leaf size=96

$$-\frac{d(a+b\operatorname{sech}^{-1}(cx))}{x} + ex(a+b\operatorname{sech}^{-1}(cx)) + \frac{bd\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{x} + \frac{be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sin^{-1}(cx)}{c}$$

[Out] (b*d*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/x - (d*(a + b*ArcSech[c*x]))/x + e*x*(a + b*ArcSech[c*x]) + (b*e*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x])/c

Rubi [A] time = 0.0658553, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {14, 6301, 451, 216}

$$-\frac{d(a+b\operatorname{sech}^{-1}(cx))}{x} + ex(a+b\operatorname{sech}^{-1}(cx)) + \frac{bd\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{x} + \frac{be\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sin^{-1}(cx)}{c}$$

Antiderivative was successfully verified.

[In] Int[((d + e*x^2)*(a + b*ArcSech[c*x]))/x^2,x]

[Out] (b*d*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/x - (d*(a + b*ArcSech[c*x]))/x + e*x*(a + b*ArcSech[c*x]) + (b*e*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x])/c

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 6301

Int[((a_) + ArcSech[(c_)*(x_)])*(b_)*((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

Rule 451

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1]))

Rule 216

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)(a + b \operatorname{sech}^{-1}(cx))}{x^2} dx &= -\frac{d(a + b \operatorname{sech}^{-1}(cx))}{x} + ex(a + b \operatorname{sech}^{-1}(cx)) + \left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{-d + ex^2}{x^2\sqrt{1-c^2x^2}} \\ &= \frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{x} - \frac{d(a + b \operatorname{sech}^{-1}(cx))}{x} + ex(a + b \operatorname{sech}^{-1}(cx)) + \left(be\sqrt{\frac{1}{1+cx}} \right) \\ &= \frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{x} - \frac{d(a + b \operatorname{sech}^{-1}(cx))}{x} + ex(a + b \operatorname{sech}^{-1}(cx)) + \frac{be\sqrt{\frac{1}{1+cx}}}{x} \end{aligned}$$

Mathematica [A] time = 0.238545, size = 107, normalized size = 1.11

$$-\frac{ad}{x} + aex - \frac{be\sqrt{\frac{1-cx}{cx+1}}\sqrt{1-c^2x^2}\sin^{-1}(cx)}{c(cx-1)} + bd\left(c + \frac{1}{x}\right)\sqrt{\frac{1-cx}{cx+1}} - \frac{bd\operatorname{sech}^{-1}(cx)}{x} + bex\operatorname{sech}^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[((d + e*x^2)*(a + b*ArcSech[c*x]))/x^2,x]

[Out] -((a*d)/x) + a*e*x + b*d*(c + x^(-1))*Sqrt[(1 - c*x)/(1 + c*x)] - (b*d*ArcSech[c*x])/x + b*e*x*ArcSech[c*x] - (b*e*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*ArcSin[c*x])/(c*(-1 + c*x))

Maple [A] time = 0.21, size = 114, normalized size = 1.2

$$c\left(\frac{a}{c^2}\left(cx e - \frac{cd}{x}\right) + \frac{b}{c^2}\left(\operatorname{arcsech}(cx)cx e - \frac{\operatorname{arcsech}(cx)cd}{x} + \sqrt{-\frac{cx-1}{cx}}\sqrt{\frac{cx+1}{cx}}\left(c^2d\sqrt{-c^2x^2+1} + \arcsin(cx)cx e\right)\frac{1}{\sqrt{-c^2x^2+1}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)*(a+b*arcsech(c*x))/x^2,x)

[Out] c*(a/c^2*(c*x*e-c*d/x)+b/c^2*(arcsech(c*x)*c*x*e-arcsech(c*x)*c*d/x+(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*(c^2*d*(-c^2*x^2+1)^(1/2)+arcsin(c*x)*c*x*e)/(-c^2*x^2+1)^(1/2)))

Maxima [A] time = 1.00703, size = 89, normalized size = 0.93

$$\left(c\sqrt{\frac{1}{c^2x^2}-1} - \frac{\operatorname{ar} \operatorname{sech}(cx)}{x}\right)bd + aex + \frac{\left(cx \operatorname{ar} \operatorname{sech}(cx) - \arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)\right)be}{c} - \frac{ad}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^2,x, algorithm="maxima")

[Out] (c*sqrt(1/(c^2*x^2) - 1) - arcsech(c*x)/x)*b*d + a*e*x + (c*x*arcsech(c*x) - arctan(sqrt(1/(c^2*x^2) - 1)))*b*e/c - a*d/x

Fricas [B] time = 2.08469, size = 401, normalized size = 4.18

$$bc^2 dx \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2}} + acex^2 - 2bex \arctan\left(\frac{cx \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2}} - 1}{cx}\right) - acd + (bcd - bce)x \log\left(\frac{cx \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2}} - 1}{x}\right) + (bcex^2 - bcd + (bcd - bce)x) \log\left(\frac{cx \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2}} + 1}{cx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^2,x, algorithm="fricas")

[Out] (b*c^2*d*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + a*c*e*x^2 - 2*b*e*x*arctan((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c*x)) - a*c*d + (b*c*d - b*c*e)*x*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + (b*c*e*x^2 - b*c*d + (b*c*d - b*c*e)*x)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/(c*x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{asech}(cx))(d + ex^2)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)*(a+b*asech(c*x))/x**2,x)

[Out] Integral((a + b*asech(c*x))*(d + e*x**2)/x**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)(b \operatorname{arsech}(cx) + a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^2,x, algorithm="giac")

[Out] integrate((e*x^2 + d)*(b*arcsech(c*x) + a)/x^2, x)

$$3.92 \quad \int \frac{(d+ex^2)(a+b\operatorname{sech}^{-1}(cx))}{x^4} dx$$

Optimal. Leaf size=126

$$\frac{d(a+b\operatorname{sech}^{-1}(cx))}{3x^3} - \frac{e(a+b\operatorname{sech}^{-1}(cx))}{x} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(2c^2d+9e)}{9x} + \frac{bd\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{9x^3}$$

[Out] (b*d*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(9*x^3) + (b*(2*c^2*d + 9*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(9*x) - (d*(a + b*ArcSech[c*x]))/(3*x^3) - (e*(a + b*ArcSech[c*x]))/x

Rubi [A] time = 0.0792224, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {14, 6301, 12, 453, 264}

$$\frac{d(a+b\operatorname{sech}^{-1}(cx))}{3x^3} - \frac{e(a+b\operatorname{sech}^{-1}(cx))}{x} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(2c^2d+9e)}{9x} + \frac{bd\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{9x^3}$$

Antiderivative was successfully verified.

[In] Int[((d + e*x^2)*(a + b*ArcSech[c*x]))/x^4, x]

[Out] (b*d*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(9*x^3) + (b*(2*c^2*d + 9*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(9*x) - (d*(a + b*ArcSech[c*x]))/(3*x^3) - (e*(a + b*ArcSech[c*x]))/x

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 6301

Int[((a_) + ArcSech[(c_)*(x_)]*(b_))*((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]) || (IGtQ[(m + 1)/2, 0] && !ILtQ[p, 0] && GtQ[m + 2*p + 3, 0]) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 453

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e^(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (

LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

Rule 264

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c *x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)(a + b \operatorname{sech}^{-1}(cx))}{x^4} dx &= -\frac{d(a + b \operatorname{sech}^{-1}(cx))}{3x^3} - \frac{e(a + b \operatorname{sech}^{-1}(cx))}{x} + \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{-d - 3e}{3x^4 \sqrt{1+cx}} dx \\ &= -\frac{d(a + b \operatorname{sech}^{-1}(cx))}{3x^3} - \frac{e(a + b \operatorname{sech}^{-1}(cx))}{x} + \frac{1}{3} \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{-d - 3e}{x^4 \sqrt{1+cx}} dx \\ &= \frac{bd \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{9x^3} - \frac{d(a + b \operatorname{sech}^{-1}(cx))}{3x^3} - \frac{e(a + b \operatorname{sech}^{-1}(cx))}{x} + \frac{1}{9} \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \right) \int \frac{-d - 3e}{x^4 \sqrt{1+cx}} dx \\ &= \frac{bd \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{9x^3} + \frac{b(2c^2d + 9e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{9x} - \frac{d(a + b \operatorname{sech}^{-1}(cx))}{3x^3} - \frac{e(a + b \operatorname{sech}^{-1}(cx))}{x} \end{aligned}$$

Mathematica [A] time = 0.104381, size = 76, normalized size = 0.6

$$\frac{-3a(d + 3ex^2) + b \sqrt{\frac{1-cx}{cx+1}}(cx+1)(2c^2dx^2 + d + 9ex^2) - 3b \operatorname{sech}^{-1}(cx)(d + 3ex^2)}{9x^3}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e*x^2)*(a + b*ArcSech[c*x]))/x^4, x]

[Out] (-3*a*(d + 3*e*x^2) + b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(d + 2*c^2*d*x^2 + 9*e*x^2) - 3*b*(d + 3*e*x^2)*ArcSech[c*x])/(9*x^3)

Maple [A] time = 0.184, size = 123, normalized size = 1.

$$c^3 \left(\frac{a}{c^2} \left(-\frac{e}{cx} - \frac{d}{3cx^3} \right) + \frac{b}{c^2} \left(-\frac{\operatorname{arcsech}(cx)e}{cx} - \frac{\operatorname{arcsech}(cx)d}{3cx^3} + \frac{2c^4dx^2 + 9c^2x^2e + c^2d}{9c^2x^2} \sqrt{\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)*(a+b*arcsech(c*x))/x^4, x)

[Out] c^3*(a/c^2*(-e/c/x-1/3/c*d/x^3)+b/c^2*(-arcsech(c*x)*e/c/x-1/3*arcsech(c*x)/c*d/x^3+1/9*(-(c*x-1)/c/x)^(1/2)/c^2/x^2*((c*x+1)/c/x)^(1/2)*(2*c^4*d*x^2+9*c^2*e*x^2+c^2*d)))

Maxima [A] time = 0.99585, size = 123, normalized size = 0.98

$$\left(c \sqrt{\frac{1}{c^2x^2} - 1} - \frac{\operatorname{arosech}(cx)}{x} \right) be + \frac{1}{9} bd \left(\frac{c^4 \left(\frac{1}{c^2x^2} - 1 \right)^{\frac{3}{2}} + 3c^4 \sqrt{\frac{1}{c^2x^2} - 1}}{c} - \frac{3 \operatorname{arosech}(cx)}{x^3} \right) - \frac{ae}{x} - \frac{ad}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^4,x, algorithm="maxima")

[Out] (c*sqrt(1/(c^2*x^2) - 1) - arcsech(c*x)/x)*b*e + 1/9*b*d*((c^4*(1/(c^2*x^2) - 1)^(3/2) + 3*c^4*sqrt(1/(c^2*x^2) - 1))/c - 3*arcsech(c*x)/x^3) - a*e/x - 1/3*a*d/x^3

Fricas [A] time = 1.75543, size = 236, normalized size = 1.87

$$\frac{9 a e x^2 + 3 a d + 3 (3 b e x^2 + b d) \log \left(\frac{c x \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2} + 1}}{c x} \right) - (b c d x + (2 b c^3 d + 9 b c e) x^3) \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2}}}{9 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^4,x, algorithm="fricas")

[Out] -1/9*(9*a*e*x^2 + 3*a*d + 3*(3*b*e*x^2 + b*d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (b*c*d*x + (2*b*c^3*d + 9*b*c*e)*x^3)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/x^3

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{asech}(cx))(d + ex^2)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)*(a+b*asech(c*x))/x**4,x)

[Out] Integral((a + b*asech(c*x))*(d + e*x**2)/x**4, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)(b \operatorname{arsech}(cx) + a)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^4,x, algorithm="giac")

[Out] integrate((e*x^2 + d)*(b*arcsech(c*x) + a)/x^4, x)

$$3.93 \quad \int \frac{(d+ex^2)(a+b\operatorname{sech}^{-1}(cx))}{x^6} dx$$

Optimal. Leaf size=183

$$\frac{d(a+b\operatorname{sech}^{-1}(cx))}{5x^5} - \frac{e(a+b\operatorname{sech}^{-1}(cx))}{3x^3} + \frac{2bc^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(12c^2d+25e)}{225x} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{225x^3}$$

[Out] (b*d*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(25*x^5) + (b*(12*c^2*d + 25*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(225*x^3) + (2*b*c^2*(12*c^2*d + 25*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(225*x) - (d*(a + b*ArcSech[c*x]))/(5*x^5) - (e*(a + b*ArcSech[c*x]))/(3*x^3)

Rubi [A] time = 0.0994742, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {14, 6301, 12, 453, 271, 264}

$$\frac{d(a+b\operatorname{sech}^{-1}(cx))}{5x^5} - \frac{e(a+b\operatorname{sech}^{-1}(cx))}{3x^3} + \frac{2bc^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(12c^2d+25e)}{225x} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{225x^3}$$

Antiderivative was successfully verified.

[In] Int[((d + e*x^2)*(a + b*ArcSech[c*x]))/x^6, x]

[Out] (b*d*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(25*x^5) + (b*(12*c^2*d + 25*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(225*x^3) + (2*b*c^2*(12*c^2*d + 25*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(225*x) - (d*(a + b*ArcSech[c*x]))/(5*x^5) - (e*(a + b*ArcSech[c*x]))/(3*x^3)

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 6301

Int[((a_) + ArcSech[(c_)*(x_)]*(b_))*((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 453

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),

$x] + \text{Dist}[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), \text{Int}[(e*x)^{(m + n)}*(a + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, p\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& (\text{IntegerQ}[n] \mid\mid \text{GtQ}[e, 0]) \&\& ((\text{GtQ}[n, 0] \&\& \text{LtQ}[m, -1]) \mid\mid (\text{LtQ}[n, 0] \&\& \text{GtQ}[m + n, -1])) \&\& !\text{ILtQ}[p, -1]$

Rule 271

$\text{Int}[(x_)^{(m_*)}*((a_) + (b_)*(x_)^{(n_*)})^{(p_)}, x_Symbol] := \text{Simp}[(x^{(m + 1)}*(a + b*x^n)^{(p + 1)})/(a*(m + 1)), x] - \text{Dist}[(b*(m + n*(p + 1) + 1))/(a*(m + 1)), \text{Int}[x^{(m + n)}*(a + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, m, n, p\}, x] \&\& \text{ILtQ}[\text{Simplify}[(m + 1)/n + p + 1], 0] \&\& \text{NeQ}[m, -1]$

Rule 264

$\text{Int}[(c_)*(x_)^{(m_*)}*((a_) + (b_)*(x_)^{(n_*)})^{(p_)}, x_Symbol] := \text{Simp}[(c*x)^{(m + 1)}*(a + b*x^n)^{(p + 1)})/(a*c*(m + 1)), x] /; \text{FreeQ}\{a, b, c, m, n, p\}, x] \&\& \text{EqQ}[(m + 1)/n + p + 1, 0] \&\& \text{NeQ}[m, -1]$

Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)(a + b \operatorname{sech}^{-1}(cx))}{x^6} dx &= -\frac{d(a + b \operatorname{sech}^{-1}(cx))}{5x^5} - \frac{e(a + b \operatorname{sech}^{-1}(cx))}{3x^3} + \left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{-3d - 5ex^2}{15x^6\sqrt{1-c^2x^2}} dx \\ &= -\frac{d(a + b \operatorname{sech}^{-1}(cx))}{5x^5} - \frac{e(a + b \operatorname{sech}^{-1}(cx))}{3x^3} + \frac{1}{15} \left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{-3d - 5ex^2}{x^6\sqrt{1-c^2x^2}} dx \\ &= \frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{25x^5} - \frac{d(a + b \operatorname{sech}^{-1}(cx))}{5x^5} - \frac{e(a + b \operatorname{sech}^{-1}(cx))}{3x^3} + \frac{1}{75} \left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\right) \int \frac{-3d - 5ex^2}{x^6\sqrt{1-c^2x^2}} dx \\ &= \frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{25x^5} + \frac{b(12c^2d + 25e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{225x^3} - \frac{d(a + b \operatorname{sech}^{-1}(cx))}{5x^5} - \frac{e(a + b \operatorname{sech}^{-1}(cx))}{3x^3} \\ &= \frac{bd\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{25x^5} + \frac{b(12c^2d + 25e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{225x^3} + \frac{2bc^2(12c^2d + 25e)}{225c^3x^3} - \frac{d(a + b \operatorname{sech}^{-1}(cx))}{5x^5} - \frac{e(a + b \operatorname{sech}^{-1}(cx))}{3x^3} \end{aligned}$$

Mathematica [A] time = 0.164759, size = 101, normalized size = 0.55

$$\frac{-15a(3d + 5ex^2) + b\sqrt{\frac{1-cx}{cx+1}}(cx+1)(3d(8c^4x^4 + 4c^2x^2 + 3) + 25ex^2(2c^2x^2 + 1)) - 15b \operatorname{sech}^{-1}(cx)(3d + 5ex^2)}{225x^5}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e*x^2)*(a + b*ArcSech[c*x]))/x^6, x]

[Out] (-15*a*(3*d + 5*e*x^2) + b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(25*e*x^2*(1 + 2*c^2*x^2) + 3*d*(3 + 4*c^2*x^2 + 8*c^4*x^4)) - 15*b*(3*d + 5*e*x^2)*ArcSech[c*x])/(225*x^5)

Maple [A] time = 0.184, size = 142, normalized size = 0.8

$$c^5 \left(\frac{a}{c^2} \left(-\frac{d}{5c^3x^5} - \frac{e}{3c^3x^3} \right) + \frac{b}{c^2} \left(-\frac{\operatorname{arcsech}(cx)d}{5c^3x^5} - \frac{\operatorname{arcsech}(cx)e}{3c^3x^3} + \frac{24c^6dx^4 + 50c^4ex^4 + 12c^4dx^2 + 25c^2x^2e + 9c^2d}{225c^4x^4} \right) \sqrt{\frac{1-cx}{1+cx}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)*(a+b*arcsech(c*x))/x^6,x)

[Out] $c^5*(a/c^2*(-1/5/c^3*d/x^5-1/3*e/c^3/x^3)+b/c^2*(-1/5*arcsech(c*x)/c^3*d/x^5-1/3*arcsech(c*x)*e/c^3/x^3+1/225*(-(c*x-1)/c/x)^{(1/2)}/c^4/x^4*((c*x+1)/c/x)^{(1/2)}*(24*c^6*d*x^4+50*c^4*e*x^4+12*c^4*d*x^2+25*c^2*e*x^2+9*c^2*d))$

Maxima [A] time = 1.00069, size = 178, normalized size = 0.97

$$\frac{1}{75}bd \left(\frac{3c^6 \left(\frac{1}{c^2x^2} - 1 \right)^{\frac{5}{2}} + 10c^6 \left(\frac{1}{c^2x^2} - 1 \right)^{\frac{3}{2}} + 15c^6 \sqrt{\frac{1}{c^2x^2} - 1}}{c} - \frac{15 \operatorname{ar} \operatorname{sech}(cx)}{x^5} \right) + \frac{1}{9}be \left(\frac{c^4 \left(\frac{1}{c^2x^2} - 1 \right)^{\frac{3}{2}} + 3c^4 \sqrt{\frac{1}{c^2x^2} - 1}}{c} - \frac{3 \operatorname{ar} \operatorname{sech}(cx)}{x^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^6,x, algorithm="maxima")

[Out] $1/75*b*d*((3*c^6*(1/(c^2*x^2) - 1)^{(5/2)} + 10*c^6*(1/(c^2*x^2) - 1)^{(3/2)} + 15*c^6*\sqrt{1/(c^2*x^2) - 1})/c - 15*arcsech(c*x)/x^5) + 1/9*b*e*((c^4*(1/(c^2*x^2) - 1)^{(3/2)} + 3*c^4*\sqrt{1/(c^2*x^2) - 1})/c - 3*arcsech(c*x)/x^3) - 1/3*a*e/x^3 - 1/5*a*d/x^5$

Fricas [A] time = 1.89353, size = 297, normalized size = 1.62

$$\frac{75 a e x^2 + 45 a d + 15 (5 b e x^2 + 3 b d) \log \left(\frac{c x \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2}} + 1}{c x} \right) - (2 (12 b c^5 d + 25 b c^3 e) x^5 + 9 b c d x + (12 b c^3 d + 25 b c e) x^3) \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2}}}{225 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^6,x, algorithm="fricas")

[Out] $-1/225*(75*a*e*x^2 + 45*a*d + 15*(5*b*e*x^2 + 3*b*d)*\log((c*x*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)} + 1)/(c*x)) - (2*(12*b*c^5*d + 25*b*c^3*e)*x^5 + 9*b*c*d*x + (12*b*c^3*d + 25*b*c*e)*x^3)*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)})/x^5$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{asech}(cx))(d + ex^2)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)*(a+b*asech(c*x))/x**6,x)

[Out] Integral((a + b*asech(c*x))*(d + e*x**2)/x**6, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)(b \operatorname{ar} \operatorname{sech}(cx) + a)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^6,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arcsech(c*x) + a)/x^6, x)
```

$$3.94 \quad \int \frac{(d+ex^2)(a+b\operatorname{sech}^{-1}(cx))}{x^8} dx$$

Optimal. Leaf size=238

$$\frac{d(a+b\operatorname{sech}^{-1}(cx))}{7x^7} - \frac{e(a+b\operatorname{sech}^{-1}(cx))}{5x^5} + \frac{8bc^4\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(30c^2d+49e)}{3675x} + \frac{4bc^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{3675x}$$

[Out] (b*d*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(49*x^7) + (b*(30*c^2*d + 49*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(1225*x^5) + (4*b*c^2*(30*c^2*d + 49*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(3675*x^3) + (8*b*c^4*(30*c^2*d + 49*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(3675*x) - (d*(a + b*ArcSech[c*x]))/(7*x^7) - (e*(a + b*ArcSech[c*x]))/(5*x^5)

Rubi [A] time = 0.121574, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {14, 6301, 12, 453, 271, 264}

$$\frac{d(a+b\operatorname{sech}^{-1}(cx))}{7x^7} - \frac{e(a+b\operatorname{sech}^{-1}(cx))}{5x^5} + \frac{8bc^4\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(30c^2d+49e)}{3675x} + \frac{4bc^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{3675x}$$

Antiderivative was successfully verified.

[In] Int[((d + e*x^2)*(a + b*ArcSech[c*x]))/x^8, x]

[Out] (b*d*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(49*x^7) + (b*(30*c^2*d + 49*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(1225*x^5) + (4*b*c^2*(30*c^2*d + 49*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(3675*x^3) + (8*b*c^4*(30*c^2*d + 49*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(3675*x) - (d*(a + b*ArcSech[c*x]))/(7*x^7) - (e*(a + b*ArcSech[c*x]))/(5*x^5)

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 6301

Int[((a_) + ArcSech[(c_)*(x_)])*(b_)*((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m + 1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && ILtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)(a + b \operatorname{sech}^{-1}(cx))}{x^8} dx &= -\frac{d(a + b \operatorname{sech}^{-1}(cx))}{7x^7} - \frac{e(a + b \operatorname{sech}^{-1}(cx))}{5x^5} + \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{-5d - 7ex^2}{35x^8 \sqrt{1-c^2x^2}} dx \\ &= -\frac{d(a + b \operatorname{sech}^{-1}(cx))}{7x^7} - \frac{e(a + b \operatorname{sech}^{-1}(cx))}{5x^5} + \frac{1}{35} \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{-5d - 7ex^2}{x^8 \sqrt{1-c^2x^2}} dx \\ &= \frac{bd \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{49x^7} - \frac{d(a + b \operatorname{sech}^{-1}(cx))}{7x^7} - \frac{e(a + b \operatorname{sech}^{-1}(cx))}{5x^5} + \frac{1}{245} \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{-5d - 7ex^2}{x^8 \sqrt{1-c^2x^2}} dx \\ &= \frac{bd \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{49x^7} + \frac{b(30c^2d + 49e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{1225x^5} - \frac{d(a + b \operatorname{sech}^{-1}(cx))}{7x^7} - \frac{e(a + b \operatorname{sech}^{-1}(cx))}{5x^5} \\ &= \frac{bd \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{49x^7} + \frac{b(30c^2d + 49e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{1225x^5} + \frac{4bc^2(30c^2d + 49e)}{1225x^5} - \frac{d(a + b \operatorname{sech}^{-1}(cx))}{7x^7} - \frac{e(a + b \operatorname{sech}^{-1}(cx))}{5x^5} \\ &= \frac{bd \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{49x^7} + \frac{b(30c^2d + 49e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{1225x^5} + \frac{4bc^2(30c^2d + 49e)}{1225x^5} - \frac{d(a + b \operatorname{sech}^{-1}(cx))}{7x^7} - \frac{e(a + b \operatorname{sech}^{-1}(cx))}{5x^5} \end{aligned}$$

Mathematica [A] time = 0.200744, size = 117, normalized size = 0.49

$$\frac{-105a(5d + 7ex^2) + b \sqrt{\frac{1-cx}{cx+1}}(cx+1)(15d(16c^6x^6 + 8c^4x^4 + 6c^2x^2 + 5) + 49ex^2(8c^4x^4 + 4c^2x^2 + 3)) - 105b \operatorname{sech}^{-1}(cx)}{3675x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcSech[c*x]))/x^8,x]
```

```
[Out] (-105*a*(5*d + 7*e*x^2) + b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(49*e*x^2*(3 + 4*c^2*x^2 + 8*c^4*x^4) + 15*d*(5 + 6*c^2*x^2 + 8*c^4*x^4 + 16*c^6*x^6)) - 105*b*(5*d + 7*e*x^2)*ArcSech[c*x])/(3675*x^7)
```

Maple [A] time = 0.191, size = 160, normalized size = 0.7

$$c^7 \left(\frac{a}{c^2} \left(-\frac{d}{7c^5x^7} - \frac{e}{5c^5x^5} \right) + \frac{b}{c^2} \left(-\frac{\operatorname{arcsech}(cx)d}{7c^5x^7} - \frac{\operatorname{arcsech}(cx)e}{5c^5x^5} + \frac{240c^8dx^6 + 392c^6ex^6 + 120c^6dx^4 + 196c^4ex^4 + 3675c^6x^6}{3675c^6x^6} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)*(a+b*arcsech(c*x))/x^8,x)

[Out] $c^7*(a/c^2*(-1/7/c^5*d/x^7-1/5*e/c^5/x^5)+b/c^2*(-1/7*arcsech(c*x)/c^5*d/x^7-1/5*arcsech(c*x)*e/c^5/x^5+1/3675*(-(c*x-1)/c/x)^(1/2)/c^6/x^6*((c*x+1)/c/x)^(1/2)*(240*c^8*d*x^6+392*c^6*e*x^6+120*c^6*d*x^4+196*c^4*e*x^4+90*c^4*d*x^2+147*c^2*e*x^2+75*c^2*d)))$

Maxima [A] time = 1.02732, size = 223, normalized size = 0.94

$$\frac{1}{245} bd \left(\frac{5c^8 \left(\frac{1}{c^2x^2} - 1 \right)^{\frac{7}{2}} + 21c^8 \left(\frac{1}{c^2x^2} - 1 \right)^{\frac{5}{2}} + 35c^8 \left(\frac{1}{c^2x^2} - 1 \right)^{\frac{3}{2}} + 35c^8 \sqrt{\frac{1}{c^2x^2} - 1}}{c} - \frac{35 \operatorname{arcsch}(cx)}{x^7} \right) + \frac{1}{75} be \left(\frac{3c^6 \left(\frac{1}{c^2x^2} - 1 \right)^{\frac{5}{2}} + 15c^6 \sqrt{\frac{1}{c^2x^2} - 1}}{c} - \frac{15 \operatorname{arcsch}(cx)}{x^5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^8,x, algorithm="maxima")

[Out] $1/245*b*d*((5*c^8*(1/(c^2*x^2) - 1)^(7/2) + 21*c^8*(1/(c^2*x^2) - 1)^(5/2) + 35*c^8*(1/(c^2*x^2) - 1)^(3/2) + 35*c^8*sqrt(1/(c^2*x^2) - 1))/c - 35*arcsech(c*x)/x^7) + 1/75*b*e*((3*c^6*(1/(c^2*x^2) - 1)^(5/2) + 10*c^6*(1/(c^2*x^2) - 1)^(3/2) + 15*c^6*sqrt(1/(c^2*x^2) - 1))/c - 15*arcsech(c*x)/x^5) - 1/5*a*e/x^5 - 1/7*a*d/x^7$

Fricas [A] time = 1.95465, size = 352, normalized size = 1.48

$$\frac{735 aex^2 + 525 ad + 105 (7bex^2 + 5bd) \log \left(\frac{cx \sqrt{-\frac{c^2x^2-1}{c^2x^2}} + 1}{cx} \right) - (8(30bc^7d + 49bc^5e)x^7 + 4(30bc^5d + 49bc^3e)x^5 + 75b^2c^3d + 75b^2c^3e)x^3}{3675x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^8,x, algorithm="fricas")

[Out] $-1/3675*(735*a*e*x^2 + 525*a*d + 105*(7*b*e*x^2 + 5*b*d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (8*(30*b*c^7*d + 49*b*c^5*e)*x^7 + 4*(30*b*c^5*d + 49*b*c^3*e)*x^5 + 75*b*c*d*x + 3*(30*b*c^3*d + 49*b*c^3*e)*x^3)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/x^7$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{asech}(cx))(d + ex^2)}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)*(a+b*asech(c*x))/x**8,x)

[Out] Integral((a + b*asech(c*x))*(d + e*x**2)/x**8, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)(b \operatorname{ar} \operatorname{sech}(cx) + a)}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^8,x, algorithm="giac")

[Out] integrate((e*x^2 + d)*(b*arcsech(c*x) + a)/x^8, x)

3.95 $\int x^5 (d + ex^2) (a + b \operatorname{sech}^{-1}(cx)) dx$

Optimal. Leaf size=232

$$\frac{1}{6}dx^6(a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{8}ex^8(a + b \operatorname{sech}^{-1}(cx)) - \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(1-c^2x^2)^{5/2}(4c^2d+9e)}{120c^8} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(1-c^2x^2)^{3/2}}{72c^8}$$

```
[Out] -(b*(4*c^2*d + 3*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/
(24*c^8) + (b*(8*c^2*d + 9*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*(1 - c^2*x^
2)^(3/2))/(72*c^8) - (b*(4*c^2*d + 9*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*
(1 - c^2*x^2)^(5/2))/(120*c^8) + (b*e*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*(1
- c^2*x^2)^(7/2))/(56*c^8) + (d*x^6*(a + b*ArcSech[c*x]))/6 + (e*x^8*(a +
b*ArcSech[c*x]))/8
```

Rubi [A] time = 0.16383, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {14, 6301, 12, 446, 77}

$$\frac{1}{6}dx^6(a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{8}ex^8(a + b \operatorname{sech}^{-1}(cx)) - \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(1-c^2x^2)^{5/2}(4c^2d+9e)}{120c^8} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(1-c^2x^2)^{3/2}}{72c^8}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] -(b*(4*c^2*d + 3*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/
(24*c^8) + (b*(8*c^2*d + 9*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*(1 - c^2*x^
2)^(3/2))/(72*c^8) - (b*(4*c^2*d + 9*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*
(1 - c^2*x^2)^(5/2))/(120*c^8) + (b*e*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*(1
- c^2*x^2)^(7/2))/(56*c^8) + (d*x^6*(a + b*ArcSech[c*x]))/6 + (e*x^8*(a +
b*ArcSech[c*x]))/8
```

Rule 14

```
Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_
+ (b_)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]
```

Rule 6301

```
Int[((a_) + ArcSech[(c_)*(x_)]*(b_))*((f_)*(x_))^(m_)*((d_) + (e_)*(
x_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p +
3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 446

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 77

```
Int[((a_) + (b_)*(x_))*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
&& ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
c, d, e, f])))
```

Rubi steps

$$\begin{aligned}
\int x^5 (d + ex^2) (a + b \operatorname{sech}^{-1}(cx)) dx &= \frac{1}{6} dx^6 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{8} ex^8 (a + b \operatorname{sech}^{-1}(cx)) + \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{x^5}{24} dx \\
&= \frac{1}{6} dx^6 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{8} ex^8 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{24} \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int x^5 dx \\
&= \frac{1}{6} dx^6 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{8} ex^8 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{48} \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \operatorname{Subst} \int x^5 dx \\
&= \frac{1}{6} dx^6 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{8} ex^8 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{48} \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \operatorname{Subst} \int x^5 dx \\
&= -\frac{b(4c^2d + 3e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{24c^8} + \frac{b(8c^2d + 9e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} (1-c^2x^2)}{72c^8}
\end{aligned}$$

Mathematica [A] time = 0.209894, size = 126, normalized size = 0.54

$$\frac{1}{24} ax^6 (4d + 3ex^2) - \frac{b \sqrt{\frac{1-cx}{cx+1}} (cx+1) (c^6 (84dx^4 + 45ex^6) + 2c^4 (56dx^2 + 27ex^4) + 8c^2 (28d + 9ex^2) + 144e)}{2520c^8} + \frac{1}{24} bx^6 \operatorname{sech}^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] (a*x^6*(4*d + 3*e*x^2))/24 - (b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(144*e
+ 8*c^2*(28*d + 9*e*x^2) + 2*c^4*(56*d*x^2 + 27*e*x^4) + c^6*(84*d*x^4 + 45
*e*x^6)))/(2520*c^8) + (b*x^6*(4*d + 3*e*x^2)*ArcSech[c*x])/24
```

Maple [A] time = 0.185, size = 150, normalized size = 0.7

$$\frac{1}{c^6} \left(\frac{a}{c^2} \left(\frac{ec^8x^8}{8} + \frac{c^8x^6d}{6} \right) + \frac{b}{c^2} \left(\frac{\operatorname{arcsech}(cx) ec^8x^8}{8} + \frac{\operatorname{arcsech}(cx) c^8x^6d}{6} - \frac{cx(45c^6ex^6 + 84c^6dx^4 + 54c^4ex^4 + 112c^4dx^2)}{2520} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x^2+d)*(a+b*arcsech(c*x)),x)
```


[Out] $1/c^6*(a/c^2*(1/8*e*c^8*x^8+1/6*c^8*x^6*d)+b/c^2*(1/8*\operatorname{arcsech}(c*x)*e*c^8*x^8+1/6*\operatorname{arcsech}(c*x)*c^8*x^6*d-1/2520*(-(c*x-1)/c/x)^{(1/2)}*c*x*((c*x+1)/c/x)^{(1/2)}*(45*c^6*e*x^6+84*c^6*d*x^4+54*c^4*e*x^4+112*c^4*d*x^2+72*c^2*e*x^2+224*c^2*d+144*e)))$

Maxima [A] time = 1.00133, size = 239, normalized size = 1.03

$$\frac{1}{8} aex^8 + \frac{1}{6} adx^6 + \frac{1}{90} \left(15x^6 \operatorname{arsh}(cx) - \frac{3c^4x^5\left(\frac{1}{c^2x^2} - 1\right)^{\frac{5}{2}} - 10c^2x^3\left(\frac{1}{c^2x^2} - 1\right)^{\frac{3}{2}} + 15x\sqrt{\frac{1}{c^2x^2} - 1}}{c^5} \right) bd + \frac{1}{280} \left(35x^8 \operatorname{arsh}(cx) - \frac{35c^4x^5\left(\frac{1}{c^2x^2} - 1\right)^{\frac{5}{2}} - 10c^2x^3\left(\frac{1}{c^2x^2} - 1\right)^{\frac{3}{2}} + 15x\sqrt{\frac{1}{c^2x^2} - 1}}{c^5} \right) bde$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="maxima")`

[Out] $1/8*a*e*x^8 + 1/6*a*d*x^6 + 1/90*(15*x^6*\operatorname{arcsech}(c*x) - (3*c^4*x^5*(1/(c^2*x^2) - 1)^{(5/2)} - 10*c^2*x^3*(1/(c^2*x^2) - 1)^{(3/2)} + 15*x*\sqrt{1/(c^2*x^2) - 1})/c^5)*b*d + 1/280*(35*x^8*\operatorname{arcsech}(c*x) + (5*c^6*x^7*(1/(c^2*x^2) - 1)^{(7/2)} - 21*c^4*x^5*(1/(c^2*x^2) - 1)^{(5/2)} + 35*c^2*x^3*(1/(c^2*x^2) - 1)^{(3/2)} - 35*x*\sqrt{1/(c^2*x^2) - 1})/c^7)*b*e$

Fricas [A] time = 2.05388, size = 381, normalized size = 1.64

$$\frac{315ac^7ex^8 + 420ac^7dx^6 + 105(3bc^7ex^8 + 4bc^7dx^6) \log\left(\frac{cx\sqrt{\frac{c^2x^2-1}{c^2x^2}+1}}{cx}\right) - (45bc^6ex^7 + 6(14bc^6d + 9bc^4e)x^5 + 8(14bc^2d + 9b^2e)x^3 + 16(14bc^2d + 9b^2e)x)\sqrt{-(c^2x^2-1)/(c^2x^2)}}{2520c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="fricas")`

[Out] $1/2520*(315*a*c^7*e*x^8 + 420*a*c^7*d*x^6 + 105*(3*b*c^7*e*x^8 + 4*b*c^7*d*x^6)*\log((c*x*\sqrt{-(c^2*x^2-1)/(c^2*x^2)}+1)/(c*x)) - (45*b*c^6*e*x^7 + 6*(14*b*c^6*d + 9*b*c^4*e)*x^5 + 8*(14*b*c^4*d + 9*b*c^2*e)*x^3 + 16*(14*b*c^2*d + 9*b^2*e)*x)*\sqrt{-(c^2*x^2-1)/(c^2*x^2)})/c^7$

Sympy [A] time = 69.7732, size = 228, normalized size = 0.98

$$\left\{ \begin{array}{l} \frac{adx^6}{6} + \frac{aex^8}{8} + \frac{bdx^6 \operatorname{asech}(cx)}{8} + \frac{bex^8 \operatorname{asech}(cx)}{8} - \frac{bdx^4\sqrt{-c^2x^2+1}}{30c^2} - \frac{bex^6\sqrt{-c^2x^2+1}}{56c^2} - \frac{2bdx^2\sqrt{-c^2x^2+1}}{45c^4} - \frac{3bex^4\sqrt{-c^2x^2+1}}{140c^4} - \frac{4bd\sqrt{-c^2x^2+1}}{45c^6} \\ (a + \infty b) \left(\frac{dx^6}{6} + \frac{ex^8}{8} \right) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(e*x**2+d)*(a+b*asech(c*x)),x)`

[Out] `Piecewise((a*d*x**6/6 + a*e*x**8/8 + b*d*x**6*asech(c*x)/6 + b*e*x**8*asech(c*x)/8 - b*d*x**4*sqrt(-c**2*x**2 + 1)/(30*c**2) - b*e*x**6*sqrt(-c**2*x**2 + 1)/(56*c**2) - 2*b*d*x**2*sqrt(-c**2*x**2 + 1)/(45*c**4) - 3*b*e*x**4*sqrt(-c**2*x**2 + 1)/(140*c**4) - 4*b*d*sqrt(-c**2*x**2 + 1)/(45*c**6), True)`

```

qrt(-c**2*x**2 + 1)/(140*c**4) - 4*b*d*sqrt(-c**2*x**2 + 1)/(45*c**6) - b*e
*x**2*sqrt(-c**2*x**2 + 1)/(35*c**6) - 2*b*e*sqrt(-c**2*x**2 + 1)/(35*c**8)
, Ne(c, 0)), ((a + oo*b)*(d*x**6/6 + e*x**8/8), True))

```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + d)(b \operatorname{ar} \operatorname{sech}(cx) + a)x^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arcsech(c*x) + a)*x^5, x)
```

3.96 $\int x^3 (d + ex^2) (a + b \operatorname{sech}^{-1}(cx)) dx$

Optimal. Leaf size=180

$$\frac{1}{4}dx^4(a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{6}ex^6(a + b \operatorname{sech}^{-1}(cx)) + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(1-c^2x^2)^{3/2}(3c^2d+4e)}{36c^6} - \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{12c^6}$$

```
[Out] -(b*(3*c^2*d + 2*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/
(12*c^6) + (b*(3*c^2*d + 4*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*(1 - c^2*x^
2)^(3/2))/(36*c^6) - (b*e*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*(1 - c^2*x^2)^(
5/2))/(30*c^6) + (d*x^4*(a + b*ArcSech[c*x]))/4 + (e*x^6*(a + b*ArcSech[c*
x]))/6
```

Rubi [A] time = 0.134929, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {14, 6301, 12, 446, 77}

$$\frac{1}{4}dx^4(a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{6}ex^6(a + b \operatorname{sech}^{-1}(cx)) + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(1-c^2x^2)^{3/2}(3c^2d+4e)}{36c^6} - \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{12c^6}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] -(b*(3*c^2*d + 2*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/
(12*c^6) + (b*(3*c^2*d + 4*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*(1 - c^2*x^
2)^(3/2))/(36*c^6) - (b*e*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*(1 - c^2*x^2)^(
5/2))/(30*c^6) + (d*x^4*(a + b*ArcSech[c*x]))/4 + (e*x^6*(a + b*ArcSech[c*
x]))/6
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_
+ (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)])*(b_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p +
3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_.)*(v_)] /; FreeQ[b, x]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

Rubi steps

$$\int x^3 (d + ex^2) (a + b \operatorname{sech}^{-1}(cx)) dx = \frac{1}{4} dx^4 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{6} ex^6 (a + b \operatorname{sech}^{-1}(cx)) + \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{x^3 (3d + 2ex^2)}{12c^6} dx$$

$$= \frac{1}{4} dx^4 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{6} ex^6 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{12} \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{x^3 (3d + 2ex^2)}{12c^6} dx$$

$$= \frac{1}{4} dx^4 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{6} ex^6 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{24} \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \operatorname{Subst} \left(\int \frac{x^3 (3d + 2ex^2)}{12c^6} dx, x, \sqrt{1+cx} \right)$$

$$= \frac{1}{4} dx^4 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{6} ex^6 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{24} \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \operatorname{Subst} \left(\int \frac{x^3 (3d + 2ex^2)}{12c^6} dx, x, \sqrt{1+cx} \right)$$

$$= -\frac{b(3c^2d + 2e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{12c^6} + \frac{b(3c^2d + 4e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} (1-c^2x^2)}{36c^6}$$

Mathematica [A] time = 0.178137, size = 106, normalized size = 0.59

$$\frac{1}{180} \left(15ax^4 (3d + 2ex^2) - \frac{b \sqrt{\frac{1-cx}{cx+1}} (cx + 1) (3c^4 (5dx^2 + 2ex^4) + c^2 (30d + 8ex^2) + 16e)}{c^6} + 15bx^4 \operatorname{sech}^{-1}(cx) (3d + 2ex^2) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3*(d + e*x^2)*(a + b*ArcSech[c*x]), x]

[Out] (15*a*x^4*(3*d + 2*e*x^2) - (b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(16*e + c^2*(30*d + 8*e*x^2) + 3*c^4*(5*d*x^2 + 2*e*x^4)))/c^6 + 15*b*x^4*(3*d + 2*e*x^2)*ArcSech[c*x])/180

Maple [A] time = 0.183, size = 132, normalized size = 0.7

$$\frac{1}{c^4} \left(\frac{a}{c^2} \left(\frac{c^6 x^6 e}{6} + \frac{x^4 c^6 d}{4} \right) + \frac{b}{c^2} \left(\frac{\operatorname{arcsech}(cx) c^6 x^6 e}{6} + \frac{\operatorname{arcsech}(cx) c^6 x^4 d}{4} - \frac{cx (6 c^4 ex^4 + 15 c^4 dx^2 + 8 c^2 x^2 e + 30 c^2 d + 16 e)}{180} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(e*x^2+d)*(a+b*arcsech(c*x)), x)

[Out] 1/c^4*(a/c^2*(1/6*c^6*x^6*e+1/4*x^4*c^6*d)+b/c^2*(1/6*arcsech(c*x)*c^6*x^6*e+1/4*arcsech(c*x)*c^6*x^4*d-1/180*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2))

$$1/2)*(6*c^4*e*x^4+15*c^4*d*x^2+8*c^2*e*x^2+30*c^2*d+16*e))$$

Maxima [A] time = 1.00878, size = 186, normalized size = 1.03

$$\frac{1}{6} a e x^6 + \frac{1}{4} a d x^4 + \frac{1}{12} \left(3 x^4 \operatorname{arsech}(c x) + \frac{c^2 x^3 \left(\frac{1}{c^2 x^2} - 1 \right)^{\frac{3}{2}} - 3 x \sqrt{\frac{1}{c^2 x^2} - 1}}{c^3} \right) b d + \frac{1}{90} \left(15 x^6 \operatorname{arsech}(c x) - \frac{3 c^4 x^5 \left(\frac{1}{c^2 x^2} - 1 \right)^{\frac{3}{2}}}{c^3} \right) b e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="maxima")

[Out] 1/6*a*e*x^6 + 1/4*a*d*x^4 + 1/12*(3*x^4*arcsech(c*x) + (c^2*x^3*(1/(c^2*x^2) - 1)^(3/2) - 3*x*sqrt(1/(c^2*x^2) - 1))/c^3)*b*d + 1/90*(15*x^6*arcsech(c*x) - (3*c^4*x^5*(1/(c^2*x^2) - 1)^(5/2) - 10*c^2*x^3*(1/(c^2*x^2) - 1)^(3/2) + 15*x*sqrt(1/(c^2*x^2) - 1))/c^5)*b*e

Fricas [A] time = 1.97318, size = 325, normalized size = 1.81

$$\frac{30 a c^5 e x^6 + 45 a c^5 d x^4 + 15 (2 b c^5 e x^6 + 3 b c^5 d x^4) \log \left(\frac{c x \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2} + 1}}{c x} \right) - (6 b c^4 e x^5 + (15 b c^4 d + 8 b c^2 e) x^3 + 2 (15 b c^2 d + 8 b e) x) \sqrt{-(c^2 x^2 - 1)/(c^2 x^2)}}{180 c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="fricas")

[Out] 1/180*(30*a*c^5*e*x^6 + 45*a*c^5*d*x^4 + 15*(2*b*c^5*e*x^6 + 3*b*c^5*d*x^4)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (6*b*c^4*e*x^5 + (15*b*c^4*d + 8*b*c^2*e)*x^3 + 2*(15*b*c^2*d + 8*b*e)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/c^5

Sympy [A] time = 14.9681, size = 177, normalized size = 0.98

$$\left\{ \begin{array}{l} \frac{a d x^4}{4} + \frac{a e x^6}{6} + \frac{b d x^4 \operatorname{asech}(c x)}{4} + \frac{b e x^6 \operatorname{asech}(c x)}{6} - \frac{b d x^2 \sqrt{-c^2 x^2 + 1}}{12 c^2} - \frac{b e x^4 \sqrt{-c^2 x^2 + 1}}{30 c^2} - \frac{b d \sqrt{-c^2 x^2 + 1}}{6 c^4} - \frac{2 b e x^2 \sqrt{-c^2 x^2 + 1}}{45 c^4} - \frac{4 b e \sqrt{-c^2 x^2 + 1}}{45 c^6} \\ (a + \infty b) \left(\frac{d x^4}{4} + \frac{e x^6}{6} \right) \end{array} \right. \quad \text{for } \text{ot}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(e*x**2+d)*(a+b*asech(c*x)),x)

[Out] Piecewise((a*d*x**4/4 + a*e*x**6/6 + b*d*x**4*asech(c*x)/4 + b*e*x**6*asech(c*x)/6 - b*d*x**2*sqrt(-c**2*x**2 + 1)/(12*c**2) - b*e*x**4*sqrt(-c**2*x**2 + 1)/(30*c**2) - b*d*sqrt(-c**2*x**2 + 1)/(6*c**4) - 2*b*e*x**2*sqrt(-c**2*x**2 + 1)/(45*c**4) - 4*b*e*sqrt(-c**2*x**2 + 1)/(45*c**6), Ne(c, 0)), ((a + oo*b)*(d*x**4/4 + e*x**6/6), True))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + d)(b \operatorname{ar} \operatorname{sech}(cx) + a)x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arcsech(c*x) + a)*x^3, x)
```

3.97 $\int x (d + ex^2) (a + b \operatorname{sech}^{-1}(cx)) dx$

Optimal. Leaf size=164

$$\frac{(d + ex^2)^2 (a + b \operatorname{sech}^{-1}(cx))}{4e} - \frac{bd^2 \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \tanh^{-1}(\sqrt{1-c^2x^2})}{4e} - \frac{b \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \sqrt{1-c^2x^2} (2c^2d + e)}{4c^4} + \frac{be \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \sqrt{1-c^2x^2}}{4c^4}$$

[Out] $-(b*(2*c^2*d + e)*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[1 - c^2*x^2])/(4*c^4) + (b*e*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*(1 - c^2*x^2)^{(3/2)})/(12*c^4) + ((d + e*x^2)^2*(a + b*\operatorname{ArcSech}[c*x]))/(4*e) - (b*d^2*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{ArcTanh}[\operatorname{Sqrt}[1 - c^2*x^2]])/(4*e)$

Rubi [A] time = 0.193181, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {6299, 517, 446, 88, 63, 208}

$$\frac{(d + ex^2)^2 (a + b \operatorname{sech}^{-1}(cx))}{4e} - \frac{bd^2 \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \tanh^{-1}(\sqrt{1-c^2x^2})}{4e} - \frac{b \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \sqrt{1-c^2x^2} (2c^2d + e)}{4c^4} + \frac{be \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \sqrt{1-c^2x^2}}{4c^4}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x*(d + e*x^2)*(a + b*\operatorname{ArcSech}[c*x]), x]$

[Out] $-(b*(2*c^2*d + e)*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[1 - c^2*x^2])/(4*c^4) + (b*e*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*(1 - c^2*x^2)^{(3/2)})/(12*c^4) + ((d + e*x^2)^2*(a + b*\operatorname{ArcSech}[c*x]))/(4*e) - (b*d^2*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{ArcTanh}[\operatorname{Sqrt}[1 - c^2*x^2]])/(4*e)$

Rule 6299

$\operatorname{Int}[(a + \operatorname{ArcSech}[c*x])*(b*x)*(d + e*x^2)^p, x_Symbol] \rightarrow \operatorname{Simp}[(d + e*x^2)^{p+1}*(a + b*\operatorname{ArcSech}[c*x])]/(2*e*(p+1)), x] + \operatorname{Dist}[(b*\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[1/(1 + c*x)])/(2*e*(p+1)), \operatorname{Int}[(d + e*x^2)^{p+1}/(x*\operatorname{Sqrt}[1 - c*x]*\operatorname{Sqrt}[1 + c*x]), x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, p\}, x] \&\& \operatorname{NeQ}[p, -1]$

Rule 517

$\operatorname{Int}[(u + (c + d*x^n)^{q_1})^{p_1}*(a_1 + b_1*x)^{non2_1}]^{p_2}, x_Symbol] \rightarrow \operatorname{Int}[u*(a_1*a_2 + b_1*b_2*x^n)^p*(c + d*x^n)^q, x] /; \operatorname{FreeQ}\{a_1, b_1, a_2, b_2, c, d, n, p, q\}, x] \&\& \operatorname{EqQ}[non2_1, n/2] \&\& \operatorname{EqQ}[a_2*b_1 + a_1*b_2, 0] \&\& (\operatorname{IntegerQ}[p] \mid\mid (\operatorname{GtQ}[a_1, 0] \&\& \operatorname{GtQ}[a_2, 0]))$

Rule 446

$\operatorname{Int}[(x + (a + b*x^n)^{p_1})^{m_1}*(c + d*x^n)^{q_1}, x_Symbol] \rightarrow \operatorname{Dist}[1/n, \operatorname{Subst}[\operatorname{Int}[x^{(\operatorname{Simplify}[(m+1)/n) - 1}*(a + b*x)^p*(c + d*x)^q}, x], x, x^n], x] /; \operatorname{FreeQ}\{a, b, c, d, m, n, p, q\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{IntegerQ}[\operatorname{Simplify}[(m+1)/n]]$

Rule 88

$\operatorname{Int}[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p], x]$

$x)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, p\}, x] \ \&\& \ \text{IntegersQ}\{m, n\} \ \&\& \ (\text{IntegerQ}\{p\} \ || \ (\text{GtQ}\{m, 0\} \ \&\& \ \text{GeQ}\{n, -1\}))$

Rule 63

$\text{Int}[(a_.) + (b_.)*(x_)^m]*((c_.) + (d_.)*(x_)^n), x_Symbol] \ :> \ \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{p*(m+1)-1}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{1/p}], x]] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 208

$\text{Int}[(a_.) + (b_.)*(x_)^2]^{-1}, x_Symbol] \ :> \ \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$

Rubi steps

$$\begin{aligned} \int x(d+ex^2)(a+b\operatorname{sech}^{-1}(cx)) dx &= \frac{(d+ex^2)^2(a+b\operatorname{sech}^{-1}(cx))}{4e} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{(d+ex^2)^2}{x\sqrt{1-cx}\sqrt{1+cx}} dx}{4e} \\ &= \frac{(d+ex^2)^2(a+b\operatorname{sech}^{-1}(cx))}{4e} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{(d+ex^2)^2}{x\sqrt{1-c^2x^2}} dx}{4e} \\ &= \frac{(d+ex^2)^2(a+b\operatorname{sech}^{-1}(cx))}{4e} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \operatorname{Subst}\left(\int \frac{(d+ex^2)^2}{x\sqrt{1-c^2x}} dx, x, x^2\right)}{8e} \\ &= \frac{(d+ex^2)^2(a+b\operatorname{sech}^{-1}(cx))}{4e} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \operatorname{Subst}\left(\int \left(\frac{e(2c^2d+e)}{c^2\sqrt{1-c^2x}} + \frac{d^2}{x\sqrt{1-c^2x}}\right) dx, x, x^2\right)}{8e} \\ &= -\frac{b(2c^2d+e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{4c^4} + \frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}(1-c^2x^2)^{3/2}}{12c^4} + \frac{(d+ex^2)^2}{4e} \\ &= -\frac{b(2c^2d+e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{4c^4} + \frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}(1-c^2x^2)^{3/2}}{12c^4} + \frac{(d+ex^2)^2}{4e} \\ &= -\frac{b(2c^2d+e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{4c^4} + \frac{be\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}(1-c^2x^2)^{3/2}}{12c^4} + \frac{(d+ex^2)^2}{4e} \end{aligned}$$

Mathematica [A] time = 0.125561, size = 85, normalized size = 0.52

$$\frac{1}{12} \left(3ax^2(2d+ex^2) - \frac{b\sqrt{\frac{1-cx}{cx+1}}(cx+1)(c^2(6d+ex^2)+2e)}{c^4} + 3bx^2\operatorname{sech}^{-1}(cx)(2d+ex^2) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x*(d + e*x^2)*(a + b*ArcSech[c*x]), x]

[Out] (3*a*x^2*(2*d + e*x^2) - (b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(2*e + c^2*(6*d + e*x^2)))/c^4 + 3*b*x^2*(2*d + e*x^2)*ArcSech[c*x])/12

Maple [A] time = 0.178, size = 113, normalized size = 0.7

$$\frac{1}{c^2} \left(\frac{a}{c^2} \left(\frac{c^4 x^4 e}{4} + \frac{x^2 c^4 d}{2} \right) + \frac{b}{c^2} \left(\frac{\operatorname{arcsech}(cx) c^4 x^4 e}{4} + \frac{\operatorname{arcsech}(cx) c^4 x^2 d}{2} - \frac{cx (c^2 x^2 e + 6 c^2 d + 2 e)}{12} \sqrt{-\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(e*x^2+d)*(a+b*arcsech(c*x)),x)

[Out] 1/c^2*(a/c^2*(1/4*c^4*x^4*e+1/2*x^2*c^4*d)+b/c^2*(1/4*arcsech(c*x)*c^4*x^4*e+1/2*arcsech(c*x)*c^4*x^2*d-1/12*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)*(c^2*e*x^2+6*c^2*d+2*e)))

Maxima [A] time = 0.998229, size = 130, normalized size = 0.79

$$\frac{1}{4} a e x^4 + \frac{1}{2} a d x^2 + \frac{1}{2} \left(x^2 \operatorname{arsh}(cx) - \frac{x \sqrt{\frac{1}{c^2 x^2} - 1}}{c} \right) b d + \frac{1}{12} \left(3 x^4 \operatorname{arsh}(cx) + \frac{c^2 x^3 \left(\frac{1}{c^2 x^2} - 1 \right)^{\frac{3}{2}} - 3 x \sqrt{\frac{1}{c^2 x^2} - 1}}{c^3} \right) b e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="maxima")

[Out] 1/4*a*e*x^4 + 1/2*a*d*x^2 + 1/2*(x^2*arcsech(c*x) - x*sqrt(1/(c^2*x^2) - 1)/c)*b*d + 1/12*(3*x^4*arcsech(c*x) + (c^2*x^3*(1/(c^2*x^2) - 1)^(3/2) - 3*x*sqrt(1/(c^2*x^2) - 1))/c^3)*b*e

Fricas [A] time = 2.0441, size = 269, normalized size = 1.64

$$\frac{3 a c^3 e x^4 + 6 a c^3 d x^2 + 3 (b c^3 e x^4 + 2 b c^3 d x^2) \log \left(\frac{c x \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2} + 1}}{c x} \right) - (b c^2 e x^3 + 2 (3 b c^2 d + b e) x) \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2}}}{12 c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="fricas")

[Out] 1/12*(3*a*c^3*e*x^4 + 6*a*c^3*d*x^2 + 3*(b*c^3*e*x^4 + 2*b*c^3*d*x^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (b*c^2*e*x^3 + 2*(3*b*c^2*d + b*e)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/c^3

Sympy [A] time = 4.15568, size = 126, normalized size = 0.77

$$\begin{cases} \frac{a d x^2}{2} + \frac{a e x^4}{4} + \frac{b d x^2 \operatorname{asech}(cx)}{2} + \frac{b e x^4 \operatorname{asech}(cx)}{4} - \frac{b d \sqrt{-c^2 x^2 + 1}}{2 c^2} - \frac{b e x^2 \sqrt{-c^2 x^2 + 1}}{12 c^2} - \frac{b e \sqrt{-c^2 x^2 + 1}}{6 c^4} & \text{for } c \neq 0 \\ (a + \infty b) \left(\frac{d x^2}{2} + \frac{e x^4}{4} \right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)*(a+b*asech(c*x)),x)
```

```
[Out] Piecewise((a*d*x**2/2 + a*e*x**4/4 + b*d*x**2*asech(c*x)/2 + b*e*x**4*asech(c*x)/4 - b*d*sqrt(-c**2*x**2 + 1)/(2*c**2) - b*e*x**2*sqrt(-c**2*x**2 + 1)/(12*c**2) - b*e*sqrt(-c**2*x**2 + 1)/(6*c**4), Ne(c, 0)), ((a + oo*b)*(d*x**2/2 + e*x**4/4), True))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + d)(b \operatorname{ar} \operatorname{sech}(cx) + a)x dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arcsech(c*x) + a)*x, x)
```

$$3.98 \quad \int \frac{(d+ex^2)(a+b\operatorname{sech}^{-1}(cx))}{x} dx$$

Optimal. Leaf size=296

$$\frac{ibd\sqrt{1-\frac{1}{c^2x^2}}\operatorname{PolyLog}\left(2, e^{2i\operatorname{csc}^{-1}(cx)}\right)}{2\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}} - d\log\left(\frac{1}{x}\right)(a+b\operatorname{sech}^{-1}(cx)) + \frac{1}{2}ex^2(a+b\operatorname{sech}^{-1}(cx)) + \frac{ibd\sqrt{1-\frac{1}{c^2x^2}}\operatorname{csc}^{-1}(cx)}{2\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}$$

```
[Out] -(b*e*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x)/(2*c) + ((I/2)*b*d*Sqrt[1 - 1/(c^2*x^2)]*ArcCsc[c*x]^2)/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) + (e*x^2*(a + b*ArcSech[c*x]))/2 - (b*d*Sqrt[1 - 1/(c^2*x^2)]*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])])/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) + (b*d*Sqrt[1 - 1/(c^2*x^2)]*ArcCsc[c*x]*Log[x^(-1)])/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) - d*(a + b*ArcSech[c*x])*Log[x^(-1)] + ((I/2)*b*d*Sqrt[1 - 1/(c^2*x^2)]*PolyLog[2, E^((2*I)*ArcCsc[c*x])])/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])
```

Rubi [A] time = 0.875166, antiderivative size = 296, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.632$, Rules used = {6303, 14, 5790, 6742, 95, 2328, 2326, 4625, 3717, 2190, 2279, 2391}

$$\frac{ibd\sqrt{1-\frac{1}{c^2x^2}}\operatorname{PolyLog}\left(2, e^{2i\operatorname{csc}^{-1}(cx)}\right)}{2\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}} - d\log\left(\frac{1}{x}\right)(a+b\operatorname{sech}^{-1}(cx)) + \frac{1}{2}ex^2(a+b\operatorname{sech}^{-1}(cx)) + \frac{ibd\sqrt{1-\frac{1}{c^2x^2}}\operatorname{csc}^{-1}(cx)}{2\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcSech[c*x]))/x, x]
```

```
[Out] -(b*e*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x)/(2*c) + ((I/2)*b*d*Sqrt[1 - 1/(c^2*x^2)]*ArcCsc[c*x]^2)/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) + (e*x^2*(a + b*ArcSech[c*x]))/2 - (b*d*Sqrt[1 - 1/(c^2*x^2)]*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])])/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) + (b*d*Sqrt[1 - 1/(c^2*x^2)]*ArcCsc[c*x]*Log[x^(-1)])/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) - d*(a + b*ArcSech[c*x])*Log[x^(-1)] + ((I/2)*b*d*Sqrt[1 - 1/(c^2*x^2)]*PolyLog[2, E^((2*I)*ArcCsc[c*x])])/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])
```

Rule 6303

```
Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcCosh[x/c])^n)/x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegersQ[m, p]
```

Rule 14

```
Int[(u_.)*((c_.)*(x_.))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_) + (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))*((f_.)*(x_.))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
```

$c^2*d + e, 0]$ && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rule 95

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]

Rule 2328

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] := Dist[Sqrt[1 + (e1*e2*x^2)/(d1*d2)]/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(a + b*Log[c*x^n])/Sqrt[1 + (e1*e2*x^2)/(d1*d2)], x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 + d1*e2, 0]

Rule 2326

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] := Simp[(ArcSin[Rt[-e, 2]*x]/Sqrt[d]]*(a + b*Log[c*x^n])/Rt[-e, 2], x] - Dist[(b*n)/Rt[-e, 2], Int[ArcSin[Rt[-e, 2]*x]/Sqrt[d]]/x, x] /; FreeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]

Rule 4625

Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] := Subst[Int[(a + b*x)^n/Tan[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]

Rule 3717

Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] := Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[(c + d*x)^m*E^(2*I*k*Pi)*E^(2*I*(e + f*x))]/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]

Rule 2190

Int[(((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/((b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

Int[Log[(a_) + (b_.)*((F_)^(e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rubi steps

$$\begin{aligned}
 \int \frac{(d + ex^2)(a + b \operatorname{sech}^{-1}(cx))}{x} dx &= -\operatorname{Subst} \left(\int \frac{(e + dx^2)(a + b \cosh^{-1}(\frac{x}{c}))}{x^3} dx, x, \frac{1}{x} \right) \\
 &= \frac{1}{2} ex^2 (a + b \operatorname{sech}^{-1}(cx)) - d (a + b \operatorname{sech}^{-1}(cx)) \log \left(\frac{1}{x} \right) + \frac{b \operatorname{Subst} \left(\int \frac{-\frac{e}{2x^2} + d \log(x)}{\sqrt{-1 + \frac{x}{c}} \sqrt{1 + \frac{x}{c}}} dx \right)}{c} \\
 &= \frac{1}{2} ex^2 (a + b \operatorname{sech}^{-1}(cx)) - d (a + b \operatorname{sech}^{-1}(cx)) \log \left(\frac{1}{x} \right) + \frac{b \operatorname{Subst} \left(\int \left(-\frac{e}{2x^2 \sqrt{-1 + \frac{x}{c}}} \right) dx \right)}{c} \\
 &= \frac{1}{2} ex^2 (a + b \operatorname{sech}^{-1}(cx)) - d (a + b \operatorname{sech}^{-1}(cx)) \log \left(\frac{1}{x} \right) + \frac{(bd) \operatorname{Subst} \left(\int \frac{\log(x)}{\sqrt{-1 + \frac{x}{c}} \sqrt{1 + \frac{x}{c}}} dx \right)}{c} \\
 &= -\frac{be \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}}{2c} + \frac{1}{2} ex^2 (a + b \operatorname{sech}^{-1}(cx)) - d (a + b \operatorname{sech}^{-1}(cx)) \log \left(\frac{1}{x} \right) \\
 &= -\frac{be \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}}{2c} + \frac{1}{2} ex^2 (a + b \operatorname{sech}^{-1}(cx)) + \frac{bd \sqrt{1 - \frac{1}{c^2 x^2}} \operatorname{csc}^{-1}(cx) \log \left(\frac{1}{x} \right)}{\sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}} \\
 &= -\frac{be \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}}{2c} + \frac{1}{2} ex^2 (a + b \operatorname{sech}^{-1}(cx)) + \frac{bd \sqrt{1 - \frac{1}{c^2 x^2}} \operatorname{csc}^{-1}(cx) \log \left(\frac{1}{x} \right)}{\sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}} \\
 &= -\frac{be \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}}{2c} + \frac{ibd \sqrt{1 - \frac{1}{c^2 x^2}} \operatorname{csc}^{-1}(cx)^2}{2 \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}} + \frac{1}{2} ex^2 (a + b \operatorname{sech}^{-1}(cx)) + \dots \\
 &= -\frac{be \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}}{2c} + \frac{ibd \sqrt{1 - \frac{1}{c^2 x^2}} \operatorname{csc}^{-1}(cx)^2}{2 \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}} + \frac{1}{2} ex^2 (a + b \operatorname{sech}^{-1}(cx)) - \dots \\
 &= -\frac{be \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}}{2c} + \frac{ibd \sqrt{1 - \frac{1}{c^2 x^2}} \operatorname{csc}^{-1}(cx)^2}{2 \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}} + \frac{1}{2} ex^2 (a + b \operatorname{sech}^{-1}(cx)) - \dots \\
 &= -\frac{be \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}}{2c} + \frac{ibd \sqrt{1 - \frac{1}{c^2 x^2}} \operatorname{csc}^{-1}(cx)^2}{2 \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}} + \frac{1}{2} ex^2 (a + b \operatorname{sech}^{-1}(cx)) - \dots
 \end{aligned}$$

Mathematica [A] time = 0.291598, size = 98, normalized size = 0.33

$$\frac{1}{2} \left(bd \operatorname{PolyLog} \left(2, -e^{-2 \operatorname{sech}^{-1}(cx)} \right) + 2ad \log(x) + aex^2 - \frac{be \sqrt{\frac{1-cx}{cx+1}} (cx+1)}{c^2} - bd \operatorname{sech}^{-1}(cx) \left(\operatorname{sech}^{-1}(cx) + 2 \log \left(e^{-2 \operatorname{sech}^{-1}(cx)} \right) \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[((d + e*x^2)*(a + b*ArcSech[c*x]))/x,x]

[Out] $(aex^2 - (b\sqrt{(1-cx)/(1+cx)}(1+cx))/c^2 + bex^2\text{ArcSech}[cx] - b\text{d}\text{ArcSech}[cx](\text{ArcSech}[cx] + 2\text{Log}[1 + E^{(-2\text{ArcSech}[cx])}])) + 2a\text{d}\text{Log}[x] + b\text{d}\text{PolyLog}[2, -E^{(-2\text{ArcSech}[cx])}])]/2$

Maple [A] time = 0.365, size = 166, normalized size = 0.6

$$\frac{aex^2}{2} + \ln(cx)ad + \frac{b(\text{arcsech}(cx))^2 d}{2} - \frac{bex}{2c} \sqrt{\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} + \frac{b\text{arcsech}(cx)x^2e}{2} + \frac{be}{2c^2} - b\text{d}\text{arcsech}(cx) \ln\left(1 + \left(\frac{1}{c}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d)*(a+b*arcsech(c*x))/x,x)`

[Out] $1/2*a*e*x^2 + \ln(c*x)*a*d + 1/2*b*\text{arcsech}(c*x)^2*d - 1/2*b/c*(-(c*x-1)/c/x)^{(1/2)}*((c*x+1)/c/x)^{(1/2)}*x*e + 1/2*b*\text{arcsech}(c*x)*x^2*e + 1/2/c^2*b*e - b*d*\text{arcsech}(c*x)*\ln(1+(1/c/x+(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2}))^2) - 1/2*b*d*\text{polylog}(2, -(1/c/x+(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2}))^2)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}aex^2 + ad \log(x) + \int bex \log\left(\sqrt{\frac{1}{cx} + 1}\sqrt{\frac{1}{cx} - 1} + \frac{1}{cx}\right) + \frac{bd \log\left(\sqrt{\frac{1}{cx} + 1}\sqrt{\frac{1}{cx} - 1} + \frac{1}{cx}\right)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)*(a+b*arcsech(c*x))/x,x, algorithm="maxima")`

[Out] $1/2*a*e*x^2 + a*d*\log(x) + \text{integrate}(b*e*x*\log(\sqrt{1/(c*x) + 1}*\sqrt{1/(c*x) - 1} + 1/(c*x)) + b*d*\log(\sqrt{1/(c*x) + 1}*\sqrt{1/(c*x) - 1} + 1/(c*x)))/x, x)$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{aex^2 + ad + (bex^2 + bd)\text{arsech}(cx)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)*(a+b*arcsech(c*x))/x,x, algorithm="fricas")`

[Out] `integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsech(c*x))/x, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \text{asech}(cx))(d + ex^2)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)*(a+b*asech(c*x))/x,x)

[Out] Integral((a + b*asech(c*x))*(d + e*x**2)/x, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)(b \operatorname{ar} \operatorname{sech}(cx) + a)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x,x, algorithm="giac")

[Out] integrate((e*x^2 + d)*(b*arcsech(c*x) + a)/x, x)

$$3.99 \quad \int \frac{(d+ex^2)(a+b\operatorname{sech}^{-1}(cx))}{x^3} dx$$

Optimal. Leaf size=309

$$\frac{ibe\sqrt{1-\frac{1}{c^2x^2}}\operatorname{PolyLog}\left(2, e^{2i\operatorname{csc}^{-1}(cx)}\right)}{2\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}} - \frac{d(a+b\operatorname{sech}^{-1}(cx))}{2x^2} - e\log\left(\frac{1}{x}\right)(a+b\operatorname{sech}^{-1}(cx)) + \frac{1}{4}bc^2d\operatorname{sech}^{-1}(cx) + \frac{ibe\sqrt{1-\frac{1}{c^2x^2}}}{2\sqrt{\frac{1}{cx}-1}}$$

[Out] (b*c*d*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])/(4*x) + ((I/2)*b*e*Sqrt[1 - 1/(c^2*x^2)]*ArcCsc[c*x]^2)/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) + (b*c^2*d*ArcSech[c*x])/4 - (d*(a + b*ArcSech[c*x]))/(2*x^2) - (b*e*Sqrt[1 - 1/(c^2*x^2)]*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])])/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) + (b*e*Sqrt[1 - 1/(c^2*x^2)]*ArcCsc[c*x]*Log[x^(-1)])/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) - e*(a + b*ArcSech[c*x])*Log[x^(-1)] + ((I/2)*b*e*Sqrt[1 - 1/(c^2*x^2)]*PolyLog[2, E^((2*I)*ArcCsc[c*x])])/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])

Rubi [A] time = 0.780008, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 14, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$, Rules used = {6303, 14, 5790, 12, 6742, 90, 52, 2328, 2326, 4625, 3717, 2190, 2279, 2391}

$$\frac{ibe\sqrt{1-\frac{1}{c^2x^2}}\operatorname{PolyLog}\left(2, e^{2i\operatorname{csc}^{-1}(cx)}\right)}{2\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}} - \frac{d(a+b\operatorname{sech}^{-1}(cx))}{2x^2} - e\log\left(\frac{1}{x}\right)(a+b\operatorname{sech}^{-1}(cx)) + \frac{1}{4}bc^2d\operatorname{sech}^{-1}(cx) + \frac{ibe\sqrt{1-\frac{1}{c^2x^2}}}{2\sqrt{\frac{1}{cx}-1}}$$

Antiderivative was successfully verified.

[In] Int[((d + e*x^2)*(a + b*ArcSech[c*x]))/x^3, x]

[Out] (b*c*d*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])/(4*x) + ((I/2)*b*e*Sqrt[1 - 1/(c^2*x^2)]*ArcCsc[c*x]^2)/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) + (b*c^2*d*ArcSech[c*x])/4 - (d*(a + b*ArcSech[c*x]))/(2*x^2) - (b*e*Sqrt[1 - 1/(c^2*x^2)]*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])])/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) + (b*e*Sqrt[1 - 1/(c^2*x^2)]*ArcCsc[c*x]*Log[x^(-1)])/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) - e*(a + b*ArcSech[c*x])*Log[x^(-1)] + ((I/2)*b*e*Sqrt[1 - 1/(c^2*x^2)]*PolyLog[2, E^((2*I)*ArcCsc[c*x])])/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])

Rule 6303

Int[((a_.) + ArcSech[(c_.)*(x_)])*(b_.))^ (n_.)*(x_)^ (m_.)*((d_.) + (e_.)*(x_)^2)^ (p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*(a + b*ArcCosh[x/c])^n]/x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegersQ[m, p]

Rule 14

Int[(u_)*((c_.)*(x_))^ (m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_))] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]

Rule 5790


```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(2*((c_.) + (d_.)*(x_))^(n_.))*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] := Simp[ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b - d, 0] && GtQ[a, 0]
```

Rule 2328

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] := Dist[Sqrt[1 + (e1*e2*x^2)/(d1*d2)]/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(a + b*Log[c*x^n])/Sqrt[1 + (e1*e2*x^2)/(d1*d2)], x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 + d1*e2, 0]
```

Rule 2326

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol] := Simp[(ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/Rt[-e, 2], x] - Dist[(b*n)/Rt[-e, 2], Int[ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]/x, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 4625

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] := Subst[Int[(a + b*x)^n/Tan[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol] := Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_))*((c_) + (d_)*(x_))^(m_)]/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d + ex^2)(a + b \operatorname{sech}^{-1}(cx))}{x^3} dx &= -\operatorname{Subst} \left(\int \frac{(e + dx^2)(a + b \cosh^{-1}(\frac{x}{c}))}{x} dx, x, \frac{1}{x} \right) \\
&= -\frac{d(a + b \operatorname{sech}^{-1}(cx))}{2x^2} - e(a + b \operatorname{sech}^{-1}(cx)) \log\left(\frac{1}{x}\right) + \frac{b \operatorname{Subst} \left(\int \frac{dx^2 + 2e \log(x)}{2\sqrt{-1 + \frac{x}{c}} \sqrt{1 + \frac{x}{c}}} dx \right)}{c} \\
&= -\frac{d(a + b \operatorname{sech}^{-1}(cx))}{2x^2} - e(a + b \operatorname{sech}^{-1}(cx)) \log\left(\frac{1}{x}\right) + \frac{b \operatorname{Subst} \left(\int \frac{dx^2 + 2e \log(x)}{\sqrt{-1 + \frac{x}{c}} \sqrt{1 + \frac{x}{c}}} dx \right)}{2c} \\
&= -\frac{d(a + b \operatorname{sech}^{-1}(cx))}{2x^2} - e(a + b \operatorname{sech}^{-1}(cx)) \log\left(\frac{1}{x}\right) + \frac{b \operatorname{Subst} \left(\int \left(\frac{dx^2}{\sqrt{-1 + \frac{x}{c}} \sqrt{1 + \frac{x}{c}}} \right) dx \right)}{2c} \\
&= -\frac{d(a + b \operatorname{sech}^{-1}(cx))}{2x^2} - e(a + b \operatorname{sech}^{-1}(cx)) \log\left(\frac{1}{x}\right) + \frac{(bd) \operatorname{Subst} \left(\int \frac{x^2}{\sqrt{-1 + \frac{x}{c}} \sqrt{1 + \frac{x}{c}}} dx \right)}{2c} \\
&= \frac{bcd \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}}{4x} - \frac{d(a + b \operatorname{sech}^{-1}(cx))}{2x^2} - e(a + b \operatorname{sech}^{-1}(cx)) \log\left(\frac{1}{x}\right) + \frac{1}{4} \\
&= \frac{bcd \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}}{4x} + \frac{1}{4} bc^2 d \operatorname{sech}^{-1}(cx) - \frac{d(a + b \operatorname{sech}^{-1}(cx))}{2x^2} + \frac{be \sqrt{1 - \frac{1}{c^2 x^2}}}{\sqrt{-1 + \frac{1}{cx}}} \\
&= \frac{bcd \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}}{4x} + \frac{1}{4} bc^2 d \operatorname{sech}^{-1}(cx) - \frac{d(a + b \operatorname{sech}^{-1}(cx))}{2x^2} + \frac{be \sqrt{1 - \frac{1}{c^2 x^2}}}{\sqrt{-1 + \frac{1}{cx}}} \\
&= \frac{bcd \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}}{4x} + \frac{ibe \sqrt{1 - \frac{1}{c^2 x^2}} \operatorname{csc}^{-1}(cx)^2}{2\sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}} + \frac{1}{4} bc^2 d \operatorname{sech}^{-1}(cx) - \frac{d(a + b \operatorname{sech}^{-1}(cx))}{2x^2} \\
&= \frac{bcd \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}}{4x} + \frac{ibe \sqrt{1 - \frac{1}{c^2 x^2}} \operatorname{csc}^{-1}(cx)^2}{2\sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}} + \frac{1}{4} bc^2 d \operatorname{sech}^{-1}(cx) - \frac{d(a + b \operatorname{sech}^{-1}(cx))}{2x^2} \\
&= \frac{bcd \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}}{4x} + \frac{ibe \sqrt{1 - \frac{1}{c^2 x^2}} \operatorname{csc}^{-1}(cx)^2}{2\sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}} + \frac{1}{4} bc^2 d \operatorname{sech}^{-1}(cx) - \frac{d(a + b \operatorname{sech}^{-1}(cx))}{2x^2} \\
&= \frac{bcd \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}}{4x} + \frac{ibe \sqrt{1 - \frac{1}{c^2 x^2}} \operatorname{csc}^{-1}(cx)^2}{2\sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}} + \frac{1}{4} bc^2 d \operatorname{sech}^{-1}(cx) - \frac{d(a + b \operatorname{sech}^{-1}(cx))}{2x^2}
\end{aligned}$$

Mathematica [A] time = 0.591147, size = 149, normalized size = 0.48

$$\frac{1}{4} \left(2be \operatorname{PolyLog} \left(2, -e^{-2 \operatorname{sech}^{-1}(cx)} \right) - \frac{2ad}{x^2} + 4ae \log(x) - \frac{bd \sqrt{\frac{1-cx}{cx+1}} \left(-c^2 x^2 + c^2 x^2 \sqrt{1 - c^2 x^2} \tanh^{-1} \left(\sqrt{1 - c^2 x^2} \right) + 1 \right)}{x^2 (cx - 1)} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[((d + e*x^2)*(a + b*ArcSech[c*x]))/x^3, x]

```
[Out] ((-2*a*d)/x^2 - (2*b*d*ArcSech[c*x])/x^2 - (b*d*Sqrt[(1 - c*x)/(1 + c*x)]*(1 - c^2*x^2 + c^2*x^2*Sqrt[1 - c^2*x^2])*ArcTanh[Sqrt[1 - c^2*x^2]]))/(x^2*(-1 + c*x)) - 2*b*e*ArcSech[c*x]*(ArcSech[c*x] + 2*Log[1 + E^(-2*ArcSech[c*x])]) + 4*a*e*Log[x] + 2*b*e*PolyLog[2, -E^(-2*ArcSech[c*x])]/4
```

Maple [A] time = 0.334, size = 170, normalized size = 0.6

$$ae \ln(cx) - \frac{ad}{2x^2} + \frac{b(\operatorname{arcsech}(cx))^2 e}{2} + \frac{bcd}{4x} \sqrt{\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} + \frac{bc^2 d \operatorname{arcsech}(cx)}{4} - \frac{b \operatorname{arcsech}(cx) d}{2x^2} - b e \operatorname{arcsech}(cx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arcsech(c*x))/x^3,x)
```

```
[Out] a*e*ln(c*x)-1/2/x^2*a*d+1/2*b*arcsech(c*x)^2*e+1/4*c*b*d/x*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)+1/4*b*c^2*d*arcsech(c*x)-1/2*b*arcsech(c*x)*d/x^2-b*e*arcsech(c*x)*ln(1+(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)-1/2*b*e*polylog(2,-(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{8} bd \left(\frac{2c^4 x \sqrt{\frac{1}{c^2 x^2} - 1}}{c^2 x^2 \left(\frac{1}{c^2 x^2} - 1\right) - 1} - c^3 \log\left(cx \sqrt{\frac{1}{c^2 x^2} - 1} + 1\right) + c^3 \log\left(cx \sqrt{\frac{1}{c^2 x^2} - 1} - 1\right) \right) + \frac{4 \operatorname{arsech}(cx)}{x^2} + be \int \frac{\log\left(\sqrt{\frac{1}{cx}} + 1 \sqrt{\frac{1}{cx} - 1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] -1/8*b*d*((2*c^4*x*sqrt(1/(c^2*x^2) - 1)/(c^2*x^2*(1/(c^2*x^2) - 1) - 1) - c^3*log(c*x*sqrt(1/(c^2*x^2) - 1) + 1) + c^3*log(c*x*sqrt(1/(c^2*x^2) - 1) - 1))/c + 4*arcsech(c*x)/x^2) + b*e*integrate(log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/x, x) + a*e*log(x) - 1/2*a*d/x^2
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{aex^2 + ad + (bex^2 + bd) \operatorname{arsech}(cx)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsech(c*x))/x^3, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{asech}(cx))(d + ex^2)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)*(a+b*asech(c*x))/x**3,x)

[Out] Integral((a + b*asech(c*x))*(d + e*x**2)/x**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)(b \operatorname{arasech}(cx) + a)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(a+b*arcsech(c*x))/x^3,x, algorithm="giac")

[Out] integrate((e*x^2 + d)*(b*arcsech(c*x) + a)/x^3, x)

3.100 $\int x^2 (d + ex^2)^2 (a + b \operatorname{sech}^{-1}(cx)) dx$

Optimal. Leaf size=275

$$\frac{1}{3}d^2x^3(a + b \operatorname{sech}^{-1}(cx)) + \frac{2}{5}dex^5(a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{7}e^2x^7(a + b \operatorname{sech}^{-1}(cx)) - \frac{bx\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(280c^4d^2 + 1680c^6)}{1680c^6}$$

[Out] $-(b*(280*c^4*d^2 + 252*c^2*d*e + 75*e^2)*x*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[1 - c^2*x^2])/(1680*c^6) - (b*e*(84*c^2*d + 25*e)*x^3*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[1 - c^2*x^2])/(840*c^4) - (b*e^2*x^5*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[1 - c^2*x^2])/(42*c^2) + (d^2*x^3*(a + b*\operatorname{ArcSech}[c*x]))/3 + (2*d*e*x^5*(a + b*\operatorname{ArcSech}[c*x]))/5 + (e^2*x^7*(a + b*\operatorname{ArcSech}[c*x]))/7 + (b*(280*c^4*d^2 + 252*c^2*d*e + 75*e^2)*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{ArcSin}[c*x])/(1680*c^7)$

Rubi [A] time = 0.234584, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {270, 6301, 12, 1267, 459, 321, 216}

$$\frac{1}{3}d^2x^3(a + b \operatorname{sech}^{-1}(cx)) + \frac{2}{5}dex^5(a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{7}e^2x^7(a + b \operatorname{sech}^{-1}(cx)) - \frac{bx\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(280c^4d^2 + 1680c^6)}{1680c^6}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^2*(d + e*x^2)^2*(a + b*\operatorname{ArcSech}[c*x]), x]$

[Out] $-(b*(280*c^4*d^2 + 252*c^2*d*e + 75*e^2)*x*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[1 - c^2*x^2])/(1680*c^6) - (b*e*(84*c^2*d + 25*e)*x^3*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[1 - c^2*x^2])/(840*c^4) - (b*e^2*x^5*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[1 - c^2*x^2])/(42*c^2) + (d^2*x^3*(a + b*\operatorname{ArcSech}[c*x]))/3 + (2*d*e*x^5*(a + b*\operatorname{ArcSech}[c*x]))/5 + (e^2*x^7*(a + b*\operatorname{ArcSech}[c*x]))/7 + (b*(280*c^4*d^2 + 252*c^2*d*e + 75*e^2)*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{ArcSin}[c*x])/(1680*c^7)$

Rule 270

$\operatorname{Int}[(c_*)*(x_*)^{(m_*)}((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(c*x)^m*(a + b*x^n)^p, x], x] /; \operatorname{FreeQ}\{a, b, c, m, n, x\} \&\& \operatorname{IGtQ}[p, 0]$

Rule 6301

$\operatorname{Int}[(a_*) + \operatorname{ArcSech}(c_*)*(x_*)*(b_*)]*((f_*)*(x_*)^{(m_*)}*((d_*) + (e_*)*(x_*)^2)^{(p_*)}, x_Symbol] \rightarrow \operatorname{With}\{u = \operatorname{IntHide}[(f*x)^m*(d + e*x^2)^p, x]\}, \operatorname{Dist}[a + b*\operatorname{ArcSech}[c*x], u, x] + \operatorname{Dist}[b*\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[1/(1 + c*x)], \operatorname{Int}[\operatorname{SimplifyIntegrand}[u/(x*\operatorname{Sqrt}[1 - c*x]*\operatorname{Sqrt}[1 + c*x]), x], x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, m, p, x\} \&\& ((\operatorname{IGtQ}[p, 0] \&\& !(\operatorname{ILtQ}[(m - 1)/2, 0] \&\& \operatorname{GtQ}[m + 2*p + 3, 0])) \|\| (\operatorname{IGtQ}[(m + 1)/2, 0] \&\& !(\operatorname{ILtQ}[p, 0] \&\& \operatorname{GtQ}[m + 2*p + 3, 0])) \|\| (\operatorname{ILtQ}[(m + 2*p + 1)/2, 0] \&\& !\operatorname{ILtQ}[(m - 1)/2, 0]))$

Rule 12

$\operatorname{Int}(a_*(u_), x_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /; \operatorname{FreeQ}[a, x] \&\& !\operatorname{MatchQ}[u, (b_)*(v_)] /; \operatorname{FreeQ}[b, x]$

Rule 1267

Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]

Rule 459

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]

Rule 321

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 216

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rubi steps

$$\begin{aligned}
 \int x^2 (d + ex^2)^2 (a + b \operatorname{sech}^{-1}(cx)) dx &= \frac{1}{3} d^2 x^3 (a + b \operatorname{sech}^{-1}(cx)) + \frac{2}{5} dex^5 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{7} e^2 x^7 (a + b \operatorname{sech}^{-1}(cx)) \\
 &= \frac{1}{3} d^2 x^3 (a + b \operatorname{sech}^{-1}(cx)) + \frac{2}{5} dex^5 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{7} e^2 x^7 (a + b \operatorname{sech}^{-1}(cx)) \\
 &= -\frac{be^2 x^5 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2 x^2}}{42c^2} + \frac{1}{3} d^2 x^3 (a + b \operatorname{sech}^{-1}(cx)) + \frac{2}{5} dex^5 (a + b \operatorname{sech}^{-1}(cx)) \\
 &= -\frac{be(84c^2 d + 25e) x^3 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2 x^2}}{840c^4} - \frac{be^2 x^5 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2 x^2}}{42c^2} \\
 &= -\frac{b(280c^4 d^2 + 252c^2 de + 75e^2) x \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2 x^2}}{1680c^6} - \frac{be(84c^2 d + 25e) x^5 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2 x^2}}{42c^2} \\
 &= -\frac{b(280c^4 d^2 + 252c^2 de + 75e^2) x \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2 x^2}}{1680c^6} - \frac{be(84c^2 d + 25e) x^5 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2 x^2}}{42c^2}
 \end{aligned}$$

Mathematica [C] time = 0.45736, size = 207, normalized size = 0.75

$$\frac{16ac^7 x^3 (35d^2 + 42dex^2 + 15e^2 x^4) - bcx \sqrt{\frac{1-cx}{cx+1}} (cx+1) (8c^4 (35d^2 + 21dex^2 + 5e^2 x^4) + 2c^2 e (126d + 25ex^2) + 75e^2) + 1680c^7}{1680c^7}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(d + e*x^2)^2*(a + b*ArcSech[c*x]),x]

[Out] (16*a*c^7*x^3*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4) - b*c*x*sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(75*e^2 + 2*c^2*e*(126*d + 25*e*x^2) + 8*c^4*(35*d^2 + 21*d*e*x^2 + 5*e^2*x^4)) + 16*b*c^7*x^3*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4)*ArcSech[c*x] + I*b*(280*c^4*d^2 + 252*c^2*d*e + 75*e^2)*Log[(-2*I)*c*x + 2*sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)]/(1680*c^7)

Maple [A] time = 0.212, size = 300, normalized size = 1.1

$$\frac{1}{c^3} \left(\frac{a}{c^4} \left(\frac{e^2 c^7 x^7}{7} + \frac{2 c^7 d e x^5}{5} + \frac{x^3 c^7 d^2}{3} \right) + \frac{b}{c^4} \left(\frac{\operatorname{arcsech}(c x) e^2 c^7 x^7}{7} + \frac{2 \operatorname{arcsech}(c x) c^7 d e x^5}{5} + \frac{\operatorname{arcsech}(c x) c^7 x^3 d^2}{3} + \frac{c x}{1680} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(e*x^2+d)^2*(a+b*arcsech(c*x)),x)

[Out] 1/c^3*(a/c^4*(1/7*e^2*c^7*x^7+2/5*c^7*d*e*x^5+1/3*x^3*c^7*d^2)+b/c^4*(1/7*arcsech(c*x)*e^2*c^7*x^7+2/5*arcsech(c*x)*c^7*d*e*x^5+1/3*arcsech(c*x)*c^7*x^3*d^2+1/1680*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)*(-40*c^5*x^5*e^2*(-c^2*x^2+1)^(1/2)-168*c^5*x^3*d*e*(-c^2*x^2+1)^(1/2)-280*d^2*c^5*x*(-c^2*x^2+1)^(1/2)+280*d^2*c^4*arcsin(c*x)-50*e^2*c^3*x^3*(-c^2*x^2+1)^(1/2)-252*c^3*d*e*x*(-c^2*x^2+1)^(1/2)+252*c^2*d*e*arcsin(c*x)-75*e^2*c*x*(-c^2*x^2+1)^(1/2)+75*e^2*arcsin(c*x))/(-c^2*x^2+1)^(1/2))

Maxima [A] time = 1.51487, size = 443, normalized size = 1.61

$$\frac{1}{7} a e^2 x^7 + \frac{2}{5} a d e x^5 + \frac{1}{3} a d^2 x^3 + \frac{1}{6} \left(2 x^3 \operatorname{arosech}(c x) - \frac{\frac{\sqrt{\frac{1}{2x^2}-1}}{c^2 \left(\frac{1}{2x^2}-1 \right) + c^2} + \frac{\arctan\left(\sqrt{\frac{1}{2x^2}-1}\right)}{c^2}}{c} \right) b d^2 + \frac{1}{20} \left(8 x^5 \operatorname{arosech}(c x) - \frac{3 \left(\frac{1}{c^2 x} \right)}{c^4 \left(\frac{1}{c^2 x^2} \right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="maxima")

[Out] 1/7*a*e^2*x^7 + 2/5*a*d*e*x^5 + 1/3*a*d^2*x^3 + 1/6*(2*x^3*arcsech(c*x) - (sqrt(1/(c^2*x^2) - 1)/(c^2*(1/(c^2*x^2) - 1) + c^2) + arctan(sqrt(1/(c^2*x^2) - 1))/c^2)/c)*b*d^2 + 1/20*(8*x^5*arcsech(c*x) - ((3*(1/(c^2*x^2) - 1)^(3/2) + 5*sqrt(1/(c^2*x^2) - 1))/(c^4*(1/(c^2*x^2) - 1)^2 + 2*c^4*(1/(c^2*x^2) - 1) + c^4) + 3*arctan(sqrt(1/(c^2*x^2) - 1))/c^4)/c)*b*d*e + 1/336*(48*x^7*arcsech(c*x) - ((15*(1/(c^2*x^2) - 1)^(5/2) + 40*(1/(c^2*x^2) - 1)^(3/2) + 33*sqrt(1/(c^2*x^2) - 1))/(c^6*(1/(c^2*x^2) - 1)^3 + 3*c^6*(1/(c^2*x^2) - 1)^2 + 3*c^6*(1/(c^2*x^2) - 1) + c^6) + 15*arctan(sqrt(1/(c^2*x^2) - 1))/c^6)/c)*b*e^2

Fricas [A] time = 3.60211, size = 775, normalized size = 2.82

$$240 ac^7 e^2 x^7 + 672 ac^7 dex^5 + 560 ac^7 d^2 x^3 - 2(280 bc^4 d^2 + 252 bc^2 de + 75 be^2) \arctan\left(\frac{cx\sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2}} - 1}{cx}\right) - 16(35 bc^7 d^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="fricas")

[Out] 1/1680*(240*a*c^7*e^2*x^7 + 672*a*c^7*d*e*x^5 + 560*a*c^7*d^2*x^3 - 2*(280*b*c^4*d^2 + 252*b*c^2*d*e + 75*b*e^2)*arctan((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c*x)) - 16*(35*b*c^7*d^2 + 42*b*c^7*d*e + 15*b*c^7*e^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + 16*(15*b*c^7*e^2*x^7 + 42*b*c^7*d*e*x^5 + 35*b*c^7*d^2*x^3 - 35*b*c^7*d^2 - 42*b*c^7*d*e - 15*b*c^7*e^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (40*b*c^6*e^2*x^6 + 2*(84*b*c^6*d*e + 25*b*c^4*e^2)*x^4 + (280*b*c^6*d^2 + 252*b*c^4*d*e + 75*b*c^2*e^2)*x^2)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/c^7

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x^2 (a + b \operatorname{asech}(cx)) (d + ex^2)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(e*x**2+d)**2*(a+b*asech(c*x)),x)

[Out] Integral(x**2*(a + b*asech(c*x))*(d + e*x**2)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + d)^2 (b \operatorname{arsech}(cx) + a)x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="giac")

[Out] integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a)*x^2, x)

3.101 $\int (d + ex^2)^2 (a + b \operatorname{sech}^{-1}(cx)) dx$

Optimal. Leaf size=204

$$d^2x(a + b \operatorname{sech}^{-1}(cx)) + \frac{2}{3}dex^3(a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{5}e^2x^5(a + b \operatorname{sech}^{-1}(cx)) + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(120c^4d^2 + 40c^2de + 9e^2)}{120c^5}$$

```
[Out] -(b*e*(40*c^2*d + 9*e)*x*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(120*c^4) - (b*e^2*x^3*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(20*c^2) + d^2*x*(a + b*ArcSech[c*x]) + (2*d*e*x^3*(a + b*ArcSech[c*x]))/3 + (e^2*x^5*(a + b*ArcSech[c*x]))/5 + (b*(120*c^4*d^2 + 40*c^2*d*e + 9*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x])/(120*c^5)
```

Rubi [A] time = 0.126856, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {194, 6291, 12, 1159, 388, 216}

$$d^2x(a + b \operatorname{sech}^{-1}(cx)) + \frac{2}{3}dex^3(a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{5}e^2x^5(a + b \operatorname{sech}^{-1}(cx)) + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(120c^4d^2 + 40c^2de + 9e^2)}{120c^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*(a + b*ArcSech[c*x]),x]
```

```
[Out] -(b*e*(40*c^2*d + 9*e)*x*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(120*c^4) - (b*e^2*x^3*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(20*c^2) + d^2*x*(a + b*ArcSech[c*x]) + (2*d*e*x^3*(a + b*ArcSech[c*x]))/3 + (e^2*x^5*(a + b*ArcSech[c*x]))/5 + (b*(120*c^4*d^2 + 40*c^2*d*e + 9*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x])/(120*c^5)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6291

```
Int[((a_.) + ArcSech[(c_.)*(x_)])*(b_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1159

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Simp[(c^p*x^(4*p - 1)*(d + e*x^2)^(q + 1))/(e*(4*p + 2*q + 1)), x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p + 2*q + 1)*x^(4*p)], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
```

*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && !LtQ[q, -1]

Rule 388

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rubi steps

$$\begin{aligned} \int (d + ex^2)^2 (a + b \operatorname{sech}^{-1}(cx)) dx &= d^2 x (a + b \operatorname{sech}^{-1}(cx)) + \frac{2}{3} dex^3 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{5} e^2 x^5 (a + b \operatorname{sech}^{-1}(cx)) + \\ &= d^2 x (a + b \operatorname{sech}^{-1}(cx)) + \frac{2}{3} dex^3 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{5} e^2 x^5 (a + b \operatorname{sech}^{-1}(cx)) + \\ &= -\frac{be^2 x^3 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2 x^2}}{20c^2} + d^2 x (a + b \operatorname{sech}^{-1}(cx)) + \frac{2}{3} dex^3 (a + b \operatorname{sech}^{-1}(cx)) + \\ &= -\frac{be(40c^2 d + 9e)x \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2 x^2}}{120c^4} - \frac{be^2 x^3 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2 x^2}}{20c^2} + \\ &= -\frac{be(40c^2 d + 9e)x \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2 x^2}}{120c^4} - \frac{be^2 x^3 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2 x^2}}{20c^2} + \end{aligned}$$

Mathematica [C] time = 0.27851, size = 174, normalized size = 0.85

$$\frac{8ac^5 x (15d^2 + 10dex^2 + 3e^2 x^4) + 8bc^5 x \operatorname{sech}^{-1}(cx) (15d^2 + 10dex^2 + 3e^2 x^4) + ib(120c^4 d^2 + 40c^2 de + 9e^2) \log\left(2\sqrt{\frac{1-cx}{cx+1}}\right)}{120c^5}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x^2)^2*(a + b*ArcSech[c*x]), x]

[Out] (8*a*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) - b*c*e*x*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(9*e + c^2*(40*d + 6*e*x^2)) + 8*b*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4)*ArcSech[c*x] + I*b*(120*c^4*d^2 + 40*c^2*d*e + 9*e^2)*Log[(-2*I)*c*x + 2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)]/(120*c^5)

Maple [A] time = 0.178, size = 228, normalized size = 1.1

$$\frac{1}{c} \left(\frac{a}{c^4} \left(\frac{e^2 c^5 x^5}{5} + \frac{2 c^5 d e x^3}{3} + x c^5 d^2 \right) + \frac{b}{c^4} \left(\frac{\operatorname{arcsech}(cx) e^2 c^5 x^5}{5} + \frac{2 \operatorname{arcsech}(cx) c^5 x^3 d e}{3} + \operatorname{arcsech}(cx) c^5 x d^2 + \frac{c x}{120} \sqrt{\frac{1-cx}{cx+1}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^2*(a+b*arcsech(c*x)), x)

[Out] $1/c*(a/c^4*(1/5*e^2*c^5*x^5+2/3*c^5*d*e*x^3+x*c^5*d^2)+b/c^4*(1/5*\operatorname{arcsech}(c*x)*e^2*c^5*x^5+2/3*\operatorname{arcsech}(c*x)*c^5*x^3*d*e+\operatorname{arcsech}(c*x)*c^5*x*d^2+1/120*(-(c*x-1)/c/x)^{(1/2)}*c*x*((c*x+1)/c/x)^{(1/2)}*(120*d^2*c^4*\arcsin(c*x)-6*e^2*c^3*x^3*(-c^2*x^2+1)^{(1/2)}-40*c^3*d*e*x*(-c^2*x^2+1)^{(1/2)}+40*c^2*d*e*\arcsin(c*x)-9*e^2*c*x*(-c^2*x^2+1)^{(1/2)}+9*e^2*\arcsin(c*x))/(-c^2*x^2+1)^{(1/2))}$

Maxima [A] time = 1.49882, size = 302, normalized size = 1.48

$$\frac{1}{5}ae^2x^5 + \frac{2}{3}adex^3 + \frac{1}{3} \left(2x^3 \operatorname{arsech}(cx) - \frac{\frac{\sqrt{\frac{1}{c^2x^2}-1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^2}}{c} \right) bde + \frac{1}{40} \left(8x^5 \operatorname{arsech}(cx) - \frac{3\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}} + 5\sqrt{\frac{1}{c^2x^2}-1}}{c^4\left(\frac{1}{c^2x^2}-1\right)^2 + 2c^4\left(\frac{1}{c^2x^2}-1\right) + c^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="maxima")`

[Out] $1/5*a*e^2*x^5 + 2/3*a*d*e*x^3 + 1/3*(2*x^3*\operatorname{arcsech}(c*x) - (\sqrt{1/(c^2*x^2) - 1})/(c^2*(1/(c^2*x^2) - 1) + c^2) + \arctan(\sqrt{1/(c^2*x^2) - 1})/c^2)/c * b*d*e + 1/40*(8*x^5*\operatorname{arcsech}(c*x) - ((3*(1/(c^2*x^2) - 1)^{(3/2)} + 5*\sqrt{1/(c^2*x^2) - 1})/(c^4*(1/(c^2*x^2) - 1)^2 + 2*c^4*(1/(c^2*x^2) - 1) + c^4) + 3*\arctan(\sqrt{1/(c^2*x^2) - 1})/c^4)/c)*b*e^2 + a*d^2*x + (c*x*\operatorname{arcsech}(c*x) - \arctan(\sqrt{1/(c^2*x^2) - 1}))*b*d^2/c$

Fricas [B] time = 3.00414, size = 679, normalized size = 3.33

$$24ac^5e^2x^5 + 80ac^5dex^3 + 120ac^5d^2x - 2(120bc^4d^2 + 40bc^2de + 9be^2) \arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}-1}}{cx}\right) - 8(15bc^5d^2 + 10bc^5de + 3b^2c^5e^2) \log\left(\frac{c*x*\sqrt{-(c^2*x^2-1)/(c^2*x^2)}-1}{c*x}\right) + 8(3b^2c^5e^2*x^5 + 10b^2c^5*d*e*x^3 + 15b^2c^5*d^2*x - 15b^2c^5*d^2 - 10b^2c^5*d*e - 3b^2c^5*e^2) \log\left(\frac{c*x*\sqrt{-(c^2*x^2-1)/(c^2*x^2)}+1}{c*x}\right) - (6b^2c^4*e^2*x^4 + (40b^2c^4*d*e + 9b^2c^2*e^2)*x^2)*\sqrt{-(c^2*x^2-1)/(c^2*x^2)}/c^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="fricas")`

[Out] $1/120*(24*a*c^5*e^2*x^5 + 80*a*c^5*d*e*x^3 + 120*a*c^5*d^2*x - 2*(120*b*c^4*d^2 + 40*b*c^2*d*e + 9*b*e^2)*\arctan((c*x*\sqrt{-(c^2*x^2-1)/(c^2*x^2)})-1)/(c*x)) - 8*(15*b*c^5*d^2 + 10*b*c^5*d*e + 3*b*c^5*e^2)*\log((c*x*\sqrt{-(c^2*x^2-1)/(c^2*x^2)})-1)/x) + 8*(3*b*c^5*e^2*x^5 + 10*b*c^5*d*e*x^3 + 15*b*c^5*d^2*x - 15*b*c^5*d^2 - 10*b*c^5*d*e - 3*b*c^5*e^2)*\log((c*x*\sqrt{-(c^2*x^2-1)/(c^2*x^2)})+1)/(c*x)) - (6*b*c^4*e^2*x^4 + (40*b*c^4*d*e + 9*b*c^2*e^2)*x^2)*\sqrt{-(c^2*x^2-1)/(c^2*x^2)}/c^5$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{asech}(cx)) (d + ex^2)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**2*(a+b*asech(c*x)),x)

[Out] Integral((a + b*asech(c*x))*(d + e*x**2)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + d)^2 (b \operatorname{arsech}(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="giac")

[Out] integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a), x)

$$3.102 \quad \int \frac{(d+ex^2)^2 (a+b\operatorname{sech}^{-1}(cx))}{x^2} dx$$

Optimal. Leaf size=177

$$-\frac{d^2 (a + b\operatorname{sech}^{-1}(cx))}{x} + 2dex (a + b\operatorname{sech}^{-1}(cx)) + \frac{1}{3}e^2x^3 (a + b\operatorname{sech}^{-1}(cx)) + \frac{bd^2 \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \sqrt{1-c^2x^2}}{x} + \frac{be \sqrt{\frac{1}{cx+1}} \sqrt{1-c^2x^2}}{x}$$

[Out] (b*d^2*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/x - (b*e^2*x*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(6*c^2) - (d^2*(a + b*ArcSech[c*x]))/x + 2*d*e*x*(a + b*ArcSech[c*x]) + (e^2*x^3*(a + b*ArcSech[c*x]))/3 + (b*e*(12*c^2*d + e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x])/(6*c^3)

Rubi [A] time = 0.13156, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {270, 6301, 12, 1265, 388, 216}

$$-\frac{d^2 (a + b\operatorname{sech}^{-1}(cx))}{x} + 2dex (a + b\operatorname{sech}^{-1}(cx)) + \frac{1}{3}e^2x^3 (a + b\operatorname{sech}^{-1}(cx)) + \frac{bd^2 \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \sqrt{1-c^2x^2}}{x} + \frac{be \sqrt{\frac{1}{cx+1}} \sqrt{1-c^2x^2}}{x}$$

Antiderivative was successfully verified.

[In] Int[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x^2,x]

[Out] (b*d^2*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/x - (b*e^2*x*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(6*c^2) - (d^2*(a + b*ArcSech[c*x]))/x + 2*d*e*x*(a + b*ArcSech[c*x]) + (e^2*x^3*(a + b*ArcSech[c*x]))/3 + (b*e*(12*c^2*d + e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x])/(6*c^3)

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 6301

Int[((a_.) + ArcSech[(c_.)*(x_)])*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 1265

Int[((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 +

```
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x],
Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x
- e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)^2 (a + b \operatorname{sech}^{-1}(cx))}{x^2} dx &= -\frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{x} + 2dex (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{3} e^2 x^3 (a + b \operatorname{sech}^{-1}(cx)) + \dots \\ &= -\frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{x} + 2dex (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{3} e^2 x^3 (a + b \operatorname{sech}^{-1}(cx)) + \dots \\ &= \frac{bd^2 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{x} - \frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{x} + 2dex (a + b \operatorname{sech}^{-1}(cx)) - \dots \\ &= \frac{bd^2 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{x} - \frac{be^2 x \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{6c^2} - \frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{x} \\ &= \frac{bd^2 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{x} - \frac{be^2 x \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{6c^2} - \frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{x} \end{aligned}$$

Mathematica [C] time = 0.24164, size = 158, normalized size = 0.89

$$\frac{2ac^3(-3d^2 + 6dex^2 + e^2x^4) - bc\sqrt{\frac{1-cx}{cx+1}}(cx+1)(e^2x^2 - 6c^2d^2) + 2bc^3\operatorname{sech}^{-1}(cx)(-3d^2 + 6dex^2 + e^2x^4) + ibex(12c^2d - \dots)}{6c^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x^2, x]
```

```
[Out] (- (b*c*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(-6*c^2*d^2 + e^2*x^2)) + 2*a*c^
3*(-3*d^2 + 6*d*e*x^2 + e^2*x^4) + 2*b*c^3*(-3*d^2 + 6*d*e*x^2 + e^2*x^4)*A
rcSech[c*x] + I*b*e*(12*c^2*d + e)*x*Log[(-2*I)*c*x + 2*Sqrt[(1 - c*x)/(1 +
c*x)]*(1 + c*x)])/(6*c^3*x)
```

Maple [A] time = 0.187, size = 197, normalized size = 1.1

$$c \left(\frac{a}{c^4} \left(\frac{c^3 x^3 e^2}{3} + 2 c^3 x d e - \frac{d^2 c^3}{x} \right) + \frac{b}{c^4} \left(\frac{e^2 \operatorname{arcsech}(cx) c^3 x^3}{3} + 2 \operatorname{arcsech}(cx) c^3 x d e - \frac{\operatorname{arcsech}(cx) d^2 c^3}{x} + \frac{1}{6} \sqrt{-\frac{cx-1}{cx}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d)^2*(a+b*arcsech(c*x))/x^2,x)`

[Out] $c*(a/c^4*(1/3*c^3*x^3*e^2+2*c^3*x*d*e-d^2*c^3/x)+b/c^4*(1/3*e^2*arcsech(c*x)*c^3*x^3+2*arcsech(c*x)*c^3*x*d*e-arcsech(c*x)*d^2*c^3/x+1/6*(-(c*x-1)/c/x)^{(1/2)*((c*x+1)/c/x)^{(1/2)*(6*(-c^2*x^2+1)^{(1/2)*c^4*d^2+12*arcsin(c*x)*c^3*x*d*e-c^2*x^2*e^2*(-c^2*x^2+1)^{(1/2)+arcsin(c*x)*c*x*e^2)/(-c^2*x^2+1)^{(1/2))}}$

Maxima [A] time = 1.52381, size = 205, normalized size = 1.16

$$\frac{1}{3}ae^2x^3 + \left(c\sqrt{\frac{1}{c^2x^2}-1} - \frac{\operatorname{ar}sech(cx)}{x} \right)bd^2 + \frac{1}{6} \left(2x^3 \operatorname{ar}sech(cx) - \frac{\frac{\sqrt{\frac{1}{c^2x^2}-1}}{c^2\left(\frac{1}{c^2x^2}-1\right)+c^2} + \frac{\arctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right)}{c^2}}{c} \right) be^2 + 2adex + \frac{2}{6} \left(cx \operatorname{ar}ctan\left(\sqrt{\frac{1}{c^2x^2}-1}\right) - \frac{cx}{c^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^2,x, algorithm="maxima")`

[Out] $1/3*a*e^2*x^3 + (c*\sqrt{1/(c^2*x^2) - 1} - \operatorname{ar}sech(c*x)/x)*b*d^2 + 1/6*(2*x^3*\operatorname{ar}sech(c*x) - (\sqrt{1/(c^2*x^2) - 1}/(c^2*(1/(c^2*x^2) - 1) + c^2) + \operatorname{ar}ctan(\sqrt{1/(c^2*x^2) - 1})/c^2)/c)*b*e^2 + 2*a*d*e*x + 2*(c*x*\operatorname{ar}sech(c*x) - \operatorname{ar}ctan(\sqrt{1/(c^2*x^2) - 1}))*b*d*e/c - a*d^2/x$

Fricas [B] time = 2.45177, size = 614, normalized size = 3.47

$$2ac^3e^2x^4 + 12ac^3dex^2 - 6ac^3d^2 - 2(12bc^2de + be^2)x \operatorname{ar}ctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}}{cx}\right) + 2(3bc^3d^2 - 6bc^3de - bc^3e^2)x \log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}}{cx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^2,x, algorithm="fricas")`

[Out] $1/6*(2*a*c^3*e^2*x^4 + 12*a*c^3*d*e*x^2 - 6*a*c^3*d^2 - 2*(12*b*c^2*d*e + b*e^2)*x*\operatorname{ar}ctan((c*x*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)}) - 1)/(c*x)) + 2*(3*b*c^3*d^2 - 6*b*c^3*d*e - b*c^3*e^2)*x*\log((c*x*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)}) - 1)/x) + 2*(b*c^3*e^2*x^4 + 6*b*c^3*d*e*x^2 - 3*b*c^3*d^2 + (3*b*c^3*d^2 - 6*b*c^3*d*e - b*c^3*e^2)*x)*\log((c*x*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)}) + 1)/(c*x)) + (6*b*c^4*d^2*x - b*c^2*e^2*x^3)*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)}/(c^3*x)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{asech}(cx))(d + ex^2)^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**2*(a+b*asech(c*x))/x**2,x)

[Out] Integral((a + b*asech(c*x))*(d + e*x**2)**2/x**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^2 (b \operatorname{ar} \operatorname{sech}(cx) + a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^2,x, algorithm="giac")

[Out] integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a)/x^2, x)

$$3.103 \quad \int \frac{(d+ex^2)^2 (a+b\operatorname{sech}^{-1}(cx))}{x^4} dx$$

Optimal. Leaf size=176

$$-\frac{d^2 (a + b\operatorname{sech}^{-1}(cx))}{3x^3} - \frac{2de (a + b\operatorname{sech}^{-1}(cx))}{x} + e^2 x (a + b\operatorname{sech}^{-1}(cx)) + \frac{bd^2 \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \sqrt{1-c^2x^2}}{9x^3} + \frac{2bd \sqrt{\frac{1}{cx+1}} \sqrt{cx}}{9x^3}$$

[Out] (b*d^2*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(9*x^3) + (2*b*d*(c^2*d + 9*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(9*x) - (d^2*(a + b*ArcSech[c*x]))/(3*x^3) - (2*d*e*(a + b*ArcSech[c*x]))/x + e^2*x*(a + b*ArcSech[c*x]) + (b*e^2*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x])/c

Rubi [A] time = 0.138995, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {270, 6301, 12, 1265, 451, 216}

$$-\frac{d^2 (a + b\operatorname{sech}^{-1}(cx))}{3x^3} - \frac{2de (a + b\operatorname{sech}^{-1}(cx))}{x} + e^2 x (a + b\operatorname{sech}^{-1}(cx)) + \frac{bd^2 \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \sqrt{1-c^2x^2}}{9x^3} + \frac{2bd \sqrt{\frac{1}{cx+1}} \sqrt{cx}}{9x^3}$$

Antiderivative was successfully verified.

[In] Int[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x^4,x]

[Out] (b*d^2*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(9*x^3) + (2*b*d*(c^2*d + 9*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(9*x) - (d^2*(a + b*ArcSech[c*x]))/(3*x^3) - (2*d*e*(a + b*ArcSech[c*x]))/x + e^2*x*(a + b*ArcSech[c*x]) + (b*e^2*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcSin[c*x])/c

Rule 270

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 6301

Int[((a_) + ArcSech[(c_)*(x_)])*(b_))*((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 1265

Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 +

```
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x],
Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x
- e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 451

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n
_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)^2 (a + b \operatorname{sech}^{-1}(cx))}{x^4} dx &= -\frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{3x^3} - \frac{2de (a + b \operatorname{sech}^{-1}(cx))}{x} + e^2 x (a + b \operatorname{sech}^{-1}(cx)) + \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \right) \\ &= -\frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{3x^3} - \frac{2de (a + b \operatorname{sech}^{-1}(cx))}{x} + e^2 x (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{3} \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \right) \\ &= \frac{bd^2 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{9x^3} - \frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{3x^3} - \frac{2de (a + b \operatorname{sech}^{-1}(cx))}{x} + \frac{1}{3} \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \right) \\ &= \frac{bd^2 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{9x^3} + \frac{2bd (c^2d + 9e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{9x} - \frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{3x^3} \\ &= \frac{bd^2 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{9x^3} + \frac{2bd (c^2d + 9e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{9x} - \frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{3x^3} \end{aligned}$$

Mathematica [C] time = 0.268651, size = 149, normalized size = 0.85

$$\frac{-3ac(d^2 + 6dex^2 - 3e^2x^4) + bcd\sqrt{\frac{1-cx}{cx+1}}(cx+1)(2c^2dx^2 + d + 18ex^2) - 3bc\operatorname{sech}^{-1}(cx)(d^2 + 6dex^2 - 3e^2x^4) + 9ibe^2x^3}{9cx^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x^4, x]
```

```
[Out] (b*c*d*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(d + 2*c^2*d*x^2 + 18*e*x^2) - 3
*a*c*(d^2 + 6*d*e*x^2 - 3*e^2*x^4) - 3*b*c*(d^2 + 6*d*e*x^2 - 3*e^2*x^4)*Ar
cSech[c*x] + (9*I)*b*e^2*x^3*Log[(-2*I)*c*x + 2*Sqrt[(1 - c*x)/(1 + c*x)]*(
1 + c*x)]/(9*c*x^3)
```

Maple [A] time = 0.185, size = 205, normalized size = 1.2

$$c^3 \left(\frac{a}{c^4} \left(cx^2 - 2 \frac{cde}{x} - \frac{cd^2}{3x^3} \right) + \frac{b}{c^4} \left(\operatorname{arcsech}(cx) cx^2 - 2 \frac{\operatorname{arcsech}(cx) cde}{x} - \frac{\operatorname{arcsech}(cx) d^2 c}{3x^3} + \frac{1}{9c^2x^2} \sqrt{\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d)^2*(a+b*arcsech(c*x))/x^4,x)`

[Out] $c^3*(a/c^4*(c*x*e^2-2*c*d*e/x-1/3*d^2*c/x^3)+b/c^4*(arcsech(c*x)*c*x*e^2-2*arcsech(c*x)*c*d*e/x-1/3*arcsech(c*x)*d^2*c/x^3+1/9*(-(c*x-1)/c/x)^{(1/2)}/c^2/x^2*((c*x+1)/c/x)^{(1/2)}*(2*(-c^2*x^2+1)^{(1/2)}*c^6*x^2*d^2+18*(-c^2*x^2+1)^{(1/2)}*c^4*x^2*d*e+(-c^2*x^2+1)^{(1/2)}*c^4*d^2+9*arcsin(c*x)*c^3*x^3*e^2)/(-c^2*x^2+1)^{(1/2))}$

Maxima [A] time = 0.978516, size = 181, normalized size = 1.03

$$2\left(c\sqrt{\frac{1}{c^2x^2}-1}-\frac{\operatorname{ar}sech(cx)}{x}\right)bde+ae^2x+\frac{1}{9}bd^2\left(\frac{c^4\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}}+3c^4\sqrt{\frac{1}{c^2x^2}-1}}{c}-\frac{3\operatorname{ar}sech(cx)}{x^3}\right)+\frac{(cx\operatorname{ar}sech(cx)-\arctan(\sqrt{\frac{1}{c^2x^2}-1}))}{9cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^4,x, algorithm="maxima")`

[Out] $2*(c*\sqrt{1/(c^2*x^2)-1}-arcsech(c*x)/x)*b*d*e+a*e^2*x+1/9*b*d^2*((c^4*(1/(c^2*x^2)-1)^{(3/2)}+3*c^4*\sqrt{1/(c^2*x^2)-1})/c-3*arcsech(c*x)/x^3)+(c*x*arcsech(c*x)-arctan(\sqrt{1/(c^2*x^2)-1}))*b*e^2/c-2*a*d*e/x-1/3*a*d^2/x^3$

Fricas [B] time = 2.23457, size = 590, normalized size = 3.35

$$\frac{9ace^2x^4-18be^2x^3\arctan\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{cx}\right)-18acdex^2+3(bcd^2+6bcde-3bce^2)x^3\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{x}\right)-3acd^2+3(3bcd^2+6bcde-3bce^2)}{9cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^4,x, algorithm="fricas")`

[Out] $1/9*(9*a*c*e^2*x^4-18*b*e^2*x^3*arctan((c*x*\sqrt{-(c^2*x^2-1)/(c^2*x^2)}-1)/(c*x))-18*a*c*d*e*x^2+3*(b*c*d^2+6*b*c*d*e-3*b*c*e^2)*x^3*\log((c*x*\sqrt{-(c^2*x^2-1)/(c^2*x^2)}-1)/x)-3*a*c*d^2+3*(3*b*c*e^2*x^4-6*b*c*d*e*x^2-b*c*d^2+(b*c*d^2+6*b*c*d*e-3*b*c*e^2)*x^3)*\log((c*x*\sqrt{-(c^2*x^2-1)/(c^2*x^2)}+1)/(c*x))+(b*c^2*d^2*x+2*(b*c^4*d^2+9*b*c^2*d*e)*x^3)*\sqrt{-(c^2*x^2-1)/(c^2*x^2)})/(c*x^3)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+b\operatorname{asech}(cx))(d+ex^2)^2}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*asech(c*x))/x**4,x)
```

```
[Out] Integral((a + b*asech(c*x))*(d + e*x**2)**2/x**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^2 (b \operatorname{arsech}(cx) + a)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a)/x^4, x)
```

$$3.104 \quad \int \frac{(d+ex^2)^2 (a+b\operatorname{sech}^{-1}(cx))}{x^6} dx$$

Optimal. Leaf size=213

$$-\frac{d^2 (a + b\operatorname{sech}^{-1}(cx))}{5x^5} - \frac{2de (a + b\operatorname{sech}^{-1}(cx))}{3x^3} - \frac{e^2 (a + b\operatorname{sech}^{-1}(cx))}{x} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2} (24c^4d^2 + 100c^2de + 225e^2)}{225x}$$

[Out] (b*d^2*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(25*x^5) + (2*b*d*(6*c^2*d + 25*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(225*x^3) + (b*(24*c^4*d^2 + 100*c^2*d*e + 225*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(225*x) - (d^2*(a + b*ArcSech[c*x]))/(5*x^5) - (2*d*e*(a + b*ArcSech[c*x]))/(3*x^3) - (e^2*(a + b*ArcSech[c*x]))/x

Rubi [A] time = 0.166714, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {270, 6301, 12, 1265, 453, 264}

$$-\frac{d^2 (a + b\operatorname{sech}^{-1}(cx))}{5x^5} - \frac{2de (a + b\operatorname{sech}^{-1}(cx))}{3x^3} - \frac{e^2 (a + b\operatorname{sech}^{-1}(cx))}{x} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2} (24c^4d^2 + 100c^2de + 225e^2)}{225x}$$

Antiderivative was successfully verified.

[In] Int[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x^6,x]

[Out] (b*d^2*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(25*x^5) + (2*b*d*(6*c^2*d + 25*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(225*x^3) + (b*(24*c^4*d^2 + 100*c^2*d*e + 225*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(225*x) - (d^2*(a + b*ArcSech[c*x]))/(5*x^5) - (2*d*e*(a + b*ArcSech[c*x]))/(3*x^3) - (e^2*(a + b*ArcSech[c*x]))/x

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 6301

Int[((a_.) + ArcSech[(c_.)*(x_)])*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 1265

Int[((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +

```
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x],
Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x
- e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 453

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n
_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 264

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{(d + ex^2)^2 (a + b \operatorname{sech}^{-1}(cx))}{x^6} dx &= -\frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{5x^5} - \frac{2de (a + b \operatorname{sech}^{-1}(cx))}{3x^3} - \frac{e^2 (a + b \operatorname{sech}^{-1}(cx))}{x} + \left(b \sqrt{\frac{1-cx}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \right) \\ &= -\frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{5x^5} - \frac{2de (a + b \operatorname{sech}^{-1}(cx))}{3x^3} - \frac{e^2 (a + b \operatorname{sech}^{-1}(cx))}{x} + \frac{1}{15} \left(b \sqrt{\frac{1-cx}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \right) \\ &= \frac{bd^2 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{25x^5} - \frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{5x^5} - \frac{2de (a + b \operatorname{sech}^{-1}(cx))}{3x^3} \\ &= \frac{bd^2 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{25x^5} + \frac{2bd (6c^2d + 25e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{225x^3} - \frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{5x^5} \\ &= \frac{bd^2 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{25x^5} + \frac{2bd (6c^2d + 25e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{225x^3} + \frac{b (2d^2 (a + b \operatorname{sech}^{-1}(cx)) - 2de (a + b \operatorname{sech}^{-1}(cx)) - e^2 (a + b \operatorname{sech}^{-1}(cx)))}{225x^5} \end{aligned}$$

Mathematica [A] time = 0.266362, size = 134, normalized size = 0.63

$$\frac{-15a(3d^2 + 10dex^2 + 15e^2x^4) + b\sqrt{\frac{1-cx}{cx+1}}(cx+1)(3d^2(8c^4x^4 + 4c^2x^2 + 3) + 50dex^2(2c^2x^2 + 1) + 225e^2x^4) - 15b\operatorname{sech}^{-1}(cx)}{225x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x^6, x]
```

```
[Out] (-15*a*(3*d^2 + 10*d*e*x^2 + 15*e^2*x^4) + b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 +
c*x)*(225*e^2*x^4 + 50*d*e*x^2*(1 + 2*c^2*x^2) + 3*d^2*(3 + 4*c^2*x^2 + 8*
c^4*x^4)) - 15*b*(3*d^2 + 10*d*e*x^2 + 15*e^2*x^4)*ArcSech[c*x])/(225*x^5)
```

Maple [A] time = 0.185, size = 193, normalized size = 0.9

$$c^5 \left(\frac{a}{c^4} \left(-\frac{e^2}{cx} - \frac{d^2}{5cx^5} - \frac{2de}{3cx^3} \right) + \frac{b}{c^4} \left(-\frac{\operatorname{arcsech}(cx)e^2}{cx} - \frac{\operatorname{arcsech}(cx)d^2}{5cx^5} - \frac{2\operatorname{arcsech}(cx)de}{3cx^3} + \frac{24c^8d^2x^4 + 100c^6dex^4}{225x^5} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d)^2*(a+b*arcsech(c*x))/x^6,x)`

[Out] $c^5*(a/c^4*(-e^2/c/x-1/5*d^2/c/x^5-2/3*c*d*e/x^3)+b/c^4*(-arcsech(c*x)*e^2/c/x-1/5*arcsech(c*x)*d^2/c/x^5-2/3*arcsech(c*x)/c*d*e/x^3+1/225*(-(c*x-1)/c/x)^{(1/2)}/c^4/x^4*((c*x+1)/c/x)^{(1/2)}*(24*c^8*d^2*x^4+100*c^6*d*e*x^4+12*c^6*d^2*x^2+225*c^4*e^2*x^4+50*c^4*d*e*x^2+9*c^4*d^2)))$

Maxima [A] time = 0.986755, size = 236, normalized size = 1.11

$$\left(c\sqrt{\frac{1}{c^2x^2}-1} - \frac{\operatorname{arsh}(cx)}{x} \right) be^2 + \frac{1}{75} bd^2 \left(\frac{3c^6\left(\frac{1}{c^2x^2}-1\right)^{\frac{5}{2}} + 10c^6\left(\frac{1}{c^2x^2}-1\right)^{\frac{3}{2}} + 15c^6\sqrt{\frac{1}{c^2x^2}-1} - \frac{15\operatorname{arsh}(cx)}{x^5}}{c} \right) + \frac{2}{9} bde$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^6,x, algorithm="maxima")`

[Out] $(c*\sqrt{1/(c^2*x^2)-1} - \operatorname{arsh}(c*x)/x)*b*e^2 + 1/75*b*d^2*((3*c^6*(1/(c^2*x^2)-1)^{(5/2)} + 10*c^6*(1/(c^2*x^2)-1)^{(3/2)} + 15*c^6*\sqrt{1/(c^2*x^2)-1}))/c - 15*\operatorname{arsh}(c*x)/x^5 + 2/9*b*d*e*((c^4*(1/(c^2*x^2)-1)^{(3/2)} + 3*c^4*\sqrt{1/(c^2*x^2)-1}))/c - 3*\operatorname{arsh}(c*x)/x^3 - a*e^2/x - 2/3*a*d*e/x^3 - 1/5*a*d^2/x^5$

Fricas [A] time = 1.79355, size = 385, normalized size = 1.81

$$\frac{225ae^2x^4 + 150adex^2 + 45ad^2 + 15(15be^2x^4 + 10bdex^2 + 3bd^2) \log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}+1}}{cx}\right) - ((24bc^5d^2 + 100bc^3de + 225bc^3d^2 + 25b^2c^3d^2 + 25b^2c^3d^2e)x^3) \sqrt{-(c^2x^2-1)/(c^2x^2)}}{225x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^6,x, algorithm="fricas")`

[Out] $-1/225*(225*a*e^2*x^4 + 150*a*d*e*x^2 + 45*a*d^2 + 15*(15*b*e^2*x^4 + 10*b*d*e*x^2 + 3*b*d^2)*\log((c*x*\sqrt{-(c^2*x^2-1)/(c^2*x^2)} + 1)/(c*x)) - ((24*b*c^5*d^2 + 100*b*c^3*d*e + 225*b*c^3*d^2)*x^5 + 9*b*c*d^2*x + 2*(6*b*c^3*d^2 + 25*b*c*d*e)*x^3)*\sqrt{-(c^2*x^2-1)/(c^2*x^2)})/x^5$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{asech}(cx))(d + ex^2)^2}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)**2*(a+b*asech(c*x))/x**6,x)`

[Out] Integral((a + b*asech(c*x))*(d + e*x**2)**2/x**6, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^2 (b \operatorname{arsech}(cx) + a)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^6,x, algorithm="giac")

[Out] integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a)/x^6, x)

$$3.105 \quad \int \frac{(d+ex^2)^2 (a+b\operatorname{sech}^{-1}(cx))}{x^8} dx$$

Optimal. Leaf size=281

$$-\frac{d^2(a+b\operatorname{sech}^{-1}(cx))}{7x^7} - \frac{2de(a+b\operatorname{sech}^{-1}(cx))}{5x^5} - \frac{e^2(a+b\operatorname{sech}^{-1}(cx))}{3x^3} + \frac{2bc^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(360c^4d^2+1176cd^2+1176c^2de+1176e^2d^2)}{11025x}$$

[Out] (b*d^2*Sqrt[(1+c*x)^(-1)]*Sqrt[1+c*x]*Sqrt[1-c^2*x^2])/(49*x^7) + (2*b*d*(15*c^2*d+49*e)*Sqrt[(1+c*x)^(-1)]*Sqrt[1+c*x]*Sqrt[1-c^2*x^2])/(1225*x^5) + (b*(360*c^4*d^2+1176*c^2*d*e+1225*e^2)*Sqrt[(1+c*x)^(-1)]*Sqrt[1+c*x]*Sqrt[1-c^2*x^2])/(11025*x^3) + (2*b*c^2*(360*c^4*d^2+1176*c^2*d*e+1225*e^2)*Sqrt[(1+c*x)^(-1)]*Sqrt[1+c*x]*Sqrt[1-c^2*x^2])/(11025*x) - (d^2*(a+b*ArcSech[c*x]))/(7*x^7) - (2*d*e*(a+b*ArcSech[c*x]))/(5*x^5) - (e^2*(a+b*ArcSech[c*x]))/(3*x^3)

Rubi [A] time = 0.200737, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {270, 6301, 12, 1265, 453, 271, 264}

$$-\frac{d^2(a+b\operatorname{sech}^{-1}(cx))}{7x^7} - \frac{2de(a+b\operatorname{sech}^{-1}(cx))}{5x^5} - \frac{e^2(a+b\operatorname{sech}^{-1}(cx))}{3x^3} + \frac{2bc^2\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(360c^4d^2+1176cd^2+1176c^2de+1176e^2d^2)}{11025x}$$

Antiderivative was successfully verified.

[In] Int[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x^8, x]

[Out] (b*d^2*Sqrt[(1+c*x)^(-1)]*Sqrt[1+c*x]*Sqrt[1-c^2*x^2])/(49*x^7) + (2*b*d*(15*c^2*d+49*e)*Sqrt[(1+c*x)^(-1)]*Sqrt[1+c*x]*Sqrt[1-c^2*x^2])/(1225*x^5) + (b*(360*c^4*d^2+1176*c^2*d*e+1225*e^2)*Sqrt[(1+c*x)^(-1)]*Sqrt[1+c*x]*Sqrt[1-c^2*x^2])/(11025*x^3) + (2*b*c^2*(360*c^4*d^2+1176*c^2*d*e+1225*e^2)*Sqrt[(1+c*x)^(-1)]*Sqrt[1+c*x]*Sqrt[1-c^2*x^2])/(11025*x) - (d^2*(a+b*ArcSech[c*x]))/(7*x^7) - (2*d*e*(a+b*ArcSech[c*x]))/(5*x^5) - (e^2*(a+b*ArcSech[c*x]))/(3*x^3)

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 6301

Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1+c*x]*Sqrt[1/(1+c*x)], Int[SimplifyIntegrand[u/(x*Sqrt[1-c*x]*Sqrt[1+c*x]), x], x]] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m-1)/2, 0] && GtQ[m+2*p+3, 0])) || (IGtQ[(m+1)/2, 0] && !(ILtQ[p, 0] && GtQ[m+2*p+3, 0])) || (ILtQ[(m+2*p+1)/2, 0] && !ILtQ[(m-1)/2, 0]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

[In] Integrate[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x^8,x]

[Out] (-105*a*(15*d^2 + 42*d*e*x^2 + 35*e^2*x^4) + b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(1225*e^2*x^4*(1 + 2*c^2*x^2) + 294*d*e*x^2*(3 + 4*c^2*x^2 + 8*c^4*x^4) + 45*d^2*(5 + 6*c^2*x^2 + 8*c^4*x^4 + 16*c^6*x^6)) - 105*b*(15*d^2 + 42*d*e*x^2 + 35*e^2*x^4)*ArcSech[c*x])/(11025*x^7)

Maple [A] time = 0.189, size = 225, normalized size = 0.8

$$c^7 \left(\frac{a}{c^4} \left(-\frac{d^2}{7c^3x^7} - \frac{2de}{5c^3x^5} - \frac{e^2}{3c^3x^3} \right) + \frac{b}{c^4} \left(-\frac{\operatorname{arcsech}(cx)d^2}{7c^3x^7} - \frac{2\operatorname{arcsech}(cx)de}{5c^3x^5} - \frac{\operatorname{arcsech}(cx)e^2}{3c^3x^3} + \frac{720c^{10}d^2x^6 + 2352c^8d^2x^4 + 11025c^6d^2x^2 + 11025c^4d^2}{11025c^7x^7} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^2*(a+b*arcsech(c*x))/x^8,x)

[Out] c^7*(a/c^4*(-1/7*d^2/c^3/x^7-2/5/c^3*d*e/x^5-1/3*e^2/c^3/x^3)+b/c^4*(-1/7*a*rcsech(c*x)*d^2/c^3/x^7-2/5*arcsech(c*x)/c^3*d*e/x^5-1/3*arcsech(c*x)*e^2/c^3/x^3+1/11025*(-(c*x-1)/c/x)^(1/2)/c^6/x^6*((c*x+1)/c/x)^(1/2)*(720*c^10*d^2*x^6+2352*c^8*d*e*x^6+360*c^8*d^2*x^4+2450*c^6*e^2*x^6+1176*c^6*d*e*x^4+270*c^6*d^2*x^2+1225*c^4*e^2*x^4+882*c^4*d*e*x^2+225*c^4*d^2)))

Maxima [A] time = 1.01764, size = 313, normalized size = 1.11

$$\frac{1}{245} bd^2 \left(\frac{5c^8 \left(\frac{1}{c^2x^2} - 1 \right)^{\frac{7}{2}} + 21c^8 \left(\frac{1}{c^2x^2} - 1 \right)^{\frac{5}{2}} + 35c^8 \left(\frac{1}{c^2x^2} - 1 \right)^{\frac{3}{2}} + 35c^8 \sqrt{\frac{1}{c^2x^2} - 1}}{c} - \frac{35 \operatorname{arcsch}(cx)}{x^7} \right) + \frac{2}{75} bde \left(\frac{3c^6 \left(\frac{1}{c^2x^2} - 1 \right)^{\frac{5}{2}} + 10c^6 \left(\frac{1}{c^2x^2} - 1 \right)^{\frac{3}{2}} + 15c^6 \sqrt{\frac{1}{c^2x^2} - 1}}{c} - 15 \operatorname{arcsch}(cx)/x^5 + \frac{1}{9} b e^2 \left(\frac{1}{c^2x^2} - 1 \right)^{\frac{3}{2}} + 3c^4 \sqrt{\frac{1}{c^2x^2} - 1} \right) / c - 3 \operatorname{arcsch}(cx)/x^3 - \frac{1}{3} a e^2 / x^3 - \frac{2}{5} a d e / x^5 - \frac{1}{7} a d^2 / x^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^8,x, algorithm="maxima")

[Out] 1/245*b*d^2*((5*c^8*(1/(c^2*x^2) - 1)^(7/2) + 21*c^8*(1/(c^2*x^2) - 1)^(5/2) + 35*c^8*(1/(c^2*x^2) - 1)^(3/2) + 35*c^8*sqrt(1/(c^2*x^2) - 1))/c - 35*arcsech(c*x)/x^7) + 2/75*b*d*e*((3*c^6*(1/(c^2*x^2) - 1)^(5/2) + 10*c^6*(1/(c^2*x^2) - 1)^(3/2) + 15*c^6*sqrt(1/(c^2*x^2) - 1))/c - 15*arcsech(c*x)/x^5) + 1/9*b*e^2*((c^4*(1/(c^2*x^2) - 1)^(3/2) + 3*c^4*sqrt(1/(c^2*x^2) - 1))/c - 3*arcsech(c*x)/x^3) - 1/3*a*e^2/x^3 - 2/5*a*d*e/x^5 - 1/7*a*d^2/x^7)

Fricas [A] time = 1.87113, size = 483, normalized size = 1.72

$$\frac{3675ae^2x^4 + 4410adex^2 + 1575ad^2 + 105(35be^2x^4 + 42bdex^2 + 15bd^2) \log\left(\frac{cx\sqrt{\frac{c^2x^2-1}{c^2x^2}+1}}{cx}\right) - (2(360bc^7d^2 + 1176bc^5d^2 + 11025c^3d^2))}{11025x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^8,x, algorithm="fricas")

```
[Out] -1/11025*(3675*a*e^2*x^4 + 4410*a*d*e*x^2 + 1575*a*d^2 + 105*(35*b*e^2*x^4
+ 42*b*d*e*x^2 + 15*b*d^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*
x)) - (2*(360*b*c^7*d^2 + 1176*b*c^5*d*e + 1225*b*c^3*e^2)*x^7 + (360*b*c^5
*d^2 + 1176*b*c^3*d*e + 1225*b*c*e^2)*x^5 + 225*b*c*d^2*x + 18*(15*b*c^3*d^
2 + 49*b*c*d*e)*x^3)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/x^7
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{asech}(cx))(d + ex^2)^2}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*asech(c*x))/x**8,x)
```

```
[Out] Integral((a + b*asech(c*x))*(d + e*x**2)**2/x**8, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^2 (b \operatorname{arasech}(cx) + a)}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^8,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a)/x^8, x)
```

3.106 $\int x^3 (d + ex^2)^2 (a + b \operatorname{sech}^{-1}(cx)) dx$

Optimal. Leaf size=278

$$\frac{1}{4}d^2x^4(a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{3}dex^6(a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{8}e^2x^8(a + b \operatorname{sech}^{-1}(cx)) + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(1-c^2x^2)^{3/2}(6c^4d^2 + \dots)}{72c^8}$$

[Out] $-(b*(6*c^4*d^2 + 8*c^2*d*e + 3*e^2)*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[1 - c^2*x^2])/(24*c^8) + (b*(6*c^4*d^2 + 16*c^2*d*e + 9*e^2)*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*(1 - c^2*x^2)^{(3/2)})/(72*c^8) - (b*e*(8*c^2*d + 9*e)*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*(1 - c^2*x^2)^{(5/2)})/(120*c^8) + (b*e^2*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*(1 - c^2*x^2)^{(7/2)})/(56*c^8) + (d^2*x^4*(a + b*\operatorname{ArcSech}[c*x]))/4 + (d*e*x^6*(a + b*\operatorname{ArcSech}[c*x]))/3 + (e^2*x^8*(a + b*\operatorname{ArcSech}[c*x]))/8$

Rubi [A] time = 0.237377, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {266, 43, 6301, 12, 1251, 771}

$$\frac{1}{4}d^2x^4(a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{3}dex^6(a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{8}e^2x^8(a + b \operatorname{sech}^{-1}(cx)) + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(1-c^2x^2)^{3/2}(6c^4d^2 + \dots)}{72c^8}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x^3*(d + e*x^2)^2*(a + b*\operatorname{ArcSech}[c*x]), x]$

[Out] $-(b*(6*c^4*d^2 + 8*c^2*d*e + 3*e^2)*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[1 - c^2*x^2])/(24*c^8) + (b*(6*c^4*d^2 + 16*c^2*d*e + 9*e^2)*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*(1 - c^2*x^2)^{(3/2)})/(72*c^8) - (b*e*(8*c^2*d + 9*e)*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*(1 - c^2*x^2)^{(5/2)})/(120*c^8) + (b*e^2*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*(1 - c^2*x^2)^{(7/2)})/(56*c^8) + (d^2*x^4*(a + b*\operatorname{ArcSech}[c*x]))/4 + (d*e*x^6*(a + b*\operatorname{ArcSech}[c*x]))/3 + (e^2*x^8*(a + b*\operatorname{ArcSech}[c*x]))/8$

Rule 266

$\operatorname{Int}[(x_)^{(m_.)}*((a_) + (b_.)*(x_)^{(n_.)})^{(p_)}, x_Symbol] \rightarrow \operatorname{Dist}[1/n, \operatorname{Subst}[\operatorname{Int}[x^{(\operatorname{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p}, x], x, x^n], x] /;$ $\operatorname{FreeQ}\{a, b, m, n, p\}, x] \ \&\& \ \operatorname{IntegerQ}[\operatorname{Simplify}[(m + 1)/n]]$

Rule 43

$\operatorname{Int}[(a_.) + (b_.)*(x_)^{(m_.)}*((c_.) + (d_.)*(x_)^{(n_.)}), x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, n\}, x] \ \&\& \ \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \ \operatorname{IGtQ}[m, 0] \ \&\& \ (\operatorname{IntegerQ}[n] \ \|\ (\operatorname{EqQ}[c, 0] \ \&\& \ \operatorname{LeQ}[7*m + 4*n + 4, 0]) \ \|\ \operatorname{LtQ}[9*m + 5*(n + 1), 0] \ \|\ \operatorname{GtQ}[m + n + 2, 0])$

Rule 6301

$\operatorname{Int}[(a_.) + \operatorname{ArcSech}[c_.*(x_)]*(b_.)*((f_.)*(x_)^{(m_.)}*((d_.) + (e_.)*(x_)^2)^{(p_.)}), x_Symbol] \rightarrow \operatorname{With}\{u = \operatorname{IntHide}[(f*x)^m*(d + e*x^2)^p, x]\}, \operatorname{Dist}[a + b*\operatorname{ArcSech}[c*x], u, x] + \operatorname{Dist}[b*\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[1/(1 + c*x)], \operatorname{Int}[\operatorname{SimplifyIntegrand}[u/(x*\operatorname{Sqrt}[1 - c*x]*\operatorname{Sqrt}[1 + c*x]), x], x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, e, f, m, p\}, x] \ \&\& \ (\operatorname{IGtQ}[p, 0] \ \&\& \ \operatorname{IntegerQ}[(m - 1)/2, 0] \ \&\& \ \operatorname{GtQ}[m + 2*p + 3, 0]) \ \|\ (\operatorname{IGtQ}[(m + 1)/2, 0] \ \&\& \ \operatorname{IntegerQ}[p, 0] \ \&\& \ \operatorname{GtQ}[m + 2*p + \dots, 0])$

3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 1251

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rule 771

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))

Rubi steps

$$\begin{aligned} \int x^3 (d + ex^2)^2 (a + b \operatorname{sech}^{-1}(cx)) dx &= \frac{1}{4} d^2 x^4 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{3} dex^6 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{8} e^2 x^8 (a + b \operatorname{sech}^{-1}(cx)) \\ &= \frac{1}{4} d^2 x^4 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{3} dex^6 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{8} e^2 x^8 (a + b \operatorname{sech}^{-1}(cx)) \\ &= \frac{1}{4} d^2 x^4 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{3} dex^6 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{8} e^2 x^8 (a + b \operatorname{sech}^{-1}(cx)) \\ &= \frac{1}{4} d^2 x^4 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{3} dex^6 (a + b \operatorname{sech}^{-1}(cx)) + \frac{1}{8} e^2 x^8 (a + b \operatorname{sech}^{-1}(cx)) \\ &= -\frac{b(6c^4 d^2 + 8c^2 de + 3e^2) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2 x^2}}{24c^8} + \frac{b(6c^4 d^2 + 16c^2 de + 9e^2)}{105c^8} \end{aligned}$$

Mathematica [A] time = 0.286706, size = 168, normalized size = 0.6

$$\frac{1}{24} \left(6ad^2x^4 + 8adex^6 + 3ae^2x^8 - \frac{b\sqrt{\frac{1-cx}{cx+1}}(cx+1)(3c^6(70d^2x^2 + 56dex^4 + 15e^2x^6) + c^4(420d^2 + 224dex^2 + 54e^2x^4) + 4e^2x^4) + 3c^6(70d^2x^2 + 56d*ex^4 + 15e^2x^6))}{105c^8} + b*x^4(6*d^2 + 8*d*ex^2 + 3*e^2*x^4)*\operatorname{ArcSech}[c*x] \right) / 24$$

Antiderivative was successfully verified.

[In] Integrate[x^3*(d + e*x^2)^2*(a + b*ArcSech[c*x]), x]

[Out] (6*a*d^2*x^4 + 8*a*d*e*x^6 + 3*a*e^2*x^8 - (b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(144*e^2 + 8*c^2*e*(56*d + 9*e*x^2) + c^4*(420*d^2 + 224*d*e*x^2 + 54*e^2*x^4) + 3*c^6*(70*d^2*x^2 + 56*d*e*x^4 + 15*e^2*x^6)))/(105*c^8) + b*x^4*(6*d^2 + 8*d*e*x^2 + 3*e^2*x^4)*ArcSech[c*x])/24

Maple [A] time = 0.181, size = 212, normalized size = 0.8

$$\frac{1}{c^4} \left(\frac{a}{c^4} \left(\frac{e^2 c^8 x^8}{8} + \frac{c^8 d e x^6}{3} + \frac{x^4 c^8 d^2}{4} \right) + \frac{b}{c^4} \left(\frac{\operatorname{arcsech}(cx) e^2 c^8 x^8}{8} + \frac{\operatorname{arcsech}(cx) c^8 d e x^6}{3} + \frac{\operatorname{arcsech}(cx) c^8 x^4 d^2}{4} - \frac{cx (45 c^6 e^2 x^8 + 168 c^6 d e x^6 + 210 c^6 d^2 x^4 + 54 c^4 e^2 x^4 + 224 c^4 d e x^2 + 420 c^4 d^2 x^2 + 72 c^2 e^2 x^2 + 448 c^2 d e + 144 e^2)}{2520} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(e*x^2+d)^2*(a+b*arcsech(c*x)),x)

[Out] 1/c^4*(a/c^4*(1/8*e^2*c^8*x^8+1/3*c^8*d*e*x^6+1/4*x^4*c^8*d^2)+b/c^4*(1/8*arcsech(c*x)*e^2*c^8*x^8+1/3*arcsech(c*x)*c^8*d*e*x^6+1/4*arcsech(c*x)*c^8*x^4*d^2-1/2520*(-(c*x-1)/c/x)^(1/2)*c*x*((c*x+1)/c/x)^(1/2)*(45*c^6*e^2*x^8+168*c^6*d*e*x^6+210*c^6*d^2*x^4+54*c^4*e^2*x^4+224*c^4*d*e*x^2+420*c^4*d^2*x^2+72*c^2*e^2*x^2+448*c^2*d*e+144*e^2)))

Maxima [A] time = 0.994232, size = 331, normalized size = 1.19

$$\frac{1}{8} a e^2 x^8 + \frac{1}{3} a d e x^6 + \frac{1}{4} a d^2 x^4 + \frac{1}{12} \left(3 x^4 \operatorname{arsh}(cx) + \frac{c^2 x^3 \left(\frac{1}{c^2 x^2} - 1 \right)^{\frac{3}{2}} - 3 x \sqrt{\frac{1}{c^2 x^2} - 1}}{c^3} \right) b d^2 + \frac{1}{45} \left(15 x^6 \operatorname{arsh}(cx) - \frac{3 c^4}{2520} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="maxima")

[Out] 1/8*a*e^2*x^8 + 1/3*a*d*e*x^6 + 1/4*a*d^2*x^4 + 1/12*(3*x^4*arcsech(c*x) + (c^2*x^3*(1/(c^2*x^2) - 1)^(3/2) - 3*x*sqrt(1/(c^2*x^2) - 1))/c^3)*b*d^2 + 1/45*(15*x^6*arcsech(c*x) - (3*c^4*x^5*(1/(c^2*x^2) - 1)^(5/2) - 10*c^2*x^3*(1/(c^2*x^2) - 1)^(3/2) + 15*x*sqrt(1/(c^2*x^2) - 1))/c^5)*b*d*e + 1/280*(35*x^8*arcsech(c*x) + (5*c^6*x^7*(1/(c^2*x^2) - 1)^(7/2) - 21*c^4*x^5*(1/(c^2*x^2) - 1)^(5/2) + 35*c^2*x^3*(1/(c^2*x^2) - 1)^(3/2) - 35*x*sqrt(1/(c^2*x^2) - 1))/c^7)*b*e^2

Fricas [A] time = 2.08779, size = 509, normalized size = 1.83

$$\frac{315 a c^7 e^2 x^8 + 840 a c^7 d e x^6 + 630 a c^7 d^2 x^4 + 105 (3 b c^7 e^2 x^8 + 8 b c^7 d e x^6 + 6 b c^7 d^2 x^4) \log \left(\frac{c x \sqrt{\frac{c^2 x^2 - 1}{c^2 x^2} + 1}}{c x} \right) - (45 b c^6 e^2 x^7 + 6 (2520 c^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="fricas")

[Out] 1/2520*(315*a*c^7*e^2*x^8 + 840*a*c^7*d*e*x^6 + 630*a*c^7*d^2*x^4 + 105*(3*b*c^7*e^2*x^8 + 8*b*c^7*d*e*x^6 + 6*b*c^7*d^2*x^4)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (45*b*c^6*e^2*x^7 + 6*(28*b*c^6*d*e + 9*b*c^4*e^2)*x^5 + 2*(105*b*c^6*d^2 + 112*b*c^4*d*e + 36*b*c^2*e^2)*x^3 + 4*(105*b*c^4*d^2 + 112*b*c^2*d*e + 36*b*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/c^7

Sympy [A] time = 51.1061, size = 332, normalized size = 1.19

$$\left\{ \begin{array}{l} \frac{ad^2x^4}{4} + \frac{adex^6}{3} + \frac{ae^2x^8}{8} + \frac{bd^2x^4 \operatorname{asech}(cx)}{3} + \frac{bdex^6 \operatorname{asech}(cx)}{3} + \frac{be^2x^8 \operatorname{asech}(cx)}{8} - \frac{bd^2x^2\sqrt{-c^2x^2+1}}{12c^2} - \frac{bdex^4\sqrt{-c^2x^2+1}}{15c^2} - \frac{be^2x^6\sqrt{-c^2x^2+1}}{56c^2} - b \\ (a + \infty b) \left(\frac{d^2x^4}{4} + \frac{dex^6}{3} + \frac{e^2x^8}{8} \right) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(e*x**2+d)**2*(a+b*asech(c*x)),x)

[Out] Piecewise((a*d**2*x**4/4 + a*d*e*x**6/3 + a*e**2*x**8/8 + b*d**2*x**4*asech(c*x)/4 + b*d*e*x**6*asech(c*x)/3 + b*e**2*x**8*asech(c*x)/8 - b*d**2*x**2*sqrt(-c**2*x**2 + 1)/(12*c**2) - b*d*e*x**4*sqrt(-c**2*x**2 + 1)/(15*c**2) - b*e**2*x**6*sqrt(-c**2*x**2 + 1)/(56*c**2) - b*d**2*sqrt(-c**2*x**2 + 1)/(6*c**4) - 4*b*d*e*x**2*sqrt(-c**2*x**2 + 1)/(45*c**4) - 3*b*e**2*x**4*sqrt(-c**2*x**2 + 1)/(140*c**4) - 8*b*d*e*sqrt(-c**2*x**2 + 1)/(45*c**6) - b*e**2*x**2*sqrt(-c**2*x**2 + 1)/(35*c**6) - 2*b*e**2*sqrt(-c**2*x**2 + 1)/(35*c**8), Ne(c, 0)), ((a + oo*b)*(d**2*x**4/4 + d*e*x**6/3 + e**2*x**8/8), True))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + d)^2 (b \operatorname{arsech}(cx) + a) x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="giac")

[Out] integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a)*x^3, x)

3.107 $\int x (d + ex^2)^2 (a + b \operatorname{sech}^{-1}(cx)) dx$

Optimal. Leaf size=230

$$\frac{(d + ex^2)^3 (a + b \operatorname{sech}^{-1}(cx))}{6e} - \frac{b \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \sqrt{1-c^2x^2} (3c^4d^2 + 3c^2de + e^2)}{6c^6} - \frac{bd^3 \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \tanh^{-1}(\sqrt{1-c^2x^2})}{6e}$$

[Out] $-(b*(3*c^4*d^2 + 3*c^2*d*e + e^2)*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[1 - c^2*x^2])/(6*c^6) + (b*e*(3*c^2*d + 2*e)*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*(1 - c^2*x^2)^{(3/2)})/(18*c^6) - (b*e^2*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*(1 - c^2*x^2)^{(5/2)})/(30*c^6) + ((d + e*x^2)^3*(a + b*\operatorname{ArcSech}[c*x]))/(6*e) - (b*d^3*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{ArcTanh}[\operatorname{Sqrt}[1 - c^2*x^2]])/(6*e)$

Rubi [A] time = 0.252243, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {6299, 517, 446, 88, 63, 208}

$$\frac{(d + ex^2)^3 (a + b \operatorname{sech}^{-1}(cx))}{6e} - \frac{b \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \sqrt{1-c^2x^2} (3c^4d^2 + 3c^2de + e^2)}{6c^6} - \frac{bd^3 \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \tanh^{-1}(\sqrt{1-c^2x^2})}{6e}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x*(d + e*x^2)^2*(a + b*\operatorname{ArcSech}[c*x]), x]$

[Out] $-(b*(3*c^4*d^2 + 3*c^2*d*e + e^2)*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[1 - c^2*x^2])/(6*c^6) + (b*e*(3*c^2*d + 2*e)*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*(1 - c^2*x^2)^{(3/2)})/(18*c^6) - (b*e^2*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*(1 - c^2*x^2)^{(5/2)})/(30*c^6) + ((d + e*x^2)^3*(a + b*\operatorname{ArcSech}[c*x]))/(6*e) - (b*d^3*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{ArcTanh}[\operatorname{Sqrt}[1 - c^2*x^2]])/(6*e)$

Rule 6299

$\operatorname{Int}[(a + \operatorname{ArcSech}[c*x])*(b*x)^p, x] \rightarrow \operatorname{Simp}[(d + e*x^2)^{p+1}*(a + b*\operatorname{ArcSech}[c*x])/(2*e*(p+1)), x] + \operatorname{Dist}[(b*\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[1/(1 + c*x)])/(2*e*(p+1)), \operatorname{Int}[(d + e*x^2)^{p+1}/(x*\operatorname{Sqrt}[1 - c*x]*\operatorname{Sqrt}[1 + c*x]), x] /;$ FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]

Rule 517

$\operatorname{Int}[(c + d*x^n)^q*(a + b*x^n)^p, x] \rightarrow \operatorname{Int}[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x] /;$ FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]))

Rule 446

$\operatorname{Int}[(a + b*x^n)^p*(c + d*x^n)^q, x] \rightarrow \operatorname{Dist}[1/n, \operatorname{Subst}[\operatorname{Int}[x^{(Simplify[(m+1)/n] - 1)}*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /;$ FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m+1)/n]]

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
 \int x (d + ex^2)^2 (a + b \operatorname{sech}^{-1}(cx)) dx &= \frac{(d + ex^2)^3 (a + b \operatorname{sech}^{-1}(cx))}{6e} + \frac{\left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx}\right) \int \frac{(d+ex^2)^3}{x \sqrt{1-cx} \sqrt{1+cx}} dx}{6e} \\
 &= \frac{(d + ex^2)^3 (a + b \operatorname{sech}^{-1}(cx))}{6e} + \frac{\left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx}\right) \int \frac{(d+ex^2)^3}{x \sqrt{1-c^2x^2}} dx}{6e} \\
 &= \frac{(d + ex^2)^3 (a + b \operatorname{sech}^{-1}(cx))}{6e} + \frac{\left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx}\right) \operatorname{Subst}\left(\int \frac{(d+ex^2)^3}{x \sqrt{1-c^2x}} dx, x, x^2\right)}{12e} \\
 &= \frac{(d + ex^2)^3 (a + b \operatorname{sech}^{-1}(cx))}{6e} + \frac{\left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx}\right) \operatorname{Subst}\left(\int \left(\frac{e(3c^4d^2 + 3c^2de + e^2)}{c^4 \sqrt{1-c^2x}}\right) dx, x, x^2\right)}{6e} \\
 &= -\frac{b(3c^4d^2 + 3c^2de + e^2) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{6c^6} + \frac{be(3c^2d + 2e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx}}{18c^6} \\
 &= -\frac{b(3c^4d^2 + 3c^2de + e^2) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{6c^6} + \frac{be(3c^2d + 2e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx}}{18c^6} \\
 &= -\frac{b(3c^4d^2 + 3c^2de + e^2) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{6c^6} + \frac{be(3c^2d + 2e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx}}{18c^6}
 \end{aligned}$$

Mathematica [A] time = 0.257237, size = 139, normalized size = 0.6

$$\frac{1}{6}ax^2(3d^2 + 3dex^2 + e^2x^4) - \frac{b\sqrt{\frac{1-cx}{cx+1}}(cx+1)(3c^4(15d^2 + 5dex^2 + e^2x^4) + 2c^2e(15d + 2ex^2) + 8e^2)}{90c^6} + \frac{1}{6}bx^2\operatorname{sech}^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Integrate[x*(d + e*x^2)^2*(a + b*ArcSech[c*x]), x]

[Out] (a*x^2*(3*d^2 + 3*d*e*x^2 + e^2*x^4))/6 - (b*sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(8*e^2 + 2*c^2*e*(15*d + 2*e*x^2) + 3*c^4*(15*d^2 + 5*d*e*x^2 + e^2*x

$\wedge 4)))/(90*c^6) + (b*x^2*(3*d^2 + 3*d*e*x^2 + e^2*x^4)*ArcSech[c*x])/6$

Maple [A] time = 0.175, size = 180, normalized size = 0.8

$$\frac{1}{c^2} \left(\frac{a}{c^4} \left(\frac{e^2 c^6 x^6}{6} + \frac{c^6 x^4 d e}{2} + \frac{x^2 c^6 d^2}{2} \right) + \frac{b}{c^4} \left(\frac{\operatorname{arcsech}(cx) e^2 c^6 x^6}{6} + \frac{\operatorname{arcsech}(cx) c^6 x^4 d e}{2} + \frac{\operatorname{arcsech}(cx) c^6 x^2 d^2}{2} - \frac{cx (3 c^4 e^2 x^6 + 3 b c^5 d e x^4 + 3 b^2 c^5 d^2 x^2 + 15 b^3 c^4 e^2 x^5 + 15 b^2 c^4 d e x^3 + 45 b^2 c^4 d^2 x + 8 b^3 e^2)}{90 c^5} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(e*x^2+d)^2*(a+b*arcsech(c*x)),x)`

[Out] $\frac{1}{c^2} \left(\frac{a}{c^4} \left(\frac{1}{6} e^2 c^6 x^6 + \frac{1}{2} c^6 x^4 d e + \frac{1}{2} x^2 c^6 d^2 \right) + \frac{b}{c^4} \left(\frac{1}{6} \operatorname{arcsech}(cx) e^2 c^6 x^6 + \frac{1}{2} \operatorname{arcsech}(cx) c^6 x^4 d e + \frac{1}{2} \operatorname{arcsech}(cx) c^6 x^2 d^2 - \frac{1}{90} \left(-\frac{c^2 x^3}{c^2 x^2} - 1 \right)^{\frac{3}{2}} - 3 x \sqrt{\frac{1}{c^2 x^2} - 1} \right) \right)$

Maxima [A] time = 0.991712, size = 250, normalized size = 1.09

$$\frac{1}{6} a e^2 x^6 + \frac{1}{2} a d e x^4 + \frac{1}{2} a d^2 x^2 + \frac{1}{2} \left(x^2 \operatorname{ar} \operatorname{sech}(cx) - \frac{x \sqrt{\frac{1}{c^2 x^2} - 1}}{c} \right) b d^2 + \frac{1}{6} \left(3 x^4 \operatorname{ar} \operatorname{sech}(cx) + \frac{c^2 x^3 \left(\frac{1}{c^2 x^2} - 1 \right)^{\frac{3}{2}} - 3 x \sqrt{\frac{1}{c^2 x^2} - 1}}{c^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="maxima")`

[Out] $\frac{1}{6} a e^2 x^6 + \frac{1}{2} a d e x^4 + \frac{1}{2} a d^2 x^2 + \frac{1}{2} \left(x^2 \operatorname{ar} \operatorname{sech}(cx) - \frac{x \sqrt{\frac{1}{c^2 x^2} - 1}}{c} \right) b d^2 + \frac{1}{6} \left(3 x^4 \operatorname{ar} \operatorname{sech}(cx) + \frac{c^2 x^3 \left(\frac{1}{c^2 x^2} - 1 \right)^{\frac{3}{2}} - 3 x \sqrt{\frac{1}{c^2 x^2} - 1}}{c^3} \right)$

Fricas [A] time = 2.01406, size = 413, normalized size = 1.8

$$\frac{15 a c^5 e^2 x^6 + 45 a c^5 d e x^4 + 45 a c^5 d^2 x^2 + 15 \left(b c^5 e^2 x^6 + 3 b c^5 d e x^4 + 3 b c^5 d^2 x^2 \right) \log \left(\frac{c x \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2} + 1}}{c x} \right) - \left(3 b c^4 e^2 x^5 + \left(15 b c^4 d e x^3 + 45 b c^4 d^2 x + 8 b^3 e^2 \right) x \right) \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2}}}{90 c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="fricas")`

[Out] $\frac{1}{90} \left(15 a c^5 e^2 x^6 + 45 a c^5 d e x^4 + 45 a c^5 d^2 x^2 + 15 \left(b c^5 e^2 x^6 + 3 b c^5 d e x^4 + 3 b c^5 d^2 x^2 \right) \log \left(\frac{c x \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2} + 1}}{c x} \right) - \left(3 b c^4 e^2 x^5 + \left(15 b c^4 d e x^3 + 45 b c^4 d^2 x + 8 b^3 e^2 \right) x \right) \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2}} \right) / c^5$

Sympy [A] time = 22.6777, size = 252, normalized size = 1.1

$$\left\{ \begin{array}{l} \frac{ad^2x^2}{2} + \frac{adex^4}{2} + \frac{ae^2x^6}{6} + \frac{bd^2x^2 \operatorname{arsech}(cx)}{2} + \frac{bdex^4 \operatorname{arsech}(cx)}{2} + \frac{be^2x^6 \operatorname{arsech}(cx)}{6} - \frac{bd^2\sqrt{-c^2x^2+1}}{2c^2} - \frac{bdex^2\sqrt{-c^2x^2+1}}{6c^2} - \frac{be^2x^4\sqrt{-c^2x^2+1}}{30c^2} - \frac{bde^2x^6\sqrt{-c^2x^2+1}}{30c^2} \\ (a + \infty b) \left(\frac{d^2x^2}{2} + \frac{dex^4}{2} + \frac{e^2x^6}{6} \right) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x**2+d)**2*(a+b*asech(c*x)),x)

[Out] Piecewise((a*d**2*x**2/2 + a*d*e*x**4/2 + a*e**2*x**6/6 + b*d**2*x**2*asech(c*x)/2 + b*d*e*x**4*asech(c*x)/2 + b*e**2*x**6*asech(c*x)/6 - b*d**2*sqrt(-c**2*x**2 + 1)/(2*c**2) - b*d*e*x**2*sqrt(-c**2*x**2 + 1)/(6*c**2) - b*e**2*x**4*sqrt(-c**2*x**2 + 1)/(30*c**2) - b*d*e*sqrt(-c**2*x**2 + 1)/(3*c**4) - 2*b*e**2*x**2*sqrt(-c**2*x**2 + 1)/(45*c**4) - 4*b*e**2*sqrt(-c**2*x**2 + 1)/(45*c**6), Ne(c, 0)), ((a + oo*b)*(d**2*x**2/2 + d*e*x**4/2 + e**2*x**6/6), True))

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + d)^2 (b \operatorname{arsech}(cx) + a) x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="giac")

[Out] integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a)*x, x)

$$3.108 \quad \int \frac{(d+ex^2)^2 (a+b\operatorname{sech}^{-1}(cx))}{x} dx$$

Optimal. Leaf size=370

$$\frac{ibd^2 \sqrt{1 - \frac{1}{c^2 x^2}} \operatorname{PolyLog}\left(2, e^{2i \operatorname{csc}^{-1}(cx)}\right)}{2\sqrt{\frac{1}{cx} - 1} \sqrt{\frac{1}{cx} + 1}} - d^2 \log\left(\frac{1}{x}\right) (a + b\operatorname{sech}^{-1}(cx)) + dex^2 (a + b\operatorname{sech}^{-1}(cx)) + \frac{1}{4} e^2 x^4 (a + b\operatorname{sech}^{-1}(cx))$$

```
[Out] -(b*e*(6*c^2*d + e)*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x)/(6*c^3) - (b*e^
2*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x^3)/(12*c) + ((I/2)*b*d^2*Sqrt[1 -
1/(c^2*x^2)]*ArcCsc[c*x]^2)/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) + d*e*x^
2*(a + b*ArcSech[c*x]) + (e^2*x^4*(a + b*ArcSech[c*x]))/4 - (b*d^2*Sqrt[1 -
1/(c^2*x^2)]*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])])/(Sqrt[-1 + 1/(c*x
)]*Sqrt[1 + 1/(c*x)]) + (b*d^2*Sqrt[1 - 1/(c^2*x^2)]*ArcCsc[c*x]*Log[x^(-1)
])/ (Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) - d^2*(a + b*ArcSech[c*x])*Log[x^
(-1)] + ((I/2)*b*d^2*Sqrt[1 - 1/(c^2*x^2)]*PolyLog[2, E^((2*I)*ArcCsc[c*x]
)])/ (Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])
```

Rubi [A] time = 1.09776, antiderivative size = 370, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 14, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6303, 266, 43, 5790, 6742, 454, 95, 2328, 2326, 4625, 3717, 2190, 2279, 2391}

$$\frac{ibd^2 \sqrt{1 - \frac{1}{c^2 x^2}} \operatorname{PolyLog}\left(2, e^{2i \operatorname{csc}^{-1}(cx)}\right)}{2\sqrt{\frac{1}{cx} - 1} \sqrt{\frac{1}{cx} + 1}} - d^2 \log\left(\frac{1}{x}\right) (a + b\operatorname{sech}^{-1}(cx)) + dex^2 (a + b\operatorname{sech}^{-1}(cx)) + \frac{1}{4} e^2 x^4 (a + b\operatorname{sech}^{-1}(cx))$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x,x]
```

```
[Out] -(b*e*(6*c^2*d + e)*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x)/(6*c^3) - (b*e^
2*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x^3)/(12*c) + ((I/2)*b*d^2*Sqrt[1 -
1/(c^2*x^2)]*ArcCsc[c*x]^2)/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) + d*e*x^
2*(a + b*ArcSech[c*x]) + (e^2*x^4*(a + b*ArcSech[c*x]))/4 - (b*d^2*Sqrt[1 -
1/(c^2*x^2)]*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])])/(Sqrt[-1 + 1/(c*x
)]*Sqrt[1 + 1/(c*x)]) + (b*d^2*Sqrt[1 - 1/(c^2*x^2)]*ArcCsc[c*x]*Log[x^(-1)
])/ (Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) - d^2*(a + b*ArcSech[c*x])*Log[x^
(-1)] + ((I/2)*b*d^2*Sqrt[1 - 1/(c^2*x^2)]*PolyLog[2, E^((2*I)*ArcCsc[c*x]
)])/ (Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])
```

Rule 6303

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^ (n_.)*(x_)^ (m_.)*((d_.) + (e_.)*(x_
)^2)^ (p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcCosh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rule 266

```
Int[(x_)^ (m_.)*((a_) + (b_.)*(x_)^ (n_.))^ (p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.)), x_Symbol] := Simp[(c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e^(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^(n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 2328

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(
d2_) + (e2_.)*(x_)]), x_Symbol] := Dist[Sqrt[1 + (e1*e2*x^2)/(d1*d2)]/(Sqrt
[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(a + b*Log[c*x^n])/Sqrt[1 + (e1*e2*x^2)/(
d1*d2)], x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 + d1
*e2, 0]
```

Rule 2326

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] := Simp[(ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]*(a + b*Log[c*x^n])/Rt[-e, 2], x
] - Dist[(b*n)/Rt[-e, 2], Int[ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 4625

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] := Subst[Int[(
a + b*x)^n/Tan[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol]
:> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex^2)^2 (a+b\operatorname{sech}^{-1}(cx))}{x} dx &= -\operatorname{Subst}\left(\int \frac{(e+dx^2)^2 (a+b\cosh^{-1}(\frac{x}{c}))}{x^5} dx, x, \frac{1}{x}\right) \\
&= dex^2 (a+b\operatorname{sech}^{-1}(cx)) + \frac{1}{4}e^2x^4 (a+b\operatorname{sech}^{-1}(cx)) - d^2 (a+b\operatorname{sech}^{-1}(cx)) \log\left(\frac{1}{x}\right) \\
&= dex^2 (a+b\operatorname{sech}^{-1}(cx)) + \frac{1}{4}e^2x^4 (a+b\operatorname{sech}^{-1}(cx)) - d^2 (a+b\operatorname{sech}^{-1}(cx)) \log\left(\frac{1}{x}\right) \\
&= dex^2 (a+b\operatorname{sech}^{-1}(cx)) + \frac{1}{4}e^2x^4 (a+b\operatorname{sech}^{-1}(cx)) - d^2 (a+b\operatorname{sech}^{-1}(cx)) \log\left(\frac{1}{x}\right) \\
&= -\frac{be^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x^3}{12c} + dex^2 (a+b\operatorname{sech}^{-1}(cx)) + \frac{1}{4}e^2x^4 (a+b\operatorname{sech}^{-1}(cx)) \\
&= -\frac{be(6c^2d+e)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{6c^3} - \frac{be^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x^3}{12c} + dex^2 (a+b\operatorname{sech}^{-1}(cx)) \\
&= -\frac{be(6c^2d+e)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{6c^3} - \frac{be^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x^3}{12c} + dex^2 (a+b\operatorname{sech}^{-1}(cx)) \\
&= -\frac{be(6c^2d+e)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{6c^3} - \frac{be^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x^3}{12c} + \frac{ibd^2\sqrt{1-\frac{1}{c^2x^2}}}{2\sqrt{-1+\frac{1}{cx}}} \\
&= -\frac{be(6c^2d+e)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{6c^3} - \frac{be^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x^3}{12c} + \frac{ibd^2\sqrt{1-\frac{1}{c^2x^2}}}{2\sqrt{-1+\frac{1}{cx}}} \\
&= -\frac{be(6c^2d+e)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{6c^3} - \frac{be^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x^3}{12c} + \frac{ibd^2\sqrt{1-\frac{1}{c^2x^2}}}{2\sqrt{-1+\frac{1}{cx}}} \\
&= -\frac{be(6c^2d+e)\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x}{6c^3} - \frac{be^2\sqrt{-1+\frac{1}{cx}}\sqrt{1+\frac{1}{cx}}x^3}{12c} + \frac{ibd^2\sqrt{1-\frac{1}{c^2x^2}}}{2\sqrt{-1+\frac{1}{cx}}}
\end{aligned}$$

Mathematica [A] time = 0.420813, size = 176, normalized size = 0.48

$$\frac{1}{2}bd^2\operatorname{PolyLog}\left(2, -e^{-2\operatorname{sech}^{-1}(cx)}\right) + ad^2\log(x) + adex^2 + \frac{1}{4}ae^2x^4 - \frac{bde\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{c^2} - \frac{be^2\sqrt{\frac{1-cx}{cx+1}}(cx+1)(c^2x^2+2)}{12c^4}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x,x]

[Out] a*d*e*x^2 + (a*e^2*x^4)/4 - (b*d*e*sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/c^2 - (b*e^2*sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(2 + c^2*x^2))/(12*c^4) + b*d*e*x^2*ArcSech[c*x] + (b*e^2*x^4*ArcSech[c*x])/4 - (b*d^2*ArcSech[c*x]*(ArcSech[c*x] + 2*Log[1 + E^(-2*ArcSech[c*x]))])/2 + a*d^2*Log[x] + (b*d^2*Poly

$\text{Log}[2, -E^{(-2*\text{ArcSech}[c*x])}]/2$

Maple [A] time = 0.485, size = 286, normalized size = 0.8

$$\frac{ae^2x^4}{4} + ax^2de + ad^2 \ln(cx) + \frac{b(\text{arcsech}(cx))^2 d^2}{2} - \frac{bx^3e^2}{12c} \sqrt{\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} - \frac{bxde}{c} \sqrt{\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} + \frac{b\text{arcsech}(cx)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x^2+d)^2*(a+b*arcsech(c*x))/x,x)`

[Out] $\frac{1}{4}ae^2x^4 + a*x^2*d*e + a*d^2*\ln(c*x) + \frac{1}{2}b*\text{arcsech}(c*x)^2*d^2 - \frac{1}{12}b/c*(-(c*x-1)/c/x)^{(1/2)}*((c*x+1)/c/x)^{(1/2)}*x^3*e^2 - b/c*(-(c*x-1)/c/x)^{(1/2)}*((c*x+1)/c/x)^{(1/2)}*x*d*e + \frac{1}{4}b*\text{arcsech}(c*x)*e^2*x^4 + b*\text{arcsech}(c*x)*x^2*d*e - \frac{1}{6}b/c^3*(-(c*x-1)/c/x)^{(1/2)}*((c*x+1)/c/x)^{(1/2)}*x*e^2 + b/c^2*d*e + \frac{1}{6}b/c^4*e^2 - b*d^2*\text{arcsech}(c*x)*\ln(1+(1/c/x+(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2}))^2) - \frac{1}{2}b*d^2*\text{polylog}(2, -(1/c/x+(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2}))^2)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4}ae^2x^4 + adex^2 + ad^2 \log(x) + \int be^2x^3 \log\left(\sqrt{\frac{1}{cx} + 1} \sqrt{\frac{1}{cx} - 1} + \frac{1}{cx}\right) + 2bdex \log\left(\sqrt{\frac{1}{cx} + 1} \sqrt{\frac{1}{cx} - 1} + \frac{1}{cx}\right) + \frac{bd^2 \log\left(\sqrt{\frac{1}{cx} + 1} \sqrt{\frac{1}{cx} - 1} + \frac{1}{cx}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x,x, algorithm="maxima")`

[Out] $\frac{1}{4}ae^2x^4 + a*d*e*x^2 + a*d^2*\log(x) + \int b*e^2*x^3*\log(\text{sqrt}(1/(c*x) + 1)*\text{sqrt}(1/(c*x) - 1) + 1/(c*x)) + 2*b*d*e*x*\log(\text{sqrt}(1/(c*x) + 1)*\text{sqrt}(1/(c*x) - 1) + 1/(c*x)) + b*d^2*\log(\text{sqrt}(1/(c*x) + 1)*\text{sqrt}(1/(c*x) - 1) + 1/(c*x)))/x, x$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ae^2x^4 + 2adex^2 + ad^2 + (be^2x^4 + 2bdex^2 + bd^2)\text{arsech}(cx)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x,x, algorithm="fricas")`

[Out] $\text{integral}((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^2)*\text{arcsech}(c*x))/x, x)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \text{asech}(cx))(d + ex^2)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**2*(a+b*asech(c*x))/x,x)

[Out] Integral((a + b*asech(c*x))*(d + e*x**2)**2/x, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^2 (b \operatorname{ar} \operatorname{sech}(cx) + a)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x,x, algorithm="giac")

[Out] integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a)/x, x)

$$3.109 \quad \int \frac{(d+ex^2)^2 (a+b\operatorname{sech}^{-1}(cx))}{x^3} dx$$

Optimal. Leaf size=373

$$\frac{ibde\sqrt{1-\frac{1}{c^2x^2}}\operatorname{PolyLog}\left(2,e^{2i\operatorname{csc}^{-1}(cx)}\right)}{\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}} - \frac{d^2(a+b\operatorname{sech}^{-1}(cx))}{2x^2} - 2de\log\left(\frac{1}{x}\right)(a+b\operatorname{sech}^{-1}(cx)) + \frac{1}{2}e^2x^2(a+b\operatorname{sech}^{-1}(cx))$$

```
[Out] (b*c*d^2*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])/(4*x) - (b*e^2*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x)/(2*c) + (I*b*d*e*Sqrt[1 - 1/(c^2*x^2)]*ArcCsc[c*x]^2)/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) + (b*c^2*d^2*ArcSech[c*x])/4 - (d^2*(a + b*ArcSech[c*x]))/(2*x^2) + (e^2*x^2*(a + b*ArcSech[c*x]))/2 - (2*b*d*e*Sqrt[1 - 1/(c^2*x^2)]*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])])/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) + (2*b*d*e*Sqrt[1 - 1/(c^2*x^2)]*ArcCsc[c*x]*Log[x^(-1)])/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) - 2*d*e*(a + b*ArcSech[c*x])*Log[x^(-1)] + (I*b*d*e*Sqrt[1 - 1/(c^2*x^2)]*PolyLog[2, E^((2*I)*ArcCsc[c*x])])/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])
```

Rubi [A] time = 1.04749, antiderivative size = 373, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 16, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {6303, 266, 43, 5790, 12, 6742, 95, 90, 52, 2328, 2326, 4625, 3717, 2190, 2279, 2391}

$$\frac{ibde\sqrt{1-\frac{1}{c^2x^2}}\operatorname{PolyLog}\left(2,e^{2i\operatorname{csc}^{-1}(cx)}\right)}{\sqrt{\frac{1}{cx}-1}\sqrt{\frac{1}{cx}+1}} - \frac{d^2(a+b\operatorname{sech}^{-1}(cx))}{2x^2} - 2de\log\left(\frac{1}{x}\right)(a+b\operatorname{sech}^{-1}(cx)) + \frac{1}{2}e^2x^2(a+b\operatorname{sech}^{-1}(cx))$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x^3,x]
```

```
[Out] (b*c*d^2*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])/(4*x) - (b*e^2*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x)/(2*c) + (I*b*d*e*Sqrt[1 - 1/(c^2*x^2)]*ArcCsc[c*x]^2)/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) + (b*c^2*d^2*ArcSech[c*x])/4 - (d^2*(a + b*ArcSech[c*x]))/(2*x^2) + (e^2*x^2*(a + b*ArcSech[c*x]))/2 - (2*b*d*e*Sqrt[1 - 1/(c^2*x^2)]*ArcCsc[c*x]*Log[1 - E^((2*I)*ArcCsc[c*x])])/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) + (2*b*d*e*Sqrt[1 - 1/(c^2*x^2)]*ArcCsc[c*x]*Log[x^(-1)])/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) - 2*d*e*(a + b*ArcSech[c*x])*Log[x^(-1)] + (I*b*d*e*Sqrt[1 - 1/(c^2*x^2)]*PolyLog[2, E^((2*I)*ArcCsc[c*x])])/(Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])
```

Rule 6303

```
Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))^ (n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcCosh[x/c])^n)/x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegersQ[m, p]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^ (p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x
_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_)^(p_.), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(m + 1)*(b*c - a*d)*(b*e - a*f), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(2)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)^
(p_.), x_Symbol] := Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] := Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 2328

```
Int[((a_.) + Log[(c_.)*(x_)]^(n_.))*(b_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(
d2_) + (e2_.)*(x_)]), x_Symbol] := Dist[Sqrt[1 + (e1*e2*x^2)/(d1*d2)]/(Sqrt
[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(a + b*Log[c*x^n])/Sqrt[1 + (e1*e2*x^2)/(
d1*d2)], x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 + d1
*e2, 0]
```

Rule 2326

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symbol]
:= Simp[(ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/Rt[-e, 2], x]
- Dist[(b*n)/Rt[-e, 2], Int[ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]/x, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 4625

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^n/(x_), x_Symbol] := Subst[Int[(a + b*x)^n/Tan[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol]
:= Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(d + ex^2)^2 (a + b \operatorname{sech}^{-1}(cx))}{x^3} dx &= -\operatorname{Subst} \left(\int \frac{(e + dx^2)^2 (a + b \cosh^{-1}(\frac{x}{c}))}{x^3} dx, x, \frac{1}{x} \right) \\
&= -\frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{2x^2} + \frac{1}{2} e^2 x^2 (a + b \operatorname{sech}^{-1}(cx)) - 2de (a + b \operatorname{sech}^{-1}(cx)) \log \left(\frac{d + ex^2}{2x} \right) \\
&= -\frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{2x^2} + \frac{1}{2} e^2 x^2 (a + b \operatorname{sech}^{-1}(cx)) - 2de (a + b \operatorname{sech}^{-1}(cx)) \log \left(\frac{d + ex^2}{2x} \right) \\
&= -\frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{2x^2} + \frac{1}{2} e^2 x^2 (a + b \operatorname{sech}^{-1}(cx)) - 2de (a + b \operatorname{sech}^{-1}(cx)) \log \left(\frac{d + ex^2}{2x} \right) \\
&= -\frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{2x^2} + \frac{1}{2} e^2 x^2 (a + b \operatorname{sech}^{-1}(cx)) - 2de (a + b \operatorname{sech}^{-1}(cx)) \log \left(\frac{d + ex^2}{2x} \right) \\
&= \frac{bcd^2 \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}}{4x} - \frac{be^2 \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}} x}{2c} - \frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{2x^2} + \frac{1}{2} e^2 x^2 (a + b \operatorname{sech}^{-1}(cx)) \\
&= \frac{bcd^2 \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}}{4x} - \frac{be^2 \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}} x}{2c} + \frac{1}{4} bc^2 d^2 \operatorname{sech}^{-1}(cx) - \frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{2x^2} \\
&= \frac{bcd^2 \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}}{4x} - \frac{be^2 \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}} x}{2c} + \frac{1}{4} bc^2 d^2 \operatorname{sech}^{-1}(cx) - \frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{2x^2} \\
&= \frac{bcd^2 \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}}{4x} - \frac{be^2 \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}} x}{2c} + \frac{ibde \sqrt{1 - \frac{1}{c^2 x^2}} \operatorname{csc}^{-1}(cx)^2}{\sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}} + \frac{1}{4} bc^2 d^2 \operatorname{sech}^{-1}(cx) - \frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{2x^2} \\
&= \frac{bcd^2 \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}}{4x} - \frac{be^2 \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}} x}{2c} + \frac{ibde \sqrt{1 - \frac{1}{c^2 x^2}} \operatorname{csc}^{-1}(cx)^2}{\sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}} + \frac{1}{4} bc^2 d^2 \operatorname{sech}^{-1}(cx) - \frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{2x^2} \\
&= \frac{bcd^2 \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}}{4x} - \frac{be^2 \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}} x}{2c} + \frac{ibde \sqrt{1 - \frac{1}{c^2 x^2}} \operatorname{csc}^{-1}(cx)^2}{\sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}} + \frac{1}{4} bc^2 d^2 \operatorname{sech}^{-1}(cx) - \frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{2x^2} \\
&= \frac{bcd^2 \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}}{4x} - \frac{be^2 \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}} x}{2c} + \frac{ibde \sqrt{1 - \frac{1}{c^2 x^2}} \operatorname{csc}^{-1}(cx)^2}{\sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}} + \frac{1}{4} bc^2 d^2 \operatorname{sech}^{-1}(cx) - \frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{2x^2}
\end{aligned}$$

Mathematica [A] time = 0.808271, size = 212, normalized size = 0.57

$$\frac{1}{4} \left(4bde \operatorname{PolyLog} \left(2, -e^{-2 \operatorname{sech}^{-1}(cx)} \right) - \frac{2ad^2}{x^2} + 8ade \log(x) + 2ae^2 x^2 - \frac{bd^2 \sqrt{\frac{1-cx}{cx+1}} \left(-c^2 x^2 + c^2 x^2 \sqrt{1 - c^2 x^2} \tanh^{-1} \left(\sqrt{1 - c^2 x^2} \right) \right)}{x^2 (cx - 1)} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[((d + e*x^2)^2*(a + b*ArcSech[c*x]))/x^3,x]

```
[Out] ((-2*a*d^2)/x^2 + 2*a*e^2*x^2 - (2*b*e^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x
))/c^2 - (2*b*d^2*ArcSech[c*x])/x^2 + 2*b*e^2*x^2*ArcSech[c*x] - (b*d^2*Sqr
t[(1 - c*x)/(1 + c*x)]*(1 - c^2*x^2 + c^2*x^2*Sqrt[1 - c^2*x^2]*ArcTanh[Sqr
t[1 - c^2*x^2]]))/(x^2*(-1 + c*x)) - 4*b*d*e*ArcSech[c*x]*(ArcSech[c*x] + 2
*Log[1 + E^(-2*ArcSech[c*x])]) + 8*a*d*e*Log[x] + 4*b*d*e*PolyLog[2, -E^(-2
*ArcSech[c*x])])/4
```

Maple [A] time = 0.417, size = 252, normalized size = 0.7

$$\frac{ax^2e^2}{2} + 2ade \ln(cx) - \frac{ad^2}{2x^2} + b(\operatorname{arcsech}(cx))^2 de + \frac{bcd^2}{4x} \sqrt{-\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} + \frac{bc^2d^2 \operatorname{arcsech}(cx)}{4} - \frac{b \operatorname{arcsech}(cx) d^2}{2x^2} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arcsech(c*x))/x^3,x)
```

```
[Out] 1/2*a*x^2*e^2+2*a*d*e*ln(c*x)-1/2*a*d^2/x^2+b*arcsech(c*x)^2*d*e+1/4*c*b*d^
2/x*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)+1/4*b*c^2*d^2*arcsech(c*x)-1/2
*b*arcsech(c*x)*d^2/x^2-1/2/c*b*(-(c*x-1)/c/x)^(1/2)*((c*x+1)/c/x)^(1/2)*x*
e^2+1/2*b*arcsech(c*x)*x^2*e^2+1/2*b/c^2*e^2-2*b*d*e*arcsech(c*x)*ln(1+(1/c
/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2)-b*d*e*polylog(2,-(1/c/x+(-1+1/c/x)^(
1/2)*(1+1/c/x)^(1/2))^2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}ae^2x^2 - \frac{1}{8}bd^2 \left(\frac{2c^4x\sqrt{\frac{1}{c^2x^2}-1}}{c^2x^2\left(\frac{1}{c^2x^2}-1\right)-1} - c^3 \log\left(cx\sqrt{\frac{1}{c^2x^2}-1}+1\right) + c^3 \log\left(cx\sqrt{\frac{1}{c^2x^2}-1}-1\right) \right) + \frac{4 \operatorname{ar} \operatorname{sech}(cx)}{x^2} + 2ade \log(x) - \frac{a}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*e^2*x^2 - 1/8*b*d^2*((2*c^4*x*sqrt(1/(c^2*x^2) - 1)/(c^2*x^2*(1/(c^2*
x^2) - 1) - 1) - c^3*log(c*x*sqrt(1/(c^2*x^2) - 1) + 1) + c^3*log(c*x*sqrt(
1/(c^2*x^2) - 1) - 1))/c + 4*arcsech(c*x)/x^2) + 2*a*d*e*log(x) - 1/2*a*d^2
/x^2 + integrate(b*e^2*x*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))
+ 2*b*d*e*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/x, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{ae^2x^4 + 2adex^2 + ad^2 + (be^2x^4 + 2bdex^2 + bd^2) \operatorname{ar} \operatorname{sech}(cx)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^3,x, algorithm="fricas")
```


[Out] `integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^2)*arcsech(c*x))/x^3, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{asech}(cx))(d + ex^2)^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)**2*(a+b*asech(c*x))/x**3,x)`

[Out] `Integral((a + b*asech(c*x))*(d + e*x**2)**2/x**3, x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^2 (b \operatorname{arsech}(cx) + a)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^2*(a+b*arcsech(c*x))/x^3,x, algorithm="giac")`

[Out] `integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a)/x^3, x)`

$$3.110 \quad \int \frac{x^2 (a + b \operatorname{sech}^{-1}(cx))}{d + ex^2} dx$$

Optimal. Leaf size=519

$$-\frac{b\sqrt{-d}\operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} + \frac{b\sqrt{-d}\operatorname{PolyLog}\left(2, \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} - \frac{b\sqrt{-d}\operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^{3/2}} + \frac{b\sqrt{-d}\operatorname{PolyLog}\left(2, \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^{3/2}}$$

```
[Out] (x*(a + b*ArcSech[c*x]))/e - (b*ArcTan[Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]]/(c*e) + (Sqrt[-d]*(a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])]/(Sqrt[e] - Sqrt[c^2*d + e]))/(2*e^(3/2)) - (Sqrt[-d]*(a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])]/(Sqrt[e] - Sqrt[c^2*d + e]))/(2*e^(3/2)) + (Sqrt[-d]*(a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])]/(Sqrt[e] + Sqrt[c^2*d + e]))/(2*e^(3/2)) - (Sqrt[-d]*(a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])]/(Sqrt[e] + Sqrt[c^2*d + e]))/(2*e^(3/2)) - (b*Sqrt[-d]*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e]))]/(2*e^(3/2)) + (b*Sqrt[-d]*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e]))]/(2*e^(3/2)) - (b*Sqrt[-d]*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))]/(2*e^(3/2)) + (b*Sqrt[-d]*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))]/(2*e^(3/2))
```

Rubi [A] time = 1.2733, antiderivative size = 519, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {6303, 5792, 5662, 92, 205, 5707, 5800, 5562, 2190, 2279, 2391}

$$-\frac{b\sqrt{-d}\operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} + \frac{b\sqrt{-d}\operatorname{PolyLog}\left(2, \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^{3/2}} - \frac{b\sqrt{-d}\operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^{3/2}} + \frac{b\sqrt{-d}\operatorname{PolyLog}\left(2, \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2), x]
```

```
[Out] (x*(a + b*ArcSech[c*x]))/e - (b*ArcTan[Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]]/(c*e) + (Sqrt[-d]*(a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])]/(Sqrt[e] - Sqrt[c^2*d + e]))/(2*e^(3/2)) - (Sqrt[-d]*(a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])]/(Sqrt[e] - Sqrt[c^2*d + e]))/(2*e^(3/2)) + (Sqrt[-d]*(a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])]/(Sqrt[e] + Sqrt[c^2*d + e]))/(2*e^(3/2)) - (Sqrt[-d]*(a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])]/(Sqrt[e] + Sqrt[c^2*d + e]))/(2*e^(3/2)) - (b*Sqrt[-d]*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e]))]/(2*e^(3/2)) + (b*Sqrt[-d]*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e]))]/(2*e^(3/2)) - (b*Sqrt[-d]*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))]/(2*e^(3/2)) + (b*Sqrt[-d]*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))]/(2*e^(3/2))
```

Rule 6303

```
Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))^(n_.)*(x_.)^(m_.)*((d_.) + (e_.)*(x_.)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcCosh[x/c])^n)/x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegersQ[m, p]
```

Rule 5792

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)*((f_.)*(x_.))^(m_.)*((d_.) + (e_.)*(x_.)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]

Rule 5662

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)*((d_.)*(x_.))^(m_.), x_Symbol] := Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

Rule 92

Int[1/(Sqrt[(a_.) + (b_.)*(x_.)]*Sqrt[(c_.) + (d_.)*(x_.)]*((e_.) + (f_.)*(x_.))), x_Symbol] := Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d*e - f*(b*c + a*d), 0]

Rule 205

Int[((a_.) + (b_.)*(x_.)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 5707

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_.)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (p > 0 || IGtQ[n, 0])

Rule 5800

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_.)), x_Symbol] := Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]

Rule 5562

Int[(((e_.) + (f_.)*(x_.))^(m_.)*Sinh[(c_.) + (d_.)*(x_.)]/(Cosh[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)), x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)), x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]

Rule 2190

Int[(((F_)^(g_.)*((e_.) + (f_.)*(x_.)))^(n_.)*((c_.) + (d_.)*(x_.))^(m_.))/((a_.) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x_.)))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

Int[Log[(a_.) + (b_.)*((F_)^(e_.)*((c_.) + (d_.)*(x_.)))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))]

)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rubi steps

$$\begin{aligned}
 \int \frac{x^2 (a + b \operatorname{sech}^{-1}(cx))}{d + ex^2} dx &= -\operatorname{Subst} \left(\int \frac{a + b \cosh^{-1} \left(\frac{x}{c} \right)}{x^2 (e + dx^2)} dx, x, \frac{1}{x} \right) \\
 &= -\operatorname{Subst} \left(\int \left(\frac{a + b \cosh^{-1} \left(\frac{x}{c} \right)}{ex^2} - \frac{d (a + b \cosh^{-1} \left(\frac{x}{c} \right))}{e (e + dx^2)} \right) dx, x, \frac{1}{x} \right) \\
 &= -\frac{\operatorname{Subst} \left(\int \frac{a + b \cosh^{-1} \left(\frac{x}{c} \right)}{x^2} dx, x, \frac{1}{x} \right)}{e} + \frac{d \operatorname{Subst} \left(\int \frac{a + b \cosh^{-1} \left(\frac{x}{c} \right)}{e + dx^2} dx, x, \frac{1}{x} \right)}{e} \\
 &= \frac{x (a + b \operatorname{sech}^{-1}(cx))}{e} - \frac{b \operatorname{Subst} \left(\int \frac{1}{x \sqrt{-1 + \frac{x}{c}} \sqrt{1 + \frac{x}{c}}} dx, x, \frac{1}{x} \right)}{ce} + \frac{d \operatorname{Subst} \left(\int \left(\frac{a + b \cosh^{-1} \left(\frac{x}{c} \right)}{2\sqrt{e}(\sqrt{e} - \sqrt{-dx})} + \frac{a + b}{2\sqrt{e}} \right) dx, x, \frac{1}{x} \right)}{e} \\
 &= \frac{x (a + b \operatorname{sech}^{-1}(cx))}{e} + \frac{d \operatorname{Subst} \left(\int \frac{a + b \cosh^{-1} \left(\frac{x}{c} \right)}{\sqrt{e} - \sqrt{-dx}} dx, x, \frac{1}{x} \right)}{2e^{3/2}} + \frac{d \operatorname{Subst} \left(\int \frac{a + b \cosh^{-1} \left(\frac{x}{c} \right)}{\sqrt{e} + \sqrt{-dx}} dx, x, \frac{1}{x} \right)}{2e^{3/2}} \\
 &= \frac{x (a + b \operatorname{sech}^{-1}(cx))}{e} - \frac{b \tan^{-1} \left(\sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}} \right)}{ce} + \frac{d \operatorname{Subst} \left(\int \frac{(a + bx) \sinh(x)}{\frac{\sqrt{e}}{c} - \sqrt{-d} \cosh(x)} dx, x, \operatorname{sech}^{-1} \left(\frac{x}{c} \right) \right)}{2e^{3/2}} \\
 &= \frac{x (a + b \operatorname{sech}^{-1}(cx))}{e} - \frac{b \tan^{-1} \left(\sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}} \right)}{ce} + \frac{d \operatorname{Subst} \left(\int \frac{e^x (a + bx)}{\frac{\sqrt{e}}{c} - \frac{\sqrt{c^2 d + e}}{c} - \sqrt{-d} e^x} dx, x, \operatorname{sech}^{-1} \left(\frac{x}{c} \right) \right)}{2e^{3/2}} \\
 &= \frac{x (a + b \operatorname{sech}^{-1}(cx))}{e} - \frac{b \tan^{-1} \left(\sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}} \right)}{ce} + \frac{\sqrt{-d} (a + b \operatorname{sech}^{-1}(cx)) \log \left(1 - \frac{c\sqrt{-d}}{\sqrt{e}} \right)}{2e^{3/2}} \\
 &= \frac{x (a + b \operatorname{sech}^{-1}(cx))}{e} - \frac{b \tan^{-1} \left(\sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}} \right)}{ce} + \frac{\sqrt{-d} (a + b \operatorname{sech}^{-1}(cx)) \log \left(1 - \frac{c\sqrt{-d}}{\sqrt{e}} \right)}{2e^{3/2}} \\
 &= \frac{x (a + b \operatorname{sech}^{-1}(cx))}{e} - \frac{b \tan^{-1} \left(\sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}} \right)}{ce} + \frac{\sqrt{-d} (a + b \operatorname{sech}^{-1}(cx)) \log \left(1 - \frac{c\sqrt{-d}}{\sqrt{e}} \right)}{2e^{3/2}}
 \end{aligned}$$

Mathematica [C] time = 1.4199, size = 921, normalized size = 1.77

$$4ac\sqrt{ex} - 4ac\sqrt{d} \tan^{-1} \left(\frac{\sqrt{ex}}{\sqrt{d}} \right) + b \left(4\sqrt{e} \left(cx \operatorname{sech}^{-1}(cx) - 2 \tan^{-1} \left(\tanh \left(\frac{1}{2} \operatorname{sech}^{-1}(cx) \right) \right) \right) - 2ic\sqrt{d} \left(-4i \sin^{-1} \left(\frac{\sqrt{\frac{i\sqrt{e}}{c\sqrt{d}} + 1}}{\sqrt{2}} \right) \tan^{-1} \left(\frac{\sqrt{ex}}{\sqrt{d}} \right) \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2), x]

```
[Out] (4*a*c*Sqrt[e]*x - 4*a*c*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + b*(4*Sqrt[e]
*(c*x*ArcSech[c*x] - 2*ArcTan[Tanh[ArcSech[c*x]/2]]) - (2*I)*c*Sqrt[d]*((-4
*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[((I*c*Sqrt[d]
+ Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] + ArcSech[c*x]*Log[1 + E
^(-2*ArcSech[c*x])] - ArcSech[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/
(c*Sqrt[d]*E^ArcSech[c*x])] + (2*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])
]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x
])] - ArcSech[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^Arc
Sech[c*x])] - (2*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1
+ (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + PolyLog[2,
(I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + PolyLog[2,
((-I)*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (2*I)*c*S
qrt[d]*((-4*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh(((
(-I)*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] + ArcSech[
c*x]*Log[1 + E^(-2*ArcSech[c*x])] - ArcSech[c*x]*Log[1 + (I*(-Sqrt[e] + Sqr
t[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (2*I)*ArcSin[Sqrt[1 - (I*Sqrt[
e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[
d]*E^ArcSech[c*x])] - ArcSech[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/
(c*Sqrt[d]*E^ArcSech[c*x])] - (2*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])
]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x
])] + PolyLog[2, (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])
] + PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])])
)/(4*c*e^(3/2))
```

Maple [C] time = 2.277, size = 411, normalized size = 0.8

$$\frac{ax}{e} - \frac{ad}{e} \arctan\left(\frac{x}{\sqrt{de}}\right) \frac{1}{\sqrt{de}} + \frac{bx \operatorname{arcsech}(cx)}{e} + \frac{bcd}{8e^2} \sum_{_R1=\operatorname{RootOf}(c^2d_Z^4+(2c^2d+4e)_Z^2+c^2d)} \frac{-_R1^2c^2d+4_R1^2e+c^2d}{-_R1(-_R1^2c^2d+c^2d+2e)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arcsech(c*x))/(e*x^2+d), x)
```

```
[Out] a/e*x-a*d/e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))+b*arcsech(c*x)/e*x+1/8*c*b/
e^2*d*sum((_R1^2*c^2*d+4*_R1^2*e+c^2*d)/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsec
h(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/
x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*
e)*_Z^2+c^2*d))-2/c*b/e*arctan(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))-1/8*
c*b/e^2*d*sum((_R1^2*c^2*d+c^2*d+4*e)/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(
c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-
(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)
*_Z^2+c^2*d))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d), x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^2 \operatorname{arsech}(cx) + ax^2}{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="fricas")

[Out] integral((b*x^2*arcsech(c*x) + a*x^2)/(e*x^2 + d), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 (a + b \operatorname{asech}(cx))}{d + ex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(a+b*asech(c*x))/(e*x**2+d),x)

[Out] Integral(x**2*(a + b*asech(c*x))/(d + e*x**2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsech}(cx) + a)x^2}{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)*x^2/(e*x^2 + d), x)

$$3.111 \quad \int \frac{x(a + b \operatorname{sech}^{-1}(cx))}{d + ex^2} dx$$

Optimal. Leaf size=459

$$\frac{b \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} + \frac{b \operatorname{PolyLog}\left(2, \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} + \frac{b \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e} + \frac{b \operatorname{PolyLog}\left(2, \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e}$$

```
[Out] -((a + b*ArcSech[c*x])^2/(b*e)) - ((a + b*ArcSech[c*x])*Log[1 + E^(-2*ArcSech[c*x])])/e + ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e) + ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e) + ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e) + ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e) + (b*PolyLog[2, -E^(-2*ArcSech[c*x])])/(2*e) + (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(2*e) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e) + (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(2*e) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e)
```

Rubi [A] time = 1.22889, antiderivative size = 441, normalized size of antiderivative = 0.96, number of steps used = 26, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {6303, 5792, 5660, 3718, 2190, 2279, 2391, 5800, 5562}

$$\frac{b \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} + \frac{b \operatorname{PolyLog}\left(2, \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e} + \frac{b \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e} + \frac{b \operatorname{PolyLog}\left(2, \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x*(a + b*ArcSech[c*x]))/(d + e*x^2), x]
```

```
[Out] ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e) + ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e) + ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e) + ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e) - ((a + b*ArcSech[c*x])*Log[1 + E^(2*ArcSech[c*x])])/e + (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(2*e) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e) + (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(2*e) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e) - (b*PolyLog[2, -E^(2*ArcSech[c*x])])/(2*e)
```

Rule 6303

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^ (n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcCosh[x/c])^n)/x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegersQ[m, p]
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^ (n_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
```

$(f*x)^m*(d + e*x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \ \&\& \ \text{NeQ}[c^2*d + e, 0] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IntegerQ}[p] \ \&\& \ \text{IntegerQ}[m]$

Rule 5660

$\text{Int}[(a + b*x)^n/\text{Coth}[x], x], x, \text{ArcCosh}[c*x]] /; \text{FreeQ}\{a, b, c\}, x] \ \&\& \ \text{IGtQ}[n, 0]$

Rule 3718

$\text{Int}[(c + d*x)^m*\tan[(e + (Complex[0, fz])*f]*x)], x_Symbol] \rightarrow -\text{Simp}[(I*(c + d*x)^{(m+1)})/(d*(m+1)), x] + \text{Dist}[2*I, \text{Int}[(c + d*x)^m*\text{E}^{(2*(-I*e) + f*fz*x))}/(1 + \text{E}^{(2*(-I*e) + f*fz*x))}], x], x] /; \text{FreeQ}\{c, d, e, f, fz\}, x] \ \&\& \ \text{IGtQ}[m, 0]$

Rule 2190

$\text{Int}[(F)^{(g*(e + f*x))}*(c + d*x)^m]/((a + b*(F)^{(g*(e + f*x))})^n), x_Symbol] \rightarrow \text{Simp}[(c + d*x)^m*\text{Log}[1 + (b*(F)^{(g*(e + f*x))})^n/a]/(b*f*g*n*\text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^{(m-1)}*\text{Log}[1 + (b*(F)^{(g*(e + f*x))})^n/a], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n\}, x] \ \&\& \ \text{IGtQ}[m, 0]$

Rule 2279

$\text{Int}[\text{Log}[a + b*(F)^{(e*(c + d*x))}], x_Symbol] \rightarrow \text{Dist}[1/(d*e*n*\text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F)^{(e*(c + d*x))}^n], x] /; \text{FreeQ}\{F, a, b, c, d, e, n\}, x] \ \&\& \ \text{GtQ}[a, 0]$

Rule 2391

$\text{Int}[\text{Log}[(c + d*x)^m*(e + f*x)^n]/(x), x_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /; \text{FreeQ}\{c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c*d, 1]$

Rule 5800

$\text{Int}[(a + b*x)^n*\text{Sinh}[c + d*x]/(c + d*x), x_Symbol] \rightarrow \text{Subst}[\text{Int}[(a + b*x)^n*\text{Sinh}[x]/(c*d + e*\text{Cosh}[x]), x], x, \text{ArcCosh}[c*x]] /; \text{FreeQ}\{a, b, c, d, e\}, x] \ \&\& \ \text{IGtQ}[n, 0]$

Rule 5562

$\text{Int}[(e + f*x)^m*\text{Sinh}[c + d*x]/(\text{Cosh}[c + d*x]*b + a), x_Symbol] \rightarrow -\text{Simp}[(e + f*x)^{(m+1)}/(b*f*(m+1)), x] + (\text{Int}[(e + f*x)^m*\text{E}^{(c + d*x)}/(a - \text{Rt}[a^2 - b^2, 2] + b*\text{E}^{(c + d*x)}), x] + \text{Int}[(e + f*x)^m*\text{E}^{(c + d*x)}/(a + \text{Rt}[a^2 - b^2, 2] + b*\text{E}^{(c + d*x)}), x]) /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ \text{NeQ}[a^2 - b^2, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{x(a + b \operatorname{sech}^{-1}(cx))}{d + ex^2} dx &= -\operatorname{Subst} \left(\int \frac{a + b \cosh^{-1}\left(\frac{x}{c}\right)}{x(e + dx^2)} dx, x, \frac{1}{x} \right) \\
&= -\operatorname{Subst} \left(\int \left(\frac{a + b \cosh^{-1}\left(\frac{x}{c}\right)}{ex} - \frac{dx(a + b \cosh^{-1}\left(\frac{x}{c}\right))}{e(e + dx^2)} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\operatorname{Subst} \left(\int \frac{a + b \cosh^{-1}\left(\frac{x}{c}\right)}{x} dx, x, \frac{1}{x} \right)}{e} + \frac{d \operatorname{Subst} \left(\int \frac{x(a + b \cosh^{-1}\left(\frac{x}{c}\right))}{e + dx^2} dx, x, \frac{1}{x} \right)}{e} \\
&= -\frac{\operatorname{Subst} \left(\int (a + bx) \tanh(x) dx, x, \operatorname{sech}^{-1}(cx) \right)}{e} + \frac{d \operatorname{Subst} \left(\int \left(-\frac{\sqrt{-d}(a + b \cosh^{-1}\left(\frac{x}{c}\right))}{2d(\sqrt{e} - \sqrt{-dx})} + \frac{\sqrt{-d}(a + b \cosh^{-1}\left(\frac{x}{c}\right))}{2d(\sqrt{e} + \sqrt{-dx})} \right) dx, x, \frac{1}{x} \right)}{e} \\
&= \frac{(a + b \operatorname{sech}^{-1}(cx))^2}{2be} - \frac{2 \operatorname{Subst} \left(\int \frac{e^{2x(a+bx)}}{1+e^{2x}} dx, x, \operatorname{sech}^{-1}(cx) \right)}{e} - \frac{\sqrt{-d} \operatorname{Subst} \left(\int \frac{a + b \cosh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e} - \sqrt{-dx}} dx, x, \frac{1}{x} \right)}{2e} \\
&= \frac{(a + b \operatorname{sech}^{-1}(cx))^2}{2be} - \frac{(a + b \operatorname{sech}^{-1}(cx)) \log(1 + e^{2 \operatorname{sech}^{-1}(cx)})}{e} + \frac{b \operatorname{Subst} \left(\int \log(1 + e^{2x}) dx, x, \operatorname{sech}^{-1}(cx) \right)}{e} \\
&= -\frac{(a + b \operatorname{sech}^{-1}(cx)) \log(1 + e^{2 \operatorname{sech}^{-1}(cx)})}{e} + \frac{b \operatorname{Subst} \left(\int \frac{\log(1+x)}{x} dx, x, e^{2 \operatorname{sech}^{-1}(cx)} \right)}{2e} - \frac{\sqrt{-d} \operatorname{Subst} \left(\int \frac{a + b \cosh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e} - \sqrt{-dx}} dx, x, \frac{1}{x} \right)}{2e} \\
&= \frac{(a + b \operatorname{sech}^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d + e}}\right)}{2e} + \frac{(a + b \operatorname{sech}^{-1}(cx)) \log\left(1 + \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d + e}}\right)}{2e} \\
&= \frac{(a + b \operatorname{sech}^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d + e}}\right)}{2e} + \frac{(a + b \operatorname{sech}^{-1}(cx)) \log\left(1 + \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d + e}}\right)}{2e} \\
&= \frac{(a + b \operatorname{sech}^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d + e}}\right)}{2e} + \frac{(a + b \operatorname{sech}^{-1}(cx)) \log\left(1 + \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d + e}}\right)}{2e} \\
&= \frac{(a + b \operatorname{sech}^{-1}(cx)) \log\left(1 - \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d + e}}\right)}{2e} + \frac{(a + b \operatorname{sech}^{-1}(cx)) \log\left(1 + \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d + e}}\right)}{2e}
\end{aligned}$$

Mathematica [C] time = 0.316926, size = 860, normalized size = 1.87

$$4ib \sin^{-1} \left(\frac{\sqrt{1 - \frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}} \right) \tanh^{-1} \left(\frac{(\sqrt{e} - ic\sqrt{d}) \tanh\left(\frac{1}{2} \operatorname{sech}^{-1}(cx)\right)}{\sqrt{dc^2 + e}} \right) + 4ib \sin^{-1} \left(\frac{\sqrt{\frac{i\sqrt{e}}{c\sqrt{d}} + 1}}{\sqrt{2}} \right) \tanh^{-1} \left(\frac{(i\sqrt{dc} + \sqrt{e}) \tanh\left(\frac{1}{2} \operatorname{sech}^{-1}(cx)\right)}{\sqrt{dc^2 + e}} \right) - 2b \operatorname{sech}^{-1}(cx)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x*(a + b*ArcSech[c*x]))/(d + e*x^2), x]

[Out] ((4*I)*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[(((-I)*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] + (4*I)*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[((I*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] - 2*b*ArcSech[c*x]*Log[1 + E^(-2*ArcSech[c*x])] + b*ArcSech[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (2*I)*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + b*ArcSech[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (2*I)*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Lo

$$g[1 + (I*(-\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e]))/(c*\text{Sqrt}[d]*E^{\text{ArcSech}[c*x]})] + b*\text{ArcSech}[c*x]*\text{Log}[1 - (I*(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e]))/(c*\text{Sqrt}[d]*E^{\text{ArcSech}[c*x]})] + (2*I)*b*\text{ArcSin}[\text{Sqrt}[1 - (I*\text{Sqrt}[e])/(\text{Sqrt}[d])]/\text{Sqrt}[2]]*\text{Log}[1 - (I*(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e]))/(c*\text{Sqrt}[d]*E^{\text{ArcSech}[c*x]})] + b*\text{ArcSech}[c*x]*\text{Log}[1 + (I*(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e]))/(c*\text{Sqrt}[d]*E^{\text{ArcSech}[c*x]})] + (2*I)*b*\text{ArcSin}[\text{Sqrt}[1 + (I*\text{Sqrt}[e])/(\text{Sqrt}[d])]/\text{Sqrt}[2]]*\text{Log}[1 + (I*(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e]))/(c*\text{Sqrt}[d]*E^{\text{ArcSech}[c*x]})] + a*\text{Log}[d + e*x^2] + b*\text{PolyLog}[2, -E^{(-2*\text{ArcSech}[c*x])}] - b*\text{PolyLog}[2, (I*(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e]))/(c*\text{Sqrt}[d]*E^{\text{ArcSech}[c*x]})] - b*\text{PolyLog}[2, (I*(-\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e]))/(c*\text{Sqrt}[d]*E^{\text{ArcSech}[c*x]})] - b*\text{PolyLog}[2, ((-I)*(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e]))/(c*\text{Sqrt}[d]*E^{\text{ArcSech}[c*x]})] - b*\text{PolyLog}[2, (I*(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e]))/(c*\text{Sqrt}[d]*E^{\text{ArcSech}[c*x]})]/(2*e)$$

Maple [C] time = 0.454, size = 513, normalized size = 1.1

$$\frac{a \ln(c^2 e x^2 + c^2 d)}{2e} + \frac{b c^2 d}{4e} \sum_{R1=\text{RootOf}(c^2 d Z^4 + (2c^2 d + 4e) Z^2 + c^2 d)} \frac{-R1^2 + 1}{-R1^2 c^2 d + c^2 d + 2e} \left(\text{arcsech}(cx) \ln \left(\frac{1}{-R1} \left(-R1 - \frac{1}{cx} - \sqrt{\dots} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a+b*arcsech(c*x))/(e*x^2+d),x)

[Out] 1/2*a/e*ln(c^2*e*x^2+c^2*d)+1/4*c^2*b*d/e*sum((R1^2+1)/(R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/R1)+dilog((R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/R1)),R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-b/e*arcsech(c*x)*ln(1+I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))-b/e*arcsech(c*x)*ln(1-I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))-b/e*dilog(1+I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))-b/e*dilog(1-I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))+1/4*b/e*sum((R1^2*c^2*d+c^2*d+4*e)/(R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/R1)+dilog((R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/R1)),R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b \int \frac{x \log \left(\sqrt{\frac{1}{cx} + 1} \sqrt{\frac{1}{cx} - 1} + \frac{1}{cx} \right)}{e x^2 + d} dx + \frac{a \log(e x^2 + d)}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="maxima")

[Out] b*integrate(x*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e*x^2 + d), x) + 1/2*a*log(e*x^2 + d)/e

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{b x \text{ar sech}(c x) + a x}{e x^2 + d}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x*arcsech(c*x) + a*x)/(e*x^2 + d), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x(a + b \operatorname{asech}(cx))}{d + ex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asech(c*x))/(e*x**2+d),x)
```

```
[Out] Integral(x*(a + b*asech(c*x))/(d + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsech}(cx) + a)x}{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x/(e*x^2 + d), x)
```

$$3.112 \quad \int \frac{a+b\operatorname{sech}^{-1}(cx)}{d+ex^2} dx$$

Optimal. Leaf size=469

$$-\frac{b\operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{b\operatorname{PolyLog}\left(2, \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{b\operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{b\operatorname{PolyLog}\left(2, \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}$$

```
[Out] ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) + ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(2*Sqrt[-d]*Sqrt[e]) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(2*Sqrt[-d]*Sqrt[e]) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e])
```

Rubi [A] time = 0.966431, antiderivative size = 469, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {6293, 5707, 5800, 5562, 2190, 2279, 2391}

$$-\frac{b\operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{b\operatorname{PolyLog}\left(2, \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{b\operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}} + \frac{b\operatorname{PolyLog}\left(2, \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2\sqrt{-d}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])/(d + e*x^2), x]
```

```
[Out] ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) + ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(2*Sqrt[-d]*Sqrt[e]) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e]) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(2*Sqrt[-d]*Sqrt[e]) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*Sqrt[-d]*Sqrt[e])
```

Rule 6293

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*(a + b*ArcCosh[x/c])^n]/x^(2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_.)), x_Symbol]
:> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x]]
/; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5562

```
Int((((e_.) + (f_.)*(x_.))^(m_.)*Sinh[(c_.) + (d_.)*(x_.)]/(Cosh[(c_.) + (d_.)
*(x_.)]*(b_.) + (a_.)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)),
x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)),
x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2190

```
Int((((F_)^((g_.)*((e_.) + (f_.)*(x_.))))^(n_.)*((c_.) + (d_.)*(x_.))^(m_.))/
((a_.) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_.))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a]/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x))
)^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_.) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_.))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_.) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + b \operatorname{sech}^{-1}(cx)}{d + ex^2} dx &= -\operatorname{Subst} \left(\int \frac{a + b \cosh^{-1}\left(\frac{x}{c}\right)}{e + dx^2} dx, x, \frac{1}{x} \right) \\
&= -\operatorname{Subst} \left(\int \left(\frac{a + b \cosh^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}(\sqrt{e} - \sqrt{-dx})} + \frac{a + b \cosh^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}(\sqrt{e} + \sqrt{-dx})} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\operatorname{Subst} \left(\int \frac{a + b \cosh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e} - \sqrt{-dx}} dx, x, \frac{1}{x} \right)}{2\sqrt{e}} - \frac{\operatorname{Subst} \left(\int \frac{a + b \cosh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e} + \sqrt{-dx}} dx, x, \frac{1}{x} \right)}{2\sqrt{e}} \\
&= -\frac{\operatorname{Subst} \left(\int \frac{(a+bx) \sinh(x)}{\frac{\sqrt{e}}{c} - \sqrt{-d} \cosh(x)} dx, x, \operatorname{sech}^{-1}(cx) \right)}{2\sqrt{e}} - \frac{\operatorname{Subst} \left(\int \frac{(a+bx) \sinh(x)}{\frac{\sqrt{e}}{c} + \sqrt{-d} \cosh(x)} dx, x, \operatorname{sech}^{-1}(cx) \right)}{2\sqrt{e}} \\
&= -\frac{\operatorname{Subst} \left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c} - \frac{\sqrt{c^2d+e}}{c} - \sqrt{-d}e^x} dx, x, \operatorname{sech}^{-1}(cx) \right)}{2\sqrt{e}} - \frac{\operatorname{Subst} \left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c} + \frac{\sqrt{c^2d+e}}{c} - \sqrt{-d}e^x} dx, x, \operatorname{sech}^{-1}(cx) \right)}{2\sqrt{e}} \\
&= \frac{(a + b \operatorname{sech}^{-1}(cx)) \log \left(1 - \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d+e}} \right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a + b \operatorname{sech}^{-1}(cx)) \log \left(1 + \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d+e}} \right)}{2\sqrt{-d}\sqrt{e}} + \frac{(a + b \operatorname{sech}^{-1}(cx)) \log \left(1 - \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d+e}} \right)}{2\sqrt{-d}\sqrt{e}} \\
&= \frac{(a + b \operatorname{sech}^{-1}(cx)) \log \left(1 - \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d+e}} \right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a + b \operatorname{sech}^{-1}(cx)) \log \left(1 + \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d+e}} \right)}{2\sqrt{-d}\sqrt{e}} + \frac{(a + b \operatorname{sech}^{-1}(cx)) \log \left(1 - \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d+e}} \right)}{2\sqrt{-d}\sqrt{e}} \\
&= \frac{(a + b \operatorname{sech}^{-1}(cx)) \log \left(1 - \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d+e}} \right)}{2\sqrt{-d}\sqrt{e}} - \frac{(a + b \operatorname{sech}^{-1}(cx)) \log \left(1 + \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d+e}} \right)}{2\sqrt{-d}\sqrt{e}} + \frac{(a + b \operatorname{sech}^{-1}(cx)) \log \left(1 - \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d+e}} \right)}{2\sqrt{-d}\sqrt{e}}
\end{aligned}$$

Mathematica [C] time = 0.370371, size = 849, normalized size = 1.81

$$2a \tan^{-1} \left(\frac{\sqrt{ex}}{\sqrt{d}} \right) - 4b \sin^{-1} \left(\frac{\sqrt{1 - \frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}} \right) \tanh^{-1} \left(\frac{(\sqrt{e} - ic\sqrt{d}) \tanh\left(\frac{1}{2} \operatorname{sech}^{-1}(cx)\right)}{\sqrt{d^2 + e}} \right) + 4b \sin^{-1} \left(\frac{\sqrt{\frac{i\sqrt{e}}{c\sqrt{d}} + 1}}{\sqrt{2}} \right) \tanh^{-1} \left(\frac{(i\sqrt{d}c + \sqrt{e}) \tanh\left(\frac{1}{2} \operatorname{sech}^{-1}(cx)\right)}{\sqrt{d^2 + e}} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*ArcSech[c*x])/(d + e*x^2), x]

[Out] (2*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - 4*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[(((-I)*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] + 4*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[(((I*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] - I*b*ArcSech[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - 2*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + I*b*ArcSech[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + 2*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + I*b*ArcSech[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - 2*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - I*b*ArcSech[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])])

$$\frac{2*d + e)}{(c*\sqrt{d}*E^{\text{ArcSech}[c*x]})] + 2*b*\text{ArcSin}[\sqrt{1 + (I*\sqrt{e})/(c*\sqrt{d})}]/\sqrt{2}]*\text{Log}[1 + (I*(\sqrt{e} + \sqrt{c^2*d + e}))/(c*\sqrt{d}*E^{\text{ArcSech}[c*x]})] - I*b*\text{PolyLog}[2, (I*(\sqrt{e} - \sqrt{c^2*d + e}))/(c*\sqrt{d}*E^{\text{ArcSech}[c*x]})] + I*b*\text{PolyLog}[2, (I*(-\sqrt{e} + \sqrt{c^2*d + e}))/(c*\sqrt{d}*E^{\text{ArcSech}[c*x]})] + I*b*\text{PolyLog}[2, ((-I)*(\sqrt{e} + \sqrt{c^2*d + e}))/(c*\sqrt{d}*E^{\text{ArcSech}[c*x]})] - I*b*\text{PolyLog}[2, (I*(\sqrt{e} + \sqrt{c^2*d + e}))/(c*\sqrt{d}*E^{\text{ArcSech}[c*x]})]]/(2*\sqrt{d}*\sqrt{e})$$

Maple [C] time = 1.344, size = 302, normalized size = 0.6

$$a \arctan\left(\frac{ex}{\sqrt{de}}\right) \frac{1}{\sqrt{de}} - \frac{bc}{2} \sum_{_R1=\text{RootOf}(c^2d_Z^4+(2c^2d+4e)_Z^2+c^2d)} \frac{-_R1}{-R1^2c^2d + c^2d + 2e} \left(\text{arcsech}(cx) \ln\left(\frac{1}{-R1} \left(-R1 - \frac{1}{c}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))/(e*x^2+d), x)

[Out] a/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))-1/2*c*b*sum(_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+1/2*c*b*sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/(e*x^2+d), x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b \text{arsech}(cx) + a}{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/(e*x^2+d), x, algorithm="fricas")

[Out] integral((b*arcsech(c*x) + a)/(e*x^2 + d), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \text{asech}(cx)}{d + ex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asech(c*x))/(e*x**2+d),x)

[Out] Integral((a + b*asech(c*x))/(d + e*x**2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{ar} \operatorname{sech}(cx) + a}{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)/(e*x^2 + d), x)

$$3.113 \quad \int \frac{a+b\operatorname{sech}^{-1}(cx)}{x(d+ex^2)} dx$$

Optimal. Leaf size=417

$$\frac{b\operatorname{PolyLog}\left(2, -\frac{c\sqrt{-de}\operatorname{sech}^{-1}(cx)}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d} - \frac{b\operatorname{PolyLog}\left(2, \frac{c\sqrt{-de}\operatorname{sech}^{-1}(cx)}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d} - \frac{b\operatorname{PolyLog}\left(2, -\frac{c\sqrt{-de}\operatorname{sech}^{-1}(cx)}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d} - \frac{b\operatorname{PolyLog}\left(2, \frac{c\sqrt{-de}\operatorname{sech}^{-1}(cx)}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d}$$

```
[Out] (a + b*ArcSech[c*x])^2/(2*b*d) - ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d) - ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d) - ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d) - ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(2*d) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(2*d) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d)
```

Rubi [A] time = 0.989292, antiderivative size = 417, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6303, 5792, 5800, 5562, 2190, 2279, 2391}

$$\frac{b\operatorname{PolyLog}\left(2, -\frac{c\sqrt{-de}\operatorname{sech}^{-1}(cx)}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d} - \frac{b\operatorname{PolyLog}\left(2, \frac{c\sqrt{-de}\operatorname{sech}^{-1}(cx)}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d} - \frac{b\operatorname{PolyLog}\left(2, -\frac{c\sqrt{-de}\operatorname{sech}^{-1}(cx)}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d} - \frac{b\operatorname{PolyLog}\left(2, \frac{c\sqrt{-de}\operatorname{sech}^{-1}(cx)}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])/(x*(d + e*x^2)), x]
```

```
[Out] (a + b*ArcSech[c*x])^2/(2*b*d) - ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d) - ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d) - ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d) - ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(2*d) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(2*d) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d)
```

Rule 6303

```
Int[((a_) + ArcSech[(c_)*(x_)])*(b_)^(n_)*(x_)^(m_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := -Subst[Int[((e + d*x^2)^p*(a + b*ArcCosh[x/c])^n)/x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegersQ[m, p]
```

Rule 5792

```
Int[((a_) + ArcCosh[(c_)*(x_)])*(b_)^(n_)*((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^n_]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Subst[Int[(a + b*x)^n*Sinh[x]/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x]]
/; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_.)*(x_)]*(b_.) + (a_)), x_Symbol]
:> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)), x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)), x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol]
:> Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol]
:> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + b \operatorname{sech}^{-1}(cx)}{x(d + ex^2)} dx &= -\operatorname{Subst} \left(\int \frac{x \left(a + b \cosh^{-1} \left(\frac{x}{c} \right) \right)}{e + dx^2} dx, x, \frac{1}{x} \right) \\
&= -\operatorname{Subst} \left(\int \left(-\frac{\sqrt{-d} \left(a + b \cosh^{-1} \left(\frac{x}{c} \right) \right)}{2d(\sqrt{e} - \sqrt{-dx})} + \frac{\sqrt{-d} \left(a + b \cosh^{-1} \left(\frac{x}{c} \right) \right)}{2d(\sqrt{e} + \sqrt{-dx})} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\operatorname{Subst} \left(\int \frac{a + b \cosh^{-1} \left(\frac{x}{c} \right)}{\sqrt{e} - \sqrt{-dx}} dx, x, \frac{1}{x} \right)}{2\sqrt{-d}} + \frac{\operatorname{Subst} \left(\int \frac{a + b \cosh^{-1} \left(\frac{x}{c} \right)}{\sqrt{e} + \sqrt{-dx}} dx, x, \frac{1}{x} \right)}{2\sqrt{-d}} \\
&= -\frac{\operatorname{Subst} \left(\int \frac{\frac{(a+bx) \sinh(x)}{c} - \sqrt{-d} \cosh(x)}{\sqrt{e} - \sqrt{-dx}} dx, x, \operatorname{sech}^{-1}(cx) \right)}{2\sqrt{-d}} + \frac{\operatorname{Subst} \left(\int \frac{\frac{(a+bx) \sinh(x)}{c} + \sqrt{-d} \cosh(x)}{\sqrt{e} + \sqrt{-dx}} dx, x, \operatorname{sech}^{-1}(cx) \right)}{2\sqrt{-d}} \\
&= \frac{(a + b \operatorname{sech}^{-1}(cx))^2}{2bd} - \frac{\operatorname{Subst} \left(\int \frac{\frac{e^x(a+bx)}{c} - \frac{\sqrt{c^2d+e}}{c} - \sqrt{-dex}}{\sqrt{e} - \sqrt{-dx}} dx, x, \operatorname{sech}^{-1}(cx) \right)}{2\sqrt{-d}} - \frac{\operatorname{Subst} \left(\int \frac{\frac{e^x(a+bx)}{c} + \frac{\sqrt{c^2d+e}}{c} - \sqrt{-dex}}{\sqrt{e} + \sqrt{-dx}} dx, x, \operatorname{sech}^{-1}(cx) \right)}{2\sqrt{-d}} \\
&= \frac{(a + b \operatorname{sech}^{-1}(cx))^2}{2bd} - \frac{(a + b \operatorname{sech}^{-1}(cx)) \log \left(1 - \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d+e}} \right)}{2d} - \frac{(a + b \operatorname{sech}^{-1}(cx)) \log \left(1 - \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e} + \sqrt{c^2d+e}} \right)}{2d} \\
&= \frac{(a + b \operatorname{sech}^{-1}(cx))^2}{2bd} - \frac{(a + b \operatorname{sech}^{-1}(cx)) \log \left(1 - \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d+e}} \right)}{2d} - \frac{(a + b \operatorname{sech}^{-1}(cx)) \log \left(1 - \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e} + \sqrt{c^2d+e}} \right)}{2d} \\
&= \frac{(a + b \operatorname{sech}^{-1}(cx))^2}{2bd} - \frac{(a + b \operatorname{sech}^{-1}(cx)) \log \left(1 - \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d+e}} \right)}{2d} - \frac{(a + b \operatorname{sech}^{-1}(cx)) \log \left(1 - \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e} + \sqrt{c^2d+e}} \right)}{2d}
\end{aligned}$$

Mathematica [C] time = 0.885412, size = 386, normalized size = 0.93

$$b \left(\operatorname{PolyLog} \left(2, -\frac{(-2\sqrt{e(c^2d+e)+c^2d+2e})e^{-2\operatorname{sech}^{-1}(cx)}}{c^2d} \right) + \operatorname{PolyLog} \left(2, -\frac{(2(\sqrt{e(c^2d+e)+e})+c^2d)e^{-2\operatorname{sech}^{-1}(cx)}}{c^2d} \right) - 2 \left(\operatorname{sech}^{-1}(cx) \left(\log \left(1 - \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e} - \sqrt{c^2d+e}} \right) + \log \left(1 - \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e} + \sqrt{c^2d+e}} \right) \right) \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*ArcSech[c*x])/(x*(d + e*x^2)), x]

[Out] (4*a*Log[x] - 2*a*Log[d + e*x^2] + b*(-2*(ArcSech[c*x]^2 + I*ArcSin[Sqrt[1 + e/(c^2*d)]]*(2*ArcTanh[(e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/Sqrt[e*(c^2*d + e)]] - Log[(2*e - 2*Sqrt[e*(c^2*d + e)] + c^2*d*(1 + E^(2*ArcSech[c*x]))])/(c^2*d*E^(2*ArcSech[c*x]))] + Log[(2*(e + Sqrt[e*(c^2*d + e)]) + c^2*d*(1 + E^(2*ArcSech[c*x]))])/(c^2*d*E^(2*ArcSech[c*x]))] + ArcSech[c*x]*(Log[(2*e - 2*Sqrt[e*(c^2*d + e)] + c^2*d*(1 + E^(2*ArcSech[c*x]))])/(c^2*d*E^(2*ArcSech[c*x]))] + Log[(2*(e + Sqrt[e*(c^2*d + e)]) + c^2*d*(1 + E^(2*ArcSech[c*x]))])/(c^2*d*E^(2*ArcSech[c*x]))])) + PolyLog[2, -((c^2*d + 2*e - 2*Sqrt[e*(c^2*d + e)])/(c^2*d*E^(2*ArcSech[c*x])))] + PolyLog[2, -((c^2*d + 2*(e + Sqrt[e*(c^2*d + e)]))/(c^2*d*E^(2*ArcSech[c*x])))])))/(4*d)

Maple [C] time = 0.605, size = 3157, normalized size = 7.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((a+b*\text{arcsech}(c*x))/x/(e*x^2+d),x)$

[Out] $\frac{1}{4}b*c^2/(c^2*d+e)*\text{polylog}(2,c^2*d*(1/c/x+(-1+1/c/x)^{1/2})*(1+1/c/x)^{1/2})^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e)-1/2*b/d*\ln(1-c^2*d*(1/c/x+(-1+1/c/x)^{1/2})*(1+1/c/x)^{1/2})^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e))*\text{arcsech}(c*x)-1/2*b*c^2/(c^2*d+e)*\text{arcsech}(c*x)^2-b/c^2/d^2*\text{arcsech}(c*x)^2*(e*(c^2*d+e))^{1/2}+1/2*b/c^2/d^2*\text{polylog}(2,c^2*d*(1/c/x+(-1+1/c/x)^{1/2})*(1+1/c/x)^{1/2})^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e))*(e*(c^2*d+e))^{1/2}+1/2*b*c^2/(c^2*d+e)*\ln(1-c^2*d*(1/c/x+(-1+1/c/x)^{1/2})*(1+1/c/x)^{1/2})^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e))*\text{arcsech}(c*x)-5/2*b/(c^2*d+e)/d*\text{arcsech}(c*x)^2*e-3/4*b/(c^2*d+e)/d*\text{polylog}(2,c^2*d*(1/c/x+(-1+1/c/x)^{1/2})*(1+1/c/x)^{1/2})^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e))*(e*(c^2*d+e))^{1/2}+5/4*b/(c^2*d+e)/d*\text{polylog}(2,c^2*d*(1/c/x+(-1+1/c/x)^{1/2})*(1+1/c/x)^{1/2})^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e))*e+2*b/c^2/d^2*e*\text{arcsech}(c*x)^2+2*b/c^4/d^3*e^2*\text{arcsech}(c*x)^2-b/c^2/d^2*e*\text{polylog}(2,c^2*d*(1/c/x+(-1+1/c/x)^{1/2})*(1+1/c/x)^{1/2})^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e))-b/c^4/d^3*e^2*\text{polylog}(2,c^2*d*(1/c/x+(-1+1/c/x)^{1/2})*(1+1/c/x)^{1/2})^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e))+b*(e*(c^2*d+e))^{1/2}/d/(c^2*d+e)*\text{arcsech}(c*x)^2+1/4*b*(e*(c^2*d+e))^{1/2}/d/(c^2*d+e)*\text{polylog}(2,c^2*d*(1/c/x+(-1+1/c/x)^{1/2})*(1+1/c/x)^{1/2})^2/(-c^2*d+2*(e*(c^2*d+e))^{1/2}-2*e))-1/2*b/d*\text{sum}((_R1^2*c^2*d+2*c^2*d+4*e)/(_R1^2*c^2*d+c^2*d+2*e)*(\text{arcsech}(c*x)*\ln((_R1-1/c/x-(-1+1/c/x)^{1/2})*(1+1/c/x)^{1/2}))/_R1)+\text{dilog}((_R1-1/c/x-(-1+1/c/x)^{1/2})*(1+1/c/x)^{1/2}))/_R1),_R1=\text{RootOf}(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-1/4*b/d*\text{polylog}(2,c^2*d*(1/c/x+(-1+1/c/x)^{1/2})*(1+1/c/x)^{1/2})^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e))+b/d*\text{arcsech}(c*x)^2-1/2*a/d*\ln(c^2*e*x^2+c^2*d)+a/d*\ln(c*x)-2*b/c^4/d^3*e^2/(c^2*d+e)*\ln(1-c^2*d*(1/c/x+(-1+1/c/x)^{1/2})*(1+1/c/x)^{1/2})^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e))*\text{arcsech}(c*x)*(e*(c^2*d+e))^{1/2}-3*b/c^2/(c^2*d+e)/d^2*\ln(1-c^2*d*(1/c/x+(-1+1/c/x)^{1/2})*(1+1/c/x)^{1/2})^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e))*\text{arcsech}(c*x)*(e*(c^2*d+e))^{1/2}*e+3*b/c^2/(c^2*d+e)/d^2*\text{arcsech}(c*x)^2*(e*(c^2*d+e))^{1/2}*e-3/2*b/c^2/(c^2*d+e)/d^2*\text{polylog}(2,c^2*d*(1/c/x+(-1+1/c/x)^{1/2})*(1+1/c/x)^{1/2})^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e))*(e*(c^2*d+e))^{1/2}*e+2*b/c^4/d^3*e^2/(c^2*d+e)*\text{arcsech}(c*x)^2*(e*(c^2*d+e))^{1/2}-2*b/c^2/d^2*e*\ln(1-c^2*d*(1/c/x+(-1+1/c/x)^{1/2})*(1+1/c/x)^{1/2})^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e))*\text{arcsech}(c*x)+b/c^2/d^2*\ln(1-c^2*d*(1/c/x+(-1+1/c/x)^{1/2})*(1+1/c/x)^{1/2})^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e))*\text{arcsech}(c*x)*(e*(c^2*d+e))^{1/2}-4*b/c^2/(c^2*d+e)/d^2*\text{arcsech}(c*x)^2*e^2-2*b/c^4/d^3*e^3/(c^2*d+e)*\text{arcsech}(c*x)^2-1/8*b*c^2/e/(c^2*d+e)*\text{polylog}(2,c^2*d*(1/c/x+(-1+1/c/x)^{1/2})*(1+1/c/x)^{1/2})^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e))*(e*(c^2*d+e))^{1/2}+5/2*b/(c^2*d+e)/d*\ln(1-c^2*d*(1/c/x+(-1+1/c/x)^{1/2})*(1+1/c/x)^{1/2})^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e))*\text{arcsech}(c*x)*e-2*b/c^4/d^3*e*\text{arcsech}(c*x)^2*(e*(c^2*d+e))^{1/2}+1/8*b*c^2*(e*(c^2*d+e))^{1/2}/e/(c^2*d+e)*\text{polylog}(2,c^2*d*(1/c/x+(-1+1/c/x)^{1/2})*(1+1/c/x)^{1/2})^2/(-c^2*d+2*(e*(c^2*d+e))^{1/2}-2*e))+1/2*b*(e*(c^2*d+e))^{1/2}/d/(c^2*d+e)*\text{arcsech}(c*x)*\ln(1-c^2*d*(1/c/x+(-1+1/c/x)^{1/2})*(1+1/c/x)^{1/2})^2/(-c^2*d+2*(e*(c^2*d+e))^{1/2}-2*e))+b/c^4/d^3*e*\text{polylog}(2,c^2*d*(1/c/x+(-1+1/c/x)^{1/2})*(1+1/c/x)^{1/2})^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e))*(e*(c^2*d+e))^{1/2}+2*b/c^2/(c^2*d+e)/d^2*\text{polylog}(2,c^2*d*(1/c/x+(-1+1/c/x)^{1/2})*(1+1/c/x)^{1/2})^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e))*e^2-2*b/c^4/d^3*e^2*\ln(1-c^2*d*(1/c/x+(-1+1/c/x)^{1/2})*(1+1/c/x)^{1/2})^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e))*\text{arcsech}(c*x)+b/c^4/d^3*e^3/(c^2*d+e)*\text{polylog}(2,c^2*d*(1/c/x+(-1+1/c/x)^{1/2})*(1+1/c/x)^{1/2})^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e))-3/2*b/(c^2*d+e)/d*\ln(1-c^2*d*(1/c/x+(-1+1/c/x)^{1/2})*(1+1/c/x)^{1/2})^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e))*\text{arcsech}(c*x)*(e*(c^2*d+e))^{1/2}-b/c^4/d^3*e^2/(c^2*d+e)*\text{polylog}(2,c^2*d*(1/c/x+(-1+1/c/x)^{1/2})*(1+1/c/x)^{1/2})^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e))*(e*(c^2*d+e))^{1/2}+4*b/c^2/(c^2*d+e)/d^2*\ln(1-c^2*d*(1/c/x+(-1+1/c/x)^{1/2})*(1+1/c/x)^{1/2})^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e))*\text{arcsech}(c*x)*e^2+1/4*b*c^2*(e*(c^2*d+e))^{1/2}/e/(c^2*d+e)*\text{arcsech}(c*x)*\ln($

$$1-c^2*d*(1/c/x+(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)})^2/(-c^2*d+2*(e*(c^2*d+e))^{(1/2)-2*e})+2*b/c^4/d^3*e*\ln(1-c^2*d*(1/c/x+(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)})^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)-2*e}))*\operatorname{arcsech}(c*x)*(e*(c^2*d+e))^{(1/2)+2*b/c^4/d^3*e^3/(c^2*d+e)*\ln(1-c^2*d*(1/c/x+(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)})^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)-2*e}))*\operatorname{arcsech}(c*x)-1/4*b*c^2/e/(c^2*d+e)*\ln(1-c^2*d*(1/c/x+(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)})^2/(-c^2*d-2*(e*(c^2*d+e))^{(1/2)-2*e}))*\operatorname{arcsech}(c*x)*(e*(c^2*d+e))^{(1/2)}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}a\left(\frac{\log(ex^2+d)}{d}-\frac{2\log(x)}{d}\right)+b\int\frac{\log\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{ex^3+dx}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d),x, algorithm="maxima")

[Out] -1/2*a*(log(e*x^2 + d)/d - 2*log(x)/d) + b*integrate(log(sqrt(1/(c*x) + 1))*sqrt(1/(c*x) - 1) + 1/(c*x))/(e*x^3 + d*x), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b\operatorname{arsech}(cx)+a}{ex^3+dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d),x, algorithm="fricas")

[Out] integral((b*arcsech(c*x) + a)/(e*x^3 + d*x), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\operatorname{asech}(cx)}{x(d+ex^2)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asech(c*x))/x/(e*x**2+d),x)

[Out] Integral((a + b*asech(c*x))/(x*(d + e*x**2)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\operatorname{arsech}(cx)+a}{(ex^2+d)x}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/((e*x^2 + d)*x), x)
```

$$3.114 \quad \int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^2(d+ex^2)} dx$$

Optimal. Leaf size=523

$$-\frac{b\sqrt{e}\operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} + \frac{b\sqrt{e}\operatorname{PolyLog}\left(2, \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} - \frac{b\sqrt{e}\operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2(-d)^{3/2}} + \frac{b\sqrt{e}\operatorname{PolyLog}\left(2, \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2(-d)^{3/2}}$$

```
[Out] (b*c*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]/d - a/(d*x) - (b*ArcSech[c*x])/(d*x) + (Sqrt[e]*(a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*(-d)^(3/2)) - (Sqrt[e]*(a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*(-d)^(3/2)) + (Sqrt[e]*(a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*(-d)^(3/2)) - (Sqrt[e]*(a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*(-d)^(3/2)) - (b*Sqrt[e]*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(2*(-d)^(3/2)) + (b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*(-d)^(3/2)) - (b*Sqrt[e]*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(2*(-d)^(3/2)) + (b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*(-d)^(3/2))
```

Rubi [A] time = 1.26103, antiderivative size = 523, normalized size of antiderivative = 1., number of steps used = 24, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {6303, 5792, 5654, 74, 5707, 5800, 5562, 2190, 2279, 2391}

$$-\frac{b\sqrt{e}\operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} + \frac{b\sqrt{e}\operatorname{PolyLog}\left(2, \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2(-d)^{3/2}} - \frac{b\sqrt{e}\operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2(-d)^{3/2}} + \frac{b\sqrt{e}\operatorname{PolyLog}\left(2, \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2(-d)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])/(x^2*(d + e*x^2)), x]
```

```
[Out] (b*c*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]/d - a/(d*x) - (b*ArcSech[c*x])/(d*x) + (Sqrt[e]*(a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*(-d)^(3/2)) - (Sqrt[e]*(a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*(-d)^(3/2)) + (Sqrt[e]*(a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*(-d)^(3/2)) - (Sqrt[e]*(a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*(-d)^(3/2)) - (b*Sqrt[e]*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(2*(-d)^(3/2)) + (b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*(-d)^(3/2)) - (b*Sqrt[e]*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(2*(-d)^(3/2)) + (b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*(-d)^(3/2))
```

Rule 6303

```
Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] :> -Subst[Int[((e + d*x^2)^p*(a + b*ArcCosh[x/c])^n)/x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegersQ[m, p]
```

Rule 5792

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]

Rule 5654

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] := Simp[x*(a + b*ArcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]

Rule 74

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]

Rule 5707

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (p > 0 || IGtQ[n, 0])

Rule 5800

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbol] := Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]

Rule 5562

Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)), x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)), x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]

Rule 2190

Int[(((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_.) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

Int[Log[(a_.) + (b_.)*((F_)^(e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_.)*((d_.) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rubi steps

$$\begin{aligned}
\int \frac{a + b \operatorname{sech}^{-1}(cx)}{x^2(d + ex^2)} dx &= -\operatorname{Subst} \left(\int \frac{x^2 \left(a + b \cosh^{-1} \left(\frac{x}{c} \right) \right)}{e + dx^2} dx, x, \frac{1}{x} \right) \\
&= -\operatorname{Subst} \left(\int \left(\frac{a + b \cosh^{-1} \left(\frac{x}{c} \right)}{d} - \frac{e \left(a + b \cosh^{-1} \left(\frac{x}{c} \right) \right)}{d(e + dx^2)} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\operatorname{Subst} \left(\int \left(a + b \cosh^{-1} \left(\frac{x}{c} \right) \right) dx, x, \frac{1}{x} \right)}{d} + \frac{e \operatorname{Subst} \left(\int \frac{a + b \cosh^{-1} \left(\frac{x}{c} \right)}{e + dx^2} dx, x, \frac{1}{x} \right)}{d} \\
&= -\frac{a}{dx} - \frac{b \operatorname{Subst} \left(\int \cosh^{-1} \left(\frac{x}{c} \right) dx, x, \frac{1}{x} \right)}{d} + \frac{e \operatorname{Subst} \left(\int \left(\frac{a + b \cosh^{-1} \left(\frac{x}{c} \right)}{2\sqrt{e}(\sqrt{e} - \sqrt{-dx})} + \frac{a + b \cosh^{-1} \left(\frac{x}{c} \right)}{2\sqrt{e}(\sqrt{e} + \sqrt{-dx})} \right) dx, x, \frac{1}{x} \right)}{d} \\
&= -\frac{a}{dx} - \frac{b \operatorname{sech}^{-1}(cx)}{dx} + \frac{b \operatorname{Subst} \left(\int \frac{x}{\sqrt{-1 + \frac{x}{c}} \sqrt{1 + \frac{x}{c}}} dx, x, \frac{1}{x} \right)}{cd} + \frac{\sqrt{e} \operatorname{Subst} \left(\int \frac{a + b \cosh^{-1} \left(\frac{x}{c} \right)}{\sqrt{e} - \sqrt{-dx}} dx, x, \frac{1}{x} \right)}{2d} \\
&= \frac{bc\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}}{d} - \frac{a}{dx} - \frac{b \operatorname{sech}^{-1}(cx)}{dx} + \frac{\sqrt{e} \operatorname{Subst} \left(\int \frac{(a + bx) \sinh(x)}{\frac{\sqrt{e}}{c} - \sqrt{-d} \cosh(x)} dx, x, \operatorname{sech}^{-1}(cx) \right)}{2d} \\
&= \frac{bc\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}}{d} - \frac{a}{dx} - \frac{b \operatorname{sech}^{-1}(cx)}{dx} + \frac{\sqrt{e} \operatorname{Subst} \left(\int \frac{e^{x(a + bx)}}{\frac{\sqrt{e}}{c} - \frac{\sqrt{c^2 d + e}}{c} - \sqrt{-d} e^x} dx, x, \operatorname{sech}^{-1}(cx) \right)}{2d} \\
&= \frac{bc\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}}{d} - \frac{a}{dx} - \frac{b \operatorname{sech}^{-1}(cx)}{dx} + \frac{\sqrt{e} (a + b \operatorname{sech}^{-1}(cx)) \log \left(1 - \frac{c\sqrt{-d} \operatorname{sech}^{-1}(cx)}{\sqrt{e} - \sqrt{c^2 d + e}} \right)}{2(-d)^{3/2}} \\
&= \frac{bc\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}}{d} - \frac{a}{dx} - \frac{b \operatorname{sech}^{-1}(cx)}{dx} + \frac{\sqrt{e} (a + b \operatorname{sech}^{-1}(cx)) \log \left(1 - \frac{c\sqrt{-d} \operatorname{sech}^{-1}(cx)}{\sqrt{e} - \sqrt{c^2 d + e}} \right)}{2(-d)^{3/2}} \\
&= \frac{bc\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}}{d} - \frac{a}{dx} - \frac{b \operatorname{sech}^{-1}(cx)}{dx} + \frac{\sqrt{e} (a + b \operatorname{sech}^{-1}(cx)) \log \left(1 - \frac{c\sqrt{-d} \operatorname{sech}^{-1}(cx)}{\sqrt{e} - \sqrt{c^2 d + e}} \right)}{2(-d)^{3/2}}
\end{aligned}$$

Mathematica [C] time = 1.7393, size = 933, normalized size = 1.78

$$-4\sqrt{ex} \tan^{-1} \left(\frac{\sqrt{ex}}{\sqrt{d}} \right) a - 4\sqrt{da} + b \left(4\sqrt{d} \sqrt{\frac{1-cx}{cx+1}} (cx+1) - 4\sqrt{d} \operatorname{sech}^{-1}(cx) - 2i\sqrt{ex} \left(-4i \sin^{-1} \left(\frac{\sqrt{\frac{i\sqrt{e}}{c\sqrt{d}} + 1}}{\sqrt{2}}} \right) \tanh^{-1} \left(\frac{i\sqrt{dc} + \sqrt{e}}{\sqrt{d}} \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*ArcSech[c*x])/(x^2*(d + e*x^2)), x]

[Out] (-4*a*Sqrt[d] - 4*a*Sqrt[e]*x*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + b*(4*Sqrt[d]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) - 4*Sqrt[d]*ArcSech[c*x] - (2*I)*Sqrt[e]*x*((-4*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]]/Sqrt[2]]*ArcTanh[((I*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] + ArcSech[c*x]*Log[1 + E^(-2*ArcSech[c*x])] - ArcSech[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (2*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]]/Sqrt[2])*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSe

```

ch[c*x]]) - ArcSech[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]
*E^ArcSech[c*x])] - (2*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]
*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + Poly
Log[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + PolyL
og[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (2*
I)*Sqrt[e]*x*((-4*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcT
anh[((( -I)*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] + Ar
cSech[c*x]*Log[1 + E^(-2*ArcSech[c*x])] - ArcSech[c*x]*Log[1 + (I*(-Sqrt[e]
+ Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (2*I)*ArcSin[Sqrt[1 - (I
*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c
*Sqrt[d]*E^ArcSech[c*x])] - ArcSech[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d +
e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (2*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sq
rt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSe
ch[c*x])] + PolyLog[2, (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech
[c*x])] + PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c
*x])])])])/(4*d^(3/2)*x)

```

Maple [C] time = 2.456, size = 372, normalized size = 0.7

$$-\frac{a}{dx} - \frac{ae}{d} \arctan\left(\frac{ex}{\sqrt{de}}\right) \frac{1}{\sqrt{de}} + \frac{bc}{d} \sqrt{-\frac{cx-1}{cx}} \sqrt{\frac{cx+1}{cx}} - \frac{b \operatorname{arcsech}(cx)}{dx} + \frac{bec}{2d} \sum_{R1=\operatorname{RootOf}(c^2d_Z^4+(2c^2d+4e)_Z^2+c^2d)} \frac{-R1^2}{-R1^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arcsech(c*x))/x^2/(e*x^2+d),x)
```

```

[Out] -a/d/x-a*e/d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))+c*b/d*(-(c*x-1)/c/x)^(1/2)
*((c*x+1)/c/x)^(1/2)-b*arcsech(c*x)/d/x+1/2*c*b*e/d*sum(_R1/(_R1^2*c^2*d+c^
2*d+2*e)*(arcsech(c*x)*ln(( _R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)
+dilog(( _R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*
_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-1/2*c*b*e/d*sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*
e)*(arcsech(c*x)*ln(( _R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)+dilog
(( _R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/_R1)),_R1=RootOf(c^2*d*_Z^4+(
2*c^2*d+4*e)*_Z^2+c^2*d))

```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b \operatorname{arcsech}(cx) + a}{ex^4 + dx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arcsech(c*x) + a)/(e*x^4 + d*x^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{arsech}(cx)}{x^2 (d + ex^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arsech(c*x))/x**2/(e*x**2+d),x)
```

```
[Out] Integral((a + b*arsech(c*x))/(x**2*(d + e*x**2)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsech}(cx) + a}{(ex^2 + d)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/((e*x^2 + d)*x^2), x)
```

$$3.115 \quad \int \frac{x^5 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^2} dx$$

Optimal. Leaf size=631

$$\frac{bd \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-de} \operatorname{sech}^{-1}(cx)}{\sqrt{e-\sqrt{c^2d+e}}}\right)}{e^3} - \frac{bd \operatorname{PolyLog}\left(2, \frac{c\sqrt{-de} \operatorname{sech}^{-1}(cx)}{\sqrt{e-\sqrt{c^2d+e}}}\right)}{e^3} - \frac{bd \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-de} \operatorname{sech}^{-1}(cx)}{\sqrt{c^2d+e+\sqrt{e}}}\right)}{e^3} - \frac{bd \operatorname{PolyLog}\left(2, \frac{c\sqrt{-de} \operatorname{sech}^{-1}(cx)}{\sqrt{c^2d+e+\sqrt{e}}}\right)}{e^3}$$

[Out] $-(b \sqrt{-1 + 1/(c*x)}) \sqrt{1 + 1/(c*x)} x / (2*c*e^2) + (d*(a + b \operatorname{ArcSech}[c*x])) / (2*e^2*(e + d/x^2)) + (x^2*(a + b \operatorname{ArcSech}[c*x])) / (2*e^2) + (2*d*(a + b \operatorname{ArcSech}[c*x])^2) / (b*e^3) - (b*d \sqrt{-1 + 1/(c^2*x^2)}) \operatorname{ArcTanh}[\sqrt{c^2*d + e} / (c \sqrt{e} \sqrt{-1 + 1/(c^2*x^2)}) x] / (2*e^{5/2} \sqrt{c^2*d + e} \sqrt{-1 + 1/(c*x)}) \sqrt{1 + 1/(c*x)} + (2*d*(a + b \operatorname{ArcSech}[c*x]) \operatorname{Log}[1 + E^{(-2*\operatorname{ArcSech}[c*x])}]) / e^3 - (d*(a + b \operatorname{ArcSech}[c*x]) \operatorname{Log}[1 - (c \sqrt{-d} * E^{\operatorname{ArcSech}[c*x]}) / (\sqrt{e} - \sqrt{c^2*d + e})]) / e^3 - (d*(a + b \operatorname{ArcSech}[c*x]) \operatorname{Log}[1 + (c \sqrt{-d} * E^{\operatorname{ArcSech}[c*x]}) / (\sqrt{e} - \sqrt{c^2*d + e})]) / e^3 - (d*(a + b \operatorname{ArcSech}[c*x]) \operatorname{Log}[1 - (c \sqrt{-d} * E^{\operatorname{ArcSech}[c*x]}) / (\sqrt{e} + \sqrt{c^2*d + e})]) / e^3 - (d*(a + b \operatorname{ArcSech}[c*x]) \operatorname{Log}[1 + (c \sqrt{-d} * E^{\operatorname{ArcSech}[c*x]}) / (\sqrt{e} + \sqrt{c^2*d + e})]) / e^3 - (b*d \operatorname{PolyLog}[2, -E^{(-2*\operatorname{ArcSech}[c*x])}]) / e^3 - (b*d \operatorname{PolyLog}[2, -(c \sqrt{-d} * E^{\operatorname{ArcSech}[c*x]}) / (\sqrt{e} - \sqrt{c^2*d + e})]) / e^3 - (b*d \operatorname{PolyLog}[2, (c \sqrt{-d} * E^{\operatorname{ArcSech}[c*x]}) / (\sqrt{e} - \sqrt{c^2*d + e})]) / e^3 - (b*d \operatorname{PolyLog}[2, -(c \sqrt{-d} * E^{\operatorname{ArcSech}[c*x]}) / (\sqrt{e} + \sqrt{c^2*d + e})]) / e^3 - (b*d \operatorname{PolyLog}[2, (c \sqrt{-d} * E^{\operatorname{ArcSech}[c*x]}) / (\sqrt{e} + \sqrt{c^2*d + e})]) / e^3$

Rubi [A] time = 1.54794, antiderivative size = 611, normalized size of antiderivative = 0.97, number of steps used = 32, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {6303, 5792, 5662, 95, 5660, 3718, 2190, 2279, 2391, 5788, 519, 377, 208, 5800, 5562}

$$\frac{bd \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-de} \operatorname{sech}^{-1}(cx)}{\sqrt{e-\sqrt{c^2d+e}}}\right)}{e^3} - \frac{bd \operatorname{PolyLog}\left(2, \frac{c\sqrt{-de} \operatorname{sech}^{-1}(cx)}{\sqrt{e-\sqrt{c^2d+e}}}\right)}{e^3} - \frac{bd \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-de} \operatorname{sech}^{-1}(cx)}{\sqrt{c^2d+e+\sqrt{e}}}\right)}{e^3} - \frac{bd \operatorname{PolyLog}\left(2, \frac{c\sqrt{-de} \operatorname{sech}^{-1}(cx)}{\sqrt{c^2d+e+\sqrt{e}}}\right)}{e^3}$$

Warning: Unable to verify antiderivative.

[In] $\operatorname{Int}[(x^5*(a + b \operatorname{ArcSech}[c*x])) / (d + e*x^2)^2, x]$

[Out] $-(b \sqrt{-1 + 1/(c*x)}) \sqrt{1 + 1/(c*x)} x / (2*c*e^2) + (d*(a + b \operatorname{ArcSech}[c*x])) / (2*e^2*(e + d/x^2)) + (x^2*(a + b \operatorname{ArcSech}[c*x])) / (2*e^2) - (b*d \sqrt{-1 + 1/(c^2*x^2)}) \operatorname{ArcTanh}[\sqrt{c^2*d + e} / (c \sqrt{e} \sqrt{-1 + 1/(c^2*x^2)}) x] / (2*e^{5/2} \sqrt{c^2*d + e} \sqrt{-1 + 1/(c*x)}) \sqrt{1 + 1/(c*x)} - (d*(a + b \operatorname{ArcSech}[c*x]) \operatorname{Log}[1 - (c \sqrt{-d} * E^{\operatorname{ArcSech}[c*x]}) / (\sqrt{e} - \sqrt{c^2*d + e})]) / e^3 - (d*(a + b \operatorname{ArcSech}[c*x]) \operatorname{Log}[1 + (c \sqrt{-d} * E^{\operatorname{ArcSech}[c*x]}) / (\sqrt{e} - \sqrt{c^2*d + e})]) / e^3 - (d*(a + b \operatorname{ArcSech}[c*x]) \operatorname{Log}[1 - (c \sqrt{-d} * E^{\operatorname{ArcSech}[c*x]}) / (\sqrt{e} + \sqrt{c^2*d + e})]) / e^3 - (d*(a + b \operatorname{ArcSech}[c*x]) \operatorname{Log}[1 + (c \sqrt{-d} * E^{\operatorname{ArcSech}[c*x]}) / (\sqrt{e} + \sqrt{c^2*d + e})]) / e^3 + (2*d*(a + b \operatorname{ArcSech}[c*x]) \operatorname{Log}[1 + E^{(2*\operatorname{ArcSech}[c*x])}]) / e^3 - (b*d \operatorname{PolyLog}[2, -(c \sqrt{-d} * E^{\operatorname{ArcSech}[c*x]}) / (\sqrt{e} - \sqrt{c^2*d + e})]) / e^3 - (b*d \operatorname{PolyLog}[2, (c \sqrt{-d} * E^{\operatorname{ArcSech}[c*x]}) / (\sqrt{e} - \sqrt{c^2*d + e})]) / e^3 - (b*d \operatorname{PolyLog}[2, -(c \sqrt{-d} * E^{\operatorname{ArcSech}[c*x]}) / (\sqrt{e} + \sqrt{c^2*d + e})]) / e^3 - (b*d \operatorname{PolyLog}[2, (c \sqrt{-d} * E^{\operatorname{ArcSech}[c*x]}) / (\sqrt{e} + \sqrt{c^2*d + e})]) / e^3$

$$\frac{e^{3n}}{e^3} - \frac{(b*d*PolyLog[2, (c*sqrt[-d]*E^{ArcSech[c*x]})/(sqrt[e] + sqrt[c^2*d + e])])}{e^3} + \frac{(b*d*PolyLog[2, -E^{(2*ArcSech[c*x])})}{e^3}$$

Rule 6303

$$\text{Int}[(a + \text{ArcSech}[c*x])*(b*x)^n*(d + (e*x)^2)^p, x_Symbol] \rightarrow -\text{Subst}[\text{Int}[(e + d*x^2)^p*(a + b*\text{ArcCosh}[x/c])^n/x^{m+2*(p+1)}, x], x, 1/x] /; \text{FreeQ}\{a, b, c, d, e, n, x\} \&\& \text{IGtQ}[n, 0] \&\& \text{IntegersQ}[m, p]$$

Rule 5792

$$\text{Int}[(a + \text{ArcCosh}[c*x])*(b*x)^n*(f*x)^m*(d + (e*x)^2)^p, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*\text{ArcCosh}[c*x])^n*(f*x)^m*(d + e*x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, x\} \&\& \text{NeQ}[c^2*d + e, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{IntegerQ}[p] \&\& \text{IntegerQ}[m]$$

Rule 5662

$$\text{Int}[(a + \text{ArcCosh}[c*x])*(b*x)^n*(d*x)^m, x_Symbol] \rightarrow \text{Simp}[(d*x)^{m+1}*(a + b*\text{ArcCosh}[c*x])^n/(d*(m+1)), x] - \text{Dist}[(b*c*n)/(d*(m+1)), \text{Int}[(d*x)^{m+1}*(a + b*\text{ArcCosh}[c*x])^{n-1}/(\text{sqrt}[-1 + c*x]*\text{sqrt}[1 + c*x]), x], x] /; \text{FreeQ}\{a, b, c, d, m, x\} \&\& \text{IGtQ}[n, 0] \&\& \text{NeQ}[m, -1]$$

Rule 95

$$\text{Int}[(a + (b*x)^m*(c + (d*x)^n)*(e + (f*x)^p)), x_Symbol] \rightarrow \text{Simp}[(b*(a + b*x)^{m+1}*(c + d*x)^{n+1}*(e + f*x)^{p+1})/(m+1)*(b*c - a*d)*(b*e - a*f), x] /; \text{FreeQ}\{a, b, c, d, e, f, m, n, p, x\} \&\& \text{EqQ}[\text{Simplify}[m + n + p + 3], 0] \&\& \text{EqQ}[a*d*f*(m+1) + b*c*f*(n+1) + b*d*e*(p+1), 0] \&\& \text{NeQ}[m, -1]$$

Rule 5660

$$\text{Int}[(a + \text{ArcCosh}[c*x])*(b*x)^n/(x), x_Symbol] \rightarrow \text{Subst}[\text{Int}[(a + b*x)^n/\text{Coth}[x], x], x, \text{ArcCosh}[c*x]] /; \text{FreeQ}\{a, b, c, x\} \&\& \text{IGtQ}[n, 0]$$

Rule 3718

$$\text{Int}[(c + (d*x)^m*\tan[(e + (\text{Complex}[0, fz])*(f*x)]), x_Symbol] \rightarrow -\text{Simp}[(I*(c + d*x)^{m+1})/(d*(m+1)), x] + \text{Dist}[2*I, \text{Int}[(c + d*x)^m*E^{(2*(-I*e) + f*fz*x))}/(1 + E^{(2*(-I*e) + f*fz*x))}, x], x] /; \text{FreeQ}\{c, d, e, f, fz, x\} \&\& \text{IGtQ}[m, 0]$$

Rule 2190

$$\text{Int}[(F^{(g*(e + (f*x)^n))})^n*(c + (d*x)^m)/((a + (b*x)^n*(F^{(g*(e + (f*x)^n))})^n)), x_Symbol] \rightarrow \text{Simp}[(c + d*x)^m*\text{Log}[1 + (b*(F^{(g*(e + f*x))})^n)/a]/(b*f*g*n*\text{Log}[F]), x] - \text{Dist}[(d*m)/(b*f*g*n*\text{Log}[F]), \text{Int}[(c + d*x)^{m-1}*\text{Log}[1 + (b*(F^{(g*(e + f*x))})^n)/a], x], x] /; \text{FreeQ}\{F, a, b, c, d, e, f, g, n, x\} \&\& \text{IGtQ}[m, 0]$$

Rule 2279

$$\text{Int}[\text{Log}[a + (b*x)^n*(F^{(e*(c + d*x))})], x_Symbol] \rightarrow \text{Dist}[1/(d*e*n*\text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b*x]/x, x], x, (F^{(e*(c + d*x))})]$$

)ⁿ], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rule 5788

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x]))/(2*e*(p + 1)), x] - Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] && NeQ[p, -1]

Rule 519

Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_) * ((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] := Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && !(EqQ[n, 2] && IGtQ[q, 0])

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 5800

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbol] := Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]

Rule 5562

Int((((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_.)*(x_)])*(b_.) + (a_)), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)), x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)), x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x^5 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^2} dx &= -\operatorname{Subst} \left(\int \frac{a + b \cosh^{-1} \left(\frac{x}{c} \right)}{x^3 (e + dx^2)^2} dx, x, \frac{1}{x} \right) \\
&= -\operatorname{Subst} \left(\int \left(\frac{a + b \cosh^{-1} \left(\frac{x}{c} \right)}{e^2 x^3} - \frac{2d (a + b \cosh^{-1} \left(\frac{x}{c} \right))}{e^3 x} + \frac{d^2 x (a + b \cosh^{-1} \left(\frac{x}{c} \right))}{e^2 (e + dx^2)^2} + \frac{2d^2}{e^2 (e + dx^2)^2} \right) dx, x, \frac{1}{x} \right) \\
&= \frac{(2d) \operatorname{Subst} \left(\int \frac{a + b \cosh^{-1} \left(\frac{x}{c} \right)}{x} dx, x, \frac{1}{x} \right)}{e^3} - \frac{(2d^2) \operatorname{Subst} \left(\int \frac{x (a + b \cosh^{-1} \left(\frac{x}{c} \right))}{e + dx^2} dx, x, \frac{1}{x} \right)}{e^3} - \frac{\operatorname{Subst} \left(\int \frac{2d^2}{e^2 (e + dx^2)^2} dx, x, \frac{1}{x} \right)}{e^3} \\
&= \frac{d (a + b \operatorname{sech}^{-1}(cx))}{2e^2 \left(e + \frac{d}{x^2} \right)} + \frac{x^2 (a + b \operatorname{sech}^{-1}(cx))}{2e^2} + \frac{(2d) \operatorname{Subst} \left(\int (a + bx) \tanh(x) dx, x, \operatorname{sech}^{-1}(cx) \right)}{e^3} \\
&= -\frac{b \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}}{2ce^2} + \frac{d (a + b \operatorname{sech}^{-1}(cx))}{2e^2 \left(e + \frac{d}{x^2} \right)} + \frac{x^2 (a + b \operatorname{sech}^{-1}(cx))}{2e^2} - \frac{d (a + b \operatorname{sech}^{-1}(cx))}{be^3} \\
&= -\frac{b \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}}{2ce^2} + \frac{d (a + b \operatorname{sech}^{-1}(cx))}{2e^2 \left(e + \frac{d}{x^2} \right)} + \frac{x^2 (a + b \operatorname{sech}^{-1}(cx))}{2e^2} - \frac{d (a + b \operatorname{sech}^{-1}(cx))}{be^3} \\
&= -\frac{b \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}}{2ce^2} + \frac{d (a + b \operatorname{sech}^{-1}(cx))}{2e^2 \left(e + \frac{d}{x^2} \right)} + \frac{x^2 (a + b \operatorname{sech}^{-1}(cx))}{2e^2} - \frac{bd \sqrt{-1 + \frac{1}{c^2 x^2}}}{2e^{5/2} \sqrt{c^2 d + 2e}} \\
&= -\frac{b \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}}{2ce^2} + \frac{d (a + b \operatorname{sech}^{-1}(cx))}{2e^2 \left(e + \frac{d}{x^2} \right)} + \frac{x^2 (a + b \operatorname{sech}^{-1}(cx))}{2e^2} - \frac{bd \sqrt{-1 + \frac{1}{c^2 x^2}}}{2e^{5/2} \sqrt{c^2 d + 2e}} \\
&= -\frac{b \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}}{2ce^2} + \frac{d (a + b \operatorname{sech}^{-1}(cx))}{2e^2 \left(e + \frac{d}{x^2} \right)} + \frac{x^2 (a + b \operatorname{sech}^{-1}(cx))}{2e^2} - \frac{bd \sqrt{-1 + \frac{1}{c^2 x^2}}}{2e^{5/2} \sqrt{c^2 d + 2e}} \\
&= -\frac{b \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}}{2ce^2} + \frac{d (a + b \operatorname{sech}^{-1}(cx))}{2e^2 \left(e + \frac{d}{x^2} \right)} + \frac{x^2 (a + b \operatorname{sech}^{-1}(cx))}{2e^2} - \frac{bd \sqrt{-1 + \frac{1}{c^2 x^2}}}{2e^{5/2} \sqrt{c^2 d + 2e}}
\end{aligned}$$

Mathematica [C] time = 4.81581, size = 1278, normalized size = 2.03

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(x^5*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]

[Out] $-\frac{(-2*a*e*x^2 + (2*a*d^2))/(d + e*x^2) + 4*a*d*\operatorname{Log}[d + e*x^2] + b*((2*e*\operatorname{Sqrt}[(1 - c*x)/(1 + c*x)])/c^2 + (2*e*x*\operatorname{Sqrt}[(1 - c*x)/(1 + c*x)])/c - 2*e*x^2*A$

$$\begin{aligned} & \operatorname{rcSech}[c*x] + (d^{3/2}*\operatorname{ArcSech}[c*x]) / (\operatorname{Sqrt}[d] - I*\operatorname{Sqrt}[e]*x) + (d^{3/2}*\operatorname{ArcSech}[c*x]) / (\operatorname{Sqrt}[d] + I*\operatorname{Sqrt}[e]*x) + (16*I)*d*\operatorname{ArcSin}[\operatorname{Sqrt}[1 - (I*\operatorname{Sqrt}[e]) / (c*\operatorname{Sqrt}[d])]] / \operatorname{Sqrt}[2]] * \operatorname{ArcTanh}[((-I)*c*\operatorname{Sqrt}[d] + \operatorname{Sqrt}[e])* \operatorname{Tanh}[\operatorname{ArcSech}[c*x] / 2]] / \operatorname{Sqrt}[c^2*d + e]] + (16*I)*d*\operatorname{ArcSin}[\operatorname{Sqrt}[1 + (I*\operatorname{Sqrt}[e]) / (c*\operatorname{Sqrt}[d])]] / \operatorname{Sqrt}[2]] * \operatorname{ArcTanh}[(I*c*\operatorname{Sqrt}[d] + \operatorname{Sqrt}[e])* \operatorname{Tanh}[\operatorname{ArcSech}[c*x] / 2]] / \operatorname{Sqrt}[c^2*d + e]] - 8*d*\operatorname{ArcSech}[c*x]*\operatorname{Log}[1 + E^{(-2*\operatorname{ArcSech}[c*x])}] + 4*d*\operatorname{ArcSech}[c*x]*\operatorname{Log}[1 + (I*(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[c^2*d + e])) / (c*\operatorname{Sqrt}[d]*E^{\operatorname{ArcSech}[c*x]})] - (8*I)*d*\operatorname{ArcSin}[\operatorname{Sqrt}[1 + (I*\operatorname{Sqrt}[e]) / (c*\operatorname{Sqrt}[d])]] / \operatorname{Sqrt}[2]] * \operatorname{Log}[1 + (I*(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[c^2*d + e])) / (c*\operatorname{Sqrt}[d]*E^{\operatorname{ArcSech}[c*x]})] + 4*d*\operatorname{ArcSech}[c*x]*\operatorname{Log}[1 + (I*(-\operatorname{Sqrt}[e] + \operatorname{Sqrt}[c^2*d + e])) / (c*\operatorname{Sqrt}[d]*E^{\operatorname{ArcSech}[c*x]})] - (8*I)*d*\operatorname{ArcSin}[\operatorname{Sqrt}[1 - (I*\operatorname{Sqrt}[e]) / (c*\operatorname{Sqrt}[d])]] / \operatorname{Sqrt}[2]] * \operatorname{Log}[1 + (I*(-\operatorname{Sqrt}[e] + \operatorname{Sqrt}[c^2*d + e])) / (c*\operatorname{Sqrt}[d]*E^{\operatorname{ArcSech}[c*x]})] + 4*d*\operatorname{ArcSech}[c*x]*\operatorname{Log}[1 - (I*(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[c^2*d + e])) / (c*\operatorname{Sqrt}[d]*E^{\operatorname{ArcSech}[c*x]})] + (8*I)*d*\operatorname{ArcSin}[\operatorname{Sqrt}[1 - (I*\operatorname{Sqrt}[e]) / (c*\operatorname{Sqrt}[d])]] / \operatorname{Sqrt}[2]] * \operatorname{Log}[1 - (I*(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[c^2*d + e])) / (c*\operatorname{Sqrt}[d]*E^{\operatorname{ArcSech}[c*x]})] + 4*d*\operatorname{ArcSech}[c*x]*\operatorname{Log}[1 + (I*(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[c^2*d + e])) / (c*\operatorname{Sqrt}[d]*E^{\operatorname{ArcSech}[c*x]})] + (8*I)*d*\operatorname{ArcSin}[\operatorname{Sqrt}[1 + (I*\operatorname{Sqrt}[e]) / (c*\operatorname{Sqrt}[d])]] / \operatorname{Sqrt}[2]] * \operatorname{Log}[1 + (I*(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[c^2*d + e])) / (c*\operatorname{Sqrt}[d]*E^{\operatorname{ArcSech}[c*x]})] + 2*d*\operatorname{Log}[x] - 2*d*\operatorname{Log}[1 + \operatorname{Sqrt}[(1 - c*x) / (1 + c*x)]] + c*x*\operatorname{Sqrt}[(1 - c*x) / (1 + c*x)] + (d*\operatorname{Sqrt}[e]*\operatorname{Log}[(2*I)*\operatorname{Sqrt}[e]*(\operatorname{Sqrt}[d]*\operatorname{Sqrt}[(1 - c*x) / (1 + c*x)]*(1 + c*x) + (\operatorname{Sqrt}[d]*\operatorname{Sqrt}[e] + I*c^2*d*x) / \operatorname{Sqrt}[c^2*d + e])]) / (I*\operatorname{Sqrt}[d] + \operatorname{Sqrt}[e]*x)) / \operatorname{Sqrt}[c^2*d + e] + (d*\operatorname{Sqrt}[e]*\operatorname{Log}[(2*\operatorname{Sqrt}[e]*(I*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[(1 - c*x) / (1 + c*x)]*(1 + c*x) + (I*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[e] + c^2*d*x) / \operatorname{Sqrt}[c^2*d + e])]) / ((-I)*\operatorname{Sqrt}[d] + \operatorname{Sqrt}[e]*x)) / \operatorname{Sqrt}[c^2*d + e] + 4*d*\operatorname{PolyLog}[2, -E^{(-2*\operatorname{ArcSech}[c*x])}] - 4*d*\operatorname{PolyLog}[2, (I*(\operatorname{Sqrt}[e] - \operatorname{Sqrt}[c^2*d + e])) / (c*\operatorname{Sqrt}[d]*E^{\operatorname{ArcSech}[c*x]})] - 4*d*\operatorname{PolyLog}[2, (I*(-\operatorname{Sqrt}[e] + \operatorname{Sqrt}[c^2*d + e])) / (c*\operatorname{Sqrt}[d]*E^{\operatorname{ArcSech}[c*x]})] - 4*d*\operatorname{PolyLog}[2, ((-I)*(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[c^2*d + e])) / (c*\operatorname{Sqrt}[d]*E^{\operatorname{ArcSech}[c*x]})] - 4*d*\operatorname{PolyLog}[2, (I*(\operatorname{Sqrt}[e] + \operatorname{Sqrt}[c^2*d + e])) / (c*\operatorname{Sqrt}[d]*E^{\operatorname{ArcSech}[c*x]})])]) / (4*e^3) \end{aligned}$$

Maple [C] time = 0.779, size = 870, normalized size = 1.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{int}(x^5*(a+b*\operatorname{arcsech}(c*x)) / (e*x^2+d)^2, x)$

[Out] $\frac{1}{2}a*x^2/e^2 - \frac{1}{2}c^2*a/e^3*d^2/(c^2*e*x^2+c^2*d) - a/e^3*d*\ln(c^2*e*x^2+c^2*d) - \frac{1}{2}c*b/(c^2*e*x^2+c^2*d)/e*(-(c*x-1)/c/x)^{(1/2)}*((c*x+1)/c/x)^{(1/2)}*x^3 - \frac{1}{2}c*b/(c^2*e*x^2+c^2*d)/e^2*(-(c*x-1)/c/x)^{(1/2)}*((c*x+1)/c/x)^{(1/2)}*x*d + \frac{1}{2}c^2*b/(c^2*e*x^2+c^2*d)/e*\operatorname{arcsech}(c*x)*x^4 + c^2*b/(c^2*e*x^2+c^2*d)/e^2*\operatorname{arcsech}(c*x)*d*x^2 + \frac{1}{2}b/(c^2*e*x^2+c^2*d)/e*x^2 + \frac{1}{2}b/(c^2*e*x^2+c^2*d)/e^2*d + \frac{1}{2}b*(e*(c^2*d+e))^{(1/2)}/e^3/(c^2*d+e)*d*\operatorname{arctanh}(1/4*(2*c^2*d*(1/c/x+(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)}))^2 + 2*c^2*d+4*e)/(c^2*d*e+e^2)^{(1/2)}) - \frac{1}{2}b/e^3*d*\sum((_R1^2*c^2*d+c^2*d+4*e)/(_R1^2*c^2*d+c^2*d+2*e)*(\operatorname{arcsech}(c*x)*\ln((_R1-1/c/x-(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)})/_R1)+\operatorname{dilog}((_R1-1/c/x-(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)})/_R1)), _R1=\operatorname{RootOf}(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+2*b/e^3*d*\operatorname{arcsech}(c*x)*\ln(1+I*(1/c/x+(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)})))+2*b/e^3*d*\operatorname{arcsech}(c*x)*\ln(1-I*(1/c/x+(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)})))+2*b/e^3*d*\operatorname{dilog}(1+I*(1/c/x+(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)}))+2*b/e^3*d*\operatorname{dilog}(1-I*(1/c/x+(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)}))- \frac{1}{2}c^2*b/e^3*d^2*\sum((_R1^2+1)/(_R1^2*c^2*d+c^2*d+2*e)*(\operatorname{arcsech}(c*x)*\ln((_R1-1/c/x-(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)})/_R1)+\operatorname{dilog}((_R1-1/c/x-(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)})/_R1)), _R1=\operatorname{RootOf}(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2} a \left(\frac{d^2}{e^4 x^2 + d e^3} - \frac{x^2}{e^2} + \frac{2 d \log(e x^2 + d)}{e^3} \right) + b \int \frac{x^5 \log \left(\sqrt{\frac{1}{c x} + 1} \sqrt{\frac{1}{c x} - 1} + \frac{1}{c x} \right)}{e^2 x^4 + 2 d e x^2 + d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="maxima")

[Out] -1/2*a*(d^2/(e^4*x^2 + d*e^3) - x^2/e^2 + 2*d*log(e*x^2 + d)/e^3) + b*integrate(x^5*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e^2*x^4 + 2*d*e*x^2 + d^2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{b x^5 \operatorname{arsech}(c x) + a x^5}{e^2 x^4 + 2 d e x^2 + d^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="fricas")

[Out] integral((b*x^5*arcsech(c*x) + a*x^5)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*(a+b*asech(c*x))/(e*x**2+d)**2,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsech}(c x) + a) x^5}{(e x^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)*x^5/(e*x^2 + d)^2, x)

$$3.116 \quad \int \frac{x^3 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^2} dx$$

Optimal. Leaf size=580

$$\frac{b \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2} + \frac{b \operatorname{PolyLog}\left(2, \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2} + \frac{b \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^2} + \frac{b \operatorname{PolyLog}\left(2, \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^2}$$

```
[Out] -(a + b*ArcSech[c*x])/(2*e*(e + d/x^2)) - (a + b*ArcSech[c*x])^2/(b*e^2) +
(b*Sqrt[-1 + 1/(c^2*x^2)]*ArcTanh[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[-1 + 1/(c
^2*x^2)]*x)]/(2*e^(3/2)*Sqrt[c^2*d + e]*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x
)]) - ((a + b*ArcSech[c*x])*Log[1 + E^(-2*ArcSech[c*x])])/e^2 + ((a + b*Arc
Sech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])
])/(2*e^2) + ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt
[e] - Sqrt[c^2*d + e])])/2e^2 + ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d
]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/2e^2 + ((a + b*ArcSech[c
*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/2e
^2) + (b*PolyLog[2, -E^(-2*ArcSech[c*x])])/(2*e^2) + (b*PolyLog[2, -(c*Sqr
t[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e^2) + (b*PolyLog[2
, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e^2) + (b*Po
lyLog[2, -(c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^
2) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])
])/(2*e^2)
```

Rubi [A] time = 1.46331, antiderivative size = 562, normalized size of antiderivative = 0.97, number of steps used = 30, number of rules used = 13, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {6303, 5792, 5660, 3718, 2190, 2279, 2391, 5788, 519, 377, 208, 5800, 5562}

$$\frac{b \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2} + \frac{b \operatorname{PolyLog}\left(2, \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^2} + \frac{b \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^2} + \frac{b \operatorname{PolyLog}\left(2, \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^2}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^3*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -(a + b*ArcSech[c*x])/(2*e*(e + d/x^2)) + (b*Sqrt[-1 + 1/(c^2*x^2)]*ArcTanh
[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[-1 + 1/(c^2*x^2)]*x)]/(2*e^(3/2)*Sqrt[c^2
*d + e]*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) + ((a + b*ArcSech[c*x])*Log[1
- (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/2e^2 + ((a
+ b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d
+ e])])/2e^2 + ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x]
)/(Sqrt[e] + Sqrt[c^2*d + e])])/2e^2 + ((a + b*ArcSech[c*x])*Log[1 + (c*
Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/2e^2 - ((a + b*Ar
cSech[c*x])*Log[1 + E^(2*ArcSech[c*x])])/e^2 + (b*PolyLog[2, -(c*Sqrt[-d]*
E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e^2) + (b*PolyLog[2, (c*S
qrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e^2) + (b*PolyLog[
2, -(c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^2) + (
b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e
```

$^2) - (b \cdot \text{PolyLog}[2, -E^{(2 \cdot \text{ArcSech}[c \cdot x])}] / (2 \cdot e^2)$

Rule 6303

$\text{Int}[(a + \text{ArcSech}[c \cdot x] \cdot b)^{n_1} \cdot x^{m_1} \cdot ((d + e) \cdot x^2)^{p_1}, x_Symbol] \rightarrow -\text{Subst}[\text{Int}[(e + d \cdot x^2)^p \cdot (a + b \cdot \text{ArcCosh}[x/c])^n / x^{m+2(p+1)}, x], x, 1/x] /;$ FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegersQ[m, p]

Rule 5792

$\text{Int}[(a + \text{ArcCosh}[c \cdot x] \cdot b)^{n_1} \cdot (f \cdot x)^{m_1} \cdot ((d + e) \cdot x^2)^{p_1}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b \cdot \text{ArcCosh}[c \cdot x])^n \cdot (f \cdot x)^m \cdot (d + e \cdot x^2)^p, x], x] /;$ FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2 \cdot d + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]

Rule 5660

$\text{Int}[(a + \text{ArcCosh}[c \cdot x] \cdot b)^{n_1} / (x), x_Symbol] \rightarrow \text{Subst}[\text{Int}[(a + b \cdot x)^n / \text{Coth}[x], x], x, \text{ArcCosh}[c \cdot x]] /;$ FreeQ[{a, b, c}, x] && IGtQ[n, 0]

Rule 3718

$\text{Int}[(c + (d \cdot x)^{m_1}) \cdot \tan[(e + (\text{Complex}[0, fz]) \cdot f \cdot x)], x_Symbol] \rightarrow -\text{Simp}[(I \cdot (c + d \cdot x)^{m+1}) / (d \cdot (m+1)), x] + \text{Dist}[2 \cdot I, \text{Int}[(c + d \cdot x)^m \cdot E^{(2 \cdot (-I \cdot e) + f \cdot fz \cdot x))} / (1 + E^{(2 \cdot (-I \cdot e) + f \cdot fz \cdot x))}), x], x] /;$ FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

Rule 2190

$\text{Int}[(F)^{(g \cdot (e + f \cdot x))^{n_1} \cdot ((c + (d \cdot x)^{m_1}) / ((a + b \cdot (F)^{(g \cdot (e + f \cdot x))^{n_1}}))^{n_1}), x_Symbol] \rightarrow \text{Simp}[(c + d \cdot x)^m \cdot \text{Log}[1 + (b \cdot (F^{(g \cdot (e + f \cdot x))^{n_1}}) / a)] / (b \cdot f \cdot g \cdot n \cdot \text{Log}[F]), x] - \text{Dist}[(d \cdot m) / (b \cdot f \cdot g \cdot n \cdot \text{Log}[F]), \text{Int}[(c + d \cdot x)^{m-1} \cdot \text{Log}[1 + (b \cdot (F^{(g \cdot (e + f \cdot x))^{n_1}}) / a)], x], x] /;$ FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

$\text{Int}[\text{Log}[(a + (b \cdot (F)^{(e \cdot (c + d \cdot x))^{n_1}})], x_Symbol] \rightarrow \text{Dist}[1 / (d \cdot e \cdot n \cdot \text{Log}[F]), \text{Subst}[\text{Int}[\text{Log}[a + b \cdot x] / x, x], x, (F^{(e \cdot (c + d \cdot x))^{n_1}})], x] /;$ FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

$\text{Int}[\text{Log}[(c + (d + e \cdot x^n))] / (x), x_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c \cdot e \cdot x^n)] / n, x] /;$ FreeQ[{c, d, e, n}, x] && EqQ[c \cdot d, 1]

Rule 5788

$\text{Int}[(a + \text{ArcCosh}[c \cdot x] \cdot b) \cdot x \cdot ((d + e) \cdot x^2)^{p_1}, x_Symbol] \rightarrow \text{Simp}[(d + e \cdot x^2)^{p+1} \cdot (a + b \cdot \text{ArcCosh}[c \cdot x]) / (2 \cdot e \cdot (p+1)), x] - \text{Dist}[(b \cdot c) / (2 \cdot e \cdot (p+1)), \text{Int}[(d + e \cdot x^2)^{p+1} / (\text{Sqrt}[1 + c \cdot x] \cdot \text{Sqrt}[-1 + c \cdot x]), x], x] /;$ FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2 \cdot d + e, 0] && NeQ[p, -1]

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] := Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p]]/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] := Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d
_.)*(x_)]*(b_.) + (a_)), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^2} dx &= -\operatorname{Subst} \left(\int \frac{a + b \cosh^{-1} \left(\frac{x}{c} \right)}{x (e + dx^2)^2} dx, x, \frac{1}{x} \right) \\
&= -\operatorname{Subst} \left(\int \left(\frac{a + b \cosh^{-1} \left(\frac{x}{c} \right)}{e^2 x} - \frac{dx (a + b \cosh^{-1} \left(\frac{x}{c} \right))}{e (e + dx^2)^2} - \frac{dx (a + b \cosh^{-1} \left(\frac{x}{c} \right))}{e^2 (e + dx^2)} \right) dx, x, \right. \\
&= -\frac{\operatorname{Subst} \left(\int \frac{a + b \cosh^{-1} \left(\frac{x}{c} \right)}{x} dx, x, \frac{1}{x} \right)}{e^2} + \frac{d \operatorname{Subst} \left(\int \frac{x (a + b \cosh^{-1} \left(\frac{x}{c} \right))}{e + dx^2} dx, x, \frac{1}{x} \right)}{e^2} + \frac{d \operatorname{Subst} \left(\int \frac{x}{e + dx^2} dx, x, \frac{1}{x} \right)}{e^2} \\
&= -\frac{a + b \operatorname{sech}^{-1}(cx)}{2e \left(e + \frac{d}{x^2} \right)} - \frac{\operatorname{Subst} \left(\int (a + bx) \tanh(x) dx, x, \operatorname{sech}^{-1}(cx) \right)}{e^2} + \frac{d \operatorname{Subst} \left(\int \left(-\frac{\sqrt{-d}}{2} \right) dx, x, \frac{1}{x} \right)}{e^2} \\
&= -\frac{a + b \operatorname{sech}^{-1}(cx)}{2e \left(e + \frac{d}{x^2} \right)} + \frac{(a + b \operatorname{sech}^{-1}(cx))^2}{2be^2} - \frac{2 \operatorname{Subst} \left(\int \frac{e^{2x(a+bx)}}{1+e^{2x}} dx, x, \operatorname{sech}^{-1}(cx) \right)}{e^2} - \frac{\sqrt{-d}}{2e^2} \\
&= -\frac{a + b \operatorname{sech}^{-1}(cx)}{2e \left(e + \frac{d}{x^2} \right)} + \frac{(a + b \operatorname{sech}^{-1}(cx))^2}{2be^2} - \frac{(a + b \operatorname{sech}^{-1}(cx)) \log \left(1 + e^{2 \operatorname{sech}^{-1}(cx)} \right)}{e^2} + \frac{b \sqrt{-d}}{2e^2} \\
&= -\frac{a + b \operatorname{sech}^{-1}(cx)}{2e \left(e + \frac{d}{x^2} \right)} + \frac{b \sqrt{-1 + \frac{1}{c^2 x^2}} \tanh^{-1} \left(\frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{-1 + \frac{1}{c^2 x^2}} x} \right)}{2e^{3/2} \sqrt{c^2 d + e} \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}} - \frac{(a + b \operatorname{sech}^{-1}(cx)) \log \left(1 + e^{2 \operatorname{sech}^{-1}(cx)} \right)}{e^2} \\
&= -\frac{a + b \operatorname{sech}^{-1}(cx)}{2e \left(e + \frac{d}{x^2} \right)} + \frac{b \sqrt{-1 + \frac{1}{c^2 x^2}} \tanh^{-1} \left(\frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{-1 + \frac{1}{c^2 x^2}} x} \right)}{2e^{3/2} \sqrt{c^2 d + e} \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}} + \frac{(a + b \operatorname{sech}^{-1}(cx)) \log \left(1 - e^{2 \operatorname{sech}^{-1}(cx)} \right)}{2e^2} \\
&= -\frac{a + b \operatorname{sech}^{-1}(cx)}{2e \left(e + \frac{d}{x^2} \right)} + \frac{b \sqrt{-1 + \frac{1}{c^2 x^2}} \tanh^{-1} \left(\frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{-1 + \frac{1}{c^2 x^2}} x} \right)}{2e^{3/2} \sqrt{c^2 d + e} \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}} + \frac{(a + b \operatorname{sech}^{-1}(cx)) \log \left(1 - e^{2 \operatorname{sech}^{-1}(cx)} \right)}{2e^2} \\
&= -\frac{a + b \operatorname{sech}^{-1}(cx)}{2e \left(e + \frac{d}{x^2} \right)} + \frac{b \sqrt{-1 + \frac{1}{c^2 x^2}} \tanh^{-1} \left(\frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{-1 + \frac{1}{c^2 x^2}} x} \right)}{2e^{3/2} \sqrt{c^2 d + e} \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}} + \frac{(a + b \operatorname{sech}^{-1}(cx)) \log \left(1 - e^{2 \operatorname{sech}^{-1}(cx)} \right)}{2e^2}
\end{aligned}$$

Mathematica [C] time = 1.21561, size = 1208, normalized size = 2.08

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(x^3*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]

[Out] ((2*a*d)/(d + e*x^2) + (b*Sqrt[d]*ArcSech[c*x])/(Sqrt[d] - I*Sqrt[e]*x) + (b*Sqrt[d]*ArcSech[c*x])/(Sqrt[d] + I*Sqrt[e]*x) + (8*I)*b*ArcSin[Sqrt[1 - (

```

I*Sqrt[e]]/(c*Sqrt[d])/Sqrt[2]]*ArcTanh[(((-I)*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] + (8*I)*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[((I*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] - 4*b*ArcSech[c*x]*Log[1 + E^(-2*ArcSech[c*x])] + 2*b*ArcSech[c*x]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (4*I)*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + 2*b*ArcSech[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (4*I)*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + 2*b*ArcSech[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (4*I)*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + 2*b*ArcSech[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (4*I)*b*ArcSin[Sqrt[1 + (I*Sqrt[e])/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + 2*b*Log[x] + 2*a*Log[d + e*x^2] - 2*b*Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]] + (b*Sqrt[e]*Log[(2*I)*Sqrt[e]*(Sqrt[d]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + (Sqrt[d]*Sqrt[e] + I*c^2*d*x)/Sqrt[c^2*d + e]))/(I*Sqrt[d] + Sqrt[e]*x))/Sqrt[c^2*d + e] + (b*Sqrt[e]*Log[(2*Sqrt[e]*(I*Sqrt[d]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + (I*Sqrt[d]*Sqrt[e] + c^2*d*x)/Sqrt[c^2*d + e]))/((-I)*Sqrt[d] + Sqrt[e]*x))/Sqrt[c^2*d + e] + 2*b*PolyLog[2, -E^(-2*ArcSech[c*x])] - 2*b*PolyLog[2, (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - 2*b*PolyLog[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - 2*b*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - 2*b*PolyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])])]/(4*e^2)

```

Maple [C] time = 0.461, size = 661, normalized size = 1.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^3*(a+b*\text{arcsech}(c*x))/(e*x^2+d)^2,x)$

```

[Out] 1/2*c^2*a/e^2*d/(c^2*e*x^2+c^2*d)+1/2*a/e^2*ln(c^2*e*x^2+c^2*d)-1/2*c^2*b*x^2*arcsech(c*x)/(c^2*e*x^2+c^2*d)/e-1/2*b*(e*(c^2*d+e))^(1/2)/e^2/(c^2*d+e)*arctanh(1/4*(2*c^2*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))^2+2*c^2*d+4*e)/(c^2*d+e+e^2)^(1/2))+1/4*b/e^2*sum((R1^2*c^2*d+c^2*d+4*e)/(R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/R1)+dilog((R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/R1)),R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-b/e^2*arcsech(c*x)*ln(1+I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))-b/e^2*arcsech(c*x)*ln(1-I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))-b/e^2*dilog(1+I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))-b/e^2*dilog(1-I*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))+1/4*c^2*b/e^2*d*sum((R1^2+1)/(R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/R1)+dilog((R1-1/c/x-(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/R1)),R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))

```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2}a\left(\frac{d}{e^3x^2 + de^2} + \frac{\log(ex^2 + d)}{e^2}\right) + b\int \frac{x^3 \log\left(\sqrt{\frac{1}{cx} + 1}\sqrt{\frac{1}{cx} - 1} + \frac{1}{cx}\right)}{e^2x^4 + 2dex^2 + d^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="maxima")

[Out] 1/2*a*(d/(e^3*x^2 + d*e^2) + log(e*x^2 + d)/e^2) + b*integrate(x^3*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e^2*x^4 + 2*d*e*x^2 + d^2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^3 \operatorname{arsech}(cx) + ax^3}{e^2x^4 + 2dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="fricas")

[Out] integral((b*x^3*arcsech(c*x) + a*x^3)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(a+b*asech(c*x))/(e*x**2+d)**2,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsech}(cx) + a)x^3}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)*x^3/(e*x^2 + d)^2, x)

$$3.117 \quad \int \frac{x(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^2} dx$$

Optimal. Leaf size=147

$$-\frac{a+b\operatorname{sech}^{-1}(cx)}{2e(d+ex^2)} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{2de} - \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{c^2d+e}}\right)}{2d\sqrt{e}\sqrt{c^2d+e}}$$

[Out] $-(a + b*\operatorname{ArcSech}[c*x])/(2*e*(d + e*x^2)) + (b*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{ArcTanh}[\operatorname{Sqrt}[1 - c^2*x^2]])/(2*d*e) - (b*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{ArcTanh}[(\operatorname{Sqrt}[e]*\operatorname{Sqrt}[1 - c^2*x^2])/\operatorname{Sqrt}[c^2*d + e]])/(2*d*\operatorname{Sqrt}[e]*\operatorname{Sqrt}[c^2*d + e])$

Rubi [A] time = 0.244675, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {6299, 517, 446, 86, 63, 208}

$$-\frac{a+b\operatorname{sech}^{-1}(cx)}{2e(d+ex^2)} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{2de} - \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{c^2d+e}}\right)}{2d\sqrt{e}\sqrt{c^2d+e}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(x*(a + b*\operatorname{ArcSech}[c*x]))/(d + e*x^2)^2, x]$

[Out] $-(a + b*\operatorname{ArcSech}[c*x])/(2*e*(d + e*x^2)) + (b*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{ArcTanh}[\operatorname{Sqrt}[1 - c^2*x^2]])/(2*d*e) - (b*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{ArcTanh}[(\operatorname{Sqrt}[e]*\operatorname{Sqrt}[1 - c^2*x^2])/\operatorname{Sqrt}[c^2*d + e]])/(2*d*\operatorname{Sqrt}[e]*\operatorname{Sqrt}[c^2*d + e])$

Rule 6299

$\operatorname{Int}[(a_.) + \operatorname{ArcSech}[(c_.)*(x_.)]*(b_.)]*(x_.)*((d_.) + (e_.)*(x_.)^2)^{(p_.)}, x_Symbol] \rightarrow \operatorname{Simp}[(d + e*x^2)^{(p + 1)}*(a + b*\operatorname{ArcSech}[c*x])]/(2*e*(p + 1)), x] + \operatorname{Dist}[(b*\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[1/(1 + c*x)])]/(2*e*(p + 1)), \operatorname{Int}[(d + e*x^2)^{(p + 1)}]/(x*\operatorname{Sqrt}[1 - c*x]*\operatorname{Sqrt}[1 + c*x]), x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, p\}, x] \&\& \operatorname{NeQ}[p, -1]$

Rule 517

$\operatorname{Int}[(u_.)*((c_.) + (d_.)*(x_.)^{(n_.)})^{(q_.)}*((a1_.) + (b1_.)*(x_.)^{(non2_.)})^{(p_.)}*((a2_.) + (b2_.)*(x_.)^{(non2_.)})^{(p_.)}, x_Symbol] \rightarrow \operatorname{Int}[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x] /; \operatorname{FreeQ}\{a1, b1, a2, b2, c, d, n, p, q\}, x] \&\& \operatorname{EqQ}[non2, n/2] \&\& \operatorname{EqQ}[a2*b1 + a1*b2, 0] \&\& (\operatorname{IntegerQ}[p] \|\| (\operatorname{GtQ}[a1, 0] \&\& \operatorname{GtQ}[a2, 0]))$

Rule 446

$\operatorname{Int}[(x_.)^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}*((c_.) + (d_.)*(x_.)^{(n_.)})^{(q_.)}, x_Symbol] \rightarrow \operatorname{Dist}[1/n, \operatorname{Subst}[\operatorname{Int}[x^{(\operatorname{Simplify}[(m + 1)/n] - 1)}*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; \operatorname{FreeQ}\{a, b, c, d, m, n, p, q\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{IntegerQ}[\operatorname{Simplify}[(m + 1)/n]]$

Rule 86

Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && !IntegerQ[p]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
 \int \frac{x(a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^2} dx &= -\frac{a + b \operatorname{sech}^{-1}(cx)}{2e(d + ex^2)} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}(d+ex^2)} dx}{2e} \\
 &= -\frac{a + b \operatorname{sech}^{-1}(cx)}{2e(d + ex^2)} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{1}{x\sqrt{1-c^2x^2}(d+ex^2)} dx}{2e} \\
 &= -\frac{a + b \operatorname{sech}^{-1}(cx)}{2e(d + ex^2)} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-c^2x}(d+ex)} dx, x, x^2\right)}{4e} \\
 &= -\frac{a + b \operatorname{sech}^{-1}(cx)}{2e(d + ex^2)} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1-c^2x}(d+ex)} dx, x, x^2\right)}{4d} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1-c^2x^2}} dx, x, x^2\right)}{4d} \\
 &= -\frac{a + b \operatorname{sech}^{-1}(cx)}{2e(d + ex^2)} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \operatorname{Subst}\left(\int \frac{1}{d + \frac{e}{c^2} - \frac{ex^2}{c^2}} dx, x, \sqrt{1-c^2x^2}\right)}{2c^2d} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1-c^2x^2}} dx, x, \sqrt{1-c^2x^2}\right)}{2c^2d} \\
 &= -\frac{a + b \operatorname{sech}^{-1}(cx)}{2e(d + ex^2)} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx} \tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{2de} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx} \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{d+ex}}{\sqrt{1-cx}}\right)}{2d\sqrt{e}\sqrt{c^2d+e}}
 \end{aligned}$$

Mathematica [C] time = 0.963418, size = 345, normalized size = 2.35

$$\frac{2a}{d+ex^2} + \frac{b\sqrt{e} \log\left(\frac{c^2d^3/2\sqrt{ex+ide} + \frac{de\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{ex-i\sqrt{d}\sqrt{e}}}{b}\right)}{d\sqrt{c^2d+e}} + \frac{b\sqrt{e} \log\left(\frac{de+ic^2d^3/2\sqrt{ex} + \frac{de\sqrt{\frac{1-cx}{cx+1}}(cx+1)}{ex+i\sqrt{d}\sqrt{e}}}{b}\right)}{d\sqrt{c^2d+e}} + \frac{2b \operatorname{sech}^{-1}(cx)}{d+ex^2} - \frac{2b \log\left(cx\sqrt{\frac{1-cx}{cx+1}} + \sqrt{\frac{1-cx}{cx+1}}\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]

```
[Out] -((2*a)/(d + e*x^2) + (2*b*ArcSech[c*x])/(d + e*x^2) + (2*b*Log[x])/d - (2*
b*Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]])/d + (
b*Sqrt[e]*Log[(4*((I*d*e + c^2*d^(3/2)*Sqrt[e]*x)/(Sqrt[c^2*d + e]*(Sqrt[d]
+ I*Sqrt[e]*x)) + (d*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/((-I)*Sqrt[d]*
Sqrt[e] + e*x)))/b])/d + (b*Sqrt[e]*Log[(4*((d*e + I*c^2*
d^(3/2)*Sqrt[e]*x)/(Sqrt[c^2*d + e]*(I*Sqrt[d] + Sqrt[e]*x)) + (d*e*Sqrt[(1
- c*x)/(1 + c*x)]*(1 + c*x))/(I*Sqrt[d]*Sqrt[e] + e*x)))/b])/d + (d*Sqrt[c^2*d
+ e]))/(4*e)
```

Maple [B] time = 0.283, size = 844, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arcsech(c*x))/(e*x^2+d)^2,x)
```

```
[Out] -1/2*c^2*a/e/(c^2*e*x^2+c^2*d)-1/2*c^2*b/e/(c^2*e*x^2+c^2*d)*arcsech(c*x)-1
/2*c^3*b*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)/(-c^2*x^2+1)^(1/2)/((-c
^2*d*e)^(1/2)+e)/((-c^2*d*e)^(1/2)-e)*arctanh(1/(-c^2*x^2+1)^(1/2))+1/4*c^3
*b*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)/(-c^2*x^2+1)^(1/2)/((-c^2*d*
e)^(1/2)+e)/((-c^2*d*e)^(1/2)-e)/((c^2*d+e)/e)^(1/2)*ln(2*((-c^2*x^2+1)^(1/2
))*((c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*x*e+(-c^2*d*e)^(1/2)))
+1/4*c^3*b*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)/(-c^2*x^2+1)^(1/2)/((-c
^2*d*e)^(1/2)+e)/((-c^2*d*e)^(1/2)-e)/((c^2*d+e)/e)^(1/2)*ln(2*(-(c^2*x^2+
1)^(1/2))*((c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(-c*x*e+(-c^2*d*e)^(
1/2))) -1/2*c*b*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)/(-c^2*x^2+1)^(1/2
)/d/((-c^2*d*e)^(1/2)+e)/((-c^2*d*e)^(1/2)-e)*arctanh(1/(-c^2*x^2+1)^(1/2))
*e+1/4*c*b*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)/(-c^2*x^2+1)^(1/2)/d/
((-c^2*d*e)^(1/2)+e)/((-c^2*d*e)^(1/2)-e)/((c^2*d+e)/e)^(1/2)*ln(2*((-c^2*x
^2+1)^(1/2))*((c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*x*e+(-c^2*d*
e)^(1/2))) *e+1/4*c*b*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)/(-c^2*x^2+1)
^(1/2)/d/((-c^2*d*e)^(1/2)+e)/((-c^2*d*e)^(1/2)-e)/((c^2*d+e)/e)^(1/2)*ln(2*
(-(-c^2*x^2+1)^(1/2))*((c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(-c*x*e+
(-c^2*d*e)^(1/2))) *e
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2} \left(2c^2 \int \frac{x^3}{2(c^2d^2x^2 + (c^2dex^2 - de)x^2 + (c^2d^2x^2 + (c^2dex^2 - de)x^2 - d^2)\sqrt{cx+1}\sqrt{-cx+1} - d^2)} dx + \frac{x^2 \log(\sqrt{cx+1}\sqrt{-cx+1} - d^2)}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*c^2*integrate(1/2*x^3/(c^2*d^2*x^2 + (c^2*d*e*x^2 - d*e)*x^2 + (c^2*
d^2*x^2 + (c^2*d*e*x^2 - d*e)*x^2 - d^2)*sqrt(c*x + 1)*sqrt(-c*x + 1) - d^2
), x) + (x^2*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1) - x^2*log(c) - x^2*log(x
))/d + (d*e*x^2 + d^2) - 2*integrate(1/2*x/(c^2*d^2*x^2 + (c^2*d*e*x^2 - d*e)*
x^2 - d^2), x))*b - 1/2*a/(e^2*x^2 + d*e)
```

Fricas [B] time = 1.97457, size = 1265, normalized size = 8.61

$$\frac{2ac^2d^2 + 2ade - \sqrt{c^2de + e^2}(bex^2 + bd) \log \left(\frac{c^4d^2 + 4c^2de - (c^4de + 2c^2e^2)x^2 + 4(c^3de + ce^2)x \sqrt{-\frac{c^2x^2 - 1}{c^2x^2}} + 4e^2 + 2(c^2ex^2 - c^2d - (c^3d + 2ce)x \sqrt{-\frac{c^2x^2 - 1}{c^2x^2}})}{ex^2 + d} \right)}{4(c^2d^3e + d^2e^2 + (c^2d^2e^2 + d^2e^3)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="fricas")

[Out] [-1/4*(2*a*c^2*d^2 + 2*a*d*e - sqrt(c^2*d*e + e^2)*(b*e*x^2 + b*d)*log((c^4*d^2 + 4*c^2*d*e - (c^4*d*e + 2*c^2*e^2)*x^2 + 4*(c^3*d*e + c*e^2)*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 4*e^2 + 2*(c^2*e*x^2 - c^2*d - (c^3*d + 2*c*e)*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 2*e)*sqrt(c^2*d*e + e^2))/(e*x^2 + d)) + 2*(b*c^2*d^2 + b*d*e + (b*c^2*d*e + b*e^2)*x^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + 2*(b*c^2*d^2 + b*d*e)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/(c^2*d^3*e + d^2*e^2 + (c^2*d^2*e^2 + d*e^3)*x^2), -1/2*(a*c^2*d^2 + a*d*e + sqrt(-c^2*d*e - e^2)*(b*e*x^2 + b*d)*arctan((sqrt(-c^2*d*e - e^2)*c*d*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - sqrt(-c^2*d*e - e^2)*(e*x^2 + d))/((c^2*d*e + e^2)*x^2)) + (b*c^2*d^2 + b*d*e + (b*c^2*d*e + b*e^2)*x^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + (b*c^2*d^2 + b*d*e)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)))/(c^2*d^3*e + d^2*e^2 + (c^2*d^2*e^2 + d*e^3)*x^2)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*asech(c*x))/(e*x**2+d)**2,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{ar} \operatorname{sech}(cx) + a)x}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)*x/(e*x^2 + d)^2, x)

$$3.118 \quad \int \frac{a+b\operatorname{sech}^{-1}(cx)}{x(d+ex^2)^2} dx$$

Optimal. Leaf size=542

$$\frac{b\operatorname{PolyLog}\left(2, -\frac{c\sqrt{-de}\operatorname{sech}^{-1}(cx)}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2} - \frac{b\operatorname{PolyLog}\left(2, \frac{c\sqrt{-de}\operatorname{sech}^{-1}(cx)}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2} - \frac{b\operatorname{PolyLog}\left(2, -\frac{c\sqrt{-de}\operatorname{sech}^{-1}(cx)}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^2} - \frac{b\operatorname{PolyLog}\left(2, \frac{c\sqrt{-de}\operatorname{sech}^{-1}(cx)}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^2}$$

```
[Out] -(e*(a + b*ArcSech[c*x]))/(2*d^2*(e + d/x^2)) + (a + b*ArcSech[c*x])^2/(2*b
*d^2) + (b*Sqrt[e]*Sqrt[-1 + 1/(c^2*x^2)]*ArcTanh[Sqrt[c^2*d + e]/(c*Sqrt[e
]*Sqrt[-1 + 1/(c^2*x^2)]*x)]/(2*d^2*Sqrt[c^2*d + e]*Sqrt[-1 + 1/(c*x)]*Sqr
t[1 + 1/(c*x)])) - ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])
/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^2) - ((a + b*ArcSech[c*x])*Log[1 + (c*S
qrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^2) - ((a + b*Arc
Sech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])
/(2*d^2) - ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt
[e] + Sqrt[c^2*d + e])])/(2*d^2) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*
x])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(2*d^2) - (b*PolyLog[2, (c*Sqrt[-d]*E^Ar
cSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^2) - (b*PolyLog[2, -((c*Sqrt
[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(2*d^2) - (b*PolyLog[2,
(c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d^2)
```

Rubi [A] time = 1.36288, antiderivative size = 542, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {6303, 5792, 5788, 519, 377, 208, 5800, 5562, 2190, 2279, 2391}

$$\frac{b\operatorname{PolyLog}\left(2, -\frac{c\sqrt{-de}\operatorname{sech}^{-1}(cx)}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2} - \frac{b\operatorname{PolyLog}\left(2, \frac{c\sqrt{-de}\operatorname{sech}^{-1}(cx)}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^2} - \frac{b\operatorname{PolyLog}\left(2, -\frac{c\sqrt{-de}\operatorname{sech}^{-1}(cx)}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^2} - \frac{b\operatorname{PolyLog}\left(2, \frac{c\sqrt{-de}\operatorname{sech}^{-1}(cx)}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^2), x]
```

```
[Out] -(e*(a + b*ArcSech[c*x]))/(2*d^2*(e + d/x^2)) + (a + b*ArcSech[c*x])^2/(2*b
*d^2) + (b*Sqrt[e]*Sqrt[-1 + 1/(c^2*x^2)]*ArcTanh[Sqrt[c^2*d + e]/(c*Sqrt[e
]*Sqrt[-1 + 1/(c^2*x^2)]*x)]/(2*d^2*Sqrt[c^2*d + e]*Sqrt[-1 + 1/(c*x)]*Sqr
t[1 + 1/(c*x)])) - ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])
/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^2) - ((a + b*ArcSech[c*x])*Log[1 + (c*S
qrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^2) - ((a + b*Arc
Sech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])
/(2*d^2) - ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt
[e] + Sqrt[c^2*d + e])])/(2*d^2) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*
x])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(2*d^2) - (b*PolyLog[2, (c*Sqrt[-d]*E^Ar
cSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^2) - (b*PolyLog[2, -((c*Sqrt
[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(2*d^2) - (b*PolyLog[2,
(c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d^2)
```

Rule 6303

```
Int[((a_.) + ArcSech[(c_.)*(x_)])*(b_.)^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*(a + b*ArcCosh[x/c])^n]/
```

$x^{(m + 2*(p + 1))}$, x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegersQ[m, p]

Rule 5792

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)*((f_.)*(x_.))^(m_.)*((d_.) + (e_.)*(x_.)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]

Rule 5788

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))*(x_.)*((d_.) + (e_.)*(x_.)^2)^(p_.), x_Symbol] := Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x]))/(2*e*(p + 1)), x] - Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] && NeQ[p, -1]

Rule 519

Int[(u_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.)*((a1_.) + (b1_.)*(x_.)^(non2_.))^(p_.)*((a2_.) + (b2_.)*(x_.)^(non2_.))^(p_.), x_Symbol] := Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && !(EqQ[n, 2] && IGtQ[q, 0])

Rule 377

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)/((c_.) + (d_.)*(x_.)^(n_.)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 208

Int[((a_.) + (b_.)*(x_.)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 5800

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_.)), x_Symbol] := Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]

Rule 5562

Int((((e_.) + (f_.)*(x_.))^(m_.)*Sinh[(c_.) + (d_.)*(x_.)]/(Cosh[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)), x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)), x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]

Rule 2190

Int((((F_)^(g_.)*((e_.) + (f_.)*(x_.)))^(n_.)*((c_.) + (d_.)*(x_.))^(m_.))/((a_.) + (b_.)*(F_)^(g_.)*((e_.) + (f_.)*(x_.)))^(n_.), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n]/a)/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n]]

))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
 := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))
 ^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2,
 -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rubi steps

$$\begin{aligned}
\int \frac{a + b \operatorname{sech}^{-1}(cx)}{x(d + ex^2)^2} dx &= -\operatorname{Subst} \left(\int \frac{x^3 (a + b \cosh^{-1}(\frac{x}{c}))}{(e + dx^2)^2} dx, x, \frac{1}{x} \right) \\
&= -\operatorname{Subst} \left(\int \left(-\frac{ex (a + b \cosh^{-1}(\frac{x}{c}))}{d(e + dx^2)^2} + \frac{x (a + b \cosh^{-1}(\frac{x}{c}))}{d(e + dx^2)} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\operatorname{Subst} \left(\int \frac{x^{a+b \cosh^{-1}(\frac{x}{c})}}{e+dx^2} dx, x, \frac{1}{x} \right)}{d} + \frac{e \operatorname{Subst} \left(\int \frac{x^{a+b \cosh^{-1}(\frac{x}{c})}}{(e+dx^2)^2} dx, x, \frac{1}{x} \right)}{d} \\
&= -\frac{e(a + b \operatorname{sech}^{-1}(cx))}{2d^2 \left(e + \frac{d}{x^2} \right)} - \frac{\operatorname{Subst} \left(\int \left(-\frac{\sqrt{-d}(a+b \cosh^{-1}(\frac{x}{c}))}{2d(\sqrt{e}-\sqrt{-dx})} + \frac{\sqrt{-d}(a+b \cosh^{-1}(\frac{x}{c}))}{2d(\sqrt{e}+\sqrt{-dx})} \right) dx, x, \frac{1}{x} \right)}{d} + \dots \quad (be) S \\
&= -\frac{e(a + b \operatorname{sech}^{-1}(cx))}{2d^2 \left(e + \frac{d}{x^2} \right)} + \frac{\operatorname{Subst} \left(\int \frac{a+b \cosh^{-1}(\frac{x}{c})}{\sqrt{e}-\sqrt{-dx}} dx, x, \frac{1}{x} \right)}{2(-d)^{3/2}} - \frac{\operatorname{Subst} \left(\int \frac{a+b \cosh^{-1}(\frac{x}{c})}{\sqrt{e}+\sqrt{-dx}} dx, x, \frac{1}{x} \right)}{2(-d)^{3/2}} + \dots \\
&= -\frac{e(a + b \operatorname{sech}^{-1}(cx))}{2d^2 \left(e + \frac{d}{x^2} \right)} + \frac{\operatorname{Subst} \left(\int \frac{(a+bx) \sinh(x)}{\frac{\sqrt{e}}{c} - \sqrt{-d} \cosh(x)} dx, x, \operatorname{sech}^{-1}(cx) \right)}{2(-d)^{3/2}} - \frac{\operatorname{Subst} \left(\int \frac{(a+bx) \sinh(x)}{\frac{\sqrt{e}}{c} + \sqrt{-d} \cosh(x)} dx, x, \operatorname{sech}^{-1}(cx) \right)}{2(-d)^{3/2}} + \dots \\
&= -\frac{e(a + b \operatorname{sech}^{-1}(cx))}{2d^2 \left(e + \frac{d}{x^2} \right)} + \frac{(a + b \operatorname{sech}^{-1}(cx))^2}{2bd^2} + \frac{b\sqrt{e}\sqrt{-1 + \frac{1}{c^2x^2}} \tanh^{-1} \left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1 + \frac{1}{c^2x^2}x}} \right)}{2d^2\sqrt{c^2d+e}\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}} + \dots \\
&= -\frac{e(a + b \operatorname{sech}^{-1}(cx))}{2d^2 \left(e + \frac{d}{x^2} \right)} + \frac{(a + b \operatorname{sech}^{-1}(cx))^2}{2bd^2} + \frac{b\sqrt{e}\sqrt{-1 + \frac{1}{c^2x^2}} \tanh^{-1} \left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1 + \frac{1}{c^2x^2}x}} \right)}{2d^2\sqrt{c^2d+e}\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}} - \dots \quad (a - \\
&= -\frac{e(a + b \operatorname{sech}^{-1}(cx))}{2d^2 \left(e + \frac{d}{x^2} \right)} + \frac{(a + b \operatorname{sech}^{-1}(cx))^2}{2bd^2} + \frac{b\sqrt{e}\sqrt{-1 + \frac{1}{c^2x^2}} \tanh^{-1} \left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1 + \frac{1}{c^2x^2}x}} \right)}{2d^2\sqrt{c^2d+e}\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}} - \dots \quad (a - \\
&= -\frac{e(a + b \operatorname{sech}^{-1}(cx))}{2d^2 \left(e + \frac{d}{x^2} \right)} + \frac{(a + b \operatorname{sech}^{-1}(cx))^2}{2bd^2} + \frac{b\sqrt{e}\sqrt{-1 + \frac{1}{c^2x^2}} \tanh^{-1} \left(\frac{\sqrt{c^2d+e}}{c\sqrt{e}\sqrt{-1 + \frac{1}{c^2x^2}x}} \right)}{2d^2\sqrt{c^2d+e}\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}} - \dots \quad (a -
\end{aligned}$$

Mathematica [F] time = 40.2766, size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{sech}^{-1}(cx)}{x(d + ex^2)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^2), x]

[Out] Integrate[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^2), x]

Maple [C] time = 0.757, size = 3326, normalized size = 6.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))/x/(e*x^2+d)^2,x)

[Out]
$$-1/2*b/d^2*\ln(1-c^2*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e)*\operatorname{arcsech}(c*x)-3*b/c^2/d^3*e/(c^2*d+e)*\ln(1-c^2*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e)*\operatorname{arcsech}(c*x)*(e*(c^2*d+e))^{1/2}-2*b/c^4/d^4*e^2/(c^2*d+e)*\ln(1-c^2*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e)*\operatorname{arcsech}(c*x)*(e*(c^2*d+e))^{1/2}-1/4*b*c^2/d/e/(c^2*d+e)*\ln(1-c^2*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e)*\operatorname{arcsech}(c*x)*(e*(c^2*d+e))^{1/2}+1/4*b*c^2*(e*(c^2*d+e))^{1/2}/d/e/(c^2*d+e)*\operatorname{arcsech}(c*x)*\ln(1-c^2*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))^2/(-c^2*d+2*(e*(c^2*d+e))^{1/2}-2*e)+a/d^2*\ln(c*x)+1/2*a*c^2/d/(c^2*e*x^2+c^2*d)-1/2*b/d^2*\sum((_R1^2*c^2*d+2*c^2*d+4*e)/(_R1^2*c^2*d+c^2*d+2*e))*\operatorname{arcsech}(c*x)*\ln((_R1-1/c/x-(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2})/_R1)+\operatorname{dilog}((_R1-1/c/x-(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2})/_R1), _R1=\operatorname{RootOf}(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+b*\operatorname{arcsech}(c*x)^2/d^2-1/4*b/d^2*\operatorname{polylog}(2,c^2*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e))-1/2*a/d^2*\ln(c^2*e*x^2+c^2*d)-b/c^2/d^3*\operatorname{polylog}(2,c^2*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e))*e+2*b/c^2/d^3*e*\operatorname{arcsech}(c*x)^2+2*b/c^4/d^4*e^2*\operatorname{arcsech}(c*x)^2-b/c^2/d^3*\operatorname{arcsech}(c*x)^2*(e*(c^2*d+e))^{1/2}-b/c^4/d^4*e^2*\operatorname{polylog}(2,c^2*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e))+1/4*b*c^2/d/(c^2*d+e)*\operatorname{polylog}(2,c^2*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))^2/(-c^2*d+2*(e*(c^2*d+e))^{1/2}-2*e))-1/2*b*(e*(c^2*d+e))^{1/2}/d^2/(c^2*d+e)*\operatorname{arctanh}(1/4*(2*c^2*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))^2+2*c^2*d+4*e)/(c^2*d+e+e^2)^{1/2}))+b*(e*(c^2*d+e))^{1/2}/d^2/(c^2*d+e)*\operatorname{arcsech}(c*x)^2-5/2*b/d^2/(c^2*d+e)*\operatorname{arcsech}(c*x)^2*e+1/8*b*c^2*(e*(c^2*d+e))^{1/2}/d/e/(c^2*d+e)*\operatorname{polylog}(2,c^2*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))^2/(-c^2*d+2*(e*(c^2*d+e))^{1/2}-2*e))-3/2*b/c^2/d^3/(c^2*d+e)*\operatorname{polylog}(2,c^2*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e))*e*(c^2*d+e))^{1/2}*e-1/2*b*c^2*x^2*e*\operatorname{arcsech}(c*x)/(c^2*e*x^2+c^2*d)/d^2+4*b/c^2/d^3*e^2/(c^2*d+e)*\ln(1-c^2*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e))*\operatorname{arcsech}(c*x)-1/8*b*c^2/d/e/(c^2*d+e)*\operatorname{polylog}(2,c^2*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e))*e*(c^2*d+e))^{1/2}+2*b/c^4/d^4*e^2/(c^2*d+e)*\operatorname{arcsech}(c*x)^2*(e*(c^2*d+e))^{1/2}-b/c^4/d^4*e^2/(c^2*d+e)*\operatorname{polylog}(2,c^2*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e))*e*(c^2*d+e))^{1/2}-3/4*b/d^2/(c^2*d+e)*\operatorname{polylog}(2,c^2*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e))*e*(c^2*d+e))^{1/2}+5/4*b/d^2/(c^2*d+e)*\operatorname{polylog}(2,c^2*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e))*e+1/2*b/c^2/d^3*\operatorname{polylog}(2,c^2*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e))*e*(c^2*d+e))^{1/2}+b/c^2/d^3*\ln(1-c^2*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e))*\operatorname{arcsech}(c*x)*(e*(c^2*d+e))^{1/2}-2*b/c^2/d^3*\ln(1-c^2*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e))*\operatorname{arcsech}(c*x)*e-2*b/c^4/d^4*e^2*\ln(1-c^2*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e))*\operatorname{arcsech}(c*x)$$

$$\begin{aligned}
& +1/2*b*c^2/d/(c^2*d+e)*\ln(1-c^2*d*(1/c/x+(-1+1/c/x)^{1/2})*(1+1/c/x)^{1/2})^2 \\
& /(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e))*\operatorname{arcsech}(c*x)-2*b/c^4/d^4*e*\operatorname{arcsech}(c*x) \\
& ^2*(e*(c^2*d+e))^{1/2}+b/c^4/d^4*e*\operatorname{polylog}(2,c^2*d*(1/c/x+(-1+1/c/x)^{1/2})* \\
& (1+1/c/x)^{1/2})^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e))*(e*(c^2*d+e))^{1/2} \\
& -4*b/c^2/d^3/(c^2*d+e)*\operatorname{arcsech}(c*x)^2*e^{2-3/2*b/d^2/(c^2*d+e)*\ln(1-c^2*d*(\\
& 1/c/x+(-1+1/c/x)^{1/2})*(1+1/c/x)^{1/2})^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e)} \\
&))*\operatorname{arcsech}(c*x)*(e*(c^2*d+e))^{1/2}+1/2*b*(e*(c^2*d+e))^{1/2}/d^2/(c^2*d+e) \\
& *\operatorname{arcsech}(c*x)*\ln(1-c^2*d*(1/c/x+(-1+1/c/x)^{1/2})*(1+1/c/x)^{1/2})^2/(-c^2*d \\
& +2*(e*(c^2*d+e))^{1/2}-2*e))+5/2*b/d^2*e/(c^2*d+e)*\ln(1-c^2*d*(1/c/x+(-1+1/ \\
& c/x)^{1/2})*(1+1/c/x)^{1/2})^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e))*\operatorname{arcsech}(c \\
& *x)+b/c^4/d^4*e^3/(c^2*d+e)*\operatorname{polylog}(2,c^2*d*(1/c/x+(-1+1/c/x)^{1/2})*(1+1/c/ \\
& x)^{1/2})^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e))+2*b/c^2/d^3/(c^2*d+e)*\operatorname{polyl} \\
& \operatorname{og}(2,c^2*d*(1/c/x+(-1+1/c/x)^{1/2})*(1+1/c/x)^{1/2})^2/(-c^2*d-2*(e*(c^2*d+e) \\
&))^{1/2}-2*e))*e^{2-2*b/c^4/d^4*e^3/(c^2*d+e)*\operatorname{arcsech}(c*x)^2+2*b/c^4/d^4*e* \\
& \ln(1-c^2*d*(1/c/x+(-1+1/c/x)^{1/2})*(1+1/c/x)^{1/2})^2/(-c^2*d-2*(e*(c^2*d+e) \\
&))^{1/2}-2*e))*\operatorname{arcsech}(c*x)*(e*(c^2*d+e))^{1/2}+3*b/c^2/d^3/(c^2*d+e)*\operatorname{arcsec} \\
& \operatorname{h}(c*x)^2*(e*(c^2*d+e))^{1/2}*e+2*b/c^4/d^4*e^3/(c^2*d+e)*\ln(1-c^2*d*(1/c/x+ \\
& (-1+1/c/x)^{1/2})*(1+1/c/x)^{1/2})^2/(-c^2*d-2*(e*(c^2*d+e))^{1/2}-2*e))*\operatorname{arc} \\
& \operatorname{sech}(c*x)
\end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^2,x, algorithm="maxima")

[Out] Exception raised: RuntimeError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b \operatorname{arsech}(cx) + a}{e^2x^5 + 2dex^3 + d^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^2,x, algorithm="fricas")

[Out] integral((b*arcsech(c*x) + a)/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asech(c*x))/x/(e*x**2+d)**2,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{ar} \operatorname{sech}(cx) + a}{(ex^2 + d)^2 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/((e*x^2 + d)^2*x), x)
```

$$3.119 \quad \int \frac{x^4 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^2} dx$$

Optimal. Leaf size=840

$$\frac{x(a + b \operatorname{sech}^{-1}(cx))}{e^2} + \frac{3\sqrt{-d} \log\left(1 - \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e - \sqrt{dc^2 + e}}}\right)(a + b \operatorname{sech}^{-1}(cx))}{4e^{5/2}} - \frac{3\sqrt{-d} \log\left(\frac{\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}c}{\sqrt{e - \sqrt{dc^2 + e}}} + 1\right)(a + b \operatorname{sech}^{-1}(cx))}{4e^{5/2}}$$

```
[Out] -(d*(a + b*ArcSech[c*x]))/(4*e^2*(Sqrt[-d]*Sqrt[e] - d/x)) + (d*(a + b*ArcSech[c*x]))/(4*e^2*(Sqrt[-d]*Sqrt[e] + d/x)) + (x*(a + b*ArcSech[c*x]))/e^2 + (b*d*ArcTan[(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])]/(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])))/(2*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*e^2) + (b*d*ArcTan[(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])]/(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])))/(2*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*e^2) - (b*ArcTan[Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]])/(c*e^2) + (3*Sqrt[-d]*(a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*e^(5/2)) - (3*Sqrt[-d]*(a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*e^(5/2)) + (3*Sqrt[-d]*(a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*e^(5/2)) - (3*Sqrt[-d]*(a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*e^(5/2)) - (3*b*Sqrt[-d]*PolyLog[2, -(c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*e^(5/2)) + (3*b*Sqrt[-d]*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*e^(5/2)) - (3*b*Sqrt[-d]*PolyLog[2, -(c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*e^(5/2)) + (3*b*Sqrt[-d]*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*e^(5/2))
```

Rubi [A] time = 3.08786, antiderivative size = 840, normalized size of antiderivative = 1., number of steps used = 50, number of rules used = 13, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {6303, 5792, 5662, 92, 205, 5707, 5802, 93, 5800, 5562, 2190, 2279, 2391}

$$\frac{x(a + b \operatorname{sech}^{-1}(cx))}{e^2} + \frac{3\sqrt{-d} \log\left(1 - \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e - \sqrt{dc^2 + e}}}\right)(a + b \operatorname{sech}^{-1}(cx))}{4e^{5/2}} - \frac{3\sqrt{-d} \log\left(\frac{\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}c}{\sqrt{e - \sqrt{dc^2 + e}}} + 1\right)(a + b \operatorname{sech}^{-1}(cx))}{4e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -(d*(a + b*ArcSech[c*x]))/(4*e^2*(Sqrt[-d]*Sqrt[e] - d/x)) + (d*(a + b*ArcSech[c*x]))/(4*e^2*(Sqrt[-d]*Sqrt[e] + d/x)) + (x*(a + b*ArcSech[c*x]))/e^2 + (b*d*ArcTan[(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])]/(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])))/(2*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*e^2) + (b*d*ArcTan[(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])]/(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])))/(2*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*e^2) - (b*ArcTan[Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]])/(c*e^2) + (3*Sqrt[-d]*(a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*e^(5/2)) - (3*Sqrt[-d]*(a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*e^(5/2)) + (3*Sqrt[-d]*(a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*e^(5/2)) - (3*Sqrt[-d]*(a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*e^(5/2)) - (3*b*Sqrt[-d]*PolyLog[2, -(c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*e^(5/2)) + (3*b*Sqrt[-d]*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*e^(5/2)) - (3*b*Sqrt[-d]*PolyLog[2, -(c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*e^(5/2)) + (3*b*Sqrt[-d]*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*e^(5/2))
```

```
[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])]/(4
*e^(5/2)) - (3*Sqrt[-d]*(a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[
c*x])/(Sqrt[e] + Sqrt[c^2*d + e])]/(4*e^(5/2)) - (3*b*Sqrt[-d]*PolyLog[2,
-((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e]))]/(4*e^(5/2)) +
(3*b*Sqrt[-d]*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d
+ e]))]/(4*e^(5/2)) - (3*b*Sqrt[-d]*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x]
)/(Sqrt[e] + Sqrt[c^2*d + e]))]/(4*e^(5/2)) + (3*b*Sqrt[-d]*PolyLog[2, (c*
Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))]/(4*e^(5/2))
```

Rule 6303

```
Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))^ (n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] := -Subst[Int[((e + d*x^2)^p*(a + b*ArcCosh[x/c])^n)/
x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^ (n_.)*((f_.)*(x_)^(m_.)*((d_.) + (e
_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
(f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
+ e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^ (n_.)*((d_.)*(x_)^(m_.), x_Symbol]
:= Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
NeQ[m, -1]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_.)]*Sqrt[(c_.) + (d_.)*(x_.)]*((e_.) + (f_.)*(x_
))), x_Symbol] := Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^ (n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] := Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^ (n_.)*((d_.) + (e_.)*(x_)^(m_.), x
_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
- Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
- 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)))/((e_.) + (f_.)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.)^(n_)/((d_.) + (e_.)*(x_)), x_Symbol] := Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_)^(m_))*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_.)*(x_)])*(b_.) + (a_)), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)), x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)), x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2190

```
Int[(((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_))*((c_.) + (d_.)*(x_)^(m_))/((a_.) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x_)))^(n_)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_.) + (b_.)*((F_)^(e_.)*((c_.) + (d_.)*(x_)))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_.) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^2} dx &= -\operatorname{Subst} \left(\int \frac{a + b \cosh^{-1} \left(\frac{x}{c} \right)}{x^2 (e + dx^2)^2} dx, x, \frac{1}{x} \right) \\
&= -\operatorname{Subst} \left(\int \left(\frac{a + b \cosh^{-1} \left(\frac{x}{c} \right)}{e^2 x^2} - \frac{d (a + b \cosh^{-1} \left(\frac{x}{c} \right))}{e (e + dx^2)^2} - \frac{d (a + b \cosh^{-1} \left(\frac{x}{c} \right))}{e^2 (e + dx^2)} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\operatorname{Subst} \left(\int \frac{a + b \cosh^{-1} \left(\frac{x}{c} \right)}{x^2} dx, x, \frac{1}{x} \right)}{e^2} + \frac{d \operatorname{Subst} \left(\int \frac{a + b \cosh^{-1} \left(\frac{x}{c} \right)}{e + dx^2} dx, x, \frac{1}{x} \right)}{e^2} + \frac{d \operatorname{Subst} \left(\int \frac{a + b \cosh^{-1} \left(\frac{x}{c} \right)}{(e + dx^2)} dx, x, \frac{1}{x} \right)}{e} \\
&= \frac{x (a + b \operatorname{sech}^{-1}(cx))}{e^2} - \frac{b \operatorname{Subst} \left(\int \frac{1}{x \sqrt{-1 + \frac{x}{c}} \sqrt{1 + \frac{x}{c}}} dx, x, \frac{1}{x} \right)}{ce^2} + \frac{d \operatorname{Subst} \left(\int \left(\frac{a + b \cosh^{-1} \left(\frac{x}{c} \right)}{2\sqrt{e}(\sqrt{e} - \sqrt{-dx})} + \frac{a + b}{2\sqrt{e}} \right) dx, x, \frac{1}{x} \right)}{e^2} \\
&= \frac{x (a + b \operatorname{sech}^{-1}(cx))}{e^2} + \frac{d \operatorname{Subst} \left(\int \frac{a + b \cosh^{-1} \left(\frac{x}{c} \right)}{\sqrt{e} - \sqrt{-dx}} dx, x, \frac{1}{x} \right)}{2e^{5/2}} + \frac{d \operatorname{Subst} \left(\int \frac{a + b \cosh^{-1} \left(\frac{x}{c} \right)}{\sqrt{e} + \sqrt{-dx}} dx, x, \frac{1}{x} \right)}{2e^{5/2}} \\
&= -\frac{d (a + b \operatorname{sech}^{-1}(cx))}{4e^2 \left(\sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} + \frac{d (a + b \operatorname{sech}^{-1}(cx))}{4e^2 \left(\sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{x (a + b \operatorname{sech}^{-1}(cx))}{e^2} - \frac{b \tan^{-1} \left(\sqrt{-1 + \frac{1}{cx}} \right)}{ce^2} \\
&= -\frac{d (a + b \operatorname{sech}^{-1}(cx))}{4e^2 \left(\sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} + \frac{d (a + b \operatorname{sech}^{-1}(cx))}{4e^2 \left(\sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{x (a + b \operatorname{sech}^{-1}(cx))}{e^2} - \frac{b \tan^{-1} \left(\sqrt{-1 + \frac{1}{cx}} \right)}{ce^2} \\
&= -\frac{d (a + b \operatorname{sech}^{-1}(cx))}{4e^2 \left(\sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} + \frac{d (a + b \operatorname{sech}^{-1}(cx))}{4e^2 \left(\sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{x (a + b \operatorname{sech}^{-1}(cx))}{e^2} + \frac{bd \tan^{-1} \left(\frac{\sqrt{cd - \sqrt{-d}}}{\sqrt{cd + \sqrt{-d}}} \right)}{2\sqrt{cd - \sqrt{-d}} \sqrt{e} \sqrt{cd}} \\
&= -\frac{d (a + b \operatorname{sech}^{-1}(cx))}{4e^2 \left(\sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} + \frac{d (a + b \operatorname{sech}^{-1}(cx))}{4e^2 \left(\sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{x (a + b \operatorname{sech}^{-1}(cx))}{e^2} + \frac{bd \tan^{-1} \left(\frac{\sqrt{cd - \sqrt{-d}}}{\sqrt{cd + \sqrt{-d}}} \right)}{2\sqrt{cd - \sqrt{-d}} \sqrt{e} \sqrt{cd}} \\
&= -\frac{d (a + b \operatorname{sech}^{-1}(cx))}{4e^2 \left(\sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} + \frac{d (a + b \operatorname{sech}^{-1}(cx))}{4e^2 \left(\sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{x (a + b \operatorname{sech}^{-1}(cx))}{e^2} + \frac{bd \tan^{-1} \left(\frac{\sqrt{cd - \sqrt{-d}}}{\sqrt{cd + \sqrt{-d}}} \right)}{2\sqrt{cd - \sqrt{-d}} \sqrt{e} \sqrt{cd}} \\
&= -\frac{d (a + b \operatorname{sech}^{-1}(cx))}{4e^2 \left(\sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} + \frac{d (a + b \operatorname{sech}^{-1}(cx))}{4e^2 \left(\sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{x (a + b \operatorname{sech}^{-1}(cx))}{e^2} + \frac{bd \tan^{-1} \left(\frac{\sqrt{cd - \sqrt{-d}}}{\sqrt{cd + \sqrt{-d}}} \right)}{2\sqrt{cd - \sqrt{-d}} \sqrt{e} \sqrt{cd}} \\
&= -\frac{d (a + b \operatorname{sech}^{-1}(cx))}{4e^2 \left(\sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} + \frac{d (a + b \operatorname{sech}^{-1}(cx))}{4e^2 \left(\sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \frac{x (a + b \operatorname{sech}^{-1}(cx))}{e^2} + \frac{bd \tan^{-1} \left(\frac{\sqrt{cd - \sqrt{-d}}}{\sqrt{cd + \sqrt{-d}}} \right)}{2\sqrt{cd - \sqrt{-d}} \sqrt{e} \sqrt{cd}}
\end{aligned}$$

Mathematica [C] time = 1.49012, size = 1270, normalized size = 1.51

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]

[Out] $(4*a*\sqrt{e}*x + (2*a*d*\sqrt{e}*x)/(d + e*x^2) + 4*b*\sqrt{e}*x*\text{ArcSech}[c*x] + (b*d*\text{ArcSech}[c*x])/((-1)*\sqrt{d} + \sqrt{e}*x) + (b*d*\text{ArcSech}[c*x])/(I*\sqrt{d} + \sqrt{e}*x) - 6*a*\sqrt{d}*\text{ArcTan}[(\sqrt{e}*x)/\sqrt{d}] - (8*b*\sqrt{e}*\text{ArcTan}[\text{Tanh}[\text{ArcSech}[c*x]/2]])/c + 12*b*\sqrt{d}*\text{ArcSin}[\sqrt{1 - (I*\sqrt{e})/(c*\sqrt{d})}]/\sqrt{2}]*\text{ArcTanh}[((-1)*c*\sqrt{d} + \sqrt{e})*\text{Tanh}[\text{ArcSech}[c*x]/2]]/\sqrt{c^2*d + e}] - 12*b*\sqrt{d}*\text{ArcSin}[\sqrt{1 + (I*\sqrt{e})/(c*\sqrt{d})}]/\sqrt{2}]*\text{ArcTanh}[(I*c*\sqrt{d} + \sqrt{e})*\text{Tanh}[\text{ArcSech}[c*x]/2]]/\sqrt{c^2*d + e}] + (3*I)*b*\sqrt{d}*\text{ArcSech}[c*x]*\text{Log}[1 + (I*(\sqrt{e} - \sqrt{c^2*d + e}))/(c*\sqrt{d}*E^{\text{ArcSech}[c*x]})] + 6*b*\sqrt{d}*\text{ArcSin}[\sqrt{1 + (I*\sqrt{e})/(c*\sqrt{d})}]/\sqrt{2}]*\text{Log}[1 + (I*(\sqrt{e} - \sqrt{c^2*d + e}))/(c*\sqrt{d}*E^{\text{ArcSech}[c*x]})] - (3*I)*b*\sqrt{d}*\text{ArcSech}[c*x]*\text{Log}[1 + (I*(-\sqrt{e} + \sqrt{c^2*d + e}))/(c*\sqrt{d}*E^{\text{ArcSech}[c*x]})] - 6*b*\sqrt{d}*\text{ArcSin}[\sqrt{1 - (I*\sqrt{e})/(c*\sqrt{d})}]/\sqrt{2}]*\text{Log}[1 + (I*(-\sqrt{e} + \sqrt{c^2*d + e}))/(c*\sqrt{d}*E^{\text{ArcSech}[c*x]})] - (3*I)*b*\sqrt{d}*\text{ArcSech}[c*x]*\text{Log}[1 - (I*(\sqrt{e} + \sqrt{c^2*d + e}))/(c*\sqrt{d}*E^{\text{ArcSech}[c*x]})] + 6*b*\sqrt{d}*\text{ArcSin}[\sqrt{1 - (I*\sqrt{e})/(c*\sqrt{d})}]/\sqrt{2}]*\text{Log}[1 - (I*(\sqrt{e} + \sqrt{c^2*d + e}))/(c*\sqrt{d}*E^{\text{ArcSech}[c*x]})] + (3*I)*b*\sqrt{d}*\text{ArcSech}[c*x]*\text{Log}[1 + (I*(\sqrt{e} + \sqrt{c^2*d + e}))/(c*\sqrt{d}*E^{\text{ArcSech}[c*x]})] - 6*b*\sqrt{d}*\text{ArcSin}[\sqrt{1 + (I*\sqrt{e})/(c*\sqrt{d})}]/\sqrt{2}]*\text{Log}[1 + (I*(\sqrt{e} + \sqrt{c^2*d + e}))/(c*\sqrt{d}*E^{\text{ArcSech}[c*x]})] - (I*b*\sqrt{d}*\sqrt{e}*\text{Log}[(2*I)*\sqrt{e}*(\sqrt{d}*\sqrt{(1 - c*x)/(1 + c*x)}*(1 + c*x) + (\sqrt{d}*\sqrt{e} + I*c^2*d*x)/\sqrt{c^2*d + e}))/ (I*\sqrt{d} + \sqrt{e}*x)]/\sqrt{c^2*d + e} + (I*b*\sqrt{d}*\sqrt{e}*\text{Log}[(2*\sqrt{e}*(I*\sqrt{d}*\sqrt{(1 - c*x)/(1 + c*x)}*(1 + c*x) + (I*\sqrt{d}*\sqrt{e} + c^2*d*x)/\sqrt{c^2*d + e}))/((-1)*\sqrt{d} + \sqrt{e}*x)]/\sqrt{c^2*d + e} + (3*I)*b*\sqrt{d}*\text{PolyLog}[2, (I*(\sqrt{e} - \sqrt{c^2*d + e}))/(c*\sqrt{d}*E^{\text{ArcSech}[c*x]})] - (3*I)*b*\sqrt{d}*\text{PolyLog}[2, (I*(-\sqrt{e} + \sqrt{c^2*d + e}))/(c*\sqrt{d}*E^{\text{ArcSech}[c*x]})] - (3*I)*b*\sqrt{d}*\text{PolyLog}[2, ((-1)*(\sqrt{e} + \sqrt{c^2*d + e}))/(c*\sqrt{d}*E^{\text{ArcSech}[c*x]})] + (3*I)*b*\sqrt{d}*\text{PolyLog}[2, (I*(\sqrt{e} + \sqrt{c^2*d + e}))/(c*\sqrt{d}*E^{\text{ArcSech}[c*x]})]])/(4*e^(5/2))$

Maple [C] time = 17.249, size = 2016, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^2,x)

[Out] $a/e^2*x + 1/2*c^2*a/e^2*d*x/(c^2*e*x^2+c^2*d) - 3/2*a/e^2*d/(d*e)^(1/2)*\arctan(x*e/(d*e)^(1/2)) + c^2*b*x^3*\text{arcsech}(c*x)/(c^2*e*x^2+c^2*d)/e + 3/2*c^2*b*x*\text{arcsech}(c*x)/(c^2*e*x^2+c^2*d)/e^2*d - 1/2/c^2*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*\arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d/e^2 + 1/c^4*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*\arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^2/e^2*(e*(c^2*d+e))^(1/2) - 1/c^4*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*\arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^2/e - 1/2/c^2*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*\arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e^2/(c^2*d+e)/d*(e*(c^2*d+e))^(1/2) + 1/c^2*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*\arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e/(c^2*d+e)/d - 1/c^4*b*((c^2*d+2*(e*(c^2*d$

$$\begin{aligned}
& +e)^{(1/2)+2e} * d)^{(1/2)} * \arctan(c * d * (1/c/x + (-1+1/c/x)^{(1/2)} * (1+1/c/x)^{(1/2)} \\
&) / ((c^{2*d+2} * (e * (c^{2*d+e}))^{(1/2)+2e} * d)^{(1/2)}) / e / (c^{2*d+e} / d^{2 * (e * (c^{2*d+e} \\
&)^{(1/2)+1/c^4 * b * ((c^{2*d+2} * (e * (c^{2*d+e}))^{(1/2)+2e} * d)^{(1/2)} * \arctan(c * d * (1/c \\
& / x + (-1+1/c/x)^{(1/2)} * (1+1/c/x)^{(1/2)})) / ((c^{2*d+2} * (e * (c^{2*d+e}))^{(1/2)+2e} * d)^{(1/2)}) / (c^{2*d+e} / d^{2-1/2/c^2 * b * (-c^{2*d-2} * (e * (c^{2*d+e}))^{(1/2)+2e} * d)^{(1/2)} \\
& * \operatorname{arctanh}(c * d * (1/c/x + (-1+1/c/x)^{(1/2)} * (1+1/c/x)^{(1/2)})) / ((-c^{2*d+2} * (e * (c^{2*d+e}))^{(1/2)-2e} * d)^{(1/2)}) / d / e^{2-1/c^4 * b * (-c^{2*d-2} * (e * (c^{2*d+e}))^{(1/2)+2e} * d)^{(1/2)} * \operatorname{arctanh}(c * d * (1/c/x + (-1+1/c/x)^{(1/2)} * (1+1/c/x)^{(1/2)})) / ((-c^{2*d+2} * (e * (c^{2*d+e}))^{(1/2)-2e} * d)^{(1/2)}) / d^2 / e^{2 * (e * (c^{2*d+e}))^{(1/2)-1/c^4 * b * (-c^{2*d-2} * (e * (c^{2*d+e}))^{(1/2)+2e} * d)^{(1/2)} * \operatorname{arctanh}(c * d * (1/c/x + (-1+1/c/x)^{(1/2)} * (1+1/c/x)^{(1/2)})) / ((-c^{2*d+2} * (e * (c^{2*d+e}))^{(1/2)-2e} * d)^{(1/2)}) / d^2 / e^{1/2/c^2 * b * (-c^{2*d-2} * (e * (c^{2*d+e}))^{(1/2)+2e} * d)^{(1/2)} * \operatorname{arctanh}(c * d * (1/c/x + (-1+1/c/x)^{(1/2)} * (1+1/c/x)^{(1/2)})) / ((-c^{2*d+2} * (e * (c^{2*d+e}))^{(1/2)-2e} * d)^{(1/2)}) / e^{2/(c^{2*d+e} / d * (e * (c^{2*d+e}))^{(1/2)+1/c^2 * b * (-c^{2*d-2} * (e * (c^{2*d+e}))^{(1/2)+2e} * d)^{(1/2)} * \operatorname{arctanh}(c * d * (1/c/x + (-1+1/c/x)^{(1/2)} * (1+1/c/x)^{(1/2)})) / ((-c^{2*d+2} * (e * (c^{2*d+e}))^{(1/2)-2e} * d)^{(1/2)}) / e / (c^{2*d+e} / d + 1/c^4 * b * (-c^{2*d-2} * (e * (c^{2*d+e}))^{(1/2)+2e} * d)^{(1/2)} * \operatorname{arctanh}(c * d * (1/c/x + (-1+1/c/x)^{(1/2)} * (1+1/c/x)^{(1/2)})) / ((-c^{2*d+2} * (e * (c^{2*d+e}))^{(1/2)-2e} * d)^{(1/2)}) / e / (c^{2*d+e} / d^{2 * (e * (c^{2*d+e}))^{(1/2)+1/c^4 * b * (-c^{2*d-2} * (e * (c^{2*d+e}))^{(1/2)+2e} * d)^{(1/2)} * \operatorname{arctanh}(c * d * (1/c/x + (-1+1/c/x)^{(1/2)} * (1+1/c/x)^{(1/2)})) / ((-c^{2*d+2} * (e * (c^{2*d+e}))^{(1/2)-2e} * d)^{(1/2)}) / (c^{2*d+e} / d^{2-2/c * b / e^{2 * \arctan(1/c/x + (-1+1/c/x)^{(1/2)} * (1+1/c/x)^{(1/2)})) - 3/16 * c * b / e^{3*d} * \sum((_R1^{2*c^{2*d+c^{2*d+4*e}}) / _R1 / (_R1^{2*c^{2*d+c^{2*d+2*e}} * (\operatorname{arcsech}(c*x) * \ln((_R1-1/c/x - (-1+1/c/x)^{(1/2)} * (1+1/c/x)^{(1/2)})) / _R1) + \operatorname{dilog}((_R1-1/c/x - (-1+1/c/x)^{(1/2)} * (1+1/c/x)^{(1/2)})) / _R1)), _R1 = \operatorname{RootOf}(c^{2*d} * _Z^4 + (2*c^{2*d+4*e}) * _Z^2 + c^{2*d})) + 3/16 * c * b / e^{3*d} * \sum((_R1^{2*c^{2*d+4*e}} * _R1^{2*e+c^{2*d}}) / _R1 / (_R1^{2*c^{2*d+c^{2*d+2*e}} * (\operatorname{arcsech}(c*x) * \ln((_R1-1/c/x - (-1+1/c/x)^{(1/2)} * (1+1/c/x)^{(1/2)})) / _R1) + \operatorname{dilog}((_R1-1/c/x - (-1+1/c/x)^{(1/2)} * (1+1/c/x)^{(1/2)})) / _R1)), _R1 = \operatorname{RootOf}(c^{2*d} * _Z^4 + (2*c^{2*d+4*e}) * _Z^2 + c^{2*d}))
\end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{bx^4 \operatorname{ar} \operatorname{sech}(cx) + ax^4}{e^2x^4 + 2dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="fricas")

[Out] integral((b*x^4*arcsech(c*x) + a*x^4)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(a+b*asech(c*x))/(e*x**2+d)**2,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{ar} \operatorname{sech}(cx) + a)x^4}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)*x^4/(e*x^2 + d)^2, x)

$$3.120 \quad \int \frac{x^2 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^2} dx$$

Optimal. Leaf size=786

$$-\frac{b \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-de} \operatorname{sech}^{-1}(cx)}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-de}^{3/2}} + \frac{b \operatorname{PolyLog}\left(2, \frac{c\sqrt{-de} \operatorname{sech}^{-1}(cx)}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-de}^{3/2}} - \frac{b \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-de} \operatorname{sech}^{-1}(cx)}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4\sqrt{-de}^{3/2}} + \frac{b \operatorname{PolyLog}\left(2, \frac{c\sqrt{-de} \operatorname{sech}^{-1}(cx)}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4\sqrt{-de}^{3/2}}$$

```
[Out] (a + b*ArcSech[c*x])/(4*e*(Sqrt[-d]*Sqrt[e] - d/x)) - (a + b*ArcSech[c*x])/(4*e*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*ArcTan[(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(2*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*e) - (b*ArcTan[(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(2*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*e) + ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) - ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) + ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) - ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(4*Sqrt[-d]*e^(3/2)) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(4*Sqrt[-d]*e^(3/2)) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2))
```

Rubi [A] time = 1.57437, antiderivative size = 786, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {6303, 5707, 5802, 93, 205, 5800, 5562, 2190, 2279, 2391}

$$-\frac{b \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-de} \operatorname{sech}^{-1}(cx)}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-de}^{3/2}} + \frac{b \operatorname{PolyLog}\left(2, \frac{c\sqrt{-de} \operatorname{sech}^{-1}(cx)}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4\sqrt{-de}^{3/2}} - \frac{b \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-de} \operatorname{sech}^{-1}(cx)}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4\sqrt{-de}^{3/2}} + \frac{b \operatorname{PolyLog}\left(2, \frac{c\sqrt{-de} \operatorname{sech}^{-1}(cx)}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4\sqrt{-de}^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (a + b*ArcSech[c*x])/(4*e*(Sqrt[-d]*Sqrt[e] - d/x)) - (a + b*ArcSech[c*x])/(4*e*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*ArcTan[(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(2*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*e) - (b*ArcTan[(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(2*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*e) + ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) - ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) + ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) - ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(4*Sqrt[-d]*e^(3/2)) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(4*Sqrt[-d]*e^(3/2)) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2))
```

```

])))/(4*Sqrt[-d]*e^(3/2)) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2)) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(4*Sqrt[-d]*e^(3/2)) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*Sqrt[-d]*e^(3/2))

```

Rule 6303

```

Int[((a_.) + ArcSech[(c_.)*(x_)])*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := -Subst[Int[((e + d*x^2)^p*(a + b*ArcCosh[x/c])^n)/x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegersQ[m, p]

```

Rule 5707

```

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] && (p > 0 || IGtQ[n, 0])

```

Rule 5802

```

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^m), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x] - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n, 0] && NeQ[m, -1]

```

Rule 93

```

Int((((a_.) + (b_.)*(x_)^m)*((c_.) + (d_.)*(x_)^n))/((e_.) + (f_.)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]

```

Rule 205

```

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

```

Rule 5800

```

Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbol] := Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]

```

Rule 5562

```

Int((((e_.) + (f_.)*(x_)^m)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_.)*(x_)])*(b_.) + (a_.)), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)), x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)), x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]

```

Rule 2190

```

Int((((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_)^m))/((a_.) + (b_.)*(F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp

```

```

[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

```

Rule 2279

```

Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

```

Rule 2391

```

Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

```

Rubi steps

$$\begin{aligned}
\int \frac{x^2 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^2} dx &= -\operatorname{Subst} \left(\int \frac{a + b \cosh^{-1}\left(\frac{x}{c}\right)}{(e + dx^2)^2} dx, x, \frac{1}{x} \right) \\
&= -\operatorname{Subst} \left(\int \left(\frac{d(a + b \cosh^{-1}\left(\frac{x}{c}\right))}{4e(\sqrt{-d}\sqrt{e} - dx)^2} - \frac{d(a + b \cosh^{-1}\left(\frac{x}{c}\right))}{4e(\sqrt{-d}\sqrt{e} + dx)^2} - \frac{d(a + b \cosh^{-1}\left(\frac{x}{c}\right))}{2e(-de - d^2x^2)} \right) dx, x, \frac{1}{x} \right) \\
&= \frac{d \operatorname{Subst} \left(\int \frac{a + b \cosh^{-1}\left(\frac{x}{c}\right)}{(\sqrt{-d}\sqrt{e} - dx)^2} dx, x, \frac{1}{x} \right)}{4e} + \frac{d \operatorname{Subst} \left(\int \frac{a + b \cosh^{-1}\left(\frac{x}{c}\right)}{(\sqrt{-d}\sqrt{e} + dx)^2} dx, x, \frac{1}{x} \right)}{4e} + \frac{d \operatorname{Subst} \left(\int \frac{a + b \cosh^{-1}\left(\frac{x}{c}\right)}{-de - d^2x^2} dx, x, \frac{1}{x} \right)}{2e} \\
&= \frac{a + b \operatorname{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e} - \frac{d}{x}\right)} - \frac{a + b \operatorname{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e} + \frac{d}{x}\right)} - \frac{b \operatorname{Subst} \left(\int \frac{1}{\sqrt{-1 + \frac{x}{c}} \sqrt{1 + \frac{x}{c}} (\sqrt{-d}\sqrt{e} - dx)} dx, x, \frac{1}{x} \right)}{4ce} + \frac{b \operatorname{Subst} \left(\int \frac{1}{\sqrt{-1 + \frac{x}{c}} \sqrt{1 + \frac{x}{c}} (\sqrt{-d}\sqrt{e} + dx)} dx, x, \frac{1}{x} \right)}{4ce} \\
&= \frac{a + b \operatorname{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e} - \frac{d}{x}\right)} - \frac{a + b \operatorname{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e} + \frac{d}{x}\right)} - \frac{\operatorname{Subst} \left(\int \frac{a + b \cosh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e} - \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{4e^{3/2}} - \frac{\operatorname{Subst} \left(\int \frac{a + b \cosh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e} + \sqrt{-d}x} dx, x, \frac{1}{x} \right)}{4e^{3/2}} \\
&= \frac{a + b \operatorname{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e} - \frac{d}{x}\right)} - \frac{a + b \operatorname{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e} + \frac{d}{x}\right)} - \frac{b \tan^{-1} \left(\frac{\sqrt{cd - \sqrt{-d}\sqrt{e}} \sqrt{1 + \frac{1}{cx}}}{\sqrt{cd + \sqrt{-d}\sqrt{e}} \sqrt{-1 + \frac{1}{cx}}} \right)}{2\sqrt{cd - \sqrt{-d}\sqrt{e}} \sqrt{cd + \sqrt{-d}\sqrt{e}}} - \frac{b \tan^{-1} \left(\frac{\sqrt{cd - \sqrt{-d}\sqrt{e}} \sqrt{1 + \frac{1}{cx}}}{\sqrt{cd + \sqrt{-d}\sqrt{e}} \sqrt{-1 + \frac{1}{cx}}} \right)}{2\sqrt{cd - \sqrt{-d}\sqrt{e}} \sqrt{cd + \sqrt{-d}\sqrt{e}}} \\
&= \frac{a + b \operatorname{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e} - \frac{d}{x}\right)} - \frac{a + b \operatorname{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e} + \frac{d}{x}\right)} - \frac{b \tan^{-1} \left(\frac{\sqrt{cd - \sqrt{-d}\sqrt{e}} \sqrt{1 + \frac{1}{cx}}}{\sqrt{cd + \sqrt{-d}\sqrt{e}} \sqrt{-1 + \frac{1}{cx}}} \right)}{2\sqrt{cd - \sqrt{-d}\sqrt{e}} \sqrt{cd + \sqrt{-d}\sqrt{e}}} - \frac{b \tan^{-1} \left(\frac{\sqrt{cd - \sqrt{-d}\sqrt{e}} \sqrt{1 + \frac{1}{cx}}}{\sqrt{cd + \sqrt{-d}\sqrt{e}} \sqrt{-1 + \frac{1}{cx}}} \right)}{2\sqrt{cd - \sqrt{-d}\sqrt{e}} \sqrt{cd + \sqrt{-d}\sqrt{e}}} \\
&= \frac{a + b \operatorname{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e} - \frac{d}{x}\right)} - \frac{a + b \operatorname{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e} + \frac{d}{x}\right)} - \frac{b \tan^{-1} \left(\frac{\sqrt{cd - \sqrt{-d}\sqrt{e}} \sqrt{1 + \frac{1}{cx}}}{\sqrt{cd + \sqrt{-d}\sqrt{e}} \sqrt{-1 + \frac{1}{cx}}} \right)}{2\sqrt{cd - \sqrt{-d}\sqrt{e}} \sqrt{cd + \sqrt{-d}\sqrt{e}}} - \frac{b \tan^{-1} \left(\frac{\sqrt{cd - \sqrt{-d}\sqrt{e}} \sqrt{1 + \frac{1}{cx}}}{\sqrt{cd + \sqrt{-d}\sqrt{e}} \sqrt{-1 + \frac{1}{cx}}} \right)}{2\sqrt{cd - \sqrt{-d}\sqrt{e}} \sqrt{cd + \sqrt{-d}\sqrt{e}}} \\
&= \frac{a + b \operatorname{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e} - \frac{d}{x}\right)} - \frac{a + b \operatorname{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e} + \frac{d}{x}\right)} - \frac{b \tan^{-1} \left(\frac{\sqrt{cd - \sqrt{-d}\sqrt{e}} \sqrt{1 + \frac{1}{cx}}}{\sqrt{cd + \sqrt{-d}\sqrt{e}} \sqrt{-1 + \frac{1}{cx}}} \right)}{2\sqrt{cd - \sqrt{-d}\sqrt{e}} \sqrt{cd + \sqrt{-d}\sqrt{e}}} - \frac{b \tan^{-1} \left(\frac{\sqrt{cd - \sqrt{-d}\sqrt{e}} \sqrt{1 + \frac{1}{cx}}}{\sqrt{cd + \sqrt{-d}\sqrt{e}} \sqrt{-1 + \frac{1}{cx}}} \right)}{2\sqrt{cd - \sqrt{-d}\sqrt{e}} \sqrt{cd + \sqrt{-d}\sqrt{e}}} \\
&= \frac{a + b \operatorname{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e} - \frac{d}{x}\right)} - \frac{a + b \operatorname{sech}^{-1}(cx)}{4e\left(\sqrt{-d}\sqrt{e} + \frac{d}{x}\right)} - \frac{b \tan^{-1} \left(\frac{\sqrt{cd - \sqrt{-d}\sqrt{e}} \sqrt{1 + \frac{1}{cx}}}{\sqrt{cd + \sqrt{-d}\sqrt{e}} \sqrt{-1 + \frac{1}{cx}}} \right)}{2\sqrt{cd - \sqrt{-d}\sqrt{e}} \sqrt{cd + \sqrt{-d}\sqrt{e}}} - \frac{b \tan^{-1} \left(\frac{\sqrt{cd - \sqrt{-d}\sqrt{e}} \sqrt{1 + \frac{1}{cx}}}{\sqrt{cd + \sqrt{-d}\sqrt{e}} \sqrt{-1 + \frac{1}{cx}}} \right)}{2\sqrt{cd - \sqrt{-d}\sqrt{e}} \sqrt{cd + \sqrt{-d}\sqrt{e}}}
\end{aligned}$$

Mathematica [C] time = 1.57029, size = 1226, normalized size = 1.56

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]

[Out] ((-2*a*Sqrt[e]*x)/(d + e*x^2) + (b*ArcSech[c*x])/(I*Sqrt[d] - Sqrt[e]*x) - (b*ArcSech[c*x])/(I*Sqrt[d] + Sqrt[e]*x) + (2*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] - (4*b*ArcSin[Sqrt[1 - (I*Sqrt[e])/(c*Sqrt[d])]]/Sqrt[2])*ArcTanh[

$$\begin{aligned} &(((-I) * c * \sqrt{d} + \sqrt{e}) * \operatorname{Tanh}[\operatorname{ArcSech}[c * x] / 2]) / \sqrt{c^2 * d + e} / \sqrt{d} \\ &+ (4 * b * \operatorname{ArcSin}[\sqrt{1 + (I * \sqrt{e}) / (c * \sqrt{d})}] / \sqrt{2}] * \operatorname{ArcTanh}[(I * c * \sqrt{d} + \sqrt{e}) * \operatorname{Tanh}[\operatorname{ArcSech}[c * x] / 2]) / \sqrt{c^2 * d + e}]) / \sqrt{d} - (I * b * \operatorname{ArcSech}[c * x] * \operatorname{Log}[1 + (I * (\sqrt{e} - \sqrt{c^2 * d + e})) / (c * \sqrt{d} * E^{\operatorname{ArcSech}[c * x]})]) / \sqrt{d} - (2 * b * \operatorname{ArcSin}[\sqrt{1 + (I * \sqrt{e}) / (c * \sqrt{d})}] / \sqrt{2}] * \operatorname{Log}[1 + (I * (\sqrt{e} - \sqrt{c^2 * d + e})) / (c * \sqrt{d} * E^{\operatorname{ArcSech}[c * x]})]) / \sqrt{d} + (I * b * \operatorname{ArcSech}[c * x] * \operatorname{Log}[1 + (I * (-\sqrt{e} + \sqrt{c^2 * d + e})) / (c * \sqrt{d} * E^{\operatorname{ArcSech}[c * x]})]) / \sqrt{d} + (2 * b * \operatorname{ArcSin}[\sqrt{1 - (I * \sqrt{e}) / (c * \sqrt{d})}] / \sqrt{2}] * \operatorname{Log}[1 + (I * (-\sqrt{e} + \sqrt{c^2 * d + e})) / (c * \sqrt{d} * E^{\operatorname{ArcSech}[c * x]})]) / \sqrt{d} + (I * b * \operatorname{ArcSech}[c * x] * \operatorname{Log}[1 - (I * (\sqrt{e} + \sqrt{c^2 * d + e})) / (c * \sqrt{d} * E^{\operatorname{ArcSech}[c * x]})]) / \sqrt{d} - (2 * b * \operatorname{ArcSin}[\sqrt{1 - (I * \sqrt{e}) / (c * \sqrt{d})}] / \sqrt{2}] * \operatorname{Log}[1 - (I * (\sqrt{e} + \sqrt{c^2 * d + e})) / (c * \sqrt{d} * E^{\operatorname{ArcSech}[c * x]})]) / \sqrt{d} - (I * b * \operatorname{ArcSech}[c * x] * \operatorname{Log}[1 + (I * (\sqrt{e} + \sqrt{c^2 * d + e})) / (c * \sqrt{d} * E^{\operatorname{ArcSech}[c * x]})]) / \sqrt{d} + (2 * b * \operatorname{ArcSin}[\sqrt{1 + (I * \sqrt{e}) / (c * \sqrt{d})}] / \sqrt{2}] * \operatorname{Log}[1 + (I * (\sqrt{e} + \sqrt{c^2 * d + e})) / (c * \sqrt{d} * E^{\operatorname{ArcSech}[c * x]})]) / \sqrt{d} + (I * b * \sqrt{e} * \operatorname{Log}[(2 * I) * \sqrt{e} * (\sqrt{d} * \sqrt{(1 - c * x) / (1 + c * x)}) * (1 + c * x) + (\sqrt{d} * \sqrt{e} + I * c^2 * d * x) / \sqrt{c^2 * d + e}]) / (I * \sqrt{d} + \sqrt{e} * x)) / (\sqrt{d} * \sqrt{c^2 * d + e}) - (I * b * \sqrt{e} * \operatorname{Log}[(2 * \sqrt{e} * (I * \sqrt{d} * \sqrt{(1 - c * x) / (1 + c * x)}) * (1 + c * x) + (I * \sqrt{d} * \sqrt{e} + c^2 * d * x) / \sqrt{c^2 * d + e}]) / ((-I) * \sqrt{d} + \sqrt{e} * x)) / (\sqrt{d} * \sqrt{c^2 * d + e}) - (I * b * \operatorname{PolyLog}[2, (I * (\sqrt{e} - \sqrt{c^2 * d + e})) / (c * \sqrt{d} * E^{\operatorname{ArcSech}[c * x]})]) / \sqrt{d} + (I * b * \operatorname{PolyLog}[2, (I * (-\sqrt{e} + \sqrt{c^2 * d + e})) / (c * \sqrt{d} * E^{\operatorname{ArcSech}[c * x]})]) / \sqrt{d} + (I * b * \operatorname{PolyLog}[2, ((-I) * (\sqrt{e} + \sqrt{c^2 * d + e})) / (c * \sqrt{d} * E^{\operatorname{ArcSech}[c * x]})]) / \sqrt{d} - (I * b * \operatorname{PolyLog}[2, (I * (\sqrt{e} + \sqrt{c^2 * d + e})) / (c * \sqrt{d} * E^{\operatorname{ArcSech}[c * x]})]) / \sqrt{d}) / (4 * e^{(3/2)}) \end{aligned}$$

Maple [C] time = 2.401, size = 1880, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (x^2 * (a + b * \operatorname{arcsech}(c * x)) / (e * x^2 + d)^2, x)$

[Out]
$$\begin{aligned} &-1/2 * c^2 * a / e * x / (c^2 * e * x^2 + c^2 * d) + 1/2 * a / e / (d * e)^{(1/2)} * \arctan(x * e / (d * e)^{(1/2)}) \\ &- 1/2 * c^2 * b * \operatorname{arcsech}(c * x) / e * x / (c^2 * e * x^2 + c^2 * d) - 1/4 * c * b / e * \operatorname{sum}(_R1 / (_R1^2 * c^2 * d + c^2 * d + 2 * e) * (\operatorname{arcsech}(c * x) * \ln((_R1 - 1/c/x - (-1 + 1/c/x)^{(1/2)} * (1 + 1/c/x)^{(1/2)}) / _R1) + \operatorname{dilog}((_R1 - 1/c/x - (-1 + 1/c/x)^{(1/2)} * (1 + 1/c/x)^{(1/2)}) / _R1)), _R1 = \operatorname{RootOf}(c^2 * d * _Z^4 + (2 * c^2 * d + 4 * e) * _Z^2 + c^2 * d) + 1/2 * c^2 * b * ((c^2 * d + 2 * (e * (c^2 * d + e))^{(1/2)} + 2 * e) * d)^{(1/2)} * \arctan(c * d * (1/c/x + (-1 + 1/c/x)^{(1/2)} * (1 + 1/c/x)^{(1/2)}) / ((c^2 * d + 2 * (e * (c^2 * d + e))^{(1/2)} + 2 * e) * d)^{(1/2)}) / d^2 / e - 1/c^4 * b * ((c^2 * d + 2 * (e * (c^2 * d + e))^{(1/2)} + 2 * e) * d)^{(1/2)} * \arctan(c * d * (1/c/x + (-1 + 1/c/x)^{(1/2)} * (1 + 1/c/x)^{(1/2)}) / ((c^2 * d + 2 * (e * (c^2 * d + e))^{(1/2)} + 2 * e) * d)^{(1/2)}) / e / d^3 * (e * (c^2 * d + e))^{(1/2)} + 1/c^4 * b * ((c^2 * d + 2 * (e * (c^2 * d + e))^{(1/2)} + 2 * e) * d)^{(1/2)} * \arctan(c * d * (1/c/x + (-1 + 1/c/x)^{(1/2)} * (1 + 1/c/x)^{(1/2)}) / ((c^2 * d + 2 * (e * (c^2 * d + e))^{(1/2)} + 2 * e) * d)^{(1/2)}) / d^3 + 1/2 / c^2 * b * ((c^2 * d + 2 * (e * (c^2 * d + e))^{(1/2)} + 2 * e) * d)^{(1/2)} * \arctan(c * d * (1/c/x + (-1 + 1/c/x)^{(1/2)} * (1 + 1/c/x)^{(1/2)}) / ((c^2 * d + 2 * (e * (c^2 * d + e))^{(1/2)} + 2 * e) * d)^{(1/2)}) / e / (c^2 * d + e) / d^2 * (e * (c^2 * d + e))^{(1/2)} - 1/c^2 * b * ((c^2 * d + 2 * (e * (c^2 * d + e))^{(1/2)} + 2 * e) * d)^{(1/2)} * \arctan(c * d * (1/c/x + (-1 + 1/c/x)^{(1/2)} * (1 + 1/c/x)^{(1/2)}) / ((c^2 * d + 2 * (e * (c^2 * d + e))^{(1/2)} + 2 * e) * d)^{(1/2)}) / (c^2 * d + e) / d^2 + 1/c^4 * b * ((c^2 * d + 2 * (e * (c^2 * d + e))^{(1/2)} + 2 * e) * d)^{(1/2)} * \arctan(c * d * (1/c/x + (-1 + 1/c/x)^{(1/2)} * (1 + 1/c/x)^{(1/2)}) / ((c^2 * d + 2 * (e * (c^2 * d + e))^{(1/2)} + 2 * e) * d)^{(1/2)}) / (c^2 * d + e) / d^3 * (e * (c^2 * d + e))^{(1/2)} - 1/c^4 * b * ((c^2 * d + 2 * (e * (c^2 * d + e))^{(1/2)} + 2 * e) * d)^{(1/2)} * \arctan(c * d * (1/c/x + (-1 + 1/c/x)^{(1/2)} * (1 + 1/c/x)^{(1/2)}) / ((c^2 * d + 2 * (e * (c^2 * d + e))^{(1/2)} + 2 * e) * d)^{(1/2)}) * e / (c^2 * d + e) / d^3 + 1/2 / c^2 * b * (-c^2 * d - 2 * (e * (c^2 * d + e))^{(1/2)} + 2 * e) * d)^{(1/2)} * \operatorname{arctanh}(c * d * (1/c/x + (-1 + 1/c/x)^{(1/2)} * (1 + 1/c/x)^{(1/2)}) / ((-c^2 * d + 2 * (e * (c^2 * d + e))^{(1/2)} - 2 * e) * d)^{(1/2)}) / d^2 / e + 1/c^4 * b * (-c^2 * d - 2 * (e * (c^2 * d + e))^{(1/2)} + 2 * e) * d \end{aligned}$$

$$\begin{aligned} &)^{(1/2)} \operatorname{arctanh}(c*d*(1/c/x+(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)})) / ((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e)*d)^{(1/2)} / e/d^3*(e*(c^2*d+e))^{(1/2)}+1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)} \operatorname{arctanh}(c*d*(1/c/x+(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)})) / ((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e)*d)^{(1/2)} / d^3-1/2/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)} \operatorname{arctanh}(c*d*(1/c/x+(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)})) / ((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e)*d)^{(1/2)} / e/(c^2*d+e)/d^2*(e*(c^2*d+e))^{(1/2)}-1/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)} \operatorname{arctanh}(c*d*(1/c/x+(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)})) / ((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e)*d)^{(1/2)} / (c^2*d+e)/d^2-1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)} \operatorname{arctanh}(c*d*(1/c/x+(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)})) / ((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e)*d)^{(1/2)} / (c^2*d+e)/d^3*(e*(c^2*d+e))^{(1/2)}-1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)} \operatorname{arctanh}(c*d*(1/c/x+(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)})) / ((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e)*d)^{(1/2)} * e/(c^2*d+e)/d^3+1/4*c*b/e*sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)}))/_R1)+dilog((_R1-1/c/x-(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)}))/_R1)), _R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d)) \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{bx^2 \operatorname{arsech}(cx) + ax^2}{e^2x^4 + 2dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="fricas")

[Out] integral((b*x^2*arcsech(c*x) + a*x^2)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(a+b*asech(c*x))/(e*x**2+d)**2,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{ar} \operatorname{sech}(cx) + a)x^2}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x^2/(e*x^2 + d)^2, x)
```


$$3.121 \quad \int \frac{a+b\operatorname{sech}^{-1}(cx)}{(d+ex^2)^2} dx$$

Optimal. Leaf size=786

$$\frac{b\operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} - \frac{b\operatorname{PolyLog}\left(2, \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} + \frac{b\operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4(-d)^{3/2}\sqrt{e}} - \frac{b\operatorname{PolyLog}\left(2, \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

```
[Out] -(a + b*ArcSech[c*x])/(4*d*(Sqrt[-d]*Sqrt[e] - d/x)) + (a + b*ArcSech[c*x])
/(4*d*(Sqrt[-d]*Sqrt[e] + d/x)) + (b*ArcTan[(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*S
qrt[1 + 1/(c*x)])/(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(2*d*
Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]) + (b*ArcTan[(Sqr
t[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*
Sqrt[-1 + 1/(c*x)])])/(2*d*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]
*Sqrt[e]]) - ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqr
t[e] - Sqrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e]) + ((a + b*ArcSech[c*x])*Lo
g[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*(-d)^(3/
2)*Sqrt[e]) - ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqr
t[e] + Sqrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e]) + ((a + b*ArcSech[c*x])*L
og[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*(-d)^(3
/2)*Sqrt[e]) + (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[
c^2*d + e]))]/(4*(-d)^(3/2)*Sqrt[e]) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech
[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e]) + (b*PolyLog[2,
-((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))]/(4*(-d)^(3/2)
*Sqrt[e]) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d
+ e])])/(4*(-d)^(3/2)*Sqrt[e])
```

Rubi [A] time = 2.83591, antiderivative size = 786, normalized size of antiderivative = 1., number of steps used = 47, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {6293, 5792, 5707, 5802, 93, 205, 5800, 5562, 2190, 2279, 2391}

$$\frac{b\operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} - \frac{b\operatorname{PolyLog}\left(2, \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{4(-d)^{3/2}\sqrt{e}} + \frac{b\operatorname{PolyLog}\left(2, -\frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4(-d)^{3/2}\sqrt{e}} - \frac{b\operatorname{PolyLog}\left(2, \frac{c\sqrt{-d}e^{\operatorname{sech}^{-1}(cx)}}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])/(d + e*x^2)^2, x]
```

```
[Out] -(a + b*ArcSech[c*x])/(4*d*(Sqrt[-d]*Sqrt[e] - d/x)) + (a + b*ArcSech[c*x])
/(4*d*(Sqrt[-d]*Sqrt[e] + d/x)) + (b*ArcTan[(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*S
qrt[1 + 1/(c*x)])/(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(2*d*
Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]) + (b*ArcTan[(Sqr
t[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*
Sqrt[-1 + 1/(c*x)])])/(2*d*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]
*Sqrt[e]]) - ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqr
t[e] - Sqrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e]) + ((a + b*ArcSech[c*x])*Lo
g[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*(-d)^(3/
2)*Sqrt[e]) - ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqr
t[e] + Sqrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e]) + ((a + b*ArcSech[c*x])*L
og[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*(-d)^(3
/2)*Sqrt[e]) + (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[
c^2*d + e]))]/(4*(-d)^(3/2)*Sqrt[e]) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech
[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*(-d)^(3/2)*Sqrt[e]) + (b*PolyLog[2,
-((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))]/(4*(-d)^(3/2)
*Sqrt[e]) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d
+ e])])/(4*(-d)^(3/2)*Sqrt[e])
```

$$\frac{[c*x]}{(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])}] / (4*(-d)^{(3/2)}*\text{Sqrt}[e]) + (b*\text{PolyLog}[2, -((c*\text{Sqrt}[-d]*E^{\text{ArcSech}[c*x]})/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])))] / (4*(-d)^{(3/2)}*\text{Sqrt}[e]) - (b*\text{PolyLog}[2, (c*\text{Sqrt}[-d]*E^{\text{ArcSech}[c*x]})/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])]) / (4*(-d)^{(3/2)}*\text{Sqrt}[e])$$
Rule 6293

$$\text{Int}[(a_.) + \text{ArcSech}[c_.*x_]*b_.)^{n_.*((d_.) + (e_.*x_)^2)^{p_.)}, x_Symbol] \rightarrow -\text{Subst}[\text{Int}[(e + d*x^2)^p*(a + b*\text{ArcCosh}[x/c])^n/x^{2*(p+1)}], x, 1/x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IntegerQ}[p]$$
Rule 5792

$$\text{Int}[(a_.) + \text{ArcCosh}[c_.*x_]*b_.)^{n_.*((f_.*x_))^{m_.*((d_.) + (e_.*x_)^2)^{p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*\text{ArcCosh}[c*x])^n, (f*x)^m*(d + e*x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{NeQ}[c^2*d + e, 0] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IntegerQ}[p] \ \&\& \ \text{IntegerQ}[m]$$
Rule 5707

$$\text{Int}[(a_.) + \text{ArcCosh}[c_.*x_]*b_.)^{n_.*((d_.) + (e_.*x_)^2)^{p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*\text{ArcCosh}[c*x])^n, (d + e*x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x \ \&\& \ \text{NeQ}[c^2*d + e, 0] \ \&\& \ \text{IntegerQ}[p] \ \&\& \ (p > 0 \ || \ \text{IGtQ}[n, 0])$$
Rule 5802

$$\text{Int}[(a_.) + \text{ArcCosh}[c_.*x_]*b_.)^{n_.*((d_.) + (e_.*x_))^{m_.)}, x_Symbol] \rightarrow \text{Simp}[(d + e*x)^{m+1}*(a + b*\text{ArcCosh}[c*x])^n/(e*(m+1)), x] - \text{Dist}[(b*c*n)/(e*(m+1)), \text{Int}[(d + e*x)^{m+1}*(a + b*\text{ArcCosh}[c*x])^{n-1})/(\text{Sqrt}[-1 + c*x]*\text{Sqrt}[1 + c*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, m\}, x \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{NeQ}[m, -1]$$
Rule 93

$$\text{Int}[(a_.) + (b_.*x_)^{m_.*((c_.) + (d_.*x_))^{n_.)}]/((e_.) + (f_.*x_)), x_Symbol] \rightarrow \text{With}[q = \text{Denominator}[m], \text{Dist}[q, \text{Subst}[\text{Int}[x^{q*(m+1)} - 1]/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^{(1/q)}/(c + d*x)^{(1/q)}], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \ \&\& \ \text{EqQ}[m + n + 1, 0] \ \&\& \ \text{RationalQ}[n] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{SimplerQ}[a + b*x, c + d*x]$$
Rule 205

$$\text{Int}[(a_.) + (b_.*x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}[a/b]$$
Rule 5800

$$\text{Int}[(a_.) + \text{ArcCosh}[c_.*x_]*b_.)^{n_./((d_.) + (e_.*x_)), x_Symbol] \rightarrow \text{Subst}[\text{Int}[(a + b*x)^n*\text{Sinh}[x]/(c*d + e*\text{Cosh}[x]), x], x, \text{ArcCosh}[c*x]] /; \text{FreeQ}\{a, b, c, d, e\}, x \ \&\& \ \text{IGtQ}[n, 0]$$
Rule 5562

$$\text{Int}[(e_.) + (f_.*x_))^{m_.*\text{Sinh}[(c_.) + (d_.*x_)]}/(\text{Cosh}[(c_.) + (d_.*x_)]*b_.) + (a_)), x_Symbol] \rightarrow -\text{Simp}[(e + f*x)^{m+1}/(b*f*(m+1)), x] + (\text{Int}[(e + f*x)^m*E^{(c + d*x)}/(a - \text{Rt}[a^2 - b^2, 2] + b*E^{(c + d*x)}), x] + \text{Int}[(e + f*x)^m*E^{(c + d*x)}/(a + \text{Rt}[a^2 - b^2, 2] + b*E^{(c + d*x)}), x])$$

, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]

Rule 2190

Int[(((F_)^((g_)*(e_) + (f_)*(x_)))^((n_)*((c_) + (d_)*(x_))^(m_)))/((a_) + (b_)*((F_)^((g_)*(e_) + (f_)*(x_)))^((n_))), x_Symbol] :> Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_)))^((n_)))]], x_Symbol] :> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] :> -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rubi steps

$$\begin{aligned}
\int \frac{a + b \operatorname{sech}^{-1}(cx)}{(d + ex^2)^2} dx &= -\operatorname{Subst} \left(\int \frac{x^2 \left(a + b \cosh^{-1} \left(\frac{x}{c} \right) \right)}{(e + dx^2)^2} dx, x, \frac{1}{x} \right) \\
&= -\operatorname{Subst} \left(\int \left(-\frac{e \left(a + b \cosh^{-1} \left(\frac{x}{c} \right) \right)}{d(e + dx^2)^2} + \frac{a + b \cosh^{-1} \left(\frac{x}{c} \right)}{d(e + dx^2)} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\operatorname{Subst} \left(\int \frac{a + b \cosh^{-1} \left(\frac{x}{c} \right)}{e + dx^2} dx, x, \frac{1}{x} \right)}{d} + \frac{e \operatorname{Subst} \left(\int \frac{a + b \cosh^{-1} \left(\frac{x}{c} \right)}{(e + dx^2)^2} dx, x, \frac{1}{x} \right)}{d} \\
&= -\frac{\operatorname{Subst} \left(\int \left(\frac{a + b \cosh^{-1} \left(\frac{x}{c} \right)}{2\sqrt{e}(\sqrt{e} - \sqrt{-dx})} + \frac{a + b \cosh^{-1} \left(\frac{x}{c} \right)}{2\sqrt{e}(\sqrt{e} + \sqrt{-dx})} \right) dx, x, \frac{1}{x} \right)}{d} + \frac{e \operatorname{Subst} \left(\int \left(-\frac{d(a + b \cosh^{-1} \left(\frac{x}{c} \right))}{4e(\sqrt{-d}\sqrt{e} - dx)^2} - \frac{d(a + b \cosh^{-1} \left(\frac{x}{c} \right))}{4e(\sqrt{-d}\sqrt{e} + dx)^2} \right) dx, x, \frac{1}{x} \right)}{d} \\
&= -\left(\frac{1}{4} \operatorname{Subst} \left(\int \frac{a + b \cosh^{-1} \left(\frac{x}{c} \right)}{(\sqrt{-d}\sqrt{e} - dx)^2} dx, x, \frac{1}{x} \right) \right) - \frac{1}{4} \operatorname{Subst} \left(\int \frac{a + b \cosh^{-1} \left(\frac{x}{c} \right)}{(\sqrt{-d}\sqrt{e} + dx)^2} dx, x, \frac{1}{x} \right) - \frac{1}{2} \operatorname{Subst} \left(\int \frac{a + b \cosh^{-1} \left(\frac{x}{c} \right)}{(\sqrt{-d}\sqrt{e} - dx)(\sqrt{-d}\sqrt{e} + dx)} dx, x, \frac{1}{x} \right) \\
&= -\frac{a + b \operatorname{sech}^{-1}(cx)}{4d(\sqrt{-d}\sqrt{e} - \frac{d}{x})} + \frac{a + b \operatorname{sech}^{-1}(cx)}{4d(\sqrt{-d}\sqrt{e} + \frac{d}{x})} - \frac{1}{2} \operatorname{Subst} \left(\int \left(-\frac{a + b \cosh^{-1} \left(\frac{x}{c} \right)}{2d\sqrt{e}(\sqrt{e} - \sqrt{-dx})} - \frac{a + b \cosh^{-1} \left(\frac{x}{c} \right)}{2d\sqrt{e}(\sqrt{e} + \sqrt{-dx})} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{a + b \operatorname{sech}^{-1}(cx)}{4d(\sqrt{-d}\sqrt{e} - \frac{d}{x})} + \frac{a + b \operatorname{sech}^{-1}(cx)}{4d(\sqrt{-d}\sqrt{e} + \frac{d}{x})} + \frac{b \operatorname{Subst} \left(\int \frac{1}{d + \frac{\sqrt{-d}\sqrt{e}}{c} - \left(-d + \frac{\sqrt{-d}\sqrt{e}}{c} \right) x^2} dx, x, \frac{\sqrt{1 + \frac{1}{cx}}}{\sqrt{-1 + \frac{1}{cx}}} \right)}{2cd} - \frac{b \operatorname{Subst} \left(\int \frac{1}{d + \frac{\sqrt{-d}\sqrt{e}}{c} + \left(-d + \frac{\sqrt{-d}\sqrt{e}}{c} \right) x^2} dx, x, \frac{\sqrt{1 + \frac{1}{cx}}}{\sqrt{-1 + \frac{1}{cx}}} \right)}{2cd} \\
&= -\frac{a + b \operatorname{sech}^{-1}(cx)}{4d(\sqrt{-d}\sqrt{e} - \frac{d}{x})} + \frac{a + b \operatorname{sech}^{-1}(cx)}{4d(\sqrt{-d}\sqrt{e} + \frac{d}{x})} + \frac{b \tan^{-1} \left(\frac{\sqrt{cd - \sqrt{-d}\sqrt{e}} \sqrt{1 + \frac{1}{cx}}}{\sqrt{cd + \sqrt{-d}\sqrt{e}} \sqrt{-1 + \frac{1}{cx}}} \right)}{2d\sqrt{cd - \sqrt{-d}\sqrt{e}} \sqrt{cd + \sqrt{-d}\sqrt{e}}} + \frac{b \tan^{-1} \left(\frac{\sqrt{cd + \sqrt{-d}\sqrt{e}} \sqrt{1 + \frac{1}{cx}}}{\sqrt{cd - \sqrt{-d}\sqrt{e}} \sqrt{-1 + \frac{1}{cx}}} \right)}{2d\sqrt{cd - \sqrt{-d}\sqrt{e}} \sqrt{cd + \sqrt{-d}\sqrt{e}}} \\
&= -\frac{a + b \operatorname{sech}^{-1}(cx)}{4d(\sqrt{-d}\sqrt{e} - \frac{d}{x})} + \frac{a + b \operatorname{sech}^{-1}(cx)}{4d(\sqrt{-d}\sqrt{e} + \frac{d}{x})} + \frac{b \tan^{-1} \left(\frac{\sqrt{cd - \sqrt{-d}\sqrt{e}} \sqrt{1 + \frac{1}{cx}}}{\sqrt{cd + \sqrt{-d}\sqrt{e}} \sqrt{-1 + \frac{1}{cx}}} \right)}{2d\sqrt{cd - \sqrt{-d}\sqrt{e}} \sqrt{cd + \sqrt{-d}\sqrt{e}}} + \frac{b \tan^{-1} \left(\frac{\sqrt{cd + \sqrt{-d}\sqrt{e}} \sqrt{1 + \frac{1}{cx}}}{\sqrt{cd - \sqrt{-d}\sqrt{e}} \sqrt{-1 + \frac{1}{cx}}} \right)}{2d\sqrt{cd - \sqrt{-d}\sqrt{e}} \sqrt{cd + \sqrt{-d}\sqrt{e}}} \\
&= -\frac{a + b \operatorname{sech}^{-1}(cx)}{4d(\sqrt{-d}\sqrt{e} - \frac{d}{x})} + \frac{a + b \operatorname{sech}^{-1}(cx)}{4d(\sqrt{-d}\sqrt{e} + \frac{d}{x})} + \frac{b \tan^{-1} \left(\frac{\sqrt{cd - \sqrt{-d}\sqrt{e}} \sqrt{1 + \frac{1}{cx}}}{\sqrt{cd + \sqrt{-d}\sqrt{e}} \sqrt{-1 + \frac{1}{cx}}} \right)}{2d\sqrt{cd - \sqrt{-d}\sqrt{e}} \sqrt{cd + \sqrt{-d}\sqrt{e}}} + \frac{b \tan^{-1} \left(\frac{\sqrt{cd + \sqrt{-d}\sqrt{e}} \sqrt{1 + \frac{1}{cx}}}{\sqrt{cd - \sqrt{-d}\sqrt{e}} \sqrt{-1 + \frac{1}{cx}}} \right)}{2d\sqrt{cd - \sqrt{-d}\sqrt{e}} \sqrt{cd + \sqrt{-d}\sqrt{e}}} \\
&= -\frac{a + b \operatorname{sech}^{-1}(cx)}{4d(\sqrt{-d}\sqrt{e} - \frac{d}{x})} + \frac{a + b \operatorname{sech}^{-1}(cx)}{4d(\sqrt{-d}\sqrt{e} + \frac{d}{x})} + \frac{b \tan^{-1} \left(\frac{\sqrt{cd - \sqrt{-d}\sqrt{e}} \sqrt{1 + \frac{1}{cx}}}{\sqrt{cd + \sqrt{-d}\sqrt{e}} \sqrt{-1 + \frac{1}{cx}}} \right)}{2d\sqrt{cd - \sqrt{-d}\sqrt{e}} \sqrt{cd + \sqrt{-d}\sqrt{e}}} + \frac{b \tan^{-1} \left(\frac{\sqrt{cd + \sqrt{-d}\sqrt{e}} \sqrt{1 + \frac{1}{cx}}}{\sqrt{cd - \sqrt{-d}\sqrt{e}} \sqrt{-1 + \frac{1}{cx}}} \right)}{2d\sqrt{cd - \sqrt{-d}\sqrt{e}} \sqrt{cd + \sqrt{-d}\sqrt{e}}} \\
&= -\frac{a + b \operatorname{sech}^{-1}(cx)}{4d(\sqrt{-d}\sqrt{e} - \frac{d}{x})} + \frac{a + b \operatorname{sech}^{-1}(cx)}{4d(\sqrt{-d}\sqrt{e} + \frac{d}{x})} + \frac{b \tan^{-1} \left(\frac{\sqrt{cd - \sqrt{-d}\sqrt{e}} \sqrt{1 + \frac{1}{cx}}}{\sqrt{cd + \sqrt{-d}\sqrt{e}} \sqrt{-1 + \frac{1}{cx}}} \right)}{2d\sqrt{cd - \sqrt{-d}\sqrt{e}} \sqrt{cd + \sqrt{-d}\sqrt{e}}} + \frac{b \tan^{-1} \left(\frac{\sqrt{cd + \sqrt{-d}\sqrt{e}} \sqrt{1 + \frac{1}{cx}}}{\sqrt{cd - \sqrt{-d}\sqrt{e}} \sqrt{-1 + \frac{1}{cx}}} \right)}{2d\sqrt{cd - \sqrt{-d}\sqrt{e}} \sqrt{cd + \sqrt{-d}\sqrt{e}}}
\end{aligned}$$

Mathematica [C] time = 1.54307, size = 1216, normalized size = 1.55

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*ArcSech[c*x])/(d + e*x^2)^2,x]

[Out]
$$\begin{aligned} & ((2*a*\sqrt{d}*x)/(d + e*x^2) + (b*\sqrt{d}*ArcSech[c*x])/((-1)*\sqrt{d}*\sqrt{e} + e*x) + (b*\sqrt{d}*ArcSech[c*x])/(1*\sqrt{d}*\sqrt{e} + e*x) + (2*a*ArcTan[(\sqrt{e}*x)/\sqrt{d}])/\sqrt{e} - (4*b*ArcSin[\sqrt{1 - (1*\sqrt{e})}/(c*\sqrt{d})])/\sqrt{2})*ArcTanh[(((1)*c*\sqrt{d} + \sqrt{e})*Tanh[ArcSech[c*x]/2])/\sqrt{c^2*d + e}])/\sqrt{e} + (4*b*ArcSin[\sqrt{1 + (1*\sqrt{e})}/(c*\sqrt{d})])/\sqrt{2})*ArcTanh[(((1)*c*\sqrt{d} + \sqrt{e})*Tanh[ArcSech[c*x]/2])/\sqrt{c^2*d + e}])/\sqrt{e} - (1*b*ArcSech[c*x]*Log[1 + (1*(\sqrt{e} - \sqrt{c^2*d + e}))]/(c*\sqrt{d}*E^{ArcSech[c*x]})))/\sqrt{e} - (2*b*ArcSin[\sqrt{1 + (1*\sqrt{e})}/(c*\sqrt{d})])/\sqrt{2})*Log[1 + (1*(\sqrt{e} - \sqrt{c^2*d + e}))]/(c*\sqrt{d}*E^{ArcSech[c*x]})]/\sqrt{e} + (1*b*ArcSech[c*x]*Log[1 + (1*(-\sqrt{e} + \sqrt{c^2*d + e}))]/(c*\sqrt{d}*E^{ArcSech[c*x]})))/\sqrt{e} + (2*b*ArcSin[\sqrt{1 - (1*\sqrt{e})}/(c*\sqrt{d})])/\sqrt{2})*Log[1 + (1*(-\sqrt{e} + \sqrt{c^2*d + e}))]/(c*\sqrt{d}*E^{ArcSech[c*x]})]/\sqrt{e} + (1*b*ArcSech[c*x]*Log[1 - (1*(\sqrt{e} + \sqrt{c^2*d + e}))]/(c*\sqrt{d}*E^{ArcSech[c*x]})])/\sqrt{e} - (2*b*ArcSin[\sqrt{1 - (1*\sqrt{e})}/(c*\sqrt{d})])/\sqrt{2})*Log[1 - (1*(\sqrt{e} + \sqrt{c^2*d + e}))]/(c*\sqrt{d}*E^{ArcSech[c*x]})]/\sqrt{e} - (1*b*ArcSech[c*x]*Log[1 + (1*(\sqrt{e} + \sqrt{c^2*d + e}))]/(c*\sqrt{d}*E^{ArcSech[c*x]})])/\sqrt{e} + (2*b*ArcSin[\sqrt{1 + (1*\sqrt{e})}/(c*\sqrt{d})])/\sqrt{2})*Log[1 + (1*(\sqrt{e} + \sqrt{c^2*d + e}))]/(c*\sqrt{d}*E^{ArcSech[c*x]})]/\sqrt{e} - (1*b*Log[((2*1)*\sqrt{e}*(\sqrt{d}*\sqrt{1 - c*x})/(1 + c*x))*(1 + c*x) + (\sqrt{d}*\sqrt{e} + 1*c^2*d*x)/\sqrt{c^2*d + e}])/(1*\sqrt{d} + \sqrt{e}*x))/\sqrt{c^2*d + e} + (1*b*Log[(2*\sqrt{e}*(1*\sqrt{d}*\sqrt{1 - c*x})/(1 + c*x))*(1 + c*x) + (1*\sqrt{d}*\sqrt{e} + c^2*d*x)/\sqrt{c^2*d + e}])/(1*\sqrt{d} + \sqrt{e}*x))/\sqrt{c^2*d + e} - (1*b*PolyLog[2, (1*(\sqrt{e} - \sqrt{c^2*d + e}))]/(c*\sqrt{d}*E^{ArcSech[c*x]})])/\sqrt{e} + (1*b*PolyLog[2, (1*(-\sqrt{e} + \sqrt{c^2*d + e}))]/(c*\sqrt{d}*E^{ArcSech[c*x]})])/\sqrt{e} + (1*b*PolyLog[2, ((1)*(\sqrt{e} + \sqrt{c^2*d + e}))]/(c*\sqrt{d}*E^{ArcSech[c*x]})])/\sqrt{e} - (1*b*PolyLog[2, (1*(\sqrt{e} + \sqrt{c^2*d + e}))]/(c*\sqrt{d}*E^{ArcSech[c*x]})])/\sqrt{e}]/(4*d^(3/2)) \end{aligned}$$

Maple [C] time = 1.619, size = 1870, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))/(e*x^2+d)^2,x)

[Out]
$$\begin{aligned} & 1/2*c^2*a*x/d/(c^2*e*x^2+c^2*d)+1/2*a/d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2)) \\ & +1/2*c^2*b*arcsech(c*x)*x/d/(c^2*e*x^2+c^2*d)-1/2/c^2*b*((c^2*d+2*(e*(c^2*d \\ & +e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2)) \\ &)/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^3+1/c^4*b*((c^2*d+2*(e*(c^2 \\ & *d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1 \\ & /2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^4*(e*(c^2*d+e))^(1/2)-1 \\ & /c^4*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(1/c/x+(-1+1/ \\ & c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^ \\ & 4*e-1/2/c^2*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(1/c/x \\ & +(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1 \\ & /2))/(c^2*d+e)/d^3*(e*(c^2*d+e))^(1/2)+1/c^2*b*((c^2*d+2*(e*(c^2*d+e))^(1/2 \\ &)+2*e)*d)^(1/2)*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x)^(1/2))/((c^2*d \\ & +2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))*e/(c^2*d+e)/d^3-1/c^4*b*((c^2*d+2*(e* \\ & (c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2)*(1+1/c/x) \\ & ^1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/d^4/(c^2*d+e)*(e*(c^2* \\ & d+e))^(1/2)*e+1/c^4*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*arctan(c* \end{aligned}$$

$$d*(1/c/x+(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)})/((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}/d^4/(c^2*d+e)*e^{-1/2}/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*\operatorname{arctanh}(c*d*(1/c/x+(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)})/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e)*d)^{(1/2)})/d^3-1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*\operatorname{arctanh}(c*d*(1/c/x+(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)})/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e)*d)^{(1/2)})/d^4*(e*(c^2*d+e))^{(1/2)}-1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*\operatorname{arctanh}(c*d*(1/c/x+(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)})/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e)*d)^{(1/2)})/d^4*e+1/2/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*\operatorname{arctanh}(c*d*(1/c/x+(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)})/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e)*d)^{(1/2)})/(c^2*d+e)/d^3*(e*(c^2*d+e))^{(1/2)}+1/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*\operatorname{arctanh}(c*d*(1/c/x+(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)})/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e)*d)^{(1/2)})*e/(c^2*d+e)/d^3+1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*\operatorname{arctanh}(c*d*(1/c/x+(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)})/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e)*d)^{(1/2)})/d^4/(c^2*d+e)*e*(c^2*d+e))^{(1/2)}*e+1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*\operatorname{arctanh}(c*d*(1/c/x+(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)})/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e)*d)^{(1/2)})/d^4/(c^2*d+e)*e^{-2-1/4*c*b/d*\sum(_R1/(_R1^2*c^2*d+c^2*d+2*e))*(\operatorname{arcsech}(c*x)*\ln((_R1-1/c/x-(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)})/_R1)+\operatorname{dilog}((_R1-1/c/x-(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)})/_R1)),_R1=\operatorname{RootOf}(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+1/4*c*b/d*\sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e))*(\operatorname{arcsech}(c*x)*\ln((_R1-1/c/x-(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)})/_R1)+\operatorname{dilog}((_R1-1/c/x-(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)})/_R1)),_R1=\operatorname{RootOf}(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b \operatorname{arsech}(cx) + a}{e^2 x^4 + 2 d e x^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="fricas")

[Out] integral((b*arcsech(c*x) + a)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asech(c*x))/(e*x**2+d)**2,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{ar} \operatorname{sech}(cx) + a}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)/(e*x^2 + d)^2, x)

$$3.122 \quad \int \frac{a + b \operatorname{sech}^{-1}(cx)}{x^2(d + ex^2)^2} dx$$

Optimal. Leaf size=844

$$-\frac{a}{d^2x} - \frac{b \operatorname{sech}^{-1}(cx)}{d^2x} + \frac{e(a + b \operatorname{sech}^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e} - \frac{d}{x}\right)} - \frac{e(a + b \operatorname{sech}^{-1}(cx))}{4d^2\left(\frac{d}{x} + \sqrt{-d}\sqrt{e}\right)} - \frac{be \tan^{-1}\left(\frac{\sqrt{cd - \sqrt{-d}\sqrt{e}\sqrt{1 + \frac{1}{cx}}}}{\sqrt{cd + \sqrt{-d}\sqrt{e}\sqrt{\frac{1}{cx} - 1}}}\right)}{2d^2\sqrt{cd - \sqrt{-d}\sqrt{e}}\sqrt{cd + \sqrt{-d}\sqrt{e}}} - \frac{be \tan^{-1}\left(\frac{\sqrt{cd + \sqrt{-d}\sqrt{e}\sqrt{1 + \frac{1}{cx}}}}{\sqrt{cd - \sqrt{-d}\sqrt{e}\sqrt{\frac{1}{cx} - 1}}}\right)}{2d^2\sqrt{cd - \sqrt{-d}\sqrt{e}}}$$

[Out] (b*c*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])/d^2 - a/(d^2*x) - (b*ArcSech[c*x])/d^2 + (e*(a + b*ArcSech[c*x]))/(4*d^2*(Sqrt[-d]*Sqrt[e] - d/x)) - (e*(a + b*ArcSech[c*x]))/(4*d^2*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*e*ArcTan[(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(2*d^2*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]) - (b*e*ArcTan[(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(2*d^2*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]) - (3*Sqrt[e]*(a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*(-d)^(5/2)) + (3*Sqrt[e]*(a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*(-d)^(5/2)) - (3*Sqrt[e]*(a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*(-d)^(5/2)) + (3*Sqrt[e]*(a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*(-d)^(5/2)) + (3*b*Sqrt[e]*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(4*(-d)^(5/2)) - (3*b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*(-d)^(5/2)) + (3*b*Sqrt[e]*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(4*(-d)^(5/2)) - (3*b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*(-d)^(5/2))

Rubi [A] time = 2.95263, antiderivative size = 844, normalized size of antiderivative = 1., number of steps used = 50, number of rules used = 13, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {6303, 5792, 5654, 74, 5707, 5802, 93, 205, 5800, 5562, 2190, 2279, 2391}

$$-\frac{a}{d^2x} - \frac{b \operatorname{sech}^{-1}(cx)}{d^2x} + \frac{e(a + b \operatorname{sech}^{-1}(cx))}{4d^2\left(\sqrt{-d}\sqrt{e} - \frac{d}{x}\right)} - \frac{e(a + b \operatorname{sech}^{-1}(cx))}{4d^2\left(\frac{d}{x} + \sqrt{-d}\sqrt{e}\right)} - \frac{be \tan^{-1}\left(\frac{\sqrt{cd - \sqrt{-d}\sqrt{e}\sqrt{1 + \frac{1}{cx}}}}{\sqrt{cd + \sqrt{-d}\sqrt{e}\sqrt{\frac{1}{cx} - 1}}}\right)}{2d^2\sqrt{cd - \sqrt{-d}\sqrt{e}}\sqrt{cd + \sqrt{-d}\sqrt{e}}} - \frac{be \tan^{-1}\left(\frac{\sqrt{cd + \sqrt{-d}\sqrt{e}\sqrt{1 + \frac{1}{cx}}}}{\sqrt{cd - \sqrt{-d}\sqrt{e}\sqrt{\frac{1}{cx} - 1}}}\right)}{2d^2\sqrt{cd - \sqrt{-d}\sqrt{e}}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcSech[c*x])/(x^2*(d + e*x^2)^2), x]

[Out] (b*c*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])/d^2 - a/(d^2*x) - (b*ArcSech[c*x])/d^2 + (e*(a + b*ArcSech[c*x]))/(4*d^2*(Sqrt[-d]*Sqrt[e] - d/x)) - (e*(a + b*ArcSech[c*x]))/(4*d^2*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*e*ArcTan[(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(2*d^2*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]) - (b*e*ArcTan[(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(2*d^2*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]) - (3*Sqrt[e]*(a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*(-d)^(5/2)) + (3*Sqrt[e]*(a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*(-d)^(5/2)) - (3*Sqrt[e]*(a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*(-d)^(5/2)) + (3*Sqrt[e]*(a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*(-d)^(5/2)) + (3*b*Sqrt[e]*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(4*(-d)^(5/2)) - (3*b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(4*(-d)^(5/2)) + (3*b*Sqrt[e]*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(4*(-d)^(5/2)) - (3*b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(4*(-d)^(5/2))

$$[c*x])*\text{Log}[1 - (c*\text{Sqrt}[-d]*E^{\text{ArcSech}[c*x]})/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])]/(4*(-d)^{(5/2)}) + (3*\text{Sqrt}[e]*(a + b*\text{ArcSech}[c*x])*\text{Log}[1 + (c*\text{Sqrt}[-d]*E^{\text{ArcSech}[c*x]})/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])]/(4*(-d)^{(5/2)}) + (3*b*\text{Sqrt}[e]*\text{PolyLog}[2, -((c*\text{Sqrt}[-d]*E^{\text{ArcSech}[c*x]})/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e]))]/(4*(-d)^{(5/2)}) - (3*b*\text{Sqrt}[e]*\text{PolyLog}[2, (c*\text{Sqrt}[-d]*E^{\text{ArcSech}[c*x]})/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])]/(4*(-d)^{(5/2)}) + (3*b*\text{Sqrt}[e]*\text{PolyLog}[2, -((c*\text{Sqrt}[-d]*E^{\text{ArcSech}[c*x]})/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e]))]/(4*(-d)^{(5/2)}) - (3*b*\text{Sqrt}[e]*\text{PolyLog}[2, (c*\text{Sqrt}[-d]*E^{\text{ArcSech}[c*x]})/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])]/(4*(-d)^{(5/2)})$$

Rule 6303

$$\text{Int}[(a + \text{ArcSech}[c*x])*(b*x)^n*(d + e*x^2)^p, x_Symbol] \rightarrow -\text{Subst}[\text{Int}[(e + d*x^2)^p*(a + b*\text{ArcCosh}[x/c])^n/x^{m+2*(p+1)}, x], x, 1/x] /; \text{FreeQ}\{a, b, c, d, e, n, x\} \&\& \text{IGtQ}[n, 0] \&\& \text{IntegersQ}[m, p]$$

Rule 5792

$$\text{Int}[(a + \text{ArcCosh}[c*x])*(b*x)^n*(f*x)^m*(d + e*x^2)^p, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*\text{ArcCosh}[c*x])^n*(f*x)^m*(d + e*x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, x\} \&\& \text{NeQ}[c^2*d + e, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{IntegerQ}[p] \&\& \text{IntegerQ}[m]$$

Rule 5654

$$\text{Int}[(a + \text{ArcCosh}[c*x])*(b*x)^n, x_Symbol] \rightarrow \text{Simp}[x*(a + b*\text{ArcCosh}[c*x])^n, x] - \text{Dist}[b*c*n, \text{Int}[(x*(a + b*\text{ArcCosh}[c*x])^{n-1})/(\text{Sqrt}[-1 + c*x]*\text{Sqrt}[1 + c*x]), x], x] /; \text{FreeQ}\{a, b, c, x\} \&\& \text{GtQ}[n, 0]$$

Rule 74

$$\text{Int}[(a + (b*x)^n*(c + d*x)^n*(e + f*x)^p, x_Symbol] \rightarrow \text{Simp}[(b*(c + d*x)^{n+1}*(e + f*x)^{p+1})/(d*f*(n + p + 2)), x] /; \text{FreeQ}\{a, b, c, d, e, f, n, p, x\} \&\& \text{NeQ}[n + p + 2, 0] \&\& \text{EqQ}[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]$$

Rule 5707

$$\text{Int}[(a + \text{ArcCosh}[c*x])*(b*x)^n*(d + e*x^2)^p, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*\text{ArcCosh}[c*x])^n*(d + e*x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, n, x\} \&\& \text{NeQ}[c^2*d + e, 0] \&\& \text{IntegerQ}[p] \&\& (p > 0 \parallel \text{IGtQ}[n, 0])$$

Rule 5802

$$\text{Int}[(a + \text{ArcCosh}[c*x])*(b*x)^n*(d + e*x)^m, x_Symbol] \rightarrow \text{Simp}[(d + e*x)^{m+1}*(a + b*\text{ArcCosh}[c*x])^n/(e*(m + 1)), x] - \text{Dist}[(b*c*n)/(e*(m + 1)), \text{Int}[(d + e*x)^{m+1}*(a + b*\text{ArcCosh}[c*x])^{n-1}/(\text{Sqrt}[-1 + c*x]*\text{Sqrt}[1 + c*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, m, x\} \&\& \text{IGtQ}[n, 0] \&\& \text{NeQ}[m, -1]$$

Rule 93

$$\text{Int}[(a + (b*x)^m*(c + d*x)^n)/(e + f*x), x_Symbol] \rightarrow \text{With}\{q = \text{Denominator}[m]\}, \text{Dist}[q, \text{Subst}[\text{Int}[x^{q*(m+1)-1}/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^{1/q}/(c + d*x)^{1/q}], x] /; \text{FreeQ}\{a, b, c, d, e, f, x\} \&\& \text{EqQ}[m + n + 1, 0] \&\& \text{RationalQ}[n]$$

&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 5800

Int[((a_) + ArcCosh[(c_)*(x_)]*(b_))^(n_)/((d_) + (e_)*(x_)), x_Symbol] := Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]

Rule 5562

Int[(((e_) + (f_)*(x_))^(m_)*Sinh[(c_) + (d_)*(x_)])/(Cosh[(c_) + (d_)*(x_)]*(b_) + (a_)), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)), x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)), x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]

Rule 2190

Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rubi steps

$$\begin{aligned}
\int \frac{a + b \operatorname{sech}^{-1}(cx)}{x^2 (d + ex^2)^2} dx &= -\operatorname{Subst} \left(\int \frac{x^4 (a + b \cosh^{-1}(\frac{x}{c}))}{(e + dx^2)^2} dx, x, \frac{1}{x} \right) \\
&= -\operatorname{Subst} \left(\int \left(\frac{a + b \cosh^{-1}(\frac{x}{c})}{d^2} + \frac{e^2 (a + b \cosh^{-1}(\frac{x}{c}))}{d^2 (e + dx^2)^2} - \frac{2e (a + b \cosh^{-1}(\frac{x}{c}))}{d^2 (e + dx^2)} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\operatorname{Subst} \left(\int (a + b \cosh^{-1}(\frac{x}{c})) dx, x, \frac{1}{x} \right)}{d^2} + \frac{(2e) \operatorname{Subst} \left(\int \frac{a + b \cosh^{-1}(\frac{x}{c})}{e + dx^2} dx, x, \frac{1}{x} \right)}{d^2} - \frac{e^2 \operatorname{Subst} \left(\int \frac{1}{(e + dx^2)^2} dx, x, \frac{1}{x} \right)}{d^2} \\
&= -\frac{a}{d^2 x} - \frac{b \operatorname{Subst} \left(\int \cosh^{-1}(\frac{x}{c}) dx, x, \frac{1}{x} \right)}{d^2} + \frac{(2e) \operatorname{Subst} \left(\int \left(\frac{a + b \cosh^{-1}(\frac{x}{c})}{2\sqrt{e}(\sqrt{e} - \sqrt{-dx})} + \frac{a + b \cosh^{-1}(\frac{x}{c})}{2\sqrt{e}(\sqrt{e} + \sqrt{-dx})} \right) dx, x, \frac{1}{x} \right)}{d^2} \\
&= -\frac{a}{d^2 x} - \frac{b \operatorname{sech}^{-1}(cx)}{d^2 x} + \frac{b \operatorname{Subst} \left(\int \frac{x}{\sqrt{-1 + \frac{x}{c}} \sqrt{1 + \frac{x}{c}}} dx, x, \frac{1}{x} \right)}{cd^2} + \frac{\sqrt{e} \operatorname{Subst} \left(\int \frac{a + b \cosh^{-1}(\frac{x}{c})}{\sqrt{e} - \sqrt{-dx}} dx, x, \frac{1}{x} \right)}{d^2} \\
&= \frac{bc \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}}{d^2} - \frac{a}{d^2 x} - \frac{b \operatorname{sech}^{-1}(cx)}{d^2 x} + \frac{e (a + b \operatorname{sech}^{-1}(cx))}{4d^2 \left(\sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} - \frac{e (a + b \operatorname{sech}^{-1}(cx))}{4d^2 \left(\sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} + \dots \\
&= \frac{bc \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}}{d^2} - \frac{a}{d^2 x} - \frac{b \operatorname{sech}^{-1}(cx)}{d^2 x} + \frac{e (a + b \operatorname{sech}^{-1}(cx))}{4d^2 \left(\sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} - \frac{e (a + b \operatorname{sech}^{-1}(cx))}{4d^2 \left(\sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} - \dots \\
&= \frac{bc \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}}{d^2} - \frac{a}{d^2 x} - \frac{b \operatorname{sech}^{-1}(cx)}{d^2 x} + \frac{e (a + b \operatorname{sech}^{-1}(cx))}{4d^2 \left(\sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} - \frac{e (a + b \operatorname{sech}^{-1}(cx))}{4d^2 \left(\sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} - \dots \\
&= \frac{bc \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}}{d^2} - \frac{a}{d^2 x} - \frac{b \operatorname{sech}^{-1}(cx)}{d^2 x} + \frac{e (a + b \operatorname{sech}^{-1}(cx))}{4d^2 \left(\sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} - \frac{e (a + b \operatorname{sech}^{-1}(cx))}{4d^2 \left(\sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} - \dots \\
&= \frac{bc \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}}{d^2} - \frac{a}{d^2 x} - \frac{b \operatorname{sech}^{-1}(cx)}{d^2 x} + \frac{e (a + b \operatorname{sech}^{-1}(cx))}{4d^2 \left(\sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} - \frac{e (a + b \operatorname{sech}^{-1}(cx))}{4d^2 \left(\sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} - \dots \\
&= \frac{bc \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}}{d^2} - \frac{a}{d^2 x} - \frac{b \operatorname{sech}^{-1}(cx)}{d^2 x} + \frac{e (a + b \operatorname{sech}^{-1}(cx))}{4d^2 \left(\sqrt{-d} \sqrt{e} - \frac{d}{x} \right)} - \frac{e (a + b \operatorname{sech}^{-1}(cx))}{4d^2 \left(\sqrt{-d} \sqrt{e} + \frac{d}{x} \right)} - \dots
\end{aligned}$$

Mathematica [C] time = 1.08703, size = 1305, normalized size = 1.55

result too large to display

$$\begin{aligned} & *d+e)^{(1/2)+2*e}*d)^{(1/2)}*e^2*\operatorname{arctanh}(c*d*(1/c/x+(-1+1/c/x)^{(1/2)}*(1+1/c/x) \\ &)^{(1/2)})/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e)*d)^{(1/2)})/d^5+c*b/d^2*(-(c*x-1) \\ &)/c/x)^{(1/2)}*((c*x+1)/c/x)^{(1/2)}-1/2*b/c^2*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)}+2 \\ & *e)*d)^{(1/2)}*\operatorname{arctanh}(c*d*(1/c/x+(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)})/((-c^2*d+ \\ & 2*(e*(c^2*d+e))^{(1/2)}-2*e)*d)^{(1/2)})/d^4/(c^2*d+e)*(e*(c^2*d+e))^{(1/2)}*e-b/ \\ & c^4*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*e^2*\operatorname{arctanh}(c*d*(1/c/x+(-1 \\ & +1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)})/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e)*d)^{(1/2)} \\ &)/d^5/(c^2*d+e)*(e*(c^2*d+e))^{(1/2)}+b/c^4*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)}+2* \\ & e)*d)^{(1/2)}*e*\operatorname{arctanh}(c*d*(1/c/x+(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)})/((-c^2*d \\ & +2*(e*(c^2*d+e))^{(1/2)}-2*e)*d)^{(1/2)})/d^5*(e*(c^2*d+e))^{(1/2)}-b/c^2*(-(c^2* \\ & d-2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*\operatorname{arctanh}(c*d*(1/c/x+(-1+1/c/x)^{(1/2)}*(\\ & 1+1/c/x)^{(1/2)})/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e)*d)^{(1/2)})/d^4/(c^2*d+e) \\ & *e^2-b/c^4*(-(c^2*d-2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*e^3*\operatorname{arctanh}(c*d*(1/ \\ & c/x+(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)})/((-c^2*d+2*(e*(c^2*d+e))^{(1/2)}-2*e)*d \\ &)^{(1/2)})/d^5/(c^2*d+e)+1/2*b/c^2*((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)} \\ &)*\operatorname{arctan}(c*d*(1/c/x+(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)})/((c^2*d+2*(e*(c^2*d+e) \\ &))^{(1/2)}+2*e)*d)^{(1/2)})/d^4/(c^2*d+e)*(e*(c^2*d+e))^{(1/2)}*e+b/c^4*((c^2*d+2 \\ & *(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*e^2*\operatorname{arctan}(c*d*(1/c/x+(-1+1/c/x)^{(1/2)}*(\\ & 1+1/c/x)^{(1/2)})/((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)})/d^5/(c^2*d+e)* \\ & (e*(c^2*d+e))^{(1/2)}-b/c^4*((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*e*\operatorname{arc} \\ & \tan(c*d*(1/c/x+(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)})/((c^2*d+2*(e*(c^2*d+e))^{(1 \\ & /2)}+2*e)*d)^{(1/2)})/d^5*(e*(c^2*d+e))^{(1/2)}-b/c^2*((c^2*d+2*(e*(c^2*d+e))^{(1 \\ & /2)}+2*e)*d)^{(1/2)}*\operatorname{arctan}(c*d*(1/c/x+(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)})/((c^2 \\ & *d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)})/d^4/(c^2*d+e)*e^2-b/c^4*((c^2*d+2*(\\ & e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)}*e^3*\operatorname{arctan}(c*d*(1/c/x+(-1+1/c/x)^{(1/2)}*(1+ \\ & 1/c/x)^{(1/2)})/((c^2*d+2*(e*(c^2*d+e))^{(1/2)}+2*e)*d)^{(1/2)})/d^5/(c^2*d+e)-1/ \\ & 2*b*c^2*x*e*\operatorname{arcsech}(c*x)/d^2/(c^2*e*x^2+c^2*d) \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b \operatorname{ar} \operatorname{sech}(cx) + a}{e^2 x^6 + 2 d e x^4 + d^2 x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d)^2,x, algorithm="fricas")

[Out] integral((b*arcsech(c*x) + a)/(e^2*x^6 + 2*d*e*x^4 + d^2*x^2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asech(c*x))/x**2/(e*x**2+d)**2,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{ar} \operatorname{sech}(cx) + a}{(ex^2 + d)^2 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d)^2,x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)/((e*x^2 + d)^2*x^2), x)

$$3.123 \quad \int \frac{x^5 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^3} dx$$

Optimal. Leaf size=778

$$\frac{b \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-de} \operatorname{sech}^{-1}(cx)}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3} + \frac{b \operatorname{PolyLog}\left(2, \frac{c\sqrt{-de} \operatorname{sech}^{-1}(cx)}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3} + \frac{b \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-de} \operatorname{sech}^{-1}(cx)}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^3} + \frac{b \operatorname{PolyLog}\left(2, \frac{c\sqrt{-de} \operatorname{sech}^{-1}(cx)}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^3}$$

```
[Out] (b*d*(c^2 - x^(-2)))/(8*c*e^2*(c^2*d + e)*(e + d/x^2)*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x) - (a + b*ArcSech[c*x])/(4*e*(e + d/x^2)^2) - (a + b*ArcSech[c*x])/(2*e^2*(e + d/x^2)) - (a + b*ArcSech[c*x])^2/(b*e^3) + (b*Sqrt[-1 + 1/(c^2*x^2)]*ArcTanh[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[-1 + 1/(c^2*x^2)]*x)])/(2*e^(5/2)*Sqrt[c^2*d + e]*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) + (b*(c^2*d + 2*e)*Sqrt[-1 + 1/(c^2*x^2)]*ArcTanh[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[-1 + 1/(c^2*x^2)]*x)])/(8*e^(5/2)*(c^2*d + e)^(3/2)*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) - ((a + b*ArcSech[c*x])*Log[1 + E^(-2*ArcSech[c*x])])/e^3 + ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/ (2*e^3) + ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/ (2*e^3) + ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/ (2*e^3) + ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/ (2*e^3) + (b*PolyLog[2, -E^(-2*ArcSech[c*x])])/ (2*e^3) + (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e]))])/ (2*e^3) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/ (2*e^3) + (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/ (2*e^3) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/ (2*e^3)
```

Rubi [A] time = 1.70935, antiderivative size = 760, normalized size of antiderivative = 0.98, number of steps used = 35, number of rules used = 14, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6303, 5792, 5660, 3718, 2190, 2279, 2391, 5788, 519, 382, 377, 208, 5800, 5562}

$$\frac{b \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-de} \operatorname{sech}^{-1}(cx)}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3} + \frac{b \operatorname{PolyLog}\left(2, \frac{c\sqrt{-de} \operatorname{sech}^{-1}(cx)}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2e^3} + \frac{b \operatorname{PolyLog}\left(2, -\frac{c\sqrt{-de} \operatorname{sech}^{-1}(cx)}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^3} + \frac{b \operatorname{PolyLog}\left(2, \frac{c\sqrt{-de} \operatorname{sech}^{-1}(cx)}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2e^3}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(x^5*(a + b*ArcSech[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (b*d*(c^2 - x^(-2)))/(8*c*e^2*(c^2*d + e)*(e + d/x^2)*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x) - (a + b*ArcSech[c*x])/(4*e*(e + d/x^2)^2) - (a + b*ArcSech[c*x])/(2*e^2*(e + d/x^2)) + (b*Sqrt[-1 + 1/(c^2*x^2)]*ArcTanh[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[-1 + 1/(c^2*x^2)]*x)])/(2*e^(5/2)*Sqrt[c^2*d + e]*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) + (b*(c^2*d + 2*e)*Sqrt[-1 + 1/(c^2*x^2)]*ArcTanh[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqrt[-1 + 1/(c^2*x^2)]*x)])/(8*e^(5/2)*(c^2*d + e)^(3/2)*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) + ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/ (2*e^3) + ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/ (2*e^3)
```

- Sqrt[c^2*d + e]])/(2*e^3) + ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^3) + ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^3) - ((a + b*ArcSech[c*x])*Log[1 + E^(2*ArcSech[c*x])])/e^3 + (b*PolyLog[2, -(c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e^3) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*e^3) + (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(2*e^3) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*e^3) - (b*PolyLog[2, -E^(2*ArcSech[c*x])])/(2*e^3)

Rule 6303

Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))^n_.*(x_)^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := -Subst[Int[(e + d*x^2)^p*(a + b*ArcCosh[x/c])^n/x^(m + 2*(p + 1)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegersQ[m, p]

Rule 5792

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^n_.*((f_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]

Rule 5660

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))^n_./(x_), x_Symbol] := Subst[Int[(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]

Rule 3718

Int[((c_.) + (d_.)*(x_)^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x_Symbol] := -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[(c + d*x)^m*E^(2*(-I*e) + f*fz*x))/(1 + E^(2*(-I*e) + f*fz*x)), x], x] /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]

Rule 2190

Int[(((F_)^(g_.)*((e_.) + (f_.)*(x_)))^n_.)*((c_.) + (d_.)*(x_)^(m_.))/((a_.) + (b_.)*((F_)^(g_.)*((e_.) + (f_.)*(x_)))^n_.), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]

Rule 2279

Int[Log[(a_.) + (b_.)*((F_)^(e_.)*((c_.) + (d_.)*(x_)))^n_.], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]

Rule 2391

Int[Log[(c_.)*((d_.) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]

Rule 5788

Int[((a_.) + ArcCosh[(c_.)*(x_.)]*(b_.))*(x_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x


```
_Symbol] := Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x]))/(2*e*(p + 1)),
x] - Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[
-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
NeQ[p, -1]
```

Rule 519

```
Int[(u_)*((c_) + (d_)*(x_)^(n_))^(q_)*((a1_) + (b1_)*(x_)^(non2_))^(p
_)*((a2_) + (b2_)*(x_)^(non2_))^(p_), x_Symbol] := Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p]]/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 382

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol]
:= -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] || !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)/((c_) + (d_)*(x_)^(n_)), x_Symbol] := Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 208

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 5800

```
Int[((a_) + ArcCosh[(c_)*(x_)])*(b_)^(n_)/((d_) + (e_)*(x_)), x_Symbo
l] := Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5562

```
Int[(((e_) + (f_)*(x_))^(m_)*Sinh[(c_) + (d_)*(x_)])/(Cosh[(c_) + (d_
_)*(x_)]*(b_) + (a_)), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^3} dx &= -\operatorname{Subst} \left(\int \frac{a + b \cosh^{-1} \left(\frac{x}{c} \right)}{x (e + dx^2)^3} dx, x, \frac{1}{x} \right) \\
&= -\operatorname{Subst} \left(\int \left(\frac{a + b \cosh^{-1} \left(\frac{x}{c} \right)}{e^3 x} - \frac{dx (a + b \cosh^{-1} \left(\frac{x}{c} \right))}{e (e + dx^2)^3} - \frac{dx (a + b \cosh^{-1} \left(\frac{x}{c} \right))}{e^2 (e + dx^2)^2} - \frac{dx (a + b \cosh^{-1} \left(\frac{x}{c} \right))}{e^3} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\operatorname{Subst} \left(\int \frac{a + b \cosh^{-1} \left(\frac{x}{c} \right)}{x} dx, x, \frac{1}{x} \right)}{e^3} + \frac{d \operatorname{Subst} \left(\int \frac{x (a + b \cosh^{-1} \left(\frac{x}{c} \right))}{e + dx^2} dx, x, \frac{1}{x} \right)}{e^3} + \frac{d \operatorname{Subst} \left(\int \frac{x (a + b \cosh^{-1} \left(\frac{x}{c} \right))}{(e + dx^2)^2} dx, x, \frac{1}{x} \right)}{e^3} \\
&= -\frac{a + b \operatorname{sech}^{-1}(cx)}{4e \left(e + \frac{d}{x^2} \right)^2} - \frac{a + b \operatorname{sech}^{-1}(cx)}{2e^2 \left(e + \frac{d}{x^2} \right)} - \frac{\operatorname{Subst} \left(\int (a + bx) \tanh(x) dx, x, \operatorname{sech}^{-1}(cx) \right)}{e^3} + \frac{d \operatorname{Subst} \left(\int \frac{x (a + b \operatorname{sech}^{-1}(cx))}{e + dx^2} dx, x, \frac{1}{x} \right)}{e^3} \\
&= -\frac{a + b \operatorname{sech}^{-1}(cx)}{4e \left(e + \frac{d}{x^2} \right)^2} - \frac{a + b \operatorname{sech}^{-1}(cx)}{2e^2 \left(e + \frac{d}{x^2} \right)} + \frac{(a + b \operatorname{sech}^{-1}(cx))^2}{2be^3} - \frac{2 \operatorname{Subst} \left(\int \frac{e^{2x} (a + bx)}{1 + e^{2x}} dx, x, \operatorname{sech}^{-1}(cx) \right)}{e^3} \\
&= \frac{bd \left(c^2 - \frac{1}{x^2} \right)}{8ce^2 (c^2 d + e) \left(e + \frac{d}{x^2} \right) \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}} x} - \frac{a + b \operatorname{sech}^{-1}(cx)}{4e \left(e + \frac{d}{x^2} \right)^2} - \frac{a + b \operatorname{sech}^{-1}(cx)}{2e^2 \left(e + \frac{d}{x^2} \right)} + \frac{(a + b \operatorname{sech}^{-1}(cx))^2}{2be^3} - \frac{2 \operatorname{Subst} \left(\int \frac{e^{2x} (a + bx)}{1 + e^{2x}} dx, x, \operatorname{sech}^{-1}(cx) \right)}{e^3} \\
&= \frac{bd \left(c^2 - \frac{1}{x^2} \right)}{8ce^2 (c^2 d + e) \left(e + \frac{d}{x^2} \right) \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}} x} - \frac{a + b \operatorname{sech}^{-1}(cx)}{4e \left(e + \frac{d}{x^2} \right)^2} - \frac{a + b \operatorname{sech}^{-1}(cx)}{2e^2 \left(e + \frac{d}{x^2} \right)} + \frac{(a + b \operatorname{sech}^{-1}(cx))^2}{2be^3} - \frac{2 \operatorname{Subst} \left(\int \frac{e^{2x} (a + bx)}{1 + e^{2x}} dx, x, \operatorname{sech}^{-1}(cx) \right)}{e^3} \\
&= \frac{bd \left(c^2 - \frac{1}{x^2} \right)}{8ce^2 (c^2 d + e) \left(e + \frac{d}{x^2} \right) \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}} x} - \frac{a + b \operatorname{sech}^{-1}(cx)}{4e \left(e + \frac{d}{x^2} \right)^2} - \frac{a + b \operatorname{sech}^{-1}(cx)}{2e^2 \left(e + \frac{d}{x^2} \right)} + \frac{(a + b \operatorname{sech}^{-1}(cx))^2}{2be^3} - \frac{2 \operatorname{Subst} \left(\int \frac{e^{2x} (a + bx)}{1 + e^{2x}} dx, x, \operatorname{sech}^{-1}(cx) \right)}{e^3} \\
&= \frac{bd \left(c^2 - \frac{1}{x^2} \right)}{8ce^2 (c^2 d + e) \left(e + \frac{d}{x^2} \right) \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}} x} - \frac{a + b \operatorname{sech}^{-1}(cx)}{4e \left(e + \frac{d}{x^2} \right)^2} - \frac{a + b \operatorname{sech}^{-1}(cx)}{2e^2 \left(e + \frac{d}{x^2} \right)} + \frac{(a + b \operatorname{sech}^{-1}(cx))^2}{2be^3} - \frac{2 \operatorname{Subst} \left(\int \frac{e^{2x} (a + bx)}{1 + e^{2x}} dx, x, \operatorname{sech}^{-1}(cx) \right)}{e^3} \\
&= \frac{bd \left(c^2 - \frac{1}{x^2} \right)}{8ce^2 (c^2 d + e) \left(e + \frac{d}{x^2} \right) \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}} x} - \frac{a + b \operatorname{sech}^{-1}(cx)}{4e \left(e + \frac{d}{x^2} \right)^2} - \frac{a + b \operatorname{sech}^{-1}(cx)}{2e^2 \left(e + \frac{d}{x^2} \right)} + \frac{(a + b \operatorname{sech}^{-1}(cx))^2}{2be^3} - \frac{2 \operatorname{Subst} \left(\int \frac{e^{2x} (a + bx)}{1 + e^{2x}} dx, x, \operatorname{sech}^{-1}(cx) \right)}{e^3} \\
&= \frac{bd \left(c^2 - \frac{1}{x^2} \right)}{8ce^2 (c^2 d + e) \left(e + \frac{d}{x^2} \right) \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}} x} - \frac{a + b \operatorname{sech}^{-1}(cx)}{4e \left(e + \frac{d}{x^2} \right)^2} - \frac{a + b \operatorname{sech}^{-1}(cx)}{2e^2 \left(e + \frac{d}{x^2} \right)} + \frac{(a + b \operatorname{sech}^{-1}(cx))^2}{2be^3} - \frac{2 \operatorname{Subst} \left(\int \frac{e^{2x} (a + bx)}{1 + e^{2x}} dx, x, \operatorname{sech}^{-1}(cx) \right)}{e^3}
\end{aligned}$$

Mathematica [C] time = 7.48651, size = 2000, normalized size = 2.57

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(x^5*(a + b*ArcSech[c*x]))/(d + e*x^2)^3,x]

```
[Out] -(a*d^2)/(4*e^3*(d + e*x^2)^2) + (a*d)/(e^3*(d + e*x^2)) + (a*Log[d + e*x^2
])/ (2*e^3) + b*(-(d*((-I)*Sqrt[e]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(Sqr
t[d]*(c^2*d + e))*((-I)*Sqrt[d] + Sqrt[e]*x)) - ArcSech[c*x]/(Sqrt[e]*((-I)
*Sqrt[d] + Sqrt[e]*x)^2) + Log[x]/(d*Sqrt[e]) - Log[1 + Sqrt[(1 - c*x)/(1 +
c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]]/(d*Sqrt[e]) + ((2*c^2*d + e)*Log[(-
4*d*Sqrt[e]*Sqrt[c^2*d + e]*(Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqr
t[(1 - c*x)/(1 + c*x)] + c*Sqrt[c^2*d + e]*x*Sqrt[(1 - c*x)/(1 + c*x)])))/(
(2*c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x)))/(d*(c^2*d + e)^(3/2)))/(16*e^(
5/2)) - (d*((I*Sqrt[e]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(Sqrt[d]*(c^2*d
+ e)*(I*Sqrt[d] + Sqrt[e]*x)) - ArcSech[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]
*x)^2) + Log[x]/(d*Sqrt[e]) - Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[
(1 - c*x)/(1 + c*x)]]/(d*Sqrt[e]) + ((2*c^2*d + e)*Log[(-4*d*Sqrt[e]*Sqrt[c
^2*d + e]*(Sqrt[e] + I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[(1 - c*x)/(1 +
c*x)] + c*Sqrt[c^2*d + e]*x*Sqrt[(1 - c*x)/(1 + c*x)])))/(2*c^2*d + e)*(I*S
qrt[d] + Sqrt[e]*x)))/(d*(c^2*d + e)^(3/2)))/(16*e^(5/2)) - (((7*I)/16)*S
qrt[d]*(-ArcSech[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) + (I*(Log[x]/Sqrt[e] - Lo
g[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]]/Sqrt[e] +
Log[((2*I)*Sqrt[e]*(Sqrt[d]*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + (Sqrt[d]*
Sqrt[e] + I*c^2*d*x)/Sqrt[c^2*d + e]))/(I*Sqrt[d] + Sqrt[e]*x))/Sqrt[c^2*d
+ e])/Sqrt[d])/e^(5/2) + (((7*I)/16)*Sqrt[d]*(-ArcSech[c*x]/((-I)*Sqrt[d]
]*Sqrt[e] + e*x)) - (I*(Log[x]/Sqrt[e] - Log[1 + Sqrt[(1 - c*x)/(1 + c*x)]
+ c*x*Sqrt[(1 - c*x)/(1 + c*x)]]/Sqrt[e] + Log[(2*Sqrt[e]*(I*Sqrt[d]*Sqrt[(
1 - c*x)/(1 + c*x)]*(1 + c*x) + (I*Sqrt[d]*Sqrt[e] + c^2*d*x)/Sqrt[c^2*d +
e]))/((-I)*Sqrt[d] + Sqrt[e]*x))/Sqrt[c^2*d + e])/Sqrt[d])/e^(5/2) + (Pol
yLog[2, -E^(-2*ArcSech[c*x])] - 2*((-4*I)*ArcSin[Sqrt[1 + (I*Sqrt[e])]/(c*Sq
rt[d])]/Sqrt[2]]*ArcTanh[((I*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqr
t[c^2*d + e]] + ArcSech[c*x]*Log[1 + E^(-2*ArcSech[c*x])] - ArcSech[c*x]*Lo
g[1 + (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (2*I)*A
rcSin[Sqrt[1 + (I*Sqrt[e])]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] - Sqr
t[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - ArcSech[c*x]*Log[1 + (I*(Sqrt[e]
+ Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] - (2*I)*ArcSin[Sqrt[1 + (
I*Sqrt[e])]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c
*Sqrt[d]*E^ArcSech[c*x])] + PolyLog[2, (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*
Sqrt[d]*E^ArcSech[c*x])] + PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[c^2*d + e]))/(c
*Sqrt[d]*E^ArcSech[c*x])])/(4*e^3) - (-PolyLog[2, -E^(-2*ArcSech[c*x])] +
2*((-4*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])]/(c*Sqrt[d])]/Sqrt[2]]*ArcTanh[(((I)
*c*Sqrt[d] + Sqrt[e])*Tanh[ArcSech[c*x]/2])/Sqrt[c^2*d + e]] + ArcSech[c*x]*
Log[1 + E^(-2*ArcSech[c*x])] - ArcSech[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2
*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + (2*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])]/(
c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^
ArcSech[c*x])] - ArcSech[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sq
rt[d]*E^ArcSech[c*x])] - (2*I)*ArcSin[Sqrt[1 - (I*Sqrt[e])]/(c*Sqrt[d])]/Sqr
t[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] +
PolyLog[2, (I*(Sqrt[e] - Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])] + P
olyLog[2, (I*(Sqrt[e] + Sqrt[c^2*d + e]))/(c*Sqrt[d]*E^ArcSech[c*x])])/(4*
e^3))
```

Maple [C] time = 0.846, size = 1779, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^3,x)
```

```
[Out] -c^2*b/e^3/(c^2*d+e)*d*arcsech(c*x)*ln(1-I*(1/c/x+(-1+1/c/x)^(1/2))*(1+1/c/x
)^(1/2))+1/4*c^4*b/e/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*x^2*d-5/8*c^2*b*(e*(c^2
```

$$\begin{aligned}
 & *d+e)^{(1/2)}/e^3/(c^2*d+e)^2*d*\operatorname{arctanh}(1/4*(2*c^2*d*(1/c/x+(-1+1/c/x)^{(1/2)} \\
 & *(1+1/c/x)^{(1/2)})^2+2*c^2*d+4*e)/(c^2*d*e+e^2)^{(1/2)})-c^2*b/e^3/(c^2*d+e)* \\
 & * \operatorname{arcsech}(c*x)*\ln(1+I*(1/c/x+(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)}))-1/2*c^4*b/e/ \\
 & (c^2*d+e)/(c^2*e*x^2+c^2*d)^2*\operatorname{arcsech}(c*x)*d*x^2-3/4*c^6*b/e/(c^2*d+e)/(c^2 \\
 & *e*x^2+c^2*d)^2*\operatorname{arcsech}(c*x)*d*x^4-1/2*c^6*b/e^2/(c^2*d+e)/(c^2*e*x^2+c^2*d \\
 &)^2*\operatorname{arcsech}(c*x)*d^2*x^2-1/8*c^5*b/e^2/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*(-(c*x \\
 & -1)/c/x)^{(1/2)}*((c*x+1)/c/x)^{(1/2)}*x*d^2-b/e^2/(c^2*d+e)*\operatorname{dilog}(1+I*(1/c/x+(\\
 & -1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)}))-b/e^2/(c^2*d+e)*\operatorname{dilog}(1-I*(1/c/x+(-1+1/c/ \\
 & x)^{(1/2)}*(1+1/c/x)^{(1/2)}))+1/2*a/e^3*\ln(c^2*e*x^2+c^2*d)+1/8*c^4*b/e^2/(c^2 \\
 & *d+e)/(c^2*e*x^2+c^2*d)^2*d^2-c^2*b/e^3/(c^2*d+e)*d*\operatorname{dilog}(1+I*(1/c/x+(-1+1/ \\
 & c/x)^{(1/2)}*(1+1/c/x)^{(1/2)}))-c^2*b/e^3/(c^2*d+e)*d*\operatorname{dilog}(1-I*(1/c/x+(-1+1/c \\
 & /x)^{(1/2)}*(1+1/c/x)^{(1/2)}))+1/4*c^4*b/e^3/(c^2*d+e)*d^2*\operatorname{sum}((_R1^2+1)/(_R1^ \\
 & 2*c^2*d+c^2*d+2*e))*(\operatorname{arcsech}(c*x)*\ln((_R1-1/c/x-(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(\\
 & 1/2)})/_R1)+\operatorname{dilog}((_R1-1/c/x-(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)})/_R1)), _R1=\operatorname{Roo \\
 & tOf}(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+1/4*c^2*b/e^2/(c^2*d+e)*d*\operatorname{sum}((_R \\
 & 1^2+1)/(_R1^2*c^2*d+c^2*d+2*e))*(\operatorname{arcsech}(c*x)*\ln((_R1-1/c/x-(-1+1/c/x)^{(1/2) \\
 & *(1+1/c/x)^{(1/2)})/_R1)+\operatorname{dilog}((_R1-1/c/x-(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)})/_ \\
 & R1)), _R1=\operatorname{RootOf}(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))+1/4*c^2*b/e^3/(c^2*d+ \\
 & e)*d*\operatorname{sum}((_R1^2*c^2*d+c^2*d+4*e)/(_R1^2*c^2*d+c^2*d+2*e))*(\operatorname{arcsech}(c*x)*\ln((\\
 & _R1-1/c/x-(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)})/_R1)+\operatorname{dilog}((_R1-1/c/x-(-1+1/c/x \\
 &)^{(1/2)}*(1+1/c/x)^{(1/2)})/_R1)), _R1=\operatorname{RootOf}(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2 \\
 & *d))-3/4*c^4*b/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*\operatorname{arcsech}(c*x)*x^4+1/4*b/e^2/(c^ \\
 & 2*d+e)*\operatorname{sum}((_R1^2*c^2*d+c^2*d+4*e)/(_R1^2*c^2*d+c^2*d+2*e))*(\operatorname{arcsech}(c*x)*\ln \\
 & ((_R1-1/c/x-(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)})/_R1)+\operatorname{dilog}((_R1-1/c/x-(-1+1/c \\
 & /x)^{(1/2)}*(1+1/c/x)^{(1/2)})/_R1)), _R1=\operatorname{RootOf}(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c \\
 & ^2*d))-1/8*c^5*b/e/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*(-(c*x-1)/c/x)^{(1/2)}*((c*x \\
 & +1)/c/x)^{(1/2)}*x^3*d-1/4*c^4*a*d^2/e^3/(c^2*e*x^2+c^2*d)^2-3/4*b*(e*(c^2*d+ \\
 & e))^{(1/2)}/e^2/(c^2*d+e)^2*\operatorname{arctanh}(1/4*(2*c^2*d*(1/c/x+(-1+1/c/x)^{(1/2)}*(1+1 \\
 & /c/x)^{(1/2)})^2+2*c^2*d+4*e)/(c^2*d*e+e^2)^{(1/2)})-b/e^2/(c^2*d+e)*\operatorname{arcsech}(c* \\
 & x)*\ln(1+I*(1/c/x+(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)}))-b/e^2/(c^2*d+e)*\operatorname{arcsech} \\
 & (c*x)*\ln(1-I*(1/c/x+(-1+1/c/x)^{(1/2)}*(1+1/c/x)^{(1/2)}))+c^2*a/e^3*d/(c^2*e*x \\
 & ^2+c^2*d)+1/8*c^4*b/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*x^4
 \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4} a \left(\frac{4 d e x^2 + 3 d^2}{e^5 x^4 + 2 d e^4 x^2 + d^2 e^3} + \frac{2 \log(e x^2 + d)}{e^3} \right) + b \int \frac{x^5 \log\left(\sqrt{\frac{1}{c x} + 1} \sqrt{\frac{1}{c x} - 1} + \frac{1}{c x}\right)}{e^3 x^6 + 3 d e^2 x^4 + 3 d^2 e x^2 + d^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="maxima")

[Out] 1/4*a*((4*d*e*x^2 + 3*d^2)/(e^5*x^4 + 2*d*e^4*x^2 + d^2*e^3) + 2*log(e*x^2 + d)/e^3) + b*integrate(x^5*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b x^5 \operatorname{arsech}(c x) + a x^5}{e^3 x^6 + 3 d e^2 x^4 + 3 d^2 e x^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="fricas")

[Out] integral((b*x^5*arcsech(c*x) + a*x^5)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*(a+b*asech(c*x))/(e*x**2+d)**3,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsech}(cx) + a)x^5}{(ex^2 + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)*x^5/(e*x^2 + d)^3, x)

$$3.124 \quad \int \frac{x^3 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^3} dx$$

Optimal. Leaf size=173

$$\frac{x^4 (a + b \operatorname{sech}^{-1}(cx))}{4d(d + ex^2)^2} - \frac{b \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} (c^2d + 2e) \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{c^2d+e}}\right)}{8de^{3/2} (c^2d + e)^{3/2}} + \frac{b \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \sqrt{1-c^2x^2}}{8e(c^2d + e)(d + ex^2)}$$

[Out] (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(8*e*(c^2*d + e)*(d + e*x^2)) + (x^4*(a + b*ArcSech[c*x]))/(4*d*(d + e*x^2)^2) - (b*(c^2*d + 2*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[(Sqrt[e]*Sqrt[1 - c^2*x^2])/Sqrt[c^2*d + e]])/(8*d*e^(3/2)*(c^2*d + e)^(3/2))

Rubi [A] time = 0.189258, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {264, 6301, 12, 446, 78, 63, 208}

$$\frac{x^4 (a + b \operatorname{sech}^{-1}(cx))}{4d(d + ex^2)^2} - \frac{b \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} (c^2d + 2e) \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{c^2d+e}}\right)}{8de^{3/2} (c^2d + e)^{3/2}} + \frac{b \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \sqrt{1-c^2x^2}}{8e(c^2d + e)(d + ex^2)}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(a + b*ArcSech[c*x]))/(d + e*x^2)^3,x]

[Out] (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(8*e*(c^2*d + e)*(d + e*x^2)) + (x^4*(a + b*ArcSech[c*x]))/(4*d*(d + e*x^2)^2) - (b*(c^2*d + 2*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[(Sqrt[e]*Sqrt[1 - c^2*x^2])/Sqrt[c^2*d + e]])/(8*d*e^(3/2)*(c^2*d + e)^(3/2))

Rule 264

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

Rule 6301

Int[((a_.) + ArcSech[(c_.)*(x_)])*(b_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

Rule 12

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 446

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 78

```
Int[((a_) + (b_)*(x_))*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_
), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || (EqQ[e, 0] || (EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^3} dx &= \frac{x^4 (a + b \operatorname{sech}^{-1}(cx))}{4d (d + ex^2)^2} + \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{x^3}{4d \sqrt{1-c^2x^2} (d + ex^2)^2} dx \\
&= \frac{x^4 (a + b \operatorname{sech}^{-1}(cx))}{4d (d + ex^2)^2} + \frac{\left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{x^3}{\sqrt{1-c^2x^2} (d + ex^2)^2} dx}{4d} \\
&= \frac{x^4 (a + b \operatorname{sech}^{-1}(cx))}{4d (d + ex^2)^2} + \frac{\left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \operatorname{Subst} \left(\int \frac{x}{\sqrt{1-c^2x} (d + ex^2)} dx, x, x^2 \right)}{8d} \\
&= \frac{b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{8e (c^2d + e) (d + ex^2)} + \frac{x^4 (a + b \operatorname{sech}^{-1}(cx))}{4d (d + ex^2)^2} + \frac{\left(b (c^2d + 2e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \operatorname{Subst} \left(\int \frac{1}{\sqrt{1-c^2x} (d + ex^2)} dx, x, x^2 \right)}{16de (c^2d + e) (d + ex^2)} \\
&= \frac{b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{8e (c^2d + e) (d + ex^2)} + \frac{x^4 (a + b \operatorname{sech}^{-1}(cx))}{4d (d + ex^2)^2} - \frac{\left(b (c^2d + 2e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \operatorname{Subst} \left(\int \frac{1}{\sqrt{1-c^2x} (d + ex^2)} dx, x, x^2 \right)}{8c^2de (c^2d + e) (d + ex^2)} \\
&= \frac{b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{8e (c^2d + e) (d + ex^2)} + \frac{x^4 (a + b \operatorname{sech}^{-1}(cx))}{4d (d + ex^2)^2} - \frac{b (c^2d + 2e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \tanh^{-1} \left(\frac{x \sqrt{1-c^2x}}{\sqrt{1+cx}} \right)}{8de^{3/2} (c^2d + e)^{3/2}}
\end{aligned}$$

Mathematica [C] time = 1.48594, size = 486, normalized size = 2.81

$$\frac{\frac{8a}{d+ex^2} - \frac{4ad}{(d+ex^2)^2} + \frac{b\sqrt{e}(c^2d+2e) \log\left(\frac{16de^{3/2}\sqrt{c^2d+e}\left(cx\sqrt{\frac{1-cx}{cx+1}}\sqrt{c^2d+e} + \sqrt{\frac{1-cx}{cx+1}}\sqrt{c^2d+e} - ic^2\sqrt{dx+\sqrt{e}}\right)}{b(c^2d+2e)(\sqrt{ex-i\sqrt{d}})}\right)}{d(c^2d+e)^{3/2}}}{16e^2} + \frac{b\sqrt{e}(c^2d+2e) \log\left(\frac{16de^{3/2}\sqrt{c^2d+e}\left(cx\sqrt{\frac{1-cx}{cx+1}}\sqrt{c^2d+e} + \sqrt{\frac{1-cx}{cx+1}}\sqrt{c^2d+e} + \sqrt{ex+i\sqrt{d}}\right)}{b(c^2d+2e)(\sqrt{ex+i\sqrt{d}})}\right)}{d(c^2d+e)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcSech[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -((-4*a*d)/(d + e*x^2)^2 + (8*a)/(d + e*x^2) - (2*e*Sqrt[(1 - c*x)/(1 + c*x)]*(b + b*c*x))/((c^2*d + e)*(d + e*x^2)) + (4*b*(d + 2*e*x^2)*ArcSech[c*x])/(d + e*x^2)^2 + (4*b*Log[x])/d - (4*b*Log[1 + Sqrt[(1 - c*x)/(1 + c*x)] + c*x*Sqrt[(1 - c*x)/(1 + c*x)]])/d + (b*Sqrt[e]*(c^2*d + 2*e)*Log[(16*d*e^(3/2)*Sqrt[c^2*d + e]*(Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[(1 - c*x)/(1 + c*x]) + c*Sqrt[c^2*d + e]*x*Sqrt[(1 - c*x)/(1 + c*x]))]/(b*(c^2*d + 2*e)*((-I)*Sqrt[d] + Sqrt[e]*x)))/(d*(c^2*d + e)^(3/2)) + (b*Sqrt[e]*(c^2*d + 2*e)*Log[(16*d*e^(3/2)*Sqrt[c^2*d + e]*(Sqrt[e] + I*c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[(1 - c*x)/(1 + c*x]) + c*Sqrt[c^2*d + e]*x*Sqrt[(1 - c*x)/(1 + c*x]))]/(b*(c^2*d + 2*e)*(I*Sqrt[d] + Sqrt[e]*x)))/(d*(c^2*d + e)^(3/2)))/(16*e^2)
```

Maple [B] time = 0.362, size = 3343, normalized size = 19.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^3,x)
```

```
[Out] -1/2*c^2*a/e^2/(c^2*e*x^2+c^2*d)+1/4*c^4*a/e^2*d/(c^2*e*x^2+c^2*d)^2-1/2*c^2*b*arcsech(c*x)/e^2/(c^2*e*x^2+c^2*d)+1/4*c^4*b*arcsech(c*x)/e^2*d/(c^2*e*x^2+c^2*d)^2-1/4*c^7*b*(-(c*x-1)/c/x)^(1/2)*x^3*((c*x+1)/c/x)^(1/2)*e^2/(-c*x*e+(-c^2*d*e)^(1/2))/(c*x*e+(-c^2*d*e)^(1/2))/((-c^2*d*e)^(1/2)-e)^2/((-c^2*d*e)^(1/2)+e)^2*d/(-c^2*x^2+1)^(1/2)*arctanh(1/(-c^2*x^2+1)^(1/2))-1/4*c^7*b*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*e/(-c*x*e+(-c^2*d*e)^(1/2))/(c*x*e+(-c^2*d*e)^(1/2))/((-c^2*d*e)^(1/2)-e)^2/((-c^2*d*e)^(1/2)+e)^2*d^2/(-c^2*x^2+1)^(1/2)*arctanh(1/(-c^2*x^2+1)^(1/2))+1/16*c^7*b*(-(c*x-1)/c/x)^(1/2)*x^3*((c*x+1)/c/x)^(1/2)*e^2/(-c*x*e+(-c^2*d*e)^(1/2))/(c*x*e+(-c^2*d*e)^(1/2))/((-c^2*d*e)/e)^(1/2)/((-c^2*d*e)^(1/2)-e)^2/((-c^2*d*e)^(1/2)+e)^2*d/(-c^2*x^2+1)^(1/2)*ln(2*((-c^2*x^2+1)^(1/2)*((c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*x*e+(-c^2*d*e)^(1/2)))+1/16*c^7*b*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*e/(-c*x*e+(-c^2*d*e)^(1/2))/(c*x*e+(-c^2*d*e)^(1/2))/((-c^2*d+e)/e)^(1/2)/((-c^2*d*e)^(1/2)-e)^2/((-c^2*d*e)^(1/2)+e)^2*d^2/(-c^2*x^2+1)^(1/2)*ln(2*((-c^2*x^2+1)^(1/2)*((c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(-c*x*e+(-c^2*d*e)^(1/2)))+1/16*c^7*b*(-(c*x-1)/c/x)^(1/2)*x*((c*x+1)/c/x)^(1/2)*e/(-c*x*e+(-c^2*d*e)^(1/2))/(c*x*e+(-c^2*d*e)^(1/2))/((-c^2*d+e)/e)^(1/2)/((-c^2*d*e)^(1/2)-e)^2/((-c^2*d*e)^(1/2)+e)^2*d^2/(-c^2*x^2+1)^(1/2)*ln(2*((-c^2*x^2+1)^(1/2)*((c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(-c*x*e+(-c^2*d*e)^(1/2)))-1/2*c^5*b*(-(c*x-1)/c/x)^(1/2)*x^3*((c*x+1)/c/x)^(1/2)*e^3/(-c*x*e+(-c^2*d*e)^(1/2))/(c*x*e+(-c^2*d*e)^(1/2))/((-c^2*d*e
```


$$\begin{aligned}
&)^{(1/2)} - e)^2 / ((-c^2 d e)^{(1/2)} + e)^2 / (-c^2 x^2 + 1)^{(1/2)} * \operatorname{arctanh}(1 / (-c^2 x^2 + 1)^{(1/2)}) - 1/2 * c^5 b * (-c x - 1) / c x)^{(1/2)} * x * ((c x + 1) / c x)^{(1/2)} * e^2 / (-c x * e + (-c^2 d e)^{(1/2)}) / (c x * e + (-c^2 d e)^{(1/2)}) / ((-c^2 d e)^{(1/2)} - e)^2 / ((-c^2 d e)^{(1/2)} + e)^2 * d / (-c^2 x^2 + 1)^{(1/2)} * \operatorname{arctanh}(1 / (-c^2 x^2 + 1)^{(1/2)}) - 1/8 * c^5 b * (-c x - 1) / c x)^{(1/2)} * x * ((c x + 1) / c x)^{(1/2)} * e^2 / (-c x * e + (-c^2 d e)^{(1/2)}) / (c x * e + (-c^2 d e)^{(1/2)}) / ((-c^2 d e)^{(1/2)} - e)^2 / ((-c^2 d e)^{(1/2)} + e)^2 * d + 3/16 * c^5 b * (-c x - 1) / c x)^{(1/2)} * x^3 * ((c x + 1) / c x)^{(1/2)} * e^3 / (-c x * e + (-c^2 d e)^{(1/2)}) / (c x * e + (-c^2 d e)^{(1/2)}) / ((c^2 d e) / e)^{(1/2)} / ((-c^2 d e)^{(1/2)} - e)^2 / ((-c^2 d e)^{(1/2)} + e)^2 / (-c^2 x^2 + 1)^{(1/2)} * \ln(2 * ((-c^2 x^2 + 1)^{(1/2)} * ((c^2 d e) / e)^{(1/2)} * e + (-c^2 d e)^{(1/2)} * c x + e) / (c x * e + (-c^2 d e)^{(1/2)})) + 3/16 * c^5 b * (-c x - 1) / c x)^{(1/2)} * x * ((c x + 1) / c x)^{(1/2)} * e^2 / (-c x * e + (-c^2 d e)^{(1/2)}) / (c x * e + (-c^2 d e)^{(1/2)}) / ((c^2 d e) / e)^{(1/2)} / ((-c^2 d e)^{(1/2)} - e)^2 / ((-c^2 d e)^{(1/2)} + e)^2 * d / (-c^2 x^2 + 1)^{(1/2)} * \ln(2 * ((-c^2 x^2 + 1)^{(1/2)} * ((c^2 d e) / e)^{(1/2)} * e + (-c^2 d e)^{(1/2)} * c x + e) / (c x * e + (-c^2 d e)^{(1/2)})) + 3/16 * c^5 b * (-c x - 1) / c x)^{(1/2)} * x^3 * ((c x + 1) / c x)^{(1/2)} * e^3 / (-c x * e + (-c^2 d e)^{(1/2)}) / (c x * e + (-c^2 d e)^{(1/2)}) / ((c^2 d e) / e)^{(1/2)} / ((-c^2 d e)^{(1/2)} - e)^2 / ((-c^2 d e)^{(1/2)} + e)^2 / (-c^2 x^2 + 1)^{(1/2)} * \ln(2 * (-(-c^2 x^2 + 1)^{(1/2)} * ((c^2 d e) / e)^{(1/2)} * e + (-c^2 d e)^{(1/2)} * c x - e) / (-c x * e + (-c^2 d e)^{(1/2)})) + 3/16 * c^5 b * (-c x - 1) / c x)^{(1/2)} * x * ((c x + 1) / c x)^{(1/2)} * e^2 / (-c x * e + (-c^2 d e)^{(1/2)}) / (c x * e + (-c^2 d e)^{(1/2)}) / ((c^2 d e) / e)^{(1/2)} / ((-c^2 d e)^{(1/2)} - e)^2 / ((-c^2 d e)^{(1/2)} + e)^2 * d / (-c^2 x^2 + 1)^{(1/2)} * \ln(2 * (-(-c^2 x^2 + 1)^{(1/2)} * ((c^2 d e) / e)^{(1/2)} * e + (-c^2 d e)^{(1/2)} * c x - e) / (-c x * e + (-c^2 d e)^{(1/2)})) - 1/4 * c^3 b * (-c x - 1) / c x)^{(1/2)} * x^3 * ((c x + 1) / c x)^{(1/2)} * e^4 / (-c x * e + (-c^2 d e)^{(1/2)}) / (c x * e + (-c^2 d e)^{(1/2)}) / ((-c^2 d e)^{(1/2)} - e)^2 / ((-c^2 d e)^{(1/2)} + e)^2 * d / (-c^2 x^2 + 1)^{(1/2)} * \operatorname{arctanh}(1 / (-c^2 x^2 + 1)^{(1/2)}) - 1/4 * c^3 b * (-c x - 1) / c x)^{(1/2)} * x * ((c x + 1) / c x)^{(1/2)} * e^3 / (-c x * e + (-c^2 d e)^{(1/2)}) / (c x * e + (-c^2 d e)^{(1/2)}) / ((-c^2 d e)^{(1/2)} - e)^2 / ((-c^2 d e)^{(1/2)} + e)^2 / (-c^2 x^2 + 1)^{(1/2)} * \operatorname{arctanh}(1 / (-c^2 x^2 + 1)^{(1/2)}) - 1/8 * c^3 b * (-c x - 1) / c x)^{(1/2)} * x * ((c x + 1) / c x)^{(1/2)} * e^3 / (-c x * e + (-c^2 d e)^{(1/2)}) / (c x * e + (-c^2 d e)^{(1/2)}) / ((-c^2 d e)^{(1/2)} - e)^2 / ((-c^2 d e)^{(1/2)} + e)^2 + 1/8 * c^3 b * (-c x - 1) / c x)^{(1/2)} * x^3 * ((c x + 1) / c x)^{(1/2)} * e^4 / (-c x * e + (-c^2 d e)^{(1/2)}) / (c x * e + (-c^2 d e)^{(1/2)}) / ((c^2 d e) / e)^{(1/2)} / ((-c^2 d e)^{(1/2)} - e)^2 / ((-c^2 d e)^{(1/2)} + e)^2 * d / (-c^2 x^2 + 1)^{(1/2)} * \ln(2 * ((-c^2 x^2 + 1)^{(1/2)} * ((c^2 d e) / e)^{(1/2)} * e + (-c^2 d e)^{(1/2)} * c x + e) / (c x * e + (-c^2 d e)^{(1/2)})) + 1/8 * c^3 b * (-c x - 1) / c x)^{(1/2)} * x * ((c x + 1) / c x)^{(1/2)} * e^3 / (-c x * e + (-c^2 d e)^{(1/2)}) / (c x * e + (-c^2 d e)^{(1/2)}) / ((c^2 d e) / e)^{(1/2)} / ((-c^2 d e)^{(1/2)} - e)^2 / ((-c^2 d e)^{(1/2)} + e)^2 / (-c^2 x^2 + 1)^{(1/2)} * \ln(2 * (-(-c^2 x^2 + 1)^{(1/2)} * ((c^2 d e) / e)^{(1/2)} * e + (-c^2 d e)^{(1/2)} * c x - e) / (-c x * e + (-c^2 d e)^{(1/2)})) + 1/8 * c^3 b * (-c x - 1) / c x)^{(1/2)} * x * ((c x + 1) / c x)^{(1/2)} * e^3 / (-c x * e + (-c^2 d e)^{(1/2)}) / (c x * e + (-c^2 d e)^{(1/2)}) / ((c^2 d e) / e)^{(1/2)} / ((-c^2 d e)^{(1/2)} - e)^2 / ((-c^2 d e)^{(1/2)} + e)^2 / (-c^2 x^2 + 1)^{(1/2)} * \ln(2 * (-(-c^2 x^2 + 1)^{(1/2)} * ((c^2 d e) / e)^{(1/2)} * e + (-c^2 d e)^{(1/2)} * c x - e) / (-c x * e + (-c^2 d e)^{(1/2)}))
\end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="maxima")

[Out] Exception raised: RuntimeError

Fricas [B] time = 2.97766, size = 2743, normalized size = 15.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="fricas")

[Out] [-1/16*(4*a*c^4*d^4 + 2*(4*a - b)*c^2*d^3*e + 2*(2*a - b)*d^2*e^2 - 2*(b*c^2*d*e^3 + b*e^4)*x^4 + 4*(2*a*c^4*d^3*e + (4*a - b)*c^2*d^2*e^2 + (2*a - b)*d*e^3)*x^2 - (b*c^2*d^3 + (b*c^2*d*e^2 + 2*b*e^3)*x^4 + 2*b*d^2*e + 2*(b*c^2*d^2*e + 2*b*d*e^2)*x^2)*sqrt(c^2*d*e + e^2)*log((c^4*d^2 + 4*c^2*d*e - (c^4*d*e + 2*c^2*e^2)*x^2 + 4*(c^3*d*e + c*e^2)*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))) + 4*e^2 + 2*(c^2*e*x^2 - c^2*d - (c^3*d + 2*c*e)*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))) - 2*e)*sqrt(c^2*d*e + e^2))/(e*x^2 + d) + 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - 2*((b*c^3*d^2*e^2 + b*c*d*e^3)*x^3 + (b*c^3*d^3*e + b*c*d^2*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^4*d^5*e^2 + 2*c^2*d^4*e^3 + d^3*e^4 + (c^4*d^3*e^4 + 2*c^2*d^2*e^5 + d*e^6)*x^4 + 2*(c^4*d^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^2), -1/8*(2*a*c^4*d^4 + (4*a - b)*c^2*d^3*e + (2*a - b)*d^2*e^2 - (b*c^2*d*e^3 + b*e^4)*x^4 + 2*(2*a*c^4*d^3*e + (4*a - b)*c^2*d^2*e^2 + (2*a - b)*d*e^3)*x^2 + (b*c^2*d^3 + (b*c^2*d*e^2 + 2*b*e^3)*x^4 + 2*b*d^2*e + 2*(b*c^2*d^2*e + 2*b*d*e^2)*x^2)*sqrt(-c^2*d*e - e^2)*arctan((sqrt(-c^2*d*e - e^2)*c*d*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - sqrt(-c^2*d*e - e^2)*(e*x^2 + d))/((c^2*d*e + e^2)*x^2)) + 2*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + 2*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - ((b*c^3*d^2*e^2 + b*c*d*e^3)*x^3 + (b*c^3*d^3*e + b*c*d^2*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^4*d^5*e^2 + 2*c^2*d^4*e^3 + d^3*e^4 + (c^4*d^3*e^4 + 2*c^2*d^2*e^5 + d*e^6)*x^4 + 2*(c^4*d^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^2)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(a+b*asech(c*x))/(e*x**2+d)**3,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsech}(cx) + a)x^3}{(ex^2 + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x^3/(e*x^2 + d)^3, x)
```

$$3.125 \quad \int \frac{x(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^3} dx$$

Optimal. Leaf size=217

$$-\frac{a+b\operatorname{sech}^{-1}(cx)}{4e(d+ex^2)^2} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{4d^2e} - \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(3c^2d+2e)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{c^2d+e}}\right)}{8d^2\sqrt{e}(c^2d+e)^{3/2}} - \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}}{8d(c^2d+e)^{3/2}}$$

[Out] $-(b*\operatorname{Sqrt}[(1+c*x)^{-1}]*\operatorname{Sqrt}[1+c*x]*\operatorname{Sqrt}[1-c^2*x^2])/(8*d*(c^2*d+e)*(d+e*x^2)) - (a+b*\operatorname{ArcSech}[c*x])/(4*e*(d+e*x^2)^2) + (b*\operatorname{Sqrt}[(1+c*x)^{-1}]*\operatorname{Sqrt}[1+c*x]*\operatorname{ArcTanh}[\operatorname{Sqrt}[1-c^2*x^2]])/(4*d^2*e) - (b*(3*c^2*d+2*e)*\operatorname{Sqrt}[(1+c*x)^{-1}]*\operatorname{Sqrt}[1+c*x]*\operatorname{ArcTanh}[(\operatorname{Sqrt}[e]*\operatorname{Sqrt}[1-c^2*x^2])/\operatorname{Sqrt}[c^2*d+e]])/(8*d^2*\operatorname{Sqrt}[e]*(c^2*d+e)^{(3/2)})$

Rubi [A] time = 0.292666, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {6299, 517, 446, 103, 156, 63, 208}

$$-\frac{a+b\operatorname{sech}^{-1}(cx)}{4e(d+ex^2)^2} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{4d^2e} - \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(3c^2d+2e)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{c^2d+e}}\right)}{8d^2\sqrt{e}(c^2d+e)^{3/2}} - \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}}{8d(c^2d+e)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(x*(a+b*\operatorname{ArcSech}[c*x]))/(d+e*x^2)^3, x]$

[Out] $-(b*\operatorname{Sqrt}[(1+c*x)^{-1}]*\operatorname{Sqrt}[1+c*x]*\operatorname{Sqrt}[1-c^2*x^2])/(8*d*(c^2*d+e)*(d+e*x^2)) - (a+b*\operatorname{ArcSech}[c*x])/(4*e*(d+e*x^2)^2) + (b*\operatorname{Sqrt}[(1+c*x)^{-1}]*\operatorname{Sqrt}[1+c*x]*\operatorname{ArcTanh}[\operatorname{Sqrt}[1-c^2*x^2]])/(4*d^2*e) - (b*(3*c^2*d+2*e)*\operatorname{Sqrt}[(1+c*x)^{-1}]*\operatorname{Sqrt}[1+c*x]*\operatorname{ArcTanh}[(\operatorname{Sqrt}[e]*\operatorname{Sqrt}[1-c^2*x^2])/\operatorname{Sqrt}[c^2*d+e]])/(8*d^2*\operatorname{Sqrt}[e]*(c^2*d+e)^{(3/2)})$

Rule 6299

$\operatorname{Int}[(a + \operatorname{ArcSech}[c*x])*(d + e*x^2)^p, x] \rightarrow \operatorname{Simp}[(d + e*x^2)^{p+1}*(a + b*\operatorname{ArcSech}[c*x])/(2*e*(p+1)), x] + \operatorname{Dist}[(b*\operatorname{Sqrt}[1+c*x]*\operatorname{Sqrt}[1/(1+c*x)])/(2*e*(p+1)), \operatorname{Int}[(d + e*x^2)^{p+1}/(x*\operatorname{Sqrt}[1-c*x]*\operatorname{Sqrt}[1+c*x]), x], x] /;$ FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]

Rule 517

$\operatorname{Int}[(u + (c + d*x^n)^q)^p, x] \rightarrow \operatorname{Int}[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x] /;$ FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]))

Rule 446

$\operatorname{Int}[(x + (a + b*x^n)^q)^p, x] \rightarrow \operatorname{Dist}[1/n, \operatorname{Subst}[\operatorname{Int}[x^{(Simplify[(m+1)/n]-1)}*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /;$ FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[

$b*c - a*d, 0]$ && IntegerQ[Simplify[(m + 1)/n]]

Rule 103

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])

Rule 156

Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
\int \frac{x(a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^3} dx &= -\frac{a + b \operatorname{sech}^{-1}(cx)}{4e(d + ex^2)^2} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}(d+ex^2)^2} dx}{4e} \\
&= -\frac{a + b \operatorname{sech}^{-1}(cx)}{4e(d + ex^2)^2} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{1}{x\sqrt{1-c^2x^2}(d+ex^2)^2} dx}{4e} \\
&= -\frac{a + b \operatorname{sech}^{-1}(cx)}{4e(d + ex^2)^2} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-c^2x}(d+ex^2)} dx, x, x^2\right)}{8e} \\
&= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{8d(c^2d + e)(d + ex^2)} - \frac{a + b \operatorname{sech}^{-1}(cx)}{4e(d + ex^2)^2} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \operatorname{Subst}\left(\int \frac{c^2d+e-\frac{1}{2}c^2ex}{x\sqrt{1-c^2x}(d+ex)} dx, x, x\right)}{8de(c^2d + e)} \\
&= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{8d(c^2d + e)(d + ex^2)} - \frac{a + b \operatorname{sech}^{-1}(cx)}{4e(d + ex^2)^2} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-c^2x}} dx, x, x\right)}{8d^2e} \\
&= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{8d(c^2d + e)(d + ex^2)} - \frac{a + b \operatorname{sech}^{-1}(cx)}{4e(d + ex^2)^2} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \operatorname{Subst}\left(\int \frac{1}{\frac{1}{c^2}-x^2} dx, x, x\right)}{4c^2d^2e} \\
&= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{8d(c^2d + e)(d + ex^2)} - \frac{a + b \operatorname{sech}^{-1}(cx)}{4e(d + ex^2)^2} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx} \tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{4d^2e} - \frac{b}{4d^2e}
\end{aligned}$$

Mathematica [C] time = 1.02215, size = 486, normalized size = 2.24

$$\frac{1}{16} \left(\frac{4a}{e(d + ex^2)^2} - \frac{b(3c^2d + 2e) \log\left(\frac{16d^2\sqrt{e}\sqrt{c^2d+e}\left(cx\sqrt{\frac{1-cx}{cx+1}}\sqrt{c^2d+e} + \sqrt{\frac{1-cx}{cx+1}}\sqrt{c^2d+e} - ic^2\sqrt{dx+\sqrt{e}}\right)}{b(3c^2d+2e)(\sqrt{ex-i\sqrt{d}})}\right)}{d^2\sqrt{e}(c^2d + e)^{3/2}} - \frac{b(3c^2d + 2e) \log\left(\frac{16d^2\sqrt{e}\sqrt{c^2d+e}}{d^2\sqrt{e}(c^2d + e)^{3/2}}\right)}{d^2\sqrt{e}(c^2d + e)^{3/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x*(a + b*ArcSech[c*x]))/(d + e*x^2)^3, x]

[Out] ((-4*a)/(e*(d + e*x^2)^2) - (2*sqrt[(1 - c*x)/(1 + c*x)]*(b + b*c*x))/(d*(c^2*d + e)*(d + e*x^2)) - (4*b*ArcSech[c*x])/(e*(d + e*x^2)^2) - (4*b*Log[x])/(d^2*e) + (4*b*Log[1 + sqrt[(1 - c*x)/(1 + c*x)] + c*x*sqrt[(1 - c*x)/(1 + c*x])])/(d^2*e) - (b*(3*c^2*d + 2*e)*Log[(16*d^2*sqrt[e]*sqrt[c^2*d + e]*(sqrt[e] - I*c^2*sqrt[d]*x + sqrt[c^2*d + e]*sqrt[(1 - c*x)/(1 + c*x)] + c*sqrt[c^2*d + e]*x*sqrt[(1 - c*x)/(1 + c*x])])/(b*(3*c^2*d + 2*e)*((-I)*sqrt[d] + sqrt[e]*x)))/(d^2*sqrt[e]*(c^2*d + e)^(3/2)) - (b*(3*c^2*d + 2*e)*Log[(16*d^2*sqrt[e]*sqrt[c^2*d + e]*(sqrt[e] + I*c^2*sqrt[d]*x + sqrt[c^2*d + e]*sqrt[(1 - c*x)/(1 + c*x)] + c*sqrt[c^2*d + e]*x*sqrt[(1 - c*x)/(1 + c*x])])/(b*(3*c^2*d + 2*e)*(I*sqrt[d] + sqrt[e]*x)))/(d^2*sqrt[e]*(c^2*d + e)^(3/2)))/16

Maple [B] time = 0.354, size = 3301, normalized size = 15.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x*(a+b*\text{arcsech}(c*x))/(e*x^2+d)^3,x)$

[Out]
$$\begin{aligned} & -1/4*c^4*a/e/(c^2*e*x^2+c^2*d)^2-1/4*c^4*b/e/(c^2*e*x^2+c^2*d)^2*\text{arcsech}(c*x) \\ & -1/4*c^7*b*(-(c*x-1)/c/x)^{(1/2)}*x^3*((c*x+1)/c/x)^{(1/2)}*e^3/(-c*x*e+(-c^2*d*e)^{(1/2)}) \\ & /((c*x*e+(-c^2*d*e)^{(1/2)})/((-c^2*d*e)^{(1/2)}-e)^2/((-c^2*d*e)^{(1/2)}+e)^2/(-c^2*x^2+1)^{(1/2)} \\ & *\text{arctanh}(1/(-c^2*x^2+1)^{(1/2)})-1/4*c^7*b*(-(c*x-1)/c/x)^{(1/2)}*x*((c*x+1)/c/x)^{(1/2)}*e^2 \\ & /(-c*x*e+(-c^2*d*e)^{(1/2)})/((c*x*e+(-c^2*d*e)^{(1/2)})/((-c^2*d*e)^{(1/2)}-e)^2/((-c^2*d*e)^{(1/2)}+e)^2 \\ & *d/(-c^2*x^2+1)^{(1/2)}*\text{arctanh}(1/(-c^2*x^2+1)^{(1/2)})+3/16*c^7*b*(-(c*x-1)/c/x)^{(1/2)}*x^3 \\ & *((c*x+1)/c/x)^{(1/2)}*e^3/(-c*x*e+(-c^2*d*e)^{(1/2)})/((c*x*e+(-c^2*d*e)^{(1/2)})/((c^2*d+e)/e)^{(1/2)}) \\ & /((-c^2*d*e)^{(1/2)}-e)^2/((-c^2*d*e)^{(1/2)}+e)^2/(-c^2*x^2+1)^{(1/2)}*\ln(2*(-(c^2*x^2+1)^{(1/2)} \\ & *((c^2*d+e)/e)^{(1/2)}*e+(-c^2*d*e)^{(1/2)}*c*x-e)/(-c*x*e+(-c^2*d*e)^{(1/2)})) \\ & +3/16*c^7*b*(-(c*x-1)/c/x)^{(1/2)}*x*((c*x+1)/c/x)^{(1/2)}*e^2/(-c*x*e+(-c^2*d*e)^{(1/2)}) \\ & /((c*x*e+(-c^2*d*e)^{(1/2)})/((c^2*d+e)/e)^{(1/2)})/((-c^2*d*e)^{(1/2)}-e)^2/((-c^2*d*e)^{(1/2)}+e)^2 \\ & *d/(-c^2*x^2+1)^{(1/2)}*\ln(2*(-(c^2*x^2+1)^{(1/2)}*((c^2*d+e)/e)^{(1/2)}*e+(-c^2*d*e)^{(1/2)}*c*x-e) \\ & /(-c*x*e+(-c^2*d*e)^{(1/2)})) \\ & +3/16*c^7*b*(-(c*x-1)/c/x)^{(1/2)}*x^3*((c*x+1)/c/x)^{(1/2)}*e^3/(-c*x*e+(-c^2*d*e)^{(1/2)}) \\ & /((c*x*e+(-c^2*d*e)^{(1/2)})/((c^2*d+e)/e)^{(1/2)})/((-c^2*d*e)^{(1/2)}-e)^2/((-c^2*d*e)^{(1/2)}+e)^2 \\ & /(-c^2*x^2+1)^{(1/2)}*\ln(2*((-c^2*x^2+1)^{(1/2)}*((c^2*d+e)/e)^{(1/2)}*e+(-c^2*d*e)^{(1/2)}*c*x+e) \\ & /(-c*x*e+(-c^2*d*e)^{(1/2)})) \\ & +3/16*c^7*b*(-(c*x-1)/c/x)^{(1/2)}*x*((c*x+1)/c/x)^{(1/2)}*e^2/(-c*x*e+(-c^2*d*e)^{(1/2)}) \\ & /((c*x*e+(-c^2*d*e)^{(1/2)})/((c^2*d+e)/e)^{(1/2)})/((-c^2*d*e)^{(1/2)}-e)^2/((-c^2*d*e)^{(1/2)}+e)^2 \\ & *d/(-c^2*x^2+1)^{(1/2)}*\ln(2*((-c^2*x^2+1)^{(1/2)}*((c^2*d+e)/e)^{(1/2)}*e+(-c^2*d*e)^{(1/2)}*c*x+e) \\ & /(-c*x*e+(-c^2*d*e)^{(1/2)})) \\ & +1/8*c^5*b*(-(c*x-1)/c/x)^{(1/2)}*x*((c*x+1)/c/x)^{(1/2)}*e^3/(-c*x*e+(-c^2*d*e)^{(1/2)}) \\ & /((c*x*e+(-c^2*d*e)^{(1/2)})/((-c^2*d*e)^{(1/2)}-e)^2/((-c^2*d*e)^{(1/2)}+e)^2/(-c^2*x^2+1)^{(1/2)} \\ & *\ln(2*((-c^2*x^2+1)^{(1/2)}*((c^2*d+e)/e)^{(1/2)}*e+(-c^2*d*e)^{(1/2)}*c*x+e) \\ & /(-c*x*e+(-c^2*d*e)^{(1/2)})) \\ & +1/2*c^5*b*(-(c*x-1)/c/x)^{(1/2)}*x^3*((c*x+1)/c/x)^{(1/2)}*e^4/(-c*x*e+(-c^2*d*e)^{(1/2)}) \\ & /((c*x*e+(-c^2*d*e)^{(1/2)})/((-c^2*d*e)^{(1/2)}-e)^2/((-c^2*d*e)^{(1/2)}+e)^2/(-c^2*x^2+1)^{(1/2)} \\ & *\text{arctanh}(1/(-c^2*x^2+1)^{(1/2)}))-1/2*c^5*b*(-(c*x-1)/c/x)^{(1/2)}*x*((c*x+1)/c/x)^{(1/2)}*e^3 \\ & /(-c*x*e+(-c^2*d*e)^{(1/2)})/((c*x*e+(-c^2*d*e)^{(1/2)})/((-c^2*d*e)^{(1/2)}-e)^2/((-c^2*d*e)^{(1/2)}+e)^2 \\ & /(-c^2*x^2+1)^{(1/2)}*\text{arctanh}(1/(-c^2*x^2+1)^{(1/2)}))+5/16*c^5*b*(-(c*x-1)/c/x)^{(1/2)}*x^3 \\ & *((c*x+1)/c/x)^{(1/2)}*e^4/(-c*x*e+(-c^2*d*e)^{(1/2)})/((c*x*e+(-c^2*d*e)^{(1/2)})/((c^2*d+e)/e)^{(1/2)}) \\ & /((-c^2*d*e)^{(1/2)}-e)^2/((-c^2*d*e)^{(1/2)}+e)^2/d/(-c^2*x^2+1)^{(1/2)}*\ln(2*((-c^2*x^2+1)^{(1/2)} \\ & *((c^2*d+e)/e)^{(1/2)}*e+(-c^2*d*e)^{(1/2)}*c*x+e) \\ & /(-c*x*e+(-c^2*d*e)^{(1/2)})) \\ & +5/16*c^5*b*(-(c*x-1)/c/x)^{(1/2)}*x*((c*x+1)/c/x)^{(1/2)}*e^3/(-c*x*e+(-c^2*d*e)^{(1/2)}) \\ & /((c*x*e+(-c^2*d*e)^{(1/2)})/((c^2*d+e)/e)^{(1/2)})/((-c^2*d*e)^{(1/2)}-e)^2/((-c^2*d*e)^{(1/2)}+e)^2 \\ & /(-c^2*x^2+1)^{(1/2)}*\ln(2*((-c^2*x^2+1)^{(1/2)}*((c^2*d+e)/e)^{(1/2)}*e+(-c^2*d*e)^{(1/2)}*c*x+e) \\ & /(-c*x*e+(-c^2*d*e)^{(1/2)})) \\ & +1/8*c^3*b*e^4*(-(c*x-1)/c/x)^{(1/2)}*x*((c*x+1)/c/x)^{(1/2)}/(c*x*e+(-c^2*d*e)^{(1/2)}) \\ & /((-c^2*d*e)^{(1/2)}-e)^2/((-c^2*d*e)^{(1/2)}+e)^2/d-1/4*c^3*b*e^5*(-(c*x-1)/c/x)^{(1/2)}*x^3 \\ & *((c*x+1)/c/x)^{(1/2)}/(c*x*e+(-c^2*d*e)^{(1/2)})/(-c*x*e+(-c^2*d*e)^{(1/2)})/((-c^2*d*e)^{(1/2)}-e)^2 \\ & /((-c^2*d*e)^{(1/2)}+e)^2/d^2/(-c^2*x^2+1)^{(1/2)}*\text{arctanh}(1/(-c^2*x^2+1)^{(1/2)})-1/4*c^3*b*e^4 \\ & *(-c*x-1)/c/x)^{(1/2)}*x*((c*x+1)/c/x)^{(1/2)}/(c*x*e+(-c^2*d*e)^{(1/2)})/(-c*x*e+(-c^2*d*e)^{(1/2)}) \\ & /((-c^2*d*e)^{(1/2)}-e)^2/((-c^2*d*e)^{(1/2)}+e)^2/d/(-c^2*x^2+1)^{(1/2)}*\text{arctanh}(1/(-c^2*x^2+1)^{(1/2)}) \\ & +1/8*c^3*b*e^5*(-(c*x-1)/c/x)^{(1/2)}*x^3*((c*x+1)/c/x)^{(1/2)}/(c*x*e+(-c^2*d*e)^{(1/2)}) \\ & /((c^2*d+e)/e)^{(1/2)}/(-c*x*e+(-c^2*d*e)^{(1/2)})/((-c^2*d*e)^{(1/2)}-e)^2 \end{aligned}$$

$$\frac{2/((-c^2*d*e)^{(1/2)+e)^2/d^2/(-c^2*x^2+1)^{(1/2)*\ln(2*(-(-c^2*x^2+1)^{(1/2)*(c^2*d+e)/e)^{(1/2)*e+(-c^2*d*e)^{(1/2)*c*x-e}/(-c*x*e+(-c^2*d*e)^{(1/2))})+1/8*c^3*b*e^4*(-(c*x-1)/c/x)^{(1/2)*x*((c*x+1)/c/x)^{(1/2)/(c*x*e+(-c^2*d*e)^{(1/2))})/((c^2*d+e)/e)^{(1/2)/(-c*x*e+(-c^2*d*e)^{(1/2))})/((-c^2*d*e)^{(1/2)-e)^2/((-c^2*d*e)^{(1/2)+e)^2/d^2/(-c^2*x^2+1)^{(1/2)*\ln(2*(-(-c^2*x^2+1)^{(1/2)*((c^2*d+e)/e)^{(1/2)*e+(-c^2*d*e)^{(1/2)*c*x-e}/(-c*x*e+(-c^2*d*e)^{(1/2))})+1/8*c^3*b*e^5*(-(c*x-1)/c/x)^{(1/2)*x^3*((c*x+1)/c/x)^{(1/2)/(c*x*e+(-c^2*d*e)^{(1/2))})/((c^2*d+e)/e)^{(1/2)/(-c*x*e+(-c^2*d*e)^{(1/2))})/((-c^2*d*e)^{(1/2)-e)^2/((-c^2*d*e)^{(1/2)+e)^2/d^2/(-c^2*x^2+1)^{(1/2)*\ln(2*((-c^2*x^2+1)^{(1/2)*((c^2*d+e)/e)^{(1/2)*e+(-c^2*d*e)^{(1/2)*c*x+e}/(-c*x*e+(-c^2*d*e)^{(1/2))})+1/8*c^3*b*e^4*(-(c*x-1)/c/x)^{(1/2)*x*((c*x+1)/c/x)^{(1/2)/(c*x*e+(-c^2*d*e)^{(1/2))})/((c^2*d+e)/e)^{(1/2)/(-c*x*e+(-c^2*d*e)^{(1/2))})/((-c^2*d*e)^{(1/2)-e)^2/((-c^2*d*e)^{(1/2)+e)^2/d/(-c^2*x^2+1)^{(1/2)*\ln(2*((-c^2*x^2+1)^{(1/2)*((c^2*d+e)/e)^{(1/2)*e+(-c^2*d*e)^{(1/2)*c*x+e}/(-c*x*e+(-c^2*d*e)^{(1/2))})$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="maxima")

[Out] Exception raised: RuntimeError

Fricas [B] time = 2.89457, size = 2538, normalized size = 11.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/16*(4*a*c^4*d^4 + 2*(4*a + b)*c^2*d^3*e + 2*(2*a + b)*d^2*e^2 + 2*(b*c^2*d*e^3 + b*e^4)*x^4 + 4*(b*c^2*d^2*e^2 + b*d*e^3)*x^2 - (3*b*c^2*d^3 + (3*b*c^2*d*e^2 + 2*b*e^3)*x^4 + 2*b*d^2*e + 2*(3*b*c^2*d^2*e + 2*b*d*e^2)*x^2) \\ & *sqrt(c^2*d*e + e^2)*log((c^4*d^2 + 4*c^2*d*e - (c^4*d*e + 2*c^2*e^2)*x^2 + 4*(c^3*d*e + c*e^2)*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 4*e^2 + 2*(c^2*e*x^2 - c^2*d - (c^3*d + 2*c*e)*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 2*e)*sqrt(c^2*d*e + e^2))/(e*x^2 + d) + 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + 4*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 2*((b*c^3*d^2*e^2 + b*c*d*e^3)*x^3 + (b*c^3*d^3*e + b*c*d^2*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))/(c^4*d^6*e + 2*c^2*d^5*e^2 + d^4*e^3 + (c^4*d^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^4 + 2*(c^4*d^5*e^2 + 2*c^2*d^4*e^3 + d^3*e^4)*x^2), -1/8*(2*a*c^4*d^4 + (4*a + b)*c^2*d^3*e + (2*a + b)*d^2*e^2 + (b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^2*d^2*e^2 + b*d*e^3)*x^2 + (3*b*c^2*d^3 + (3*b*c^2*d*e^2 + 2*b*e^3)*x^4 + 2*b*d^2*e + 2*(3*b*c^2*d^2*e + 2*b*d*e^2)*x^2)*sqrt(-c^2*d*e - e^2)*arctan((sqrt(-c^2*d*e - e^2)*c*d*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - sqrt(-c^2*d*e - e^2)*(e*x^2 + d))/((c^2*d*e + e^2)*x^2)) + 2*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/x) + 2*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + ((b*$$


```
c^3*d^2*e^2 + b*c*d*e^3)*x^3 + (b*c^3*d^3*e + b*c*d^2*e^2)*x)*sqrt(-(c^2*x^
2 - 1)/(c^2*x^2)))/(c^4*d^6*e + 2*c^2*d^5*e^2 + d^4*e^3 + (c^4*d^4*e^3 + 2*
c^2*d^3*e^4 + d^2*e^5)*x^4 + 2*(c^4*d^5*e^2 + 2*c^2*d^4*e^3 + d^3*e^4)*x^2)
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*asech(c*x))/(e*x**2+d)**3,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{ar} \operatorname{sech}(cx) + a)x}{(ex^2 + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)*x/(e*x^2 + d)^3, x)

$$3.126 \quad \int \frac{a+b\operatorname{sech}^{-1}(cx)}{x(d+ex^2)^3} dx$$

Optimal. Leaf size=741

$$\frac{b\operatorname{PolyLog}\left(2, -\frac{c\sqrt{-de}\operatorname{sech}^{-1}(cx)}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3} - \frac{b\operatorname{PolyLog}\left(2, \frac{c\sqrt{-de}\operatorname{sech}^{-1}(cx)}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3} - \frac{b\operatorname{PolyLog}\left(2, -\frac{c\sqrt{-de}\operatorname{sech}^{-1}(cx)}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^3} - \frac{b\operatorname{PolyLog}\left(2, \frac{c\sqrt{-de}\operatorname{sech}^{-1}(cx)}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^3}$$

```
[Out] -(b*e*(c^2 - x^(-2)))/(8*c*d^2*(c^2*d + e)*(e + d/x^2)*Sqrt[-1 + 1/(c*x)]*S
qrt[1 + 1/(c*x)]*x) + (e^2*(a + b*ArcSech[c*x]))/(4*d^3*(e + d/x^2)^2) - (e
*(a + b*ArcSech[c*x]))/(d^3*(e + d/x^2)) + (a + b*ArcSech[c*x])^2/(2*b*d^3)
+ (b*Sqrt[e]*Sqrt[-1 + 1/(c^2*x^2)]*ArcTanh[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqr
t[-1 + 1/(c^2*x^2)]*x)]/(d^3*Sqrt[c^2*d + e]*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1
/(c*x)]) - (b*Sqrt[e]*(c^2*d + 2*e)*Sqrt[-1 + 1/(c^2*x^2)]*ArcTanh[Sqrt[c^2
*d + e]/(c*Sqrt[e]*Sqrt[-1 + 1/(c^2*x^2)]*x)]/(8*d^3*(c^2*d + e)^(3/2)*Sqr
t[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) - ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[
-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^3) - ((a + b*ArcSech
[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2
*d^3) - ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e]
+ Sqrt[c^2*d + e])])/(2*d^3) - ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^
ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d^3) - (b*PolyLog[2, -(c*Sqr
t[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^3) - (b*PolyLog[
2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^3) - (b*P
olyLog[2, -(c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d
^3) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]
)]/(2*d^3)
```

Rubi [A] time = 1.54211, antiderivative size = 741, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6303, 5792, 5788, 519, 382, 377, 208, 5800, 5562, 2190, 2279, 2391}

$$\frac{b\operatorname{PolyLog}\left(2, -\frac{c\sqrt{-de}\operatorname{sech}^{-1}(cx)}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3} - \frac{b\operatorname{PolyLog}\left(2, \frac{c\sqrt{-de}\operatorname{sech}^{-1}(cx)}{\sqrt{e}-\sqrt{c^2d+e}}\right)}{2d^3} - \frac{b\operatorname{PolyLog}\left(2, -\frac{c\sqrt{-de}\operatorname{sech}^{-1}(cx)}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^3} - \frac{b\operatorname{PolyLog}\left(2, \frac{c\sqrt{-de}\operatorname{sech}^{-1}(cx)}{\sqrt{c^2d+e}+\sqrt{e}}\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^3), x]
```

```
[Out] -(b*e*(c^2 - x^(-2)))/(8*c*d^2*(c^2*d + e)*(e + d/x^2)*Sqrt[-1 + 1/(c*x)]*S
qrt[1 + 1/(c*x)]*x) + (e^2*(a + b*ArcSech[c*x]))/(4*d^3*(e + d/x^2)^2) - (e
*(a + b*ArcSech[c*x]))/(d^3*(e + d/x^2)) + (a + b*ArcSech[c*x])^2/(2*b*d^3)
+ (b*Sqrt[e]*Sqrt[-1 + 1/(c^2*x^2)]*ArcTanh[Sqrt[c^2*d + e]/(c*Sqrt[e]*Sqr
t[-1 + 1/(c^2*x^2)]*x)]/(d^3*Sqrt[c^2*d + e]*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1
/(c*x)]) - (b*Sqrt[e]*(c^2*d + 2*e)*Sqrt[-1 + 1/(c^2*x^2)]*ArcTanh[Sqrt[c^2
*d + e]/(c*Sqrt[e]*Sqrt[-1 + 1/(c^2*x^2)]*x)]/(8*d^3*(c^2*d + e)^(3/2)*Sqr
t[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]) - ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[
-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^3) - ((a + b*ArcSech
[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2
*d^3) - ((a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e]
+ Sqrt[c^2*d + e])])/(2*d^3) - ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^
ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d^3) - (b*PolyLog[2, -(c*Sqr
t[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^3) - (b*PolyLog[
2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(2*d^3) - (b*P
olyLog[2, -(c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(2*d
^3) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]
)]/(2*d^3)
```

$$\frac{\text{ArcSech}[c*x]}{(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])}] / (2*d^3) - (b*\text{PolyLog}[2, -((c*\text{Sqrt}[-d]*E^{\text{ArcSech}[c*x]})) / (\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])]) / (2*d^3) - (b*\text{PolyLog}[2, (c*\text{Sqrt}[-d]*E^{\text{ArcSech}[c*x]})) / (\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])]) / (2*d^3) - (b*\text{PolyLog}[2, -((c*\text{Sqrt}[-d]*E^{\text{ArcSech}[c*x]})) / (\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])]) / (2*d^3) - (b*\text{PolyLog}[2, (c*\text{Sqrt}[-d]*E^{\text{ArcSech}[c*x]})) / (\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])]) / (2*d^3)$$

Rule 6303

$$\text{Int}[(a + \text{ArcSech}(c*x))*b]^n * x^m * ((d + e*x^2)^p)^{p+1}, x_{\text{Symbol}} \rightarrow -\text{Subst}[\text{Int}[(e + d*x^2)^p * (a + b*\text{ArcCosh}[x/c])^n] / x^{m+2*(p+1)}, x], x, 1/x] /; \text{FreeQ}\{a, b, c, d, e, n, x\} \&\& \text{IGtQ}[n, 0] \&\& \text{IntegersQ}[m, p]$$

Rule 5792

$$\text{Int}[(a + \text{ArcCosh}(c*x))*b]^n * (f*x)^m * (d + e*x^2)^p, x_{\text{Symbol}} \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*\text{ArcCosh}[c*x])^n * (f*x)^m * (d + e*x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, x\} \&\& \text{NeQ}[c^2*d + e, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{IntegerQ}[p] \&\& \text{IntegerQ}[m]$$

Rule 5788

$$\text{Int}[(a + \text{ArcCosh}(c*x))*b]*x * (d + e*x^2)^p, x_{\text{Symbol}} \rightarrow \text{Simp}[(d + e*x^2)^{p+1} * (a + b*\text{ArcCosh}[c*x]) / (2*e*(p+1)), x] - \text{Dist}[(b*c) / (2*e*(p+1)), \text{Int}[(d + e*x^2)^{p+1} / (\text{Sqrt}[1 + c*x]*\text{Sqrt}[-1 + c*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, p, x\} \&\& \text{NeQ}[c^2*d + e, 0] \&\& \text{NeQ}[p, -1]$$

Rule 519

$$\text{Int}[(u + (c + d*x^n)^q)^p * ((a1 + b1*x^{non2})^{p1} * (a2 + b2*x^{non2})^{p2}), x_{\text{Symbol}} \rightarrow \text{Dist}[(a1 + b1*x^{non2})^{p1} * (a2 + b2*x^{non2})^{p2} / (a1*a2 + b1*b2*x^{non2})^{p1+p2}, \text{Int}[u*(a1*a2 + b1*b2*x^{non2})^p * (c + d*x^n)^q, x], x] /; \text{FreeQ}\{a1, b1, a2, b2, c, d, n, p, q, x\} \&\& \text{EqQ}[non2, n/2] \&\& \text{EqQ}[a2*b1 + a1*b2, 0] \&\& !(\text{EqQ}[n, 2] \&\& \text{IGtQ}[q, 0])$$

Rule 382

$$\text{Int}[(a + b*x^n)^p * (c + d*x^n)^q, x_{\text{Symbol}} \rightarrow -\text{Simp}[(b*x*(a + b*x^n)^{p+1} * (c + d*x^n)^{q+1}) / (a*n*(p+1)*(b*c - a*d)), x] + \text{Dist}[(b*c + n*(p+1)*(b*c - a*d)) / (a*n*(p+1)*(b*c - a*d)), \text{Int}[(a + b*x^n)^{p+1} * (c + d*x^n)^q, x], x] /; \text{FreeQ}\{a, b, c, d, n, q, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[n*(p+q+2) + 1, 0] \&\& (\text{LtQ}[p, -1] || !\text{LtQ}[q, -1]) \&\& \text{NeQ}[p, -1]$$

Rule 377

$$\text{Int}[(a + b*x^n)^p / (c + d*x^n), x_{\text{Symbol}} \rightarrow \text{Subst}[\text{Int}[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^{1/n}] /; \text{FreeQ}\{a, b, c, d, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[n*p + 1, 0] \&\& \text{IntegerQ}[n]$$

Rule 208

$$\text{Int}[(a + b*x^2)^{-1}, x_{\text{Symbol}} \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]]) / a, x] /; \text{FreeQ}\{a, b, x\} \&\& \text{NegQ}[a/b]$$

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^n_]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x]]
]; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_.)*(x_)]*(b_.) + (a_)), x_Symbol]
:> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)), x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)), x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)), x])
]; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol]
:> Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x]
]; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x]
]; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol]
:> -Simp[PolyLog[2, -(c*e*x^n)]/n, x]
]; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + b \operatorname{sech}^{-1}(cx)}{x(d + ex^2)^3} dx &= -\operatorname{Subst} \left(\int \frac{x^5 \left(a + b \cosh^{-1} \left(\frac{x}{c} \right) \right)}{(e + dx^2)^3} dx, x, \frac{1}{x} \right) \\
&= -\operatorname{Subst} \left(\int \left(\frac{e^2 x \left(a + b \cosh^{-1} \left(\frac{x}{c} \right) \right)}{d^2 (e + dx^2)^3} - \frac{2ex \left(a + b \cosh^{-1} \left(\frac{x}{c} \right) \right)}{d^2 (e + dx^2)^2} + \frac{x \left(a + b \cosh^{-1} \left(\frac{x}{c} \right) \right)}{d^2 (e + dx^2)} \right) dx, x \right) \\
&= -\frac{\operatorname{Subst} \left(\int \frac{x^{a+b \cosh^{-1} \left(\frac{x}{c} \right)}}{e+dx^2} dx, x, \frac{1}{x} \right)}{d^2} + \frac{(2e) \operatorname{Subst} \left(\int \frac{x^{a+b \cosh^{-1} \left(\frac{x}{c} \right)}}{(e+dx^2)^2} dx, x, \frac{1}{x} \right)}{d^2} - \frac{e^2 \operatorname{Subst} \left(\int \frac{x^{a+b \cosh^{-1} \left(\frac{x}{c} \right)}}{e+dx^2} dx, x, \frac{1}{x} \right)}{d^2} \\
&= \frac{e^2 (a + b \operatorname{sech}^{-1}(cx))}{4d^3 \left(e + \frac{d}{x^2} \right)^2} - \frac{e (a + b \operatorname{sech}^{-1}(cx))}{d^3 \left(e + \frac{d}{x^2} \right)} - \frac{\operatorname{Subst} \left(\int \left(-\frac{\sqrt{-d} (a+b \cosh^{-1} \left(\frac{x}{c} \right))}{2d(\sqrt{e}-\sqrt{-dx})} + \frac{\sqrt{-d} (a+b \cosh^{-1} \left(\frac{x}{c} \right))}{2d(\sqrt{e}+\sqrt{-dx})} \right) dx, x, \frac{1}{x} \right)}{d^2} \\
&= \frac{e^2 (a + b \operatorname{sech}^{-1}(cx))}{4d^3 \left(e + \frac{d}{x^2} \right)^2} - \frac{e (a + b \operatorname{sech}^{-1}(cx))}{d^3 \left(e + \frac{d}{x^2} \right)} - \frac{\operatorname{Subst} \left(\int \frac{a+b \cosh^{-1} \left(\frac{x}{c} \right)}{\sqrt{e}-\sqrt{-dx}} dx, x, \frac{1}{x} \right)}{2(-d)^{5/2}} + \frac{\operatorname{Subst} \left(\int \frac{a+b \cosh^{-1} \left(\frac{x}{c} \right)}{\sqrt{e}+\sqrt{-dx}} dx, x, \frac{1}{x} \right)}{2(-d)^{5/2}} \\
&= -\frac{be \left(c^2 - \frac{1}{x^2} \right)}{8cd^2 (c^2d + e) \left(e + \frac{d}{x^2} \right) \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}} x} + \frac{e^2 (a + b \operatorname{sech}^{-1}(cx))}{4d^3 \left(e + \frac{d}{x^2} \right)^2} - \frac{e (a + b \operatorname{sech}^{-1}(cx))}{d^3 \left(e + \frac{d}{x^2} \right)} \\
&= -\frac{be \left(c^2 - \frac{1}{x^2} \right)}{8cd^2 (c^2d + e) \left(e + \frac{d}{x^2} \right) \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}} x} + \frac{e^2 (a + b \operatorname{sech}^{-1}(cx))}{4d^3 \left(e + \frac{d}{x^2} \right)^2} - \frac{e (a + b \operatorname{sech}^{-1}(cx))}{d^3 \left(e + \frac{d}{x^2} \right)} \\
&= -\frac{be \left(c^2 - \frac{1}{x^2} \right)}{8cd^2 (c^2d + e) \left(e + \frac{d}{x^2} \right) \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}} x} + \frac{e^2 (a + b \operatorname{sech}^{-1}(cx))}{4d^3 \left(e + \frac{d}{x^2} \right)^2} - \frac{e (a + b \operatorname{sech}^{-1}(cx))}{d^3 \left(e + \frac{d}{x^2} \right)} \\
&= -\frac{be \left(c^2 - \frac{1}{x^2} \right)}{8cd^2 (c^2d + e) \left(e + \frac{d}{x^2} \right) \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}} x} + \frac{e^2 (a + b \operatorname{sech}^{-1}(cx))}{4d^3 \left(e + \frac{d}{x^2} \right)^2} - \frac{e (a + b \operatorname{sech}^{-1}(cx))}{d^3 \left(e + \frac{d}{x^2} \right)} \\
&= -\frac{be \left(c^2 - \frac{1}{x^2} \right)}{8cd^2 (c^2d + e) \left(e + \frac{d}{x^2} \right) \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}} x} + \frac{e^2 (a + b \operatorname{sech}^{-1}(cx))}{4d^3 \left(e + \frac{d}{x^2} \right)^2} - \frac{e (a + b \operatorname{sech}^{-1}(cx))}{d^3 \left(e + \frac{d}{x^2} \right)} \\
&= -\frac{be \left(c^2 - \frac{1}{x^2} \right)}{8cd^2 (c^2d + e) \left(e + \frac{d}{x^2} \right) \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}} x} + \frac{e^2 (a + b \operatorname{sech}^{-1}(cx))}{4d^3 \left(e + \frac{d}{x^2} \right)^2} - \frac{e (a + b \operatorname{sech}^{-1}(cx))}{d^3 \left(e + \frac{d}{x^2} \right)}
\end{aligned}$$

Mathematica [F] time = 62.3782, size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{sech}^{-1}(cx)}{x(d + ex^2)^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^3), x]

[Out] Integrate[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^3), x]

Maple [C] time = 1.322, size = 5713, normalized size = 7.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))/x/(e*x^2+d)^3,x)

[Out] result too large to display

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{4} a \left(\frac{2ex^2 + 3d}{d^2e^2x^4 + 2d^3ex^2 + d^4} - \frac{2 \log(ex^2 + d)}{d^3} + \frac{4 \log(x)}{d^3} \right) + b \int \frac{\log\left(\sqrt{\frac{1}{cx} + 1} \sqrt{\frac{1}{cx} - 1} + \frac{1}{cx}\right)}{e^3x^7 + 3de^2x^5 + 3d^2ex^3 + d^3x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^3,x, algorithm="maxima")

[Out] 1/4*a*((2*e*x^2 + 3*d)/(d^2*e^2*x^4 + 2*d^3*e*x^2 + d^4) - 2*log(e*x^2 + d)/d^3 + 4*log(x)/d^3) + b*integrate(log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x)))/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b \operatorname{ar} \operatorname{sech}(cx) + a}{e^3x^7 + 3de^2x^5 + 3d^2ex^3 + d^3x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^3,x, algorithm="fricas")

[Out] integral((b*arcsech(c*x) + a)/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asech(c*x))/x/(e*x**2+d)**3,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{ar} \operatorname{sech}(cx) + a}{(ex^2 + d)^3 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/((e*x^2 + d)^3*x), x)
```

$$3.127 \quad \int \frac{x^4 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^3} dx$$

Optimal. Leaf size=1272

result too large to display

```
[Out] (b*c*Sqrt[-d]*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])/(16*e^(3/2)*(c^2*d + e)
*(Sqrt[-d]*Sqrt[e] - d/x)) + (b*c*Sqrt[-d]*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c
*x)])/(16*e^(3/2)*(c^2*d + e)*(Sqrt[-d]*Sqrt[e] + d/x)) + (Sqrt[-d]*(a + b*
ArcSech[c*x]))/(16*e^(3/2)*(Sqrt[-d]*Sqrt[e] - d/x)^2) + (3*(a + b*ArcSech[
c*x]))/(16*e^2*(Sqrt[-d]*Sqrt[e] - d/x)) - (Sqrt[-d]*(a + b*ArcSech[c*x]))/
(16*e^(3/2)*(Sqrt[-d]*Sqrt[e] + d/x)^2) - (3*(a + b*ArcSech[c*x]))/(16*e^2*
(Sqrt[-d]*Sqrt[e] + d/x)) - (3*b*ArcTan[(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[
1 + 1/(c*x)])]/(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])))/(8*Sqrt[c
*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*e^2) - (b*d*ArcTan[(Sqr
t[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])]/(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*
Sqrt[-1 + 1/(c*x)])))/(8*(c*d - Sqrt[-d]*Sqrt[e])^(3/2)*(c*d + Sqrt[-d]*Sqr
t[e])^(3/2)*e) - (3*b*ArcTan[(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)
])/ (Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(8*Sqrt[c*d - Sqrt[-
d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*e^2) - (b*d*ArcTan[(Sqrt[c*d + Sqr
t[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])]/(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1
/(c*x)])))/(8*(c*d - Sqrt[-d]*Sqrt[e])^(3/2)*(c*d + Sqrt[-d]*Sqrt[e])^(3/2)
*e) + (3*(a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e]
- Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^(5/2)) - (3*(a + b*ArcSech[c*x])*Log[1
+ (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^
(5/2)) + (3*(a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[
e] + Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^(5/2)) - (3*(a + b*ArcSech[c*x])*Log
[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*Sqrt[-d]
*e^(5/2)) - (3*b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c
^2*d + e]))]/(16*Sqrt[-d]*e^(5/2)) + (3*b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech
[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*Sqrt[-d]*e^(5/2)) - (3*b*PolyLog[2
, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))]/(16*Sqrt[-d]
*e^(5/2)) + (3*b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2
*d + e])])/(16*Sqrt[-d]*e^(5/2))
```

Rubi [A] time = 2.26335, antiderivative size = 1272, normalized size of antiderivative = 1., number of steps used = 35, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {6303, 5707, 5802, 96, 93, 205, 5800, 5562, 2190, 2279, 2391}

$$\frac{b\sqrt{-d}\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}c}}{16e^{3/2}(dc^2+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{b\sqrt{-d}\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}c}}{16e^{3/2}(dc^2+e)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)} + \frac{3(a+b\operatorname{sech}^{-1}(cx))}{16e^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{3(a+b\operatorname{sech}^{-1}(cx))}{16e^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)} + \frac{\sqrt{-d}(a+b\operatorname{sech}^{-1}(cx))}{16e^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (b*c*Sqrt[-d]*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])/(16*e^(3/2)*(c^2*d + e)
*(Sqrt[-d]*Sqrt[e] - d/x)) + (b*c*Sqrt[-d]*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c
*x)])/(16*e^(3/2)*(c^2*d + e)*(Sqrt[-d]*Sqrt[e] + d/x)) + (Sqrt[-d]*(a + b*
ArcSech[c*x]))/(16*e^(3/2)*(Sqrt[-d]*Sqrt[e] - d/x)^2) + (3*(a + b*ArcSech[
c*x]))/(16*e^2*(Sqrt[-d]*Sqrt[e] - d/x)) - (Sqrt[-d]*(a + b*ArcSech[c*x]))/
(16*e^(3/2)*(Sqrt[-d]*Sqrt[e] + d/x)^2) - (3*(a + b*ArcSech[c*x]))/(16*e^2*
```


$$\begin{aligned} & (\text{Sqrt}[-d]\text{Sqrt}[e] + d/x) - (3*b*\text{ArcTan}[(\text{Sqrt}[c*d - \text{Sqrt}[-d]\text{Sqrt}[e]]*\text{Sqrt}[1 + 1/(c*x)])]/(\text{Sqrt}[c*d + \text{Sqrt}[-d]\text{Sqrt}[e]]*\text{Sqrt}[-1 + 1/(c*x)])))/(8*\text{Sqrt}[c*d - \text{Sqrt}[-d]\text{Sqrt}[e]]*\text{Sqrt}[c*d + \text{Sqrt}[-d]\text{Sqrt}[e]]*e^2) - (b*d*\text{ArcTan}[(\text{Sqrt}[c*d - \text{Sqrt}[-d]\text{Sqrt}[e]]*\text{Sqrt}[1 + 1/(c*x)])]/(\text{Sqrt}[c*d + \text{Sqrt}[-d]\text{Sqrt}[e]]*\text{Sqrt}[-1 + 1/(c*x)])))/(8*(c*d - \text{Sqrt}[-d]\text{Sqrt}[e])^(3/2)*(c*d + \text{Sqrt}[-d]\text{Sqrt}[e])^(3/2)*e) - (3*b*\text{ArcTan}[(\text{Sqrt}[c*d + \text{Sqrt}[-d]\text{Sqrt}[e]]*\text{Sqrt}[1 + 1/(c*x)])]/(\text{Sqrt}[c*d - \text{Sqrt}[-d]\text{Sqrt}[e]]*\text{Sqrt}[-1 + 1/(c*x)])))/(8*\text{Sqrt}[c*d - \text{Sqrt}[-d]\text{Sqrt}[e]]*\text{Sqrt}[c*d + \text{Sqrt}[-d]\text{Sqrt}[e]]*e^2) - (b*d*\text{ArcTan}[(\text{Sqrt}[c*d + \text{Sqrt}[-d]\text{Sqrt}[e]]*\text{Sqrt}[1 + 1/(c*x)])]/(\text{Sqrt}[c*d - \text{Sqrt}[-d]\text{Sqrt}[e]]*\text{Sqrt}[-1 + 1/(c*x)])))/(8*(c*d - \text{Sqrt}[-d]\text{Sqrt}[e])^(3/2)*(c*d + \text{Sqrt}[-d]\text{Sqrt}[e])^(3/2)*e) + (3*(a + b*\text{ArcSech}[c*x])*Log[1 - (c*\text{Sqrt}[-d]*E^{\text{ArcSech}[c*x]})/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])])/(16*\text{Sqrt}[-d]*e^(5/2)) - (3*(a + b*\text{ArcSech}[c*x])*Log[1 + (c*\text{Sqrt}[-d]*E^{\text{ArcSech}[c*x]})/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])])/(16*\text{Sqrt}[-d]*e^(5/2)) + (3*(a + b*\text{ArcSech}[c*x])*Log[1 - (c*\text{Sqrt}[-d]*E^{\text{ArcSech}[c*x]})/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])])/(16*\text{Sqrt}[-d]*e^(5/2)) - (3*(a + b*\text{ArcSech}[c*x])*Log[1 + (c*\text{Sqrt}[-d]*E^{\text{ArcSech}[c*x]})/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])])/(16*\text{Sqrt}[-d]*e^(5/2)) - (3*b*\text{PolyLog}[2, -(c*\text{Sqrt}[-d]*E^{\text{ArcSech}[c*x]})/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])])/(16*\text{Sqrt}[-d]*e^(5/2)) + (3*b*\text{PolyLog}[2, (c*\text{Sqrt}[-d]*E^{\text{ArcSech}[c*x]})/(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e])])/(16*\text{Sqrt}[-d]*e^(5/2)) - (3*b*\text{PolyLog}[2, -(c*\text{Sqrt}[-d]*E^{\text{ArcSech}[c*x]})/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])])/(16*\text{Sqrt}[-d]*e^(5/2)) + (3*b*\text{PolyLog}[2, (c*\text{Sqrt}[-d]*E^{\text{ArcSech}[c*x]})/(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e])])/(16*\text{Sqrt}[-d]*e^(5/2)) \end{aligned}$$
Rule 6303

$$\text{Int}[(a + \text{ArcSech}(c*x))(b*x)^n(x)^m((d + e*x^2)^p), x_Symbol] \rightarrow -\text{Subst}[\text{Int}[(e + d*x^2)^p(a + b*\text{ArcCosh}[x/c])^n/x^{m+2(p+1)}, x], x, 1/x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IntegersQ}[m, p]$$
Rule 5707

$$\text{Int}[(a + \text{ArcCosh}(c*x))(b*x)^n((d + e*x^2)^p), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*\text{ArcCosh}[c*x])^n, (d + e*x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x \ \&\& \ \text{NeQ}[c^2*d + e, 0] \ \&\& \ \text{IntegerQ}[p] \ \&\& \ (p > 0 \ || \ \text{IGtQ}[n, 0])$$
Rule 5802

$$\text{Int}[(a + \text{ArcCosh}(c*x))(b*x)^n((d + e*x^2)^m), x_Symbol] \rightarrow \text{Simp}[(d + e*x)^{m+1}(a + b*\text{ArcCosh}[c*x])^n/(e*(m+1)), x] - \text{Dist}[(b*c*n)/(e*(m+1)), \text{Int}[(d + e*x)^{m+1}(a + b*\text{ArcCosh}[c*x])^{n-1})/(\text{Sqrt}[-1 + c*x]*\text{Sqrt}[1 + c*x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, m\}, x \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{NeQ}[m, -1]$$
Rule 96

$$\text{Int}[(a + (b*x)^m)((c + d*x)^n)((e + f*x)^p), x_Symbol] \rightarrow \text{Simp}[(b*(a + b*x)^{m+1}(c + d*x)^{n+1}(e + f*x)^{p+1})/((m+1)*(b*c - a*d)*(b*e - a*f)), x] + \text{Dist}[(a*d*f*(m+1) + b*c*f*(n+1) + b*d*e*(p+1))/((m+1)*(b*c - a*d)*(b*e - a*f)), \text{Int}[(a + b*x)^{m+1}(c + d*x)^n(e + f*x)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, n, p\}, x \ \&\& \ \text{EqQ}[\text{Simplify}[m + n + p + 3], 0] \ \&\& \ (\text{LtQ}[m, -1] \ || \ \text{SumSimplerQ}[m, 1])$$
Rule 93

$$\text{Int}[(a + (b*x)^m)((c + d*x)^n)/((e + f*x)^q), x_Symbol] \rightarrow \text{With}\{q = \text{Denominator}[m]\}, \text{Dist}[q, \text{Subst}[\text{Int}[x^{q*(m+1)}$$

```
- 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5800

```
Int[((a_) + ArcCosh[(c_)*(x_)]*(b_))^(n_)/((d_) + (e_)*(x_)), x_Symbol]
:= Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5562

```
Int[(((e_) + (f_)*(x_))^(m_)*Sinh[(c_) + (d_)*(x_)])/(Cosh[(c_) + (d_
)*(x_)]*(b_) + (a_)), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)*((c_) + (d_)*(x_))^(m_))/
((a_) + (b_)*((F_)^((g_)*((e_) + (f_)*(x_))))^(n_)), x_Symbol] := Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_)*((F_)^((e_)*((c_) + (d_)*(x_))))^(n_)], x_Symbol]
:= Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_)*((d_) + (e_)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^3} dx &= -\operatorname{Subst} \left(\int \frac{a + b \cosh^{-1}\left(\frac{x}{c}\right)}{(e + dx^2)^3} dx, x, \frac{1}{x} \right) \\
&= -\operatorname{Subst} \left(\int \left(-\frac{d^3 (a + b \cosh^{-1}\left(\frac{x}{c}\right))}{8(-d)^{3/2} e^{3/2} (\sqrt{-d}\sqrt{e} - dx)^3} - \frac{3d (a + b \cosh^{-1}\left(\frac{x}{c}\right))}{16e^2 (\sqrt{-d}\sqrt{e} - dx)^2} - \frac{d^3 (a + b \cosh^{-1}\left(\frac{x}{c}\right))}{8(-d)^{3/2} e^{3/2} (\sqrt{-d}\sqrt{e} - dx)} \right) dx, x, \frac{1}{x} \right) \\
&= \frac{(3d) \operatorname{Subst} \left(\int \frac{a + b \cosh^{-1}\left(\frac{x}{c}\right)}{(\sqrt{-d}\sqrt{e} - dx)^2} dx, x, \frac{1}{x} \right)}{16e^2} + \frac{(3d) \operatorname{Subst} \left(\int \frac{a + b \cosh^{-1}\left(\frac{x}{c}\right)}{(\sqrt{-d}\sqrt{e} + dx)^2} dx, x, \frac{1}{x} \right)}{16e^2} + \frac{(3d) \operatorname{Subst} \left(\int \frac{a + b \cosh^{-1}\left(\frac{x}{c}\right)}{\sqrt{-d}\sqrt{e} - dx} dx, x, \frac{1}{x} \right)}{8(-d)^{3/2} e^{3/2}} \\
&= \frac{\sqrt{-d} (a + b \operatorname{sech}^{-1}(cx))}{16e^{3/2} \left(\sqrt{-d}\sqrt{e} - \frac{d}{x}\right)^2} + \frac{3(a + b \operatorname{sech}^{-1}(cx))}{16e^2 \left(\sqrt{-d}\sqrt{e} - \frac{d}{x}\right)} - \frac{\sqrt{-d} (a + b \operatorname{sech}^{-1}(cx))}{16e^{3/2} \left(\sqrt{-d}\sqrt{e} + \frac{d}{x}\right)^2} - \frac{3(a + b \operatorname{sech}^{-1}(cx))}{16e^2 \left(\sqrt{-d}\sqrt{e} + \frac{d}{x}\right)} \\
&= \frac{bc\sqrt{-d}\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}}{16e^{3/2} (c^2d + e) \left(\sqrt{-d}\sqrt{e} - \frac{d}{x}\right)} + \frac{bc\sqrt{-d}\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}}{16e^{3/2} (c^2d + e) \left(\sqrt{-d}\sqrt{e} + \frac{d}{x}\right)} + \frac{\sqrt{-d} (a + b \operatorname{sech}^{-1}(cx))}{16e^{3/2} \left(\sqrt{-d}\sqrt{e} - \frac{d}{x}\right)^2} \\
&= \frac{bc\sqrt{-d}\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}}{16e^{3/2} (c^2d + e) \left(\sqrt{-d}\sqrt{e} - \frac{d}{x}\right)} + \frac{bc\sqrt{-d}\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}}{16e^{3/2} (c^2d + e) \left(\sqrt{-d}\sqrt{e} + \frac{d}{x}\right)} + \frac{\sqrt{-d} (a + b \operatorname{sech}^{-1}(cx))}{16e^{3/2} \left(\sqrt{-d}\sqrt{e} - \frac{d}{x}\right)^2} \\
&= \frac{bc\sqrt{-d}\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}}{16e^{3/2} (c^2d + e) \left(\sqrt{-d}\sqrt{e} - \frac{d}{x}\right)} + \frac{bc\sqrt{-d}\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}}{16e^{3/2} (c^2d + e) \left(\sqrt{-d}\sqrt{e} + \frac{d}{x}\right)} + \frac{\sqrt{-d} (a + b \operatorname{sech}^{-1}(cx))}{16e^{3/2} \left(\sqrt{-d}\sqrt{e} - \frac{d}{x}\right)^2} \\
&= \frac{bc\sqrt{-d}\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}}{16e^{3/2} (c^2d + e) \left(\sqrt{-d}\sqrt{e} - \frac{d}{x}\right)} + \frac{bc\sqrt{-d}\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}}{16e^{3/2} (c^2d + e) \left(\sqrt{-d}\sqrt{e} + \frac{d}{x}\right)} + \frac{\sqrt{-d} (a + b \operatorname{sech}^{-1}(cx))}{16e^{3/2} \left(\sqrt{-d}\sqrt{e} - \frac{d}{x}\right)^2} \\
&= \frac{bc\sqrt{-d}\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}}{16e^{3/2} (c^2d + e) \left(\sqrt{-d}\sqrt{e} - \frac{d}{x}\right)} + \frac{bc\sqrt{-d}\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}}{16e^{3/2} (c^2d + e) \left(\sqrt{-d}\sqrt{e} + \frac{d}{x}\right)} + \frac{\sqrt{-d} (a + b \operatorname{sech}^{-1}(cx))}{16e^{3/2} \left(\sqrt{-d}\sqrt{e} - \frac{d}{x}\right)^2} \\
&= \frac{bc\sqrt{-d}\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}}{16e^{3/2} (c^2d + e) \left(\sqrt{-d}\sqrt{e} - \frac{d}{x}\right)} + \frac{bc\sqrt{-d}\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}}{16e^{3/2} (c^2d + e) \left(\sqrt{-d}\sqrt{e} + \frac{d}{x}\right)} + \frac{\sqrt{-d} (a + b \operatorname{sech}^{-1}(cx))}{16e^{3/2} \left(\sqrt{-d}\sqrt{e} - \frac{d}{x}\right)^2} \\
&= \frac{bc\sqrt{-d}\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}}{16e^{3/2} (c^2d + e) \left(\sqrt{-d}\sqrt{e} - \frac{d}{x}\right)} + \frac{bc\sqrt{-d}\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}}{16e^{3/2} (c^2d + e) \left(\sqrt{-d}\sqrt{e} + \frac{d}{x}\right)} + \frac{\sqrt{-d} (a + b \operatorname{sech}^{-1}(cx))}{16e^{3/2} \left(\sqrt{-d}\sqrt{e} - \frac{d}{x}\right)^2}
\end{aligned}$$

Mathematica [C] time = 6.20064, size = 2022, normalized size = 1.59

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^3,x]

[Out] (a*d*x)/(4*e^2*(d + e*x^2)^2) - (5*a*x)/(8*e^2*(d + e*x^2)) + (3*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*Sqrt[d]*e^(5/2)) + b*(((I/16)*Sqrt[d]*((-I)*Sqrt[e

$$\begin{aligned}
&]*\text{Sqrt}[(1 - c*x)/(1 + c*x)]*(1 + c*x))/(\text{Sqrt}[d]*(c^2*d + e)*((-I)*\text{Sqrt}[d] + \\
& \text{Sqrt}[e]*x)) - \text{ArcSech}[c*x]/(\text{Sqrt}[e]*((-I)*\text{Sqrt}[d] + \text{Sqrt}[e]*x)^2) + \text{Log}[x] \\
& /(\text{d}*\text{Sqrt}[e]) - \text{Log}[1 + \text{Sqrt}[(1 - c*x)/(1 + c*x)] + c*x*\text{Sqrt}[(1 - c*x)/(1 + \\
& c*x)]]/(\text{d}*\text{Sqrt}[e]) + ((2*c^2*d + e)*\text{Log}[(-4*d*\text{Sqrt}[e]*\text{Sqrt}[c^2*d + e]*(\text{Sqrt} \\
& [e] - I*c^2*\text{Sqrt}[d]*x + \text{Sqrt}[c^2*d + e]*\text{Sqrt}[(1 - c*x)/(1 + c*x)] + c*\text{Sqrt}[\\
& c^2*d + e]*x*\text{Sqrt}[(1 - c*x)/(1 + c*x)])]/((2*c^2*d + e)*((-I)*\text{Sqrt}[d] + \text{Sqr} \\
& \text{t}[e]*x)))]/(\text{d}*(c^2*d + e)^(3/2)))/e^2 - ((I/16)*\text{Sqrt}[d]*((I*\text{Sqrt}[e]*\text{Sqrt}[(\\
& 1 - c*x)/(1 + c*x)]*(1 + c*x))/(\text{Sqrt}[d]*(c^2*d + e)*(I*\text{Sqrt}[d] + \text{Sqrt}[e]*x) \\
&) - \text{ArcSech}[c*x]/(\text{Sqrt}[e]*(I*\text{Sqrt}[d] + \text{Sqrt}[e]*x)^2) + \text{Log}[x]/(\text{d}*\text{Sqrt}[e]) - \\
& \text{Log}[1 + \text{Sqrt}[(1 - c*x)/(1 + c*x)] + c*x*\text{Sqrt}[(1 - c*x)/(1 + c*x)]]/(\text{d}*\text{Sqrt} \\
& [e]) + ((2*c^2*d + e)*\text{Log}[(-4*d*\text{Sqrt}[e]*\text{Sqrt}[c^2*d + e]*(\text{Sqrt}[e] + I*c^2*\text{Sqr} \\
& \text{t}[d]*x + \text{Sqrt}[c^2*d + e]*\text{Sqrt}[(1 - c*x)/(1 + c*x)] + c*\text{Sqrt}[c^2*d + e]*x*\text{S} \\
& \text{qrt}[(1 - c*x)/(1 + c*x)])]/((2*c^2*d + e)*(I*\text{Sqrt}[d] + \text{Sqrt}[e]*x)))]/(\text{d}*(c^ \\
& 2*d + e)^(3/2)))/e^2 + (5*(-\text{ArcSech}[c*x]/(I*\text{Sqrt}[d]*\text{Sqrt}[e] + e*x)) + (I* \\
& (\text{Log}[x]/\text{Sqrt}[e] - \text{Log}[1 + \text{Sqrt}[(1 - c*x)/(1 + c*x)] + c*x*\text{Sqrt}[(1 - c*x)/(1 \\
& + c*x)]]/\text{Sqrt}[e] + \text{Log}[(2*I)*\text{Sqrt}[e]*(\text{Sqrt}[d]*\text{Sqrt}[(1 - c*x)/(1 + c*x)]*(\\
& 1 + c*x) + (\text{Sqrt}[d]*\text{Sqrt}[e] + I*c^2*d*x)/\text{Sqrt}[c^2*d + e]))/(I*\text{Sqrt}[d] + \text{Sqr} \\
& \text{t}[e]*x)]/\text{Sqrt}[c^2*d + e]))/\text{Sqrt}[d]))/(16*e^2) + (5*(-\text{ArcSech}[c*x]/((-I)*\text{Sqr} \\
& \text{t}[d]*\text{Sqrt}[e] + e*x)) - (I*(\text{Log}[x]/\text{Sqrt}[e] - \text{Log}[1 + \text{Sqrt}[(1 - c*x)/(1 + c* \\
& x)] + c*x*\text{Sqrt}[(1 - c*x)/(1 + c*x)]]/\text{Sqrt}[e] + \text{Log}[(2*\text{Sqrt}[e]*(I*\text{Sqrt}[d]*\text{Sqr} \\
& \text{t}[(1 - c*x)/(1 + c*x)]*(1 + c*x) + (I*\text{Sqrt}[d]*\text{Sqrt}[e] + c^2*d*x)/\text{Sqrt}[c^2* \\
& d + e]))/((-I)*\text{Sqrt}[d] + \text{Sqrt}[e]*x)]/\text{Sqrt}[c^2*d + e]))/\text{Sqrt}[d]))/(16*e^2) - \\
& (((3*I)/32)*(PolyLog[2, -E^(-2*\text{ArcSech}[c*x])] - 2*((-4*I)*\text{ArcSin}[\text{Sqrt}[1 + \\
& (I*\text{Sqrt}[e])/(c*\text{Sqrt}[d])]]/\text{Sqrt}[2]]*\text{ArcTanh}[(I*c*\text{Sqrt}[d] + \text{Sqrt}[e])*\text{Tanh}[\text{Arc} \\
& \text{Sech}[c*x]/2)]/\text{Sqrt}[c^2*d + e]) + \text{ArcSech}[c*x]*\text{Log}[1 + E^(-2*\text{ArcSech}[c*x])]) \\
& - \text{ArcSech}[c*x]*\text{Log}[1 + (I*(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e]))/(c*\text{Sqrt}[d]*E^{\text{ArcSech} \\
& [c*x]})]) + (2*I)*\text{ArcSin}[\text{Sqrt}[1 + (I*\text{Sqrt}[e])/(c*\text{Sqrt}[d])]]/\text{Sqrt}[2]]*\text{Log}[1 + (\\
& I*(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e]))/(c*\text{Sqrt}[d]*E^{\text{ArcSech}[c*x]})]) - \text{ArcSech}[c*x]*\text{L} \\
& \text{og}[1 + (I*(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e]))/(c*\text{Sqrt}[d]*E^{\text{ArcSech}[c*x]})]) - (2*I)* \\
& \text{ArcSin}[\text{Sqrt}[1 + (I*\text{Sqrt}[e])/(c*\text{Sqrt}[d])]]/\text{Sqrt}[2]]*\text{Log}[1 + (I*(\text{Sqrt}[e] + \text{Sqr} \\
& \text{t}[c^2*d + e]))/(c*\text{Sqrt}[d]*E^{\text{ArcSech}[c*x]})]) + \text{PolyLog}[2, (I*(-\text{Sqrt}[e] + \text{Sqr} \\
& \text{t}[c^2*d + e]))/(c*\text{Sqrt}[d]*E^{\text{ArcSech}[c*x]})]) + \text{PolyLog}[2, ((-I)*(\text{Sqrt}[e] + \text{Sqr} \\
& \text{t}[c^2*d + e]))/(c*\text{Sqrt}[d]*E^{\text{ArcSech}[c*x]})])])]/(\text{Sqrt}[d]*e^(5/2)) - (((3*I)/3 \\
& 2)*(-\text{PolyLog}[2, -E^(-2*\text{ArcSech}[c*x])] + 2*((-4*I)*\text{ArcSin}[\text{Sqrt}[1 - (I*\text{Sqrt}[e] \\
&)/(c*\text{Sqrt}[d])]]/\text{Sqrt}[2]]*\text{ArcTanh}[(I*(-I)*c*\text{Sqrt}[d] + \text{Sqrt}[e])*\text{Tanh}[\text{ArcSech}[c \\
& *x]/2)]/\text{Sqrt}[c^2*d + e]) + \text{ArcSech}[c*x]*\text{Log}[1 + E^(-2*\text{ArcSech}[c*x])]) - \text{ArcS} \\
& \text{ech}[c*x]*\text{Log}[1 + (I*(-\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e]))/(c*\text{Sqrt}[d]*E^{\text{ArcSech}[c*x] \\
&]})]) + (2*I)*\text{ArcSin}[\text{Sqrt}[1 - (I*\text{Sqrt}[e])/(c*\text{Sqrt}[d])]]/\text{Sqrt}[2]]*\text{Log}[1 + (I*(-\text{S} \\
& \text{qrt}[e] + \text{Sqrt}[c^2*d + e]))/(c*\text{Sqrt}[d]*E^{\text{ArcSech}[c*x]})]) - \text{ArcSech}[c*x]*\text{Log}[1 \\
& - (I*(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e]))/(c*\text{Sqrt}[d]*E^{\text{ArcSech}[c*x]})]) - (2*I)*\text{ArcS} \\
& \text{in}[\text{Sqrt}[1 - (I*\text{Sqrt}[e])/(c*\text{Sqrt}[d])]]/\text{Sqrt}[2]]*\text{Log}[1 - (I*(\text{Sqrt}[e] + \text{Sqrt}[c^ \\
& 2*d + e]))/(c*\text{Sqrt}[d]*E^{\text{ArcSech}[c*x]})]) + \text{PolyLog}[2, (I*(\text{Sqrt}[e] - \text{Sqrt}[c^2*d \\
& + e]))/(c*\text{Sqrt}[d]*E^{\text{ArcSech}[c*x]})]) + \text{PolyLog}[2, (I*(\text{Sqrt}[e] + \text{Sqrt}[c^2*d \\
& + e]))/(c*\text{Sqrt}[d]*E^{\text{ArcSech}[c*x]})])])]/(\text{Sqrt}[d]*e^(5/2))
\end{aligned}$$

Maple [C] time = 3.527, size = 3455, normalized size = 2.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^4*(a+b*\text{arcsech}(c*x))/(e*x^2+d)^3, x)$

[Out] $-3/8*c^4*a/(c^2*e*x^2+c^2*d)^2/e^2*d*x-3/4*b*((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*\arctan(c*d*(1/c/x+(-1+1/c/x)^(1/2))*(1+1/c/x)^(1/2))/((c^2*d+2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e/(c^2*d+e)^2/d+3/8*b*(-(c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2)*\text{arctanh}(c*d*(1/c/x+(-1+1/c/x)^(1/2))*(1+1/c/x)^(1/2))/((c^2*d-2*(e*(c^2*d+e))^(1/2)+2*e)*d)^(1/2))/e/(c^2*d+e)^2/d$

$$\begin{aligned}
& 1/2)/((-c^2*d+2*(e*(c^2*d+e))^{1/2}-2*e)*d)^{1/2})/e^2/(c^2*d+e)/d+3/8*a/e \\
& ^2/(d*e)^{1/2}*arctan(x*e/(d*e)^{1/2})-3/4*b*(-(c^2*d-2*(e*(c^2*d+e))^{1/2} \\
& +2*e)*d)^{1/2}*arctanh(c*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))/((-c^2* \\
& d+2*(e*(c^2*d+e))^{1/2}-2*e)*d)^{1/2})/e/(c^2*d+e)^2/d+3/8*b*((c^2*d+2*(e*(\\
& c^2*d+e))^{1/2}+2*e)*d)^{1/2}*arctan(c*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{ \\
& 1/2}))/((c^2*d+2*(e*(c^2*d+e))^{1/2}+2*e)*d)^{1/2})/e^2/(c^2*d+e)/d-5/8*c^4 \\
& *b*x^3/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*arcsech(c*x)-3/16*c^3*b/e^2/(c^2*d+e)* \\
& d*sum(_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln((_R1-1/c/x-(-1+1/c/x)^{1/2} \\
& *(1+1/c/x)^{1/2}))/_R1+dilog((_R1-1/c/x-(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2} \\
&)/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^2*d))-7/4/c^2*b*(-(c^2*d \\
& -2*(e*(c^2*d+e))^{1/2}+2*e)*d)^{1/2}*arctanh(c*d*(1/c/x+(-1+1/c/x)^{1/2}*(1 \\
& +1/c/x)^{1/2}))/((-c^2*d+2*(e*(c^2*d+e))^{1/2}-2*e)*d)^{1/2})/(c^2*d+e)^2/d^ \\
& 2+3/16*c^3*b/e^2/(c^2*d+e)*d*sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x) \\
&)*ln((_R1-1/c/x-(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))/_R1+dilog((_R1-1/c/x-(-1 \\
& +1/c/x)^{1/2}*(1+1/c/x)^{1/2}))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z \\
& ^2+c^2*d))+1/c^4*b*((c^2*d+2*(e*(c^2*d+e))^{1/2}+2*e)*d)^{1/2}*arctan(c*d*(\\
& 1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))/((c^2*d+2*(e*(c^2*d+e))^{1/2}+2*e)* \\
& d)^{1/2})/(c^2*d+e)/d^3-7/4/c^2*b*((c^2*d+2*(e*(c^2*d+e))^{1/2}+2*e)*d)^{1/2} \\
& *arctan(c*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))/((c^2*d+2*(e*(c^2*d+ \\
& e))^{1/2}+2*e)*d)^{1/2})/(c^2*d+e)^2/d^2+1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^{1/2} \\
& +2*e)*d)^{1/2}*arctanh(c*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))/((- \\
& c^2*d+2*(e*(c^2*d+e))^{1/2}-2*e)*d)^{1/2})/(c^2*d+e)/d^3-3/4/c^2*b*((c^2*d+ \\
& 2*(e*(c^2*d+e))^{1/2}+2*e)*d)^{1/2}*arctan(c*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1 \\
& /c/x)^{1/2}))/((c^2*d+2*(e*(c^2*d+e))^{1/2}+2*e)*d)^{1/2})/e^2/(c^2*d+e)/d^2 \\
& *(e*(c^2*d+e))^{1/2}+3/4/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e)*d)^{1/2} \\
& *arctanh(c*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))/((-c^2*d+2*(e*(c^2*d+ \\
& e))^{1/2}-2*e)*d)^{1/2})/e^2/(c^2*d+e)/d^2*(e*(c^2*d+e))^{1/2}+1/c^4*b*(-(c \\
& ^2*d-2*(e*(c^2*d+e))^{1/2}+2*e)*d)^{1/2}*arctanh(c*d*(1/c/x+(-1+1/c/x)^{1/2} \\
&)*(1+1/c/x)^{1/2}))/((-c^2*d+2*(e*(c^2*d+e))^{1/2}-2*e)*d)^{1/2})/e/(c^2*d+e \\
&)/d^3*(e*(c^2*d+e))^{1/2}-5/4/c^2*b*(-(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e)*d)^{ \\
& 1/2}*arctanh(c*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))/((-c^2*d+2*(e*(c \\
& ^2*d+e))^{1/2}-2*e)*d)^{1/2})/(c^2*d+e)^2/e/d^2*(e*(c^2*d+e))^{1/2}-5/8*c^6 \\
& *b*x^3/e/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*arcsech(c*x)*d-3/8*c^6*b*x/e^2/(c^2* \\
& d+e)/(c^2*e*x^2+c^2*d)^2*arcsech(c*x)*d^2-5/8*c^4*a/(c^2*e*x^2+c^2*d)^2*x^3 \\
& /e+3/16*c*b/e/(c^2*d+e)*sum(1/_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln(\\
& (_R1-1/c/x-(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))/_R1+dilog((_R1-1/c/x-(-1+1/c/ \\
& x)^{1/2}*(1+1/c/x)^{1/2}))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c^ \\
& 2*d))-3/16*c*b/e/(c^2*d+e)*sum(_R1/(_R1^2*c^2*d+c^2*d+2*e)*(arcsech(c*x)*ln \\
& ((_R1-1/c/x-(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))/_R1+dilog((_R1-1/c/x-(-1+1/c \\
& /x)^{1/2}*(1+1/c/x)^{1/2}))/_R1)),_R1=RootOf(c^2*d*_Z^4+(2*c^2*d+4*e)*_Z^2+c \\
& ^2*d))-1/c^4*b*(-(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e)*d)^{1/2}*arctanh(c*d*(1/ \\
& c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))/((-c^2*d+2*(e*(c^2*d+e))^{1/2}-2*e)*d \\
&)^{1/2})/(c^2*d+e)^2*e/d^3+1/c^4*b*((c^2*d+2*(e*(c^2*d+e))^{1/2}+2*e)*d)^{1/2} \\
& *arctan(c*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))/((c^2*d+2*(e*(c^2*d \\
& +e))^{1/2}+2*e)*d)^{1/2})/(c^2*d+e)^2/d^3*(e*(c^2*d+e))^{1/2}-1/c^4*b*(-(c^ \\
& 2*d-2*(e*(c^2*d+e))^{1/2}+2*e)*d)^{1/2}*arctanh(c*d*(1/c/x+(-1+1/c/x)^{1/2} \\
& *(1+1/c/x)^{1/2}))/((-c^2*d+2*(e*(c^2*d+e))^{1/2}-2*e)*d)^{1/2})/(c^2*d+e)^2 \\
& /d^3*(e*(c^2*d+e))^{1/2}+5/4/c^2*b*((c^2*d+2*(e*(c^2*d+e))^{1/2}+2*e)*d)^{1/2} \\
& *arctan(c*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))/((c^2*d+2*(e*(c^2*d \\
& +e))^{1/2}+2*e)*d)^{1/2})/e/(c^2*d+e)/d^2-1/c^4*b*((c^2*d+2*(e*(c^2*d+e))^{1/2} \\
& +2*e)*d)^{1/2}*arctan(c*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))/((c^ \\
& 2*d+2*(e*(c^2*d+e))^{1/2}+2*e)*d)^{1/2})/(c^2*d+e)^2*e/d^3+5/4/c^2*b*(-(c^2 \\
& *d-2*(e*(c^2*d+e))^{1/2}+2*e)*d)^{1/2}*arctanh(c*d*(1/c/x+(-1+1/c/x)^{1/2}* \\
& (1+1/c/x)^{1/2}))/((-c^2*d+2*(e*(c^2*d+e))^{1/2}-2*e)*d)^{1/2})/e/(c^2*d+e)/ \\
& d^2+1/8*c^5*b*x^4/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*(-(c*x-1)/c/x)^{1/2}*((c*x+ \\
& 1)/c/x)^{1/2}-3/8*b*(-(c^2*d-2*(e*(c^2*d+e))^{1/2}+2*e)*d)^{1/2}*arctanh(c* \\
& d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))/((-c^2*d+2*(e*(c^2*d+e))^{1/2}-2 \\
& *e)*d)^{1/2})/e^2/(c^2*d+e)^2/d*(e*(c^2*d+e))^{1/2}+3/8*b*((c^2*d+2*(e*(c^2 \\
& *d+e))^{1/2}+2*e)*d)^{1/2}*arctan(c*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}
\end{aligned}$$

$$\frac{2)}{((c^2d+2*(e*(c^2d+e))^{1/2}+2e)*d)^{1/2}}/e^2/(c^2d+e)^2/d*(e*(c^2d+e))^{1/2}-3/8*c^4*b*x/e/(c^2d+e)/(c^2*e*x^2+c^2*d)^2*\operatorname{arcsech}(c*x)*d-1/c^4*b*((c^2d+2*(e*(c^2d+e))^{1/2}+2e)*d)^{1/2}*\arctan(c*d*(1/c/x+(-1+1/c/x)^{1/2})*(1+1/c/x)^{1/2}))/((c^2d+2*(e*(c^2d+e))^{1/2}+2e)*d)^{1/2}}/e/(c^2d+e)/d^3*(e*(c^2d+e))^{1/2}+5/4/c^2*b*((c^2d+2*(e*(c^2d+e))^{1/2}+2e)*d)^{1/2}*\arctan(c*d*(1/c/x+(-1+1/c/x)^{1/2})*(1+1/c/x)^{1/2}))/((c^2d+2*(e*(c^2d+e))^{1/2}+2e)*d)^{1/2}}/(c^2d+e)^2/e/d^2*(e*(c^2d+e))^{1/2}+1/8*c^5*b*x^2/e/(c^2d+e)/(c^2*e*x^2+c^2*d)^2*(-(c*x-1)/c/x)^{1/2}*((c*x+1)/c/x)^{1/2}*d$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{bx^4 \operatorname{arsech}(cx) + ax^4}{e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="fricas")

[Out] integral((b*x^4*arcsech(c*x) + a*x^4)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(a+b*asech(c*x))/(e*x**2+d)**3,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsech}(cx) + a)x^4}{(ex^2 + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x^4/(e*x^2 + d)^3, x)
```

$$3.128 \quad \int \frac{x^2 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^3} dx$$

Optimal. Leaf size=1276

result too large to display

```
[Out] (b*c*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])/(16*Sqrt[-d]*Sqrt[e]*(c^2*d + e)
*(Sqrt[-d]*Sqrt[e] - d/x)) + (b*c*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])/(16
*Sqrt[-d]*Sqrt[e]*(c^2*d + e)*(Sqrt[-d]*Sqrt[e] + d/x)) + (a + b*ArcSech[c*
x])/(16*Sqrt[-d]*Sqrt[e]*(Sqrt[-d]*Sqrt[e] - d/x)^2) + (a + b*ArcSech[c*x])
/(16*d*e*(Sqrt[-d]*Sqrt[e] - d/x)) - (a + b*ArcSech[c*x])/(16*Sqrt[-d]*Sqrt
[e]*(Sqrt[-d]*Sqrt[e] + d/x)^2) - (a + b*ArcSech[c*x])/(16*d*e*(Sqrt[-d]*Sq
rt[e] + d/x)) + (b*ArcTan[(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])]/
(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])))/(8*(c*d - Sqrt[-d]*Sqrt
[e])^(3/2)*(c*d + Sqrt[-d]*Sqrt[e])^(3/2)) - (b*ArcTan[(Sqrt[c*d - Sqrt[-d]
*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x
)])])/(8*d*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*e) + (
b*ArcTan[(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d - Sqrt[
-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(8*(c*d - Sqrt[-d]*Sqrt[e])^(3/2)*(c*d +
Sqrt[-d]*Sqrt[e])^(3/2)) - (b*ArcTan[(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[1
+ 1/(c*x)])/(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])])/(8*d*Sqrt[c
*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*e) - ((a + b*ArcSech[c*
x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(
-d)^(3/2)*e^(3/2)) + ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*
x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2)) - ((a + b*ArcSech
[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(1
6*(-d)^(3/2)*e^(3/2)) + ((a + b*ArcSech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech
[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(3/2)*e^(3/2)) + (b*PolyLog[2
, -((c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e]))])/(16*(-d)^(3/
2)*e^(3/2)) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2
*d + e])])/(16*(-d)^(3/2)*e^(3/2)) + (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcSech[
c*x])/(Sqrt[e] + Sqrt[c^2*d + e]))])/(16*(-d)^(3/2)*e^(3/2)) - (b*PolyLog[2
, (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(3/2)
e^(3/2))
```

Rubi [A] time = 4.04015, antiderivative size = 1276, normalized size of antiderivative = 1., number of steps used = 63, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6303, 5792, 5707, 5802, 96, 93, 205, 5800, 5562, 2190, 2279, 2391}

$$\frac{b\sqrt{\frac{1}{cx} - 1}\sqrt{1 + \frac{1}{cx}c}}{16\sqrt{-d}\sqrt{e}(dc^2 + e)\left(\sqrt{-d}\sqrt{e} - \frac{d}{x}\right)} + \frac{b\sqrt{\frac{1}{cx} - 1}\sqrt{1 + \frac{1}{cx}c}}{16\sqrt{-d}\sqrt{e}(dc^2 + e)\left(\frac{d}{x} + \sqrt{-d}\sqrt{e}\right)} + \frac{a + b\operatorname{sech}^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e} - \frac{d}{x}\right)} - \frac{a + b\operatorname{sech}^{-1}(cx)}{16de\left(\frac{d}{x} + \sqrt{-d}\sqrt{e}\right)} + \frac{1}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (b*c*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])/(16*Sqrt[-d]*Sqrt[e]*(c^2*d + e)
*(Sqrt[-d]*Sqrt[e] - d/x)) + (b*c*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])/(16
*Sqrt[-d]*Sqrt[e]*(c^2*d + e)*(Sqrt[-d]*Sqrt[e] + d/x)) + (a + b*ArcSech[c*
x])/(16*Sqrt[-d]*Sqrt[e]*(Sqrt[-d]*Sqrt[e] - d/x)^2) + (a + b*ArcSech[c*x])
/(16*d*e*(Sqrt[-d]*Sqrt[e] - d/x)) - (a + b*ArcSech[c*x])/(16*Sqrt[-d]*Sqrt
[e]*(Sqrt[-d]*Sqrt[e] + d/x)^2) - (a + b*ArcSech[c*x])/(16*d*e*(Sqrt[-d]*Sq
```


$$\begin{aligned} & \text{rt}[e] + d/x) + (b \cdot \text{ArcTan}[(\text{Sqrt}[c \cdot d - \text{Sqrt}[-d] \cdot \text{Sqrt}[e]] \cdot \text{Sqrt}[1 + 1/(c \cdot x)]) / \\ & (\text{Sqrt}[c \cdot d + \text{Sqrt}[-d] \cdot \text{Sqrt}[e]] \cdot \text{Sqrt}[-1 + 1/(c \cdot x)])]) / (8 \cdot (c \cdot d - \text{Sqrt}[-d] \cdot \text{Sqrt}[e]) \\ & ^{(3/2)} \cdot (c \cdot d + \text{Sqrt}[-d] \cdot \text{Sqrt}[e])^{(3/2)}) - (b \cdot \text{ArcTan}[(\text{Sqrt}[c \cdot d - \text{Sqrt}[-d] \\ & \cdot \text{Sqrt}[e]] \cdot \text{Sqrt}[1 + 1/(c \cdot x)]) / (\text{Sqrt}[c \cdot d + \text{Sqrt}[-d] \cdot \text{Sqrt}[e]] \cdot \text{Sqrt}[-1 + 1/(c \cdot x) \\ &)])]) / (8 \cdot \text{Sqrt}[c \cdot d - \text{Sqrt}[-d] \cdot \text{Sqrt}[e]] \cdot \text{Sqrt}[c \cdot d + \text{Sqrt}[-d] \cdot \text{Sqrt}[e]] \cdot e) + (\\ & b \cdot \text{ArcTan}[(\text{Sqrt}[c \cdot d + \text{Sqrt}[-d] \cdot \text{Sqrt}[e]] \cdot \text{Sqrt}[1 + 1/(c \cdot x)]) / (\text{Sqrt}[c \cdot d - \text{Sqrt}[-d] \\ & \cdot \text{Sqrt}[e]] \cdot \text{Sqrt}[-1 + 1/(c \cdot x)])]) / (8 \cdot (c \cdot d - \text{Sqrt}[-d] \cdot \text{Sqrt}[e])^{(3/2)} \cdot (c \cdot d + \\ & \text{Sqrt}[-d] \cdot \text{Sqrt}[e])^{(3/2)}) - (b \cdot \text{ArcTan}[(\text{Sqrt}[c \cdot d + \text{Sqrt}[-d] \cdot \text{Sqrt}[e]] \cdot \text{Sqrt}[1 \\ & + 1/(c \cdot x)]) / (\text{Sqrt}[c \cdot d - \text{Sqrt}[-d] \cdot \text{Sqrt}[e]] \cdot \text{Sqrt}[-1 + 1/(c \cdot x)])]) / (8 \cdot \text{Sqrt}[c \\ & \cdot d - \text{Sqrt}[-d] \cdot \text{Sqrt}[e]] \cdot \text{Sqrt}[c \cdot d + \text{Sqrt}[-d] \cdot \text{Sqrt}[e]] \cdot e) - ((a + b \cdot \text{ArcSech}[c \cdot x] \\ &) \cdot \text{Log}[1 - (c \cdot \text{Sqrt}[-d] \cdot E^{\text{ArcSech}[c \cdot x]}) / (\text{Sqrt}[e] - \text{Sqrt}[c^2 \cdot d + e])]) / (16 \cdot (-d)^{(3/2)} \cdot e^{(3/2)}) \\ & + ((a + b \cdot \text{ArcSech}[c \cdot x]) \cdot \text{Log}[1 + (c \cdot \text{Sqrt}[-d] \cdot E^{\text{ArcSech}[c \cdot x]}) / (\text{Sqrt}[e] - \text{Sqrt}[c^2 \cdot d + e])]) / (16 \cdot (-d)^{(3/2)} \cdot e^{(3/2)}) \\ & - ((a + b \cdot \text{ArcSech}[c \cdot x]) \cdot \text{Log}[1 - (c \cdot \text{Sqrt}[-d] \cdot E^{\text{ArcSech}[c \cdot x]}) / (\text{Sqrt}[e] + \text{Sqrt}[c^2 \cdot d + e])]) / (16 \cdot (-d)^{(3/2)} \cdot e^{(3/2)}) \\ & + ((a + b \cdot \text{ArcSech}[c \cdot x]) \cdot \text{Log}[1 + (c \cdot \text{Sqrt}[-d] \cdot E^{\text{ArcSech}[c \cdot x]}) / (\text{Sqrt}[e] + \text{Sqrt}[c^2 \cdot d + e])]) / (16 \cdot (-d)^{(3/2)} \cdot e^{(3/2)}) \\ & + (b \cdot \text{PolyLog}[2, -((c \cdot \text{Sqrt}[-d] \cdot E^{\text{ArcSech}[c \cdot x]}) / (\text{Sqrt}[e] - \text{Sqrt}[c^2 \cdot d + e])]) / (16 \cdot (-d)^{(3/2)} \cdot e^{(3/2)}) \\ & - (b \cdot \text{PolyLog}[2, (c \cdot \text{Sqrt}[-d] \cdot E^{\text{ArcSech}[c \cdot x]}) / (\text{Sqrt}[e] - \text{Sqrt}[c^2 \cdot d + e])]) / (16 \cdot (-d)^{(3/2)} \cdot e^{(3/2)}) \\ & + (b \cdot \text{PolyLog}[2, -((c \cdot \text{Sqrt}[-d] \cdot E^{\text{ArcSech}[c \cdot x]}) / (\text{Sqrt}[e] + \text{Sqrt}[c^2 \cdot d + e])]) / (16 \cdot (-d)^{(3/2)} \cdot e^{(3/2)}) \\ & - (b \cdot \text{PolyLog}[2, (c \cdot \text{Sqrt}[-d] \cdot E^{\text{ArcSech}[c \cdot x]}) / (\text{Sqrt}[e] + \text{Sqrt}[c^2 \cdot d + e])]) / (16 \cdot (-d)^{(3/2)} \cdot e^{(3/2)}) \end{aligned}$$
Rule 6303

$$\text{Int}[(a + \text{ArcSech}(c \cdot x)) \cdot (b \cdot x)^n \cdot (d + e \cdot x^2)^p, x_Symbol] \rightarrow -\text{Subst}[\text{Int}[(e + d \cdot x^2)^p \cdot (a + b \cdot \text{ArcCosh}[x/c])^n / x^{m+2(p+1)}, x], x, 1/x] /; \text{FreeQ}\{a, b, c, d, e, n, x\} \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IntegersQ}[m, p]$$
Rule 5792

$$\text{Int}[(a + \text{ArcCosh}(c \cdot x)) \cdot (b \cdot x)^n \cdot (f \cdot x)^m \cdot (d + e \cdot x^2)^p, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b \cdot \text{ArcCosh}[c \cdot x])^n \cdot (f \cdot x)^m \cdot (d + e \cdot x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, x\} \ \&\& \ \text{NeQ}[c^2 \cdot d + e, 0] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IntegerQ}[p] \ \&\& \ \text{IntegerQ}[m]$$
Rule 5707

$$\text{Int}[(a + \text{ArcCosh}(c \cdot x)) \cdot (b \cdot x)^n \cdot (d + e \cdot x^2)^p, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b \cdot \text{ArcCosh}[c \cdot x])^n \cdot (d + e \cdot x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, n, x\} \ \&\& \ \text{NeQ}[c^2 \cdot d + e, 0] \ \&\& \ \text{IntegerQ}[p] \ \&\& \ (p > 0 \ || \ \text{IGtQ}[n, 0])$$
Rule 5802

$$\text{Int}[(a + \text{ArcCosh}(c \cdot x)) \cdot (b \cdot x)^n \cdot (d + e \cdot x)^m, x_Symbol] \rightarrow \text{Simp}[(d + e \cdot x)^{m+1} \cdot (a + b \cdot \text{ArcCosh}[c \cdot x])^n / (e \cdot (m+1)), x] - \text{Dist}[(b \cdot c \cdot n) / (e \cdot (m+1)), \text{Int}[(d + e \cdot x)^{m+1} \cdot (a + b \cdot \text{ArcCosh}[c \cdot x])^{n-1} / (\text{Sqrt}[-1 + c \cdot x] \cdot \text{Sqrt}[1 + c \cdot x]), x], x] /; \text{FreeQ}\{a, b, c, d, e, m, x\} \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{NeQ}[m, -1]$$
Rule 96

$$\text{Int}[(a + (b \cdot x)^m) \cdot (c + d \cdot x)^n \cdot (e + f \cdot x)^p, x_Symbol] \rightarrow \text{Simp}[(b \cdot (a + b \cdot x)^{m+1} \cdot (c + d \cdot x)^{n+1} \cdot (e + f \cdot x)^{p+1}) / ((m+1) \cdot (b \cdot c - a \cdot d) \cdot (b \cdot e - a \cdot f)), x] + \text{Dist}[(a \cdot d \cdot f \cdot (m+1) + b \cdot c \cdot f \cdot (n+1) + b \cdot d \cdot e \cdot (p+1)) / ((m+1) \cdot (b \cdot c - a \cdot d) \cdot (b \cdot e - a \cdot f)), \text{Int}[(a + b \cdot x)^{m+1} \cdot (c + d \cdot x)^n \cdot (e + f \cdot x)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, n\}$$

```
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)))/((e_.) + (f_.)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)])*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbol] := Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_)^(m_.))*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_.)*(x_)])*(b_.) + (a_)), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)), x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)), x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.))*((c_.) + (d_.)*(x_)^(m_.))/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^3} dx &= -\operatorname{Subst} \left(\int \frac{x^2 (a + b \cosh^{-1}(\frac{x}{c}))}{(e + dx^2)^3} dx, x, \frac{1}{x} \right) \\
&= -\operatorname{Subst} \left(\int \left(-\frac{e (a + b \cosh^{-1}(\frac{x}{c}))}{d (e + dx^2)^3} + \frac{a + b \cosh^{-1}(\frac{x}{c})}{d (e + dx^2)^2} \right) dx, x, \frac{1}{x} \right) \\
&= -\frac{\operatorname{Subst} \left(\int \frac{a + b \cosh^{-1}(\frac{x}{c})}{(e + dx^2)^2} dx, x, \frac{1}{x} \right)}{d} + \frac{e \operatorname{Subst} \left(\int \frac{a + b \cosh^{-1}(\frac{x}{c})}{(e + dx^2)^3} dx, x, \frac{1}{x} \right)}{d} \\
&= -\frac{\operatorname{Subst} \left(\int \left(-\frac{d(a + b \cosh^{-1}(\frac{x}{c}))}{4e(\sqrt{-d}\sqrt{e-dx})^2} - \frac{d(a + b \cosh^{-1}(\frac{x}{c}))}{4e(\sqrt{-d}\sqrt{e+dx})^2} - \frac{d(a + b \cosh^{-1}(\frac{x}{c}))}{2e(-de - d^2x^2)} \right) dx, x, \frac{1}{x} \right)}{d} + \frac{e \operatorname{Subst} \left(\int \frac{a + b \cosh^{-1}(\frac{x}{c})}{(e + dx^2)^3} dx, x, \frac{1}{x} \right)}{d} \\
&= -\frac{3 \operatorname{Subst} \left(\int \frac{a + b \cosh^{-1}(\frac{x}{c})}{(\sqrt{-d}\sqrt{e-dx})^2} dx, x, \frac{1}{x} \right)}{16e} - \frac{3 \operatorname{Subst} \left(\int \frac{a + b \cosh^{-1}(\frac{x}{c})}{(\sqrt{-d}\sqrt{e+dx})^2} dx, x, \frac{1}{x} \right)}{16e} + \frac{\operatorname{Subst} \left(\int \frac{a + b \cosh^{-1}(\frac{x}{c})}{(\sqrt{-d}\sqrt{e-dx})^2} dx, x, \frac{1}{x} \right)}{16e} \\
&= \frac{a + b \operatorname{sech}^{-1}(cx)}{16\sqrt{-d}\sqrt{e} \left(\sqrt{-d}\sqrt{e} - \frac{d}{x} \right)^2} + \frac{a + b \operatorname{sech}^{-1}(cx)}{16de \left(\sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} - \frac{a + b \operatorname{sech}^{-1}(cx)}{16\sqrt{-d}\sqrt{e} \left(\sqrt{-d}\sqrt{e} + \frac{d}{x} \right)^2} - \frac{a + b \operatorname{sech}^{-1}(cx)}{16de \left(\sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} \\
&= \frac{bc\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}}{16\sqrt{-d}\sqrt{e} (c^2d + e) \left(\sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} + \frac{bc\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}}{16\sqrt{-d}\sqrt{e} (c^2d + e) \left(\sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} + \frac{a + b \operatorname{sech}^{-1}(cx)}{16\sqrt{-d}\sqrt{e} \left(\sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} \\
&= \frac{bc\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}}{16\sqrt{-d}\sqrt{e} (c^2d + e) \left(\sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} + \frac{bc\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}}{16\sqrt{-d}\sqrt{e} (c^2d + e) \left(\sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} + \frac{a + b \operatorname{sech}^{-1}(cx)}{16\sqrt{-d}\sqrt{e} \left(\sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} \\
&= \frac{bc\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}}{16\sqrt{-d}\sqrt{e} (c^2d + e) \left(\sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} + \frac{bc\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}}{16\sqrt{-d}\sqrt{e} (c^2d + e) \left(\sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} + \frac{a + b \operatorname{sech}^{-1}(cx)}{16\sqrt{-d}\sqrt{e} \left(\sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} \\
&= \frac{bc\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}}{16\sqrt{-d}\sqrt{e} (c^2d + e) \left(\sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} + \frac{bc\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}}{16\sqrt{-d}\sqrt{e} (c^2d + e) \left(\sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} + \frac{a + b \operatorname{sech}^{-1}(cx)}{16\sqrt{-d}\sqrt{e} \left(\sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} \\
&= \frac{bc\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}}{16\sqrt{-d}\sqrt{e} (c^2d + e) \left(\sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} + \frac{bc\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}}{16\sqrt{-d}\sqrt{e} (c^2d + e) \left(\sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} + \frac{a + b \operatorname{sech}^{-1}(cx)}{16\sqrt{-d}\sqrt{e} \left(\sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} \\
&= \frac{bc\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}}{16\sqrt{-d}\sqrt{e} (c^2d + e) \left(\sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} + \frac{bc\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}}{16\sqrt{-d}\sqrt{e} (c^2d + e) \left(\sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} + \frac{a + b \operatorname{sech}^{-1}(cx)}{16\sqrt{-d}\sqrt{e} \left(\sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} \\
&= \frac{bc\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}}{16\sqrt{-d}\sqrt{e} (c^2d + e) \left(\sqrt{-d}\sqrt{e} - \frac{d}{x} \right)} + \frac{bc\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}}{16\sqrt{-d}\sqrt{e} (c^2d + e) \left(\sqrt{-d}\sqrt{e} + \frac{d}{x} \right)} + \frac{a + b \operatorname{sech}^{-1}(cx)}{16\sqrt{-d}\sqrt{e} \left(\sqrt{-d}\sqrt{e} - \frac{d}{x} \right)}
\end{aligned}$$

Mathematica [C] time = 6.15685, size = 2030, normalized size = 1.59

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2)^3,x]

[Out]
$$\begin{aligned} & -(a*x)/(4*e*(d + e*x^2)^2) + (a*x)/(8*d*e*(d + e*x^2)) + (a*\text{ArcTan}[(\text{Sqrt}[e] \\ & *x)/\text{Sqrt}[d]])/(8*d^{(3/2)}*e^{(3/2)}) + b*(((-1/16)*(((-1)*\text{Sqrt}[e]*\text{Sqrt}[(1 - c* \\ & x)/(1 + c*x)]*(1 + c*x))/(\text{Sqrt}[d]*(c^2*d + e))*((-1)*\text{Sqrt}[d] + \text{Sqrt}[e]*x)) - \\ & \text{ArcSech}[c*x]/(\text{Sqrt}[e]*((-1)*\text{Sqrt}[d] + \text{Sqrt}[e]*x)^2) + \text{Log}[x]/(d*\text{Sqrt}[e]) - \\ & \text{Log}[1 + \text{Sqrt}[(1 - c*x)/(1 + c*x)] + c*x*\text{Sqrt}[(1 - c*x)/(1 + c*x)]]/(d*\text{Sqrt} \\ & [e]) + ((2*c^2*d + e)*\text{Log}[(-4*d*\text{Sqrt}[e]*\text{Sqrt}[c^2*d + e]*(\text{Sqrt}[e] - I*c^2*\text{Sqr} \\ & \text{rt}[d]*x + \text{Sqrt}[c^2*d + e]*\text{Sqrt}[(1 - c*x)/(1 + c*x)] + c*\text{Sqrt}[c^2*d + e]*x*\text{S} \\ & \text{qrt}[(1 - c*x)/(1 + c*x)])]/((2*c^2*d + e)*((-1)*\text{Sqrt}[d] + \text{Sqrt}[e]*x)))/(d* \\ & (c^2*d + e)^{(3/2)}))/(\text{Sqrt}[d]*e) + ((I/16)*((I*\text{Sqrt}[e]*\text{Sqrt}[(1 - c*x)/(1 + \\ & c*x)]*(1 + c*x))/(\text{Sqrt}[d]*(c^2*d + e)*(I*\text{Sqrt}[d] + \text{Sqrt}[e]*x)) - \text{ArcSech}[c* \\ & x]/(\text{Sqrt}[e]*(I*\text{Sqrt}[d] + \text{Sqrt}[e]*x)^2) + \text{Log}[x]/(d*\text{Sqrt}[e]) - \text{Log}[1 + \text{Sqrt} \\ & (1 - c*x)/(1 + c*x)] + c*x*\text{Sqrt}[(1 - c*x)/(1 + c*x)]]/(d*\text{Sqrt}[e]) + ((2*c^2 \\ & *d + e)*\text{Log}[(-4*d*\text{Sqrt}[e]*\text{Sqrt}[c^2*d + e]*(\text{Sqrt}[e] + I*c^2*\text{Sqrt}[d]*x + \text{Sqrt} \\ & [c^2*d + e]*\text{Sqrt}[(1 - c*x)/(1 + c*x)] + c*\text{Sqrt}[c^2*d + e]*x*\text{Sqrt}[(1 - c*x)/ \\ & (1 + c*x)])]/((2*c^2*d + e)*(I*\text{Sqrt}[d] + \text{Sqrt}[e]*x)))/(d*(c^2*d + e)^{(3/2)} \\ &))/(\text{Sqrt}[d]*e) - (-\text{ArcSech}[c*x]/(I*\text{Sqrt}[d]*\text{Sqrt}[e] + e*x)) + (I*(\text{Log}[x]/\text{S} \\ & \text{qrt}[e] - \text{Log}[1 + \text{Sqrt}[(1 - c*x)/(1 + c*x)] + c*x*\text{Sqrt}[(1 - c*x)/(1 + c*x)]] \\ & / \text{Sqrt}[e] + \text{Log}[(2*I)*\text{Sqrt}[e]*(\text{Sqrt}[d]*\text{Sqrt}[(1 - c*x)/(1 + c*x)]*(1 + c*x) \\ & + (\text{Sqrt}[d]*\text{Sqrt}[e] + I*c^2*d*x)/\text{Sqrt}[c^2*d + e]))/(I*\text{Sqrt}[d] + \text{Sqrt}[e]*x)]/ \\ & \text{Sqrt}[c^2*d + e])/ \text{Sqrt}[d])/ (16*d*e) - (-\text{ArcSech}[c*x]/((-1)*\text{Sqrt}[d]*\text{Sqrt}[e] \\ & + e*x)) - (I*(\text{Log}[x]/\text{Sqrt}[e] - \text{Log}[1 + \text{Sqrt}[(1 - c*x)/(1 + c*x)] + c*x*\text{Sqr} \\ & \text{t}[(1 - c*x)/(1 + c*x)]]/\text{Sqrt}[e] + \text{Log}[(2*\text{Sqrt}[e]*(I*\text{Sqrt}[d]*\text{Sqrt}[(1 - c*x)/ \\ & (1 + c*x)]*(1 + c*x) + (I*\text{Sqrt}[d]*\text{Sqrt}[e] + c^2*d*x)/\text{Sqrt}[c^2*d + e]))/((-I \\ &)*\text{Sqrt}[d] + \text{Sqrt}[e]*x)]/\text{Sqrt}[c^2*d + e])/ \text{Sqrt}[d])/ (16*d*e) - ((I/32)*(\text{Poly} \\ & \text{Log}[2, -E^{(-2*\text{ArcSech}[c*x])}] - 2*((-4*I)*\text{ArcSin}[\text{Sqrt}[1 + (I*\text{Sqrt}[e])/ \\ & (c*\text{Sqrt}[d])]]/\text{Sqrt}[2]]*\text{ArcTanh}[(I*c*\text{Sqrt}[d] + \text{Sqrt}[e])* \text{Tanh}[\text{ArcSech}[c*x]/2])/ \text{Sqrt} \\ & [c^2*d + e]] + \text{ArcSech}[c*x]*\text{Log}[1 + E^{(-2*\text{ArcSech}[c*x])}] - \text{ArcSech}[c*x]*\text{Log} \\ & [1 + (I*(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e]))/(c*\text{Sqrt}[d]*E^{\text{ArcSech}[c*x]})] + (2*I)*\text{Arc} \\ & \text{cSin}[\text{Sqrt}[1 + (I*\text{Sqrt}[e])/ \\ & (c*\text{Sqrt}[d])]]/\text{Sqrt}[2]]*\text{Log}[1 + (I*(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e]))/(c*\text{Sqrt}[d] \\ & *E^{\text{ArcSech}[c*x]})] - \text{ArcSech}[c*x]*\text{Log}[1 + (I*(\text{Sqrt}[e] \\ & + \text{Sqrt}[c^2*d + e]))/(c*\text{Sqrt}[d]*E^{\text{ArcSech}[c*x]})] - (2*I)*\text{ArcSin}[\text{Sqrt}[1 + (I \\ & *\text{Sqrt}[e])/ \\ & (c*\text{Sqrt}[d])]]/\text{Sqrt}[2]]*\text{Log}[1 + (I*(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e]))/(c* \\ & \text{Sqrt}[d]*E^{\text{ArcSech}[c*x]})] + \text{PolyLog}[2, (I*(-\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e]))/(c*S \\ & \text{qrt}[d]*E^{\text{ArcSech}[c*x]})] + \text{PolyLog}[2, ((-1)*(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e]))/(c* \\ & \text{Sqrt}[d]*E^{\text{ArcSech}[c*x]})]))/(d^{(3/2)}*e^{(3/2)}) - ((I/32)*(-\text{PolyLog}[2, -E^{(-2 \\ & *\text{ArcSech}[c*x])}] + 2*((-4*I)*\text{ArcSin}[\text{Sqrt}[1 - (I*\text{Sqrt}[e])/ \\ & (c*\text{Sqrt}[d])]]/\text{Sqrt}[2]]*\text{ArcTanh}[(-1)*c*\text{Sqrt}[d] + \text{Sqrt}[e])* \text{Tanh}[\text{ArcSech}[c*x]/2])/ \text{Sqrt}[c^2*d + e \\ &] + \text{ArcSech}[c*x]*\text{Log}[1 + E^{(-2*\text{ArcSech}[c*x])}] - \text{ArcSech}[c*x]*\text{Log}[1 + (I*(- \\ & \text{Sqrt}[e] + \text{Sqrt}[c^2*d + e]))/(c*\text{Sqrt}[d]*E^{\text{ArcSech}[c*x]})] + (2*I)*\text{ArcSin}[\text{Sqrt} \\ & [1 - (I*\text{Sqrt}[e])/ \\ & (c*\text{Sqrt}[d])]]/\text{Sqrt}[2]]*\text{Log}[1 + (I*(-\text{Sqrt}[e] + \text{Sqrt}[c^2*d + \\ & e]))/(c*\text{Sqrt}[d]*E^{\text{ArcSech}[c*x]})] - \text{ArcSech}[c*x]*\text{Log}[1 - (I*(\text{Sqrt}[e] + \text{Sqrt}[\\ & c^2*d + e]))/(c*\text{Sqrt}[d]*E^{\text{ArcSech}[c*x]})] - (2*I)*\text{ArcSin}[\text{Sqrt}[1 - (I*\text{Sqrt}[e] \\ &)/ \\ & (c*\text{Sqrt}[d])]]/\text{Sqrt}[2]]*\text{Log}[1 - (I*(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e]))/(c*\text{Sqrt}[d]* \\ & E^{\text{ArcSech}[c*x]})] + \text{PolyLog}[2, (I*(\text{Sqrt}[e] - \text{Sqrt}[c^2*d + e]))/(c*\text{Sqrt}[d]*E^ \\ & \text{ArcSech}[c*x] \\ &))] + \text{PolyLog}[2, (I*(\text{Sqrt}[e] + \text{Sqrt}[c^2*d + e]))/(c*\text{Sqrt}[d]*E^{\text{Arc} \\ & \text{cSech}[c*x] \\ &))]))/(d^{(3/2)}*e^{(3/2)}) \end{aligned}$$

Maple [C] time = 2.858, size = 2537, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^2*(a+b*\text{arcsech}(c*x))/(e*x^2+d)^3,x)$

[Out] $\frac{1}{8}c^4a/(c^2ex^2+c^2d)^2/dx^3 - \frac{1}{8}c^4a/(c^2ex^2+c^2d)^2/ex + \frac{1}{8}a/d/e/(d^2e)^{1/2}*\arctan(xe/(d^2e)^{1/2}) + \frac{1}{4}c^2b*((c^2d+2*(e(c^2d+e))^{1/2}+2e)*d)^{1/2}*\arctan(c*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))/((c^2d+2*(e(c^2d+e))^{1/2}+2e)*d)^{1/2})/(c^2d+e)/d^3 + \frac{1}{4}c^2b*(-(c^2d-2*(e(c^2d+e))^{1/2}+2e)*d)^{1/2}*\text{arctanh}(c*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))/((-c^2d+2*(e(c^2d+e))^{1/2}-2e)*d)^{1/2})/(c^2d+e)/d^3 + \frac{1}{8}b*((c^2d+2*(e(c^2d+e))^{1/2}+2e)*d)^{1/2}*\arctan(c*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))/((c^2d+2*(e(c^2d+e))^{1/2}+2e)*d)^{1/2})/e/(c^2d+e)/d^2 + \frac{1}{8}b*(-(c^2d-2*(e(c^2d+e))^{1/2}+2e)*d)^{1/2}*\text{arctanh}(c*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))/((-c^2d+2*(e(c^2d+e))^{1/2}-2e)*d)^{1/2})/e/(c^2d+e)/d^2 - \frac{1}{8}c^4b*x/(c^2d+e)/(c^2ex^2+c^2d)^2*\text{arcsech}(c*x) + \frac{1}{8}c^6b*x^3/(c^2d+e)/(c^2ex^2+c^2d)^2*\text{arcsech}(c*x) - \frac{1}{8}c^5b*x^2/(c^2d+e)/(c^2ex^2+c^2d)^2*(-(c*x-1)/c/x)^{1/2}*((c*x+1)/c/x)^{1/2} + \frac{1}{4}c^2b*((c^2d+2*(e(c^2d+e))^{1/2}+2e)*d)^{1/2}*\arctan(c*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))/((c^2d+2*(e(c^2d+e))^{1/2}+2e)*d)^{1/2})/(c^2d+e)^2/d^3*(e(c^2d+e))^{1/2} - \frac{1}{4}c^2b*(-(c^2d-2*(e(c^2d+e))^{1/2}+2e)*d)^{1/2}*\arctanh(c*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))/((-c^2d+2*(e(c^2d+e))^{1/2}-2e)*d)^{1/2})/(c^2d+e)^2/d^3*(e(c^2d+e))^{1/2} - \frac{1}{4}c^2b*((c^2d+2*(e(c^2d+e))^{1/2}+2e)*d)^{1/2}*\arctan(c*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))/((c^2d+2*(e(c^2d+e))^{1/2}+2e)*d)^{1/2})/(c^2d+e)^2/e/d^3 - \frac{1}{4}c^2b*(-(c^2d-2*(e(c^2d+e))^{1/2}+2e)*d)^{1/2}*\text{arctanh}(c*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))/((-c^2d+2*(e(c^2d+e))^{1/2}-2e)*d)^{1/2})/(c^2d+e)^2/e/d^3 + \frac{1}{8}b*((c^2d+2*(e(c^2d+e))^{1/2}+2e)*d)^{1/2}*\arctan(c*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))/((c^2d+2*(e(c^2d+e))^{1/2}+2e)*d)^{1/2})/(c^2d+e)^2/e/d^2*(e(c^2d+e))^{1/2} - \frac{1}{8}b*(-(c^2d-2*(e(c^2d+e))^{1/2}+2e)*d)^{1/2}*\text{arctanh}(c*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))/((-c^2d+2*(e(c^2d+e))^{1/2}-2e)*d)^{1/2})/(c^2d+e)^2/e/d^2*(e(c^2d+e))^{1/2} - \frac{1}{4}b*(-(c^2d-2*(e(c^2d+e))^{1/2}+2e)*d)^{1/2}*\text{arctanh}(c*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))/((-c^2d+2*(e(c^2d+e))^{1/2}-2e)*d)^{1/2})/(c^2d+e)^2/e/d^2*(e(c^2d+e))^{1/2} - \frac{1}{4}b*((c^2d+2*(e(c^2d+e))^{1/2}+2e)*d)^{1/2}*\arctan(c*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))/((c^2d+2*(e(c^2d+e))^{1/2}+2e)*d)^{1/2})/(c^2d+e)^2/d^2 + \frac{1}{16}c^3b/e/(c^2d+e)*\text{sum}(1/_R1/(_R1^2*c^2d+c^2d+2e)*(arcsech(c*x)*\ln((_R1-1/c/x-(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))/_R1)+\text{dilog}((_R1-1/c/x-(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))/_R1)),_R1=\text{RootOf}(c^2d*_Z^4+(2*c^2d+4e)*_Z^2+c^2d))- \frac{1}{16}c^3b/e/(c^2d+e)*\text{sum}(1/_R1/(_R1^2*c^2d+c^2d+2e)*(arcsech(c*x)*\ln((_R1-1/c/x-(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))/_R1)+\text{dilog}((_R1-1/c/x-(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))/_R1)),_R1=\text{RootOf}(c^2d*_Z^4+(2*c^2d+4e)*_Z^2+c^2d))- \frac{1}{16}c^3b/d/(c^2d+e)*\text{sum}(1/_R1/(_R1^2*c^2d+c^2d+2e)*(arcsech(c*x)*\ln((_R1-1/c/x-(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))/_R1)+\text{dilog}((_R1-1/c/x-(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))/_R1)),_R1=\text{RootOf}(c^2d*_Z^4+(2*c^2d+4e)*_Z^2+c^2d))+ \frac{1}{16}c^3b/d/(c^2d+e)*\text{sum}(1/_R1/(_R1^2*c^2d+c^2d+2e)*(arcsech(c*x)*\ln((_R1-1/c/x-(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))/_R1)+\text{dilog}((_R1-1/c/x-(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))/_R1)),_R1=\text{RootOf}(c^2d*_Z^4+(2*c^2d+4e)*_Z^2+c^2d))- \frac{1}{8}c^5b*x^4/d/e/(c^2d+e)/(c^2ex^2+c^2d)^2*(-(c*x-1)/c/x)^{1/2}*((c*x+1)/c/x)^{1/2} - \frac{1}{4}c^2b*((c^2d+2*(e(c^2d+e))^{1/2}+2e)*d)^{1/2}*\arctan(c*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))/((c^2d+2*(e(c^2d+e))^{1/2}+2e)*d)^{1/2})/e/(c^2d+e)/d^3*(e(c^2d+e))^{1/2} + \frac{1}{4}c^2b*(-(c^2d-2*(e(c^2d+e))^{1/2}+2e)*d)^{1/2}*\text{arctanh}(c*d*(1/c/x+(-1+1/c/x)^{1/2}*(1+1/c/x)^{1/2}))/((-c^2d+2*(e(c^2d+e))^{1/2}-2e)*d)^{1/2})/e/(c^2d+e)/d^3*(e(c^2d+e))^{1/2} - \frac{1}{8}c^6b*x^3/d/e/(c^2d+e)/(c^2ex^2+c^2d)^2*\text{arcsech}(c*x) + \frac{1}{8}c^4b*x^3/d/e/(c^2d+e)/(c^2ex^2+c^2d)^2*\text{arcsech}(c*x)$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^2 \operatorname{arsech}(cx) + ax^2}{e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="fricas")

[Out] integral((b*x^2*arcsech(c*x) + a*x^2)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(a+b*asech(c*x))/(e*x**2+d)**3,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsech}(cx) + a)x^2}{(ex^2 + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)*x^2/(e*x^2 + d)^3, x)

$$3.129 \quad \int \frac{a+b\operatorname{sech}^{-1}(cx)}{(d+ex^2)^3} dx$$

Optimal. Leaf size=1272

result too large to display

```
[Out] (b*c*Sqrt[e]*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])/(16*(-d)^(3/2)*(c^2*d +
e)*(Sqrt[-d]*Sqrt[e] - d/x)) + (b*c*Sqrt[e]*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(
c*x)])/(16*(-d)^(3/2)*(c^2*d + e)*(Sqrt[-d]*Sqrt[e] + d/x)) + (Sqrt[e]*(a +
b*ArcSech[c*x]))/(16*(-d)^(3/2)*(Sqrt[-d]*Sqrt[e] - d/x)^2) - (5*(a + b*Ar
cSech[c*x]))/(16*d^2*(Sqrt[-d]*Sqrt[e] - d/x)) - (Sqrt[e]*(a + b*ArcSech[c*
x]))/(16*(-d)^(3/2)*(Sqrt[-d]*Sqrt[e] + d/x)^2) + (5*(a + b*ArcSech[c*x]))/
(16*d^2*(Sqrt[-d]*Sqrt[e] + d/x)) + (5*b*ArcTan[(Sqrt[c*d - Sqrt[-d]*Sqrt[e]
]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])))/(
8*d^2*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]) - (b*e*Arc
Tan[(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d + Sqrt[-d]*S
qrt[e]]*Sqrt[-1 + 1/(c*x)])))/(8*d*(c*d - Sqrt[-d]*Sqrt[e])^(3/2)*(c*d + Sq
rt[-d]*Sqrt[e])^(3/2)) + (5*b*ArcTan[(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[1 +
1/(c*x)])/(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])))/(8*d^2*Sqrt[
c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]) - (b*e*ArcTan[(Sqrt[c
*d + Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqr
t[-1 + 1/(c*x)])))/(8*d*(c*d - Sqrt[-d]*Sqrt[e])^(3/2)*(c*d + Sqrt[-d]*Sqrt
[e])^(3/2)) + (3*(a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])/(
Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) - (3*(a + b*ArcSech[c*
x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(
-d)^(5/2)*Sqrt[e]) + (3*(a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[
c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) - (3*(a + b*Arc
Sech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d + e])])
)/(16*(-d)^(5/2)*Sqrt[e]) - (3*b*PolyLog[2, -(c*Sqrt[-d]*E^ArcSech[c*x])/(
Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e]) + (3*b*PolyLog[2, (c*
Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] - Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[
e]) - (3*b*PolyLog[2, -(c*Sqrt[-d]*E^ArcSech[c*x])/(Sqrt[e] + Sqrt[c^2*d +
e])])/(16*(-d)^(5/2)*Sqrt[e]) + (3*b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x]
)]/(Sqrt[e] + Sqrt[c^2*d + e])])/(16*(-d)^(5/2)*Sqrt[e])
```

Rubi [A] time = 4.90957, antiderivative size = 1272, normalized size of antiderivative = 1., number of steps used = 81, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6293, 5792, 5707, 5802, 96, 93, 205, 5800, 5562, 2190, 2279, 2391}

$$\frac{b\sqrt{e}\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}c}}{16(-d)^{3/2}(dc^2+e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{b\sqrt{e}\sqrt{\frac{1}{cx}-1}\sqrt{1+\frac{1}{cx}c}}{16(-d)^{3/2}(dc^2+e)\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)} - \frac{5(a+b\operatorname{sech}^{-1}(cx))}{16d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} + \frac{5(a+b\operatorname{sech}^{-1}(cx))}{16d^2\left(\frac{d}{x}+\sqrt{-d}\sqrt{e}\right)} +$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcSech[c*x])/(d + e*x^2)^3, x]

```
[Out] (b*c*Sqrt[e]*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)])/(16*(-d)^(3/2)*(c^2*d +
e)*(Sqrt[-d]*Sqrt[e] - d/x)) + (b*c*Sqrt[e]*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(
c*x)])/(16*(-d)^(3/2)*(c^2*d + e)*(Sqrt[-d]*Sqrt[e] + d/x)) + (Sqrt[e]*(a +
b*ArcSech[c*x]))/(16*(-d)^(3/2)*(Sqrt[-d]*Sqrt[e] - d/x)^2) - (5*(a + b*Ar
cSech[c*x]))/(16*d^2*(Sqrt[-d]*Sqrt[e] - d/x)) - (Sqrt[e]*(a + b*ArcSech[c*
x]))/(16*(-d)^(3/2)*(Sqrt[-d]*Sqrt[e] + d/x)^2) + (5*(a + b*ArcSech[c*x]))/
(16*d^2*(Sqrt[-d]*Sqrt[e] + d/x)) + (5*b*ArcTan[(Sqrt[c*d - Sqrt[-d]*Sqrt[e]

```

```

]]*Sqrt[1 + 1/(c*x)]/(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)])]/(
8*d^2*Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]) - (b*e*Arc
Tan[(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d + Sqrt[-d]*S
qrt[e]]*Sqrt[-1 + 1/(c*x)]))]/(8*d*(c*d - Sqrt[-d]*Sqrt[e])^(3/2)*(c*d + Sq
rt[-d]*Sqrt[e])^(3/2)) + (5*b*ArcTan[(Sqrt[c*d + Sqrt[-d]*Sqrt[e]]*Sqrt[1 +
1/(c*x)])/(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[-1 + 1/(c*x)]))]/(8*d^2*Sqrt[
c*d - Sqrt[-d]*Sqrt[e]]*Sqrt[c*d + Sqrt[-d]*Sqrt[e]]) - (b*e*ArcTan[(Sqrt[c
*d + Sqrt[-d]*Sqrt[e]]*Sqrt[1 + 1/(c*x)])/(Sqrt[c*d - Sqrt[-d]*Sqrt[e]]*Sqr
t[-1 + 1/(c*x)]))]/(8*d*(c*d - Sqrt[-d]*Sqrt[e])^(3/2)*(c*d + Sqrt[-d]*Sqrt
[e])^(3/2)) + (3*(a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[c*x])]/(
Sqrt[e] - Sqrt[c^2*d + e]))/(16*(-d)^(5/2)*Sqrt[e]) - (3*(a + b*ArcSech[c*
x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])]/(Sqrt[e] - Sqrt[c^2*d + e]))/(16*(
-d)^(5/2)*Sqrt[e]) + (3*(a + b*ArcSech[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcSech[
c*x])]/(Sqrt[e] + Sqrt[c^2*d + e]))/(16*(-d)^(5/2)*Sqrt[e]) - (3*(a + b*Arc
Sech[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcSech[c*x])]/(Sqrt[e] + Sqrt[c^2*d + e]))
/(16*(-d)^(5/2)*Sqrt[e]) - (3*b*PolyLog[2, -(c*Sqrt[-d]*E^ArcSech[c*x])]/(
Sqrt[e] - Sqrt[c^2*d + e]))/(16*(-d)^(5/2)*Sqrt[e]) + (3*b*PolyLog[2, (c*
Sqrt[-d]*E^ArcSech[c*x])]/(Sqrt[e] - Sqrt[c^2*d + e]))/(16*(-d)^(5/2)*Sqrt[
e]) - (3*b*PolyLog[2, -(c*Sqrt[-d]*E^ArcSech[c*x])]/(Sqrt[e] + Sqrt[c^2*d +
e]))/(16*(-d)^(5/2)*Sqrt[e]) + (3*b*PolyLog[2, (c*Sqrt[-d]*E^ArcSech[c*x
])/Sqrt[e] + Sqrt[c^2*d + e]]/(16*(-d)^(5/2)*Sqrt[e])

```

Rule 6293

```

Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))^ (n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] := -Subst[Int[((e + d*x^2)^p*(a + b*ArcCosh[x/c])^n)/x^(2*(p +
1))], x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p]
]

```

Rule 5792

```

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^ (n_.)*((f_.)*(x_))^(m_.)*((d_.) + (e
_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
(f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
+ e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]

```

Rule 5707

```

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^ (n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] := Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])

```

Rule 5802

```

Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^ (n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
- Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
- 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]

```

Rule 96

```

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^ (p_.), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m

```


, 1])

Rule 93

```
Int[(((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)))/((e_.) + (f_.)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)*(b_.)]^(n_))/((d_.) + (e_.)*(x_)), x_Symbol] := Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_)^(m_))*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] := -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)), x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)), x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)), x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_))*((c_.) + (d_.)*(x_)^(m_))/((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_)), x_Symbol] := Simp[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Dist[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_)], x_Symbol] := Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

Mathematica [C] time = 6.06278, size = 2015, normalized size = 1.58

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*ArcSech[c*x])/(d + e*x^2)^3,x]

[Out]
$$\begin{aligned} & (a*x)/(4*d*(d + e*x^2)^2) + (3*a*x)/(8*d^2*(d + e*x^2)) + (3*a*ArcTan[(\sqrt{e}*x)/\sqrt{d}])/(8*d^{5/2}*\sqrt{e}) + b*(((I/16)*((-I)*\sqrt{e}*\sqrt{(1 - c*x)/(1 + c*x)}*(1 + c*x))/(\sqrt{d}*(c^2*d + e)*((-I)*\sqrt{d} + \sqrt{e}*x)) \\ & - ArcSech[c*x]/(\sqrt{e}*((-I)*\sqrt{d} + \sqrt{e}*x)^2) + \text{Log}[x]/(d*\sqrt{e}) \\ & - \text{Log}[1 + \sqrt{(1 - c*x)/(1 + c*x)}] + c*x*\sqrt{(1 - c*x)/(1 + c*x)})/(d*\sqrt{e}) + ((2*c^2*d + e)*\text{Log}[(-4*d*\sqrt{e}*\sqrt{c^2*d + e}*(\sqrt{e} - I*c^2*\sqrt{d}*x + \sqrt{c^2*d + e}*\sqrt{(1 - c*x)/(1 + c*x)} + c*\sqrt{c^2*d + e}*x*\sqrt{(1 - c*x)/(1 + c*x)})]/((2*c^2*d + e)*((-I)*\sqrt{d} + \sqrt{e}*x)))/(d*(c^2*d + e)^{(3/2)})))/d^{3/2} - ((I/16)*((I*\sqrt{e}*\sqrt{(1 - c*x)/(1 + c*x)}*(1 + c*x))/(\sqrt{d}*(c^2*d + e)*(I*\sqrt{d} + \sqrt{e}*x)) - ArcSech[c*x]/(\sqrt{e}*(I*\sqrt{d} + \sqrt{e}*x)^2) + \text{Log}[x]/(d*\sqrt{e}) - \text{Log}[1 + \sqrt{(1 - c*x)/(1 + c*x)}] + c*x*\sqrt{(1 - c*x)/(1 + c*x)})/(d*\sqrt{e}) + ((2*c^2*d + e)*\text{Log}[(-4*d*\sqrt{e}*\sqrt{c^2*d + e}*(\sqrt{e} + I*c^2*\sqrt{d}*x + \sqrt{c^2*d + e}*\sqrt{(1 - c*x)/(1 + c*x)} + c*\sqrt{c^2*d + e}*x*\sqrt{(1 - c*x)/(1 + c*x)})]/((2*c^2*d + e)*(I*\sqrt{d} + \sqrt{e}*x)))/(d*(c^2*d + e)^{(3/2)})))/d^{3/2} - (3*(-ArcSech[c*x]/(I*\sqrt{d}*\sqrt{e} + e*x)) + (I*(\text{Log}[x]/\sqrt{e} - \text{Log}[1 + \sqrt{(1 - c*x)/(1 + c*x)}] + c*x*\sqrt{(1 - c*x)/(1 + c*x)})/\sqrt{e} + \text{Log}[(2*I)*\sqrt{e}*(\sqrt{d}*\sqrt{(1 - c*x)/(1 + c*x)}*(1 + c*x) + (\sqrt{d}*\sqrt{e} + I*c^2*d*x)/\sqrt{c^2*d + e}))/I*\sqrt{d} + \sqrt{e}*x])/(\sqrt{c^2*d + e}))/\sqrt{d}))/d^{3/2} - (3*(-ArcSech[c*x]/((-I)*\sqrt{d}*\sqrt{e} + e*x)) - (I*(\text{Log}[x]/\sqrt{e} - \text{Log}[1 + \sqrt{(1 - c*x)/(1 + c*x)}] + c*x*\sqrt{(1 - c*x)/(1 + c*x)})/\sqrt{e} + \text{Log}[(2*\sqrt{e}*(I*\sqrt{d}*\sqrt{(1 - c*x)/(1 + c*x)}*(1 + c*x) + (I*\sqrt{d}*\sqrt{e} + c^2*d*x)/\sqrt{c^2*d + e}))/((-I)*\sqrt{d} + \sqrt{e}*x))/\sqrt{c^2*d + e}))/\sqrt{d}))/d^{3/2} - (((3*I)/32)*(\text{PolyLog}[2, -E^{(-2*ArcSech[c*x])}] - 2*((-4*I)*ArcSin[\sqrt{1 + (I*\sqrt{e})}/(c*\sqrt{d})])/\sqrt{2}]*ArcTanh[((I*c*\sqrt{d} + \sqrt{e})*Tanh[ArcSech[c*x]/2])/(\sqrt{c^2*d + e})] + ArcSech[c*x]*\text{Log}[1 + E^{(-2*ArcSech[c*x])}] - ArcSech[c*x]*\text{Log}[1 + (I*(\sqrt{e} - \sqrt{c^2*d + e}))/(\sqrt{c^2*d + e})])/(c*\sqrt{d}*E^{ArcSech[c*x]})] - ArcSech[c*x]*\text{Log}[1 + (I*(\sqrt{e} + \sqrt{c^2*d + e}))/(\sqrt{c^2*d + e})])/(c*\sqrt{d}*E^{ArcSech[c*x]})] - (2*I)*ArcSin[\sqrt{1 + (I*\sqrt{e})}/(c*\sqrt{d})])/\sqrt{2}]*\text{Log}[1 + (I*(\sqrt{e} - \sqrt{c^2*d + e}))/(\sqrt{c^2*d + e})])/(c*\sqrt{d}*E^{ArcSech[c*x]})] + \text{PolyLog}[2, (I*(-\sqrt{e} + \sqrt{c^2*d + e}))/(\sqrt{c^2*d + e})])/(c*\sqrt{d}*E^{ArcSech[c*x]})] + \text{PolyLog}[2, ((-I)*(\sqrt{e} + \sqrt{c^2*d + e}))/(\sqrt{c^2*d + e})])/(c*\sqrt{d}*E^{ArcSech[c*x]})])))/d^{5/2}*\sqrt{e}) - (((3*I)/32)*(-\text{PolyLog}[2, -E^{(-2*ArcSech[c*x])}] + 2*((-4*I)*ArcSin[\sqrt{1 - (I*\sqrt{e})}/(c*\sqrt{d})])/\sqrt{2}]*ArcTanh[((-I)*c*\sqrt{d} + \sqrt{e})*Tanh[ArcSech[c*x]/2])/(\sqrt{c^2*d + e})] + ArcSech[c*x]*\text{Log}[1 + E^{(-2*ArcSech[c*x])}] - ArcSech[c*x]*\text{Log}[1 + (I*(-\sqrt{e} + \sqrt{c^2*d + e}))/(\sqrt{c^2*d + e})])/(c*\sqrt{d}*E^{ArcSech[c*x]})] + (2*I)*ArcSin[\sqrt{1 - (I*\sqrt{e})}/(c*\sqrt{d})])/\sqrt{2}]*\text{Log}[1 + (I*(-\sqrt{e} + \sqrt{c^2*d + e}))/(\sqrt{c^2*d + e})])/(c*\sqrt{d}*E^{ArcSech[c*x]})] - ArcSech[c*x]*\text{Log}[1 - (I*(\sqrt{e} + \sqrt{c^2*d + e}))/(\sqrt{c^2*d + e})])/(c*\sqrt{d}*E^{ArcSech[c*x]})] - (2*I)*ArcSin[\sqrt{1 - (I*\sqrt{e})}/(c*\sqrt{d})])/\sqrt{2}]*\text{Log}[1 - (I*(\sqrt{e} + \sqrt{c^2*d + e}))/(\sqrt{c^2*d + e})])/(c*\sqrt{d}*E^{ArcSech[c*x]})] + \text{PolyLog}[2, (I*(\sqrt{e} - \sqrt{c^2*d + e}))/(\sqrt{c^2*d + e})])/(c*\sqrt{d}*E^{ArcSech[c*x]})] + \text{PolyLog}[2, (I*(\sqrt{e} + \sqrt{c^2*d + e}))/(\sqrt{c^2*d + e})])/(c*\sqrt{d}*E^{ArcSech[c*x]})])))/d^{5/2}*\sqrt{e})) \end{aligned}$$

Maple [C] time = 6.369, size = 3446, normalized size = 2.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((a+b*\text{arcsech}(c*x))/(e*x^2+d)^3,x)$

[Out] $\frac{3}{8} \frac{a}{d^2} \frac{1}{(d*e)^{1/2}} \arctan\left(\frac{x}{(d*e)^{1/2}}\right) + \frac{3}{8} \frac{c^4 b x^3}{d^2 (c^2 d + e)} \frac{1}{(c^2 e x^2 + c^2 d)^2 \text{arcsech}(c x) e^2} + \frac{5}{8} \frac{c^4 b x}{d (c^2 d + e)} \frac{1}{(c^2 e x^2 + c^2 d)^2 \text{arcsech}(c x) e} + \frac{1}{c^4 b} \frac{((c^2 d + 2(e(c^2 d + e))^{1/2} + 2e) d)^{1/2} e \arctan(c d (1/c/x + (-1 + 1/c/x)^{1/2} (1 + 1/c/x)^{1/2}))}{((c^2 d + 2(e(c^2 d + e))^{1/2} + 2e) d)^{1/2}} \frac{1}{(c^2 d + e) d^5 (e(c^2 d + e))^{1/2}} - \frac{7}{4} \frac{1}{c^2 b} \frac{((c^2 d + 2(e(c^2 d + e))^{1/2} + 2e) d)^{1/2} e \arctan(c d (1/c/x + (-1 + 1/c/x)^{1/2} (1 + 1/c/x)^{1/2}))}{((c^2 d + 2(e(c^2 d + e))^{1/2} + 2e) d)^{1/2}} \frac{1}{(c^2 d + e)^2 d^4 (e(c^2 d + e))^{1/2}} - \frac{1}{c^4 b} \frac{((c^2 d + 2(e(c^2 d + e))^{1/2} + 2e) d)^{1/2} e^2 \arctan(c d (1/c/x + (-1 + 1/c/x)^{1/2} (1 + 1/c/x)^{1/2}))}{((c^2 d + 2(e(c^2 d + e))^{1/2} + 2e) d)^{1/2}} \frac{1}{(c^2 d + e)^2 d^5 (e(c^2 d + e))^{1/2}} - \frac{1}{c^4 b} \frac{(- (c^2 d - 2(e(c^2 d + e))^{1/2} + 2e) d)^{1/2} e \text{arctanh}(c d (1/c/x + (-1 + 1/c/x)^{1/2} (1 + 1/c/x)^{1/2}))}{((-c^2 d + 2(e(c^2 d + e))^{1/2} - 2e) d)^{1/2}} \frac{1}{(c^2 d + e) d^5 (e(c^2 d + e))^{1/2}} + \frac{7}{4} \frac{1}{c^2 b} \frac{(- (c^2 d - 2(e(c^2 d + e))^{1/2} + 2e) d)^{1/2} e \text{arctanh}(c d (1/c/x + (-1 + 1/c/x)^{1/2} (1 + 1/c/x)^{1/2}))}{((-c^2 d + 2(e(c^2 d + e))^{1/2} - 2e) d)^{1/2}} \frac{1}{(c^2 d + e)^2 d^4 (e(c^2 d + e))^{1/2}} + \frac{1}{c^4 b} \frac{(- (c^2 d - 2(e(c^2 d + e))^{1/2} + 2e) d)^{1/2} e^2 \text{arctanh}(c d (1/c/x + (-1 + 1/c/x)^{1/2} (1 + 1/c/x)^{1/2}))}{((-c^2 d + 2(e(c^2 d + e))^{1/2} - 2e) d)^{1/2}} \frac{1}{(c^2 d + e)^2 d^5 (e(c^2 d + e))^{1/2}} + \frac{3}{8} \frac{c^6 b x^3}{d e (c^2 d + e)} \frac{1}{(c^2 e x^2 + c^2 d)^2 \text{arcsech}(c x)} - \frac{3}{16} \frac{c^3 b}{d (c^2 d + e)} \sum \left(\frac{1}{_R1} \frac{1}{(_R1^2 c^2 d + c^2 d + 2e)} (\text{arcsech}(c x) \ln\left(\frac{_R1 - 1/c/x - (-1 + 1/c/x)^{1/2} (1 + 1/c/x)^{1/2}}{_R1}\right) + \text{dilog}\left(\frac{_R1 - 1/c/x - (-1 + 1/c/x)^{1/2} (1 + 1/c/x)^{1/2}}{_R1}\right)) \right),$

$_R1 = \text{RootOf}(c^2 d * _Z^4 + (2 * c^2 d + 4 * e) * _Z^2 + c^2 d)$

$+ \frac{3}{16} \frac{c^3 b}{d (c^2 d + e)} \sum \left(\frac{1}{_R1} \frac{1}{(_R1^2 c^2 d + c^2 d + 2e)} (\text{arcsech}(c x) \ln\left(\frac{_R1 - 1/c/x - (-1 + 1/c/x)^{1/2} (1 + 1/c/x)^{1/2}}{_R1}\right) + \text{dilog}\left(\frac{_R1 - 1/c/x - (-1 + 1/c/x)^{1/2} (1 + 1/c/x)^{1/2}}{_R1}\right)) \right),$

$_R1 = \text{RootOf}(c^2 d * _Z^4 + (2 * c^2 d + 4 * e) * _Z^2 + c^2 d)$

$- \frac{5}{8} \frac{b}{c} \frac{((c^2 d + 2(e(c^2 d + e))^{1/2} + 2e) d)^{1/2} \arctan(c d (1/c/x + (-1 + 1/c/x)^{1/2} (1 + 1/c/x)^{1/2}))}{((c^2 d + 2(e(c^2 d + e))^{1/2} + 2e) d)^{1/2}} \frac{1}{(c^2 d + e) d^3} - \frac{5}{8} \frac{b}{c} \frac{(- (c^2 d - 2(e(c^2 d + e))^{1/2} + 2e) d)^{1/2} \arctanh(c d (1/c/x + (-1 + 1/c/x)^{1/2} (1 + 1/c/x)^{1/2}))}{((-c^2 d + 2(e(c^2 d + e))^{1/2} - 2e) d)^{1/2}} \frac{1}{(c^2 d + e) d^3} + \frac{1}{4} \frac{c^4 a x}{d (c^2 e x^2 + c^2 d)^2} + \frac{3}{8} \frac{c^2 a}{d^2 x} \frac{1}{(c^2 e x^2 + c^2 d)} - \frac{7}{4} \frac{1}{c^2 b} \frac{((c^2 d + 2(e(c^2 d + e))^{1/2} + 2e) d)^{1/2} e \arctan(c d (1/c/x + (-1 + 1/c/x)^{1/2} (1 + 1/c/x)^{1/2}))}{((c^2 d + 2(e(c^2 d + e))^{1/2} + 2e) d)^{1/2}} \frac{1}{(c^2 d + e) d^4} - \frac{1}{c^4 b} \frac{((c^2 d + 2(e(c^2 d + e))^{1/2} + 2e) d)^{1/2} e^2 \arctan(c d (1/c/x + (-1 + 1/c/x)^{1/2} (1 + 1/c/x)^{1/2}))}{((c^2 d + 2(e(c^2 d + e))^{1/2} + 2e) d)^{1/2}} \frac{1}{(c^2 d + e) d^5} + \frac{9}{4} \frac{1}{c^2 b} \frac{((c^2 d + 2(e(c^2 d + e))^{1/2} + 2e) d)^{1/2} e^2 \arctan(c d (1/c/x + (-1 + 1/c/x)^{1/2} (1 + 1/c/x)^{1/2}))}{((c^2 d + 2(e(c^2 d + e))^{1/2} + 2e) d)^{1/2}} \frac{1}{(c^2 d + e)^2 d^4} + \frac{1}{c^4 b} \frac{((c^2 d + 2(e(c^2 d + e))^{1/2} + 2e) d)^{1/2} e^3 \arctan(c d (1/c/x + (-1 + 1/c/x)^{1/2} (1 + 1/c/x)^{1/2}))}{((c^2 d + 2(e(c^2 d + e))^{1/2} + 2e) d)^{1/2}} \frac{1}{(c^2 d + e)^2 d^5} - \frac{5}{4} \frac{1}{c^2 b} \frac{(- (c^2 d - 2(e(c^2 d + e))^{1/2} + 2e) d)^{1/2} e \text{arctanh}(c d (1/c/x + (-1 + 1/c/x)^{1/2} (1 + 1/c/x)^{1/2}))}{((-c^2 d + 2(e(c^2 d + e))^{1/2} - 2e) d)^{1/2}} \frac{1}{(c^2 d + e) d^4} \frac{1}{(e(c^2 d + e))^{1/2}} - \frac{7}{4} \frac{1}{c^2 b} \frac{(- (c^2 d - 2(e(c^2 d + e))^{1/2} + 2e) d)^{1/2} e \text{arctanh}(c d (1/c/x + (-1 + 1/c/x)^{1/2} (1 + 1/c/x)^{1/2}))}{((-c^2 d + 2(e(c^2 d + e))^{1/2} - 2e) d)^{1/2}} \frac{1}{(c^2 d + e) d^4} - \frac{1}{c^4 b} \frac{(- (c^2 d - 2(e(c^2 d + e))^{1/2} + 2e) d)^{1/2} e^2 \text{arctanh}(c d (1/c/x + (-1 + 1/c/x)^{1/2} (1 + 1/c/x)^{1/2}))}{((-c^2 d + 2(e(c^2 d + e))^{1/2} - 2e) d)^{1/2}} \frac{1}{(c^2 d + e) d^5} + \frac{9}{4} \frac{1}{c^2 b} \frac{(- (c^2 d - 2(e(c^2 d + e))^{1/2} + 2e) d)^{1/2} e^2 \text{arctanh}(c d (1/c/x + (-1 + 1/c/x)^{1/2} (1 + 1/c/x)^{1/2}))}{((-c^2 d + 2(e(c^2 d + e))^{1/2} - 2e) d)^{1/2}} \frac{1}{(c^2 d + e)^2 d^4} + \frac{1}{c^4 b} \frac{(- (c^2 d - 2(e(c^2 d + e))^{1/2} + 2e) d)^{1/2} e^3 \text{arctanh}(c d (1/c/x + (-1 + 1/c/x)^{1/2} (1 + 1/c/x)^{1/2}))}{((-c^2 d + 2(e(c^2 d + e))^{1/2} - 2e) d)^{1/2}} \frac{1}{(c^2 d + e)^2 d^5} + \frac{5}{4} \frac{1}{c^2 b} \frac{((c^2 d + 2(e(c^2 d + e))^{1/2} + 2e) d)^{1/2} \arctan(c d (1/c/x + (-1 + 1/c/x)^{1/2} (1 + 1/c/x)^{1/2}))}{((c^2 d + 2(e(c^2 d + e))^{1/2} + 2e) d)^{1/2}} \frac{1}{(c^2 d + e) d^4} \frac{1}{(e(c^2 d + e))^{1/2}} - \frac{5}{8} \frac{b}{c} \frac{((c^2 d + 2(e(c^2 d + e))^{1/2} + 2e) d)^{1/2} \arctan(c d (1/c/x + (-1 + 1/c/x)^{1/2} (1 + 1/c/x)^{1/2}))}{((c^2 d + 2(e(c^2 d + e))^{1/2} + 2e) d)^{1/2}} \frac{1}{(c^2 d + e)^2 d^3} \frac{1}{(e(c^2 d + e))^{1/2}}$

$$\begin{aligned} &)^{(1/2)} + 5/8 * b * (- (c^2 * d - 2 * (e * (c^2 * d + e))^{(1/2)} + 2 * e) * d)^{(1/2)} * \operatorname{arctanh}(c * d * (1/c \\ &/ x + (-1 + 1/c/x)^{(1/2)} * (1 + 1/c/x)^{(1/2)}) / ((-c^2 * d + 2 * (e * (c^2 * d + e))^{(1/2)} - 2 * e) * d) \\ &)^{(1/2)}) / (c^2 * d + e)^2 / d^3 * (e * (c^2 * d + e))^{(1/2)} + 5/4 * b * ((c^2 * d + 2 * (e * (c^2 * d + e))^{(1/2)} \\ &)^{(1/2)} + 2 * e) * d)^{(1/2)} * \operatorname{arctan}(c * d * (1/c/x + (-1 + 1/c/x)^{(1/2)} * (1 + 1/c/x)^{(1/2)}) / ((c^2 \\ &* d + 2 * (e * (c^2 * d + e))^{(1/2)} + 2 * e) * d)^{(1/2)}) / (c^2 * d + e)^2 * e / d^3 + 5/4 * b * (- (c^2 * d - 2 \\ &* (e * (c^2 * d + e))^{(1/2)} + 2 * e) * d)^{(1/2)} * \operatorname{arctanh}(c * d * (1/c/x + (-1 + 1/c/x)^{(1/2)} * (1 + 1 \\ &/ c/x)^{(1/2)}) / ((-c^2 * d + 2 * (e * (c^2 * d + e))^{(1/2)} - 2 * e) * d)^{(1/2)}) / (c^2 * d + e)^2 * e / d^3 \\ &- 3/16 * c * b / (c^2 * d + e) / d^2 * e * \operatorname{sum}(_R1 / (_R1^2 * c^2 * d + c^2 * d + 2 * e) * (\operatorname{arcsech}(c * x) * \ln \\ &((_R1 - 1/c/x - (-1 + 1/c/x)^{(1/2)} * (1 + 1/c/x)^{(1/2)}) / _R1) + \operatorname{dilog}((_R1 - 1/c/x - (-1 + 1/c \\ &/ x)^{(1/2)} * (1 + 1/c/x)^{(1/2)}) / _R1)), _R1 = \operatorname{RootOf}(c^2 * d * _Z^4 + (2 * c^2 * d + 4 * e) * _Z^2 + c \\ &^2 * d)) + 3/16 * c * b / (c^2 * d + e) / d^2 * e * \operatorname{sum}(1 / _R1 / (_R1^2 * c^2 * d + c^2 * d + 2 * e) * (\operatorname{arcsech}(\\ &c * x) * \ln((_R1 - 1/c/x - (-1 + 1/c/x)^{(1/2)} * (1 + 1/c/x)^{(1/2)}) / _R1) + \operatorname{dilog}((_R1 - 1/c/x - \\ &(-1 + 1/c/x)^{(1/2)} * (1 + 1/c/x)^{(1/2)}) / _R1)), _R1 = \operatorname{RootOf}(c^2 * d * _Z^4 + (2 * c^2 * d + 4 * e) \\ &* _Z^2 + c^2 * d)) + 5/8 * c^6 * b * x / (c^2 * d + e) / (c^2 * e * x^2 + c^2 * d)^2 * \operatorname{arcsech}(c * x) + 1/8 * c^5 \\ &* b * x^4 / d^2 / (c^2 * d + e) / (c^2 * e * x^2 + c^2 * d)^2 * ((c * x + 1) / c/x)^{(1/2)} * (- (c * x - 1) / c/x) \\ &)^{(1/2)} * e^2 + 1/8 * c^5 * b * x^2 / d / (c^2 * d + e) / (c^2 * e * x^2 + c^2 * d)^2 * ((c * x + 1) / c/x)^{(1/2)} \\ &* (- (c * x - 1) / c/x)^{(1/2)} * e \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{b \operatorname{ar} \operatorname{sech}(cx) + a}{e^3 x^6 + 3 d e^2 x^4 + 3 d^2 e x^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="fricas")

[Out] integral((b*arcsech(c*x) + a)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asech(c*x))/(e*x**2+d)**3,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{ar} \operatorname{sech}(cx) + a}{(ex^2 + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/(e*x^2 + d)^3, x)
```

3.130 $\int x^5 \sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx)) dx$

Optimal. Leaf size=447

$$\frac{d^2 (d + ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{3e^3} - \frac{2d (d + ex^2)^{5/2} (a + b \operatorname{sech}^{-1}(cx))}{5e^3} + \frac{(d + ex^2)^{7/2} (a + b \operatorname{sech}^{-1}(cx))}{7e^3} + b \sqrt{\frac{1}{cx+1}} \sqrt{cx}$$

```
[Out] (b*(23*c^4*d^2 + 12*c^2*d*e - 75*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(1680*c^6*e^2) + (b*(29*c^2*d - 25*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*(d + e*x^2)^(3/2))/(840*c^4*e^2) - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*(d + e*x^2)^(5/2))/(42*c^2*e^2) + (d^2*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/(3*e^3) - (2*d*(d + e*x^2)^(5/2)*(a + b*ArcSech[c*x]))/(5*e^3) + ((d + e*x^2)^(7/2)*(a + b*ArcSech[c*x]))/(7*e^3) - (b*(105*c^6*d^3 - 35*c^4*d^2*e + 63*c^2*d*e^2 + 75*e^3)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTan[(Sqrt[e]*Sqrt[1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(1680*c^7*e^(5/2)) - (8*b*d^(7/2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/(105*e^3)
```

Rubi [A] time = 1.39547, antiderivative size = 447, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {266, 43, 6301, 12, 1615, 154, 157, 63, 217, 203, 93, 207}

$$\frac{d^2 (d + ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{3e^3} - \frac{2d (d + ex^2)^{5/2} (a + b \operatorname{sech}^{-1}(cx))}{5e^3} + \frac{(d + ex^2)^{7/2} (a + b \operatorname{sech}^{-1}(cx))}{7e^3} + b \sqrt{\frac{1}{cx+1}} \sqrt{cx}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]),x]
```

```
[Out] (b*(23*c^4*d^2 + 12*c^2*d*e - 75*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(1680*c^6*e^2) + (b*(29*c^2*d - 25*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*(d + e*x^2)^(3/2))/(840*c^4*e^2) - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*(d + e*x^2)^(5/2))/(42*c^2*e^2) + (d^2*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/(3*e^3) - (2*d*(d + e*x^2)^(5/2)*(a + b*ArcSech[c*x]))/(5*e^3) + ((d + e*x^2)^(7/2)*(a + b*ArcSech[c*x]))/(7*e^3) - (b*(105*c^6*d^3 - 35*c^4*d^2*e + 63*c^2*d*e^2 + 75*e^3)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTan[(Sqrt[e]*Sqrt[1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(1680*c^7*e^(5/2)) - (8*b*d^(7/2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/(105*e^3)
```

Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))*((f_.)*(x_.))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.)*((e_.) + (f_.)*(x_.))^(p_.), x_Symbol] := With[{q = Expon[Px, x], k = Coeff[Px, x, Expon[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p + q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q - 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m + q + p)))*x], x], x] /; NeQ[m + n + p + q + 1, 0] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.)*((e_.) + (f_.)*(x_.))^(p_.)*((g_.) + (h_.)*(x_.)), x_Symbol] := Simp[(h*(a + b*x)^m*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p + 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1)))*x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_.))^(n_.)*((e_.) + (f_.)*(x_.))^(p_.)*((g_.) + (h_.)*(x_.)))/((a_.) + (b_.)*(x_.)), x_Symbol] := Dist[h/b, Int[(c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```


Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 93

Int[(((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_)))/((e_) + (f_)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]

Rule 207

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int x^5 \sqrt{d+ex^2} (a+b\operatorname{sech}^{-1}(cx)) dx &= \frac{d^2 (d+ex^2)^{3/2} (a+b\operatorname{sech}^{-1}(cx))}{3e^3} - \frac{2d (d+ex^2)^{5/2} (a+b\operatorname{sech}^{-1}(cx))}{5e^3} + \frac{(d+ex^2)^{7/2} (a+b\operatorname{sech}^{-1}(cx))}{7e^3} \\
 &= \frac{d^2 (d+ex^2)^{3/2} (a+b\operatorname{sech}^{-1}(cx))}{3e^3} - \frac{2d (d+ex^2)^{5/2} (a+b\operatorname{sech}^{-1}(cx))}{5e^3} + \frac{(d+ex^2)^{7/2} (a+b\operatorname{sech}^{-1}(cx))}{7e^3} \\
 &= \frac{d^2 (d+ex^2)^{3/2} (a+b\operatorname{sech}^{-1}(cx))}{3e^3} - \frac{2d (d+ex^2)^{5/2} (a+b\operatorname{sech}^{-1}(cx))}{5e^3} + \frac{(d+ex^2)^{7/2} (a+b\operatorname{sech}^{-1}(cx))}{7e^3} \\
 &= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{5/2}}{42c^2e^2} + \frac{d^2 (d+ex^2)^{3/2} (a+b\operatorname{sech}^{-1}(cx))}{3e^3} \\
 &= \frac{b(29c^2d-25e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{840c^4e^2} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{42c^2e^2} \\
 &= \frac{b(23c^4d^2+12c^2de-75e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{1680c^6e^2} + \frac{b(29c^2d-25e)}{42c^2e^2} \\
 &= \frac{b(23c^4d^2+12c^2de-75e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{1680c^6e^2} + \frac{b(29c^2d-25e)}{42c^2e^2} \\
 &= \frac{b(23c^4d^2+12c^2de-75e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{1680c^6e^2} + \frac{b(29c^2d-25e)}{42c^2e^2} \\
 &= \frac{b(23c^4d^2+12c^2de-75e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{1680c^6e^2} + \frac{b(29c^2d-25e)}{42c^2e^2} \\
 &= \frac{b(23c^4d^2+12c^2de-75e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{1680c^6e^2} + \frac{b(29c^2d-25e)}{42c^2e^2}
 \end{aligned}$$

Mathematica [A] time = 3.01421, size = 409, normalized size = 0.91

$$\frac{\sqrt{d+ex^2} \left(16ac^6 (-4d^2ex^2 + 8d^3 + 3de^2x^4 + 15e^3x^6) - be\sqrt{\frac{1-cx}{cx+1}}(cx+1)(c^4(-41d^2 + 22dex^2 + 40e^2x^4) + 2c^2e(19d + 25e)) \right)}{1680c^6e^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^5*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]),x]

[Out] (Sqrt[d + e*x^2]*(16*a*c^6*(8*d^3 - 4*d^2*e*x^2 + 3*d*e^2*x^4 + 15*e^3*x^6) - b*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(75*e^2 + 2*c^2*e*(19*d + 25*e*x^2) + c^4*(-41*d^2 + 22*d*e*x^2 + 40*e^2*x^4)) + 16*b*c^6*(8*d^3 - 4*d^2*e*x^2 + 3*d*e^2*x^4 + 15*e^3*x^6)*ArcSech[c*x]))/(1680*c^6*e^3) - (b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[-1 + c^2*x^2]*(Sqrt[c^2]*Sqrt[e]*Sqrt[c^2*d + e]*(10*5*c^6*d^3 - 35*c^4*d^2*e + 63*c^2*d*e^2 + 75*e^3)*Sqrt[(c^2*(d + e*x^2))/(c^2*d + e)]*ArcSinh[(c*Sqrt[e]*Sqrt[-1 + c^2*x^2])/(Sqrt[c^2]*Sqrt[c^2*d + e])]) + 128*c^9*d^(7/2)*Sqrt[d + e*x^2]*ArcTan[(Sqrt[d]*Sqrt[-1 + c^2*x^2])/Sqrt[d + e*x^2]]))/(1680*c^9*e^3*(-1 + c*x)*Sqrt[d + e*x^2])

Maple [F] time = 1.95, size = 0, normalized size = 0.

$$\int x^5 (a + b \operatorname{arcsech}(cx)) \sqrt{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)

[Out] int(x^5*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 15.839, size = 4373, normalized size = 9.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")

[Out] [1/6720*(128*b*c^7*d^(7/2)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt

$$\begin{aligned}
& \left(-\frac{c^2x^2 - 1}{c^2x^2} \right) + 8d^2/x^4 - (105bc^6d^3 - 35b^2c^4d^2e + 63b^2c^2de^2 + 75b^2e^3) \sqrt{-e} \log(8c^4e^2x^4 + c^4d^2 - 6c^2de + 8(c^4de - c^2e^2)x^2 - 4(2c^4ex^3 + (c^4d - c^2e)x) \sqrt{ex^2 + d} \sqrt{-e} \sqrt{-(c^2x^2 - 1)/(c^2x^2)} + e^2) + 64(15b^2c^7e^3x^6 + 3b^2c^7de^2x^4 - 4b^2c^7d^2ex^2 + 8b^2c^7d^3) \sqrt{ex^2 + d} \log((cx \sqrt{-(c^2x^2 - 1)/(c^2x^2)} + 1)/(cx)) + 4(240a^2c^7e^3x^6 + 48a^2c^7de^2x^4 - 64a^2c^7d^2ex^2 + 128a^2c^7d^3 - (40b^2c^6e^3x^5 + 2(11b^2c^6de^2 + 25b^2c^4e^3)x^3 - (41b^2c^6d^2e - 38b^2c^4de^2 - 75b^2c^2e^3)x) \sqrt{-(c^2x^2 - 1)/(c^2x^2)}) \sqrt{ex^2 + d})/(c^7e^3), \\
& 1/3360(64b^2c^7d^{7/2} \log(((c^4d^2 - 6c^2de + e^2)x^4 - 8(c^2d^2 - de)x^2 + 4((c^3d - ce)x^3 - 2cdx) \sqrt{ex^2 + d} \sqrt{d}) \sqrt{-(c^2x^2 - 1)/(c^2x^2)} + 8d^2)/x^4 - (105bc^6d^3 - 35b^2c^4d^2e + 63b^2c^2de^2 + 75b^2e^3) \sqrt{e} \arctan(1/2(2c^2ex^3 + (c^2d - e)x) \sqrt{ex^2 + d} \sqrt{e} \sqrt{-(c^2x^2 - 1)/(c^2x^2)})/(c^2e^2x^4 + (c^2de - e^2)x^2 - de)) + 32(15b^2c^7e^3x^6 + 3b^2c^7de^2x^4 - 4b^2c^7d^2ex^2 + 8b^2c^7d^3) \sqrt{ex^2 + d} \log((cx \sqrt{-(c^2x^2 - 1)/(c^2x^2)} + 1)/(cx)) + 2(240a^2c^7e^3x^6 + 48a^2c^7de^2x^4 - 64a^2c^7d^2ex^2 + 128a^2c^7d^3 - (40b^2c^6e^3x^5 + 2(11b^2c^6de^2 + 25b^2c^4e^3)x^3 - (41b^2c^6d^2e - 38b^2c^4de^2 - 75b^2c^2e^3)x) \sqrt{-(c^2x^2 - 1)/(c^2x^2)}) \sqrt{ex^2 + d})/(c^7e^3), \\
& -1/6720(256b^2c^7 \sqrt{-d} d^3 \arctan(-1/2((c^3d - ce)x^3 - 2cdx) \sqrt{ex^2 + d} \sqrt{-d} \sqrt{-(c^2x^2 - 1)/(c^2x^2)})/(c^2de^2x^4 + (c^2d^2 - de)x^2 - d^2)) + (105bc^6d^3 - 35b^2c^4d^2e + 63b^2c^2de^2 + 75b^2e^3) \sqrt{-e} \log(8c^4e^2x^4 + c^4d^2 - 6c^2de + 8(c^4de - c^2e^2)x^2 - 4(2c^4ex^3 + (c^4d - c^2e)x) \sqrt{ex^2 + d} \sqrt{-e} \sqrt{-(c^2x^2 - 1)/(c^2x^2)} + e^2) - 64(15b^2c^7e^3x^6 + 3b^2c^7de^2x^4 - 4b^2c^7d^2ex^2 + 8b^2c^7d^3) \sqrt{ex^2 + d} \log((cx \sqrt{-(c^2x^2 - 1)/(c^2x^2)} + 1)/(cx)) - 4(240a^2c^7e^3x^6 + 48a^2c^7de^2x^4 - 64a^2c^7d^2ex^2 + 128a^2c^7d^3 - (40b^2c^6e^3x^5 + 2(11b^2c^6de^2 + 25b^2c^4e^3)x^3 - (41b^2c^6d^2e - 38b^2c^4de^2 - 75b^2c^2e^3)x) \sqrt{-(c^2x^2 - 1)/(c^2x^2)}) \sqrt{ex^2 + d})/(c^7e^3), \\
& -1/3360(128b^2c^7 \sqrt{-d} d^3 \arctan(-1/2((c^3d - ce)x^3 - 2cdx) \sqrt{ex^2 + d} \sqrt{-d} \sqrt{-(c^2x^2 - 1)/(c^2x^2)})/(c^2de^2x^4 + (c^2d^2 - de)x^2 - d^2)) + (105bc^6d^3 - 35b^2c^4d^2e + 63b^2c^2de^2 + 75b^2e^3) \sqrt{e} \arctan(1/2(2c^2ex^3 + (c^2d - e)x) \sqrt{ex^2 + d} \sqrt{e} \sqrt{-(c^2x^2 - 1)/(c^2x^2)})/(c^2e^2x^4 + (c^2de - e^2)x^2 - de)) - 32(15b^2c^7e^3x^6 + 3b^2c^7de^2x^4 - 4b^2c^7d^2ex^2 + 8b^2c^7d^3) \sqrt{ex^2 + d} \log((cx \sqrt{-(c^2x^2 - 1)/(c^2x^2)} + 1)/(cx)) - 2(240a^2c^7e^3x^6 + 48a^2c^7de^2x^4 - 64a^2c^7d^2ex^2 + 128a^2c^7d^3 - (40b^2c^6e^3x^5 + 2(11b^2c^6de^2 + 25b^2c^4e^3)x^3 - (41b^2c^6d^2e - 38b^2c^4de^2 - 75b^2c^2e^3)x) \sqrt{-(c^2x^2 - 1)/(c^2x^2)}) \sqrt{ex^2 + d})/(c^7e^3)]
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*(a+b*asech(c*x))*(e*x**2+d)**(1/2), x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex^2 + d}(b \operatorname{ar} \operatorname{sech}(cx) + a)x^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)*x^5, x)
```

3.131 $\int x^3 \sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx)) dx$

Optimal. Leaf size=329

$$\frac{d(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{3e^2} + \frac{(d+ex^2)^{5/2}(a+b\operatorname{sech}^{-1}(cx))}{5e^2} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(15c^4d^2-10c^2de-9e^2)\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d-c^2x^2}}\right)}{120c^5e^{3/2}}$$

```
[Out] -(b*(c^2*d + 9*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt
[d + e*x^2])/(120*c^4*e) - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c
^2*x^2]*(d + e*x^2)^(3/2))/(20*c^2*e) - (d*(d + e*x^2)^(3/2)*(a + b*ArcSech
[c*x]))/(3*e^2) + ((d + e*x^2)^(5/2)*(a + b*ArcSech[c*x]))/(5*e^2) + (b*(15
*c^4*d^2 - 10*c^2*d*e - 9*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTan[(S
qrt[e]*Sqrt[1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(120*c^5*e^(3/2)) + (2*b*d^
(5/2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*S
qrt[1 - c^2*x^2])])/(15*e^2)
```

Rubi [A] time = 0.427168, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {266, 43, 6301, 12, 573, 154, 157, 63, 217, 203, 93, 207}

$$\frac{d(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{3e^2} + \frac{(d+ex^2)^{5/2}(a+b\operatorname{sech}^{-1}(cx))}{5e^2} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(15c^4d^2-10c^2de-9e^2)\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d-c^2x^2}}\right)}{120c^5e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]),x]
```

```
[Out] -(b*(c^2*d + 9*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt
[d + e*x^2])/(120*c^4*e) - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c
^2*x^2]*(d + e*x^2)^(3/2))/(20*c^2*e) - (d*(d + e*x^2)^(3/2)*(a + b*ArcSech
[c*x]))/(3*e^2) + ((d + e*x^2)^(5/2)*(a + b*ArcSech[c*x]))/(5*e^2) + (b*(15
*c^4*d^2 - 10*c^2*d*e - 9*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTan[(S
qrt[e]*Sqrt[1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(120*c^5*e^(3/2)) + (2*b*d^
(5/2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*S
qrt[1 - c^2*x^2])])/(15*e^2)
```

Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6301

```
Int[((a_) + ArcSech[(c_)*(x_)]*(b_))*((f_)*(x_)^(m_))*((d_) + (e_)*(
x_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{
```

$a, b, c, d, e, f, m, p, x$] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 573

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_))^(r_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 154

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[(h*(a + b*x)^m*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p + 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1)))]*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0] && IntegersQ[2*m, 2*n, 2*p]

Rule 157

Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_))) / ((a_.) + (b_.)*(x_)), x_Symbol] := Dist[h/b, Int[(c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 217

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 93

Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)) / ((e_.) + (f_.)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)]

], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int x^3 \sqrt{d+ex^2} (a+b\operatorname{sech}^{-1}(cx)) dx &= -\frac{d(d+ex^2)^{3/2} (a+b\operatorname{sech}^{-1}(cx))}{3e^2} + \frac{(d+ex^2)^{5/2} (a+b\operatorname{sech}^{-1}(cx))}{5e^2} + \left(b\sqrt{\frac{1}{1+cx}} \right. \\
 &= -\frac{d(d+ex^2)^{3/2} (a+b\operatorname{sech}^{-1}(cx))}{3e^2} + \frac{(d+ex^2)^{5/2} (a+b\operatorname{sech}^{-1}(cx))}{5e^2} + \frac{\left(b\sqrt{\frac{1}{1+cx}} \right. \\
 &= -\frac{d(d+ex^2)^{3/2} (a+b\operatorname{sech}^{-1}(cx))}{3e^2} + \frac{(d+ex^2)^{5/2} (a+b\operatorname{sech}^{-1}(cx))}{5e^2} + \frac{\left(b\sqrt{\frac{1}{1+cx}} \right. \\
 &= -\frac{b\sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} (d+ex^2)^{3/2}}{20c^2e} - \frac{d(d+ex^2)^{3/2} (a+b\operatorname{sech}^{-1}(cx))}{3e^2} + \left(b\sqrt{\frac{1}{1+cx}} \right. \\
 &= -\frac{b(c^2d+9e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \sqrt{d+ex^2}}{120c^4e} - \frac{b\sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} (a+b\operatorname{sech}^{-1}(cx))}{20c^2e} \\
 &= -\frac{b(c^2d+9e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \sqrt{d+ex^2}}{120c^4e} - \frac{b\sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} (a+b\operatorname{sech}^{-1}(cx))}{20c^2e} \\
 &= -\frac{b(c^2d+9e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \sqrt{d+ex^2}}{120c^4e} - \frac{b\sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} (a+b\operatorname{sech}^{-1}(cx))}{20c^2e} \\
 &= -\frac{b(c^2d+9e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \sqrt{d+ex^2}}{120c^4e} - \frac{b\sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} (a+b\operatorname{sech}^{-1}(cx))}{20c^2e} \\
 &= -\frac{b(c^2d+9e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \sqrt{d+ex^2}}{120c^4e} - \frac{b\sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} (a+b\operatorname{sech}^{-1}(cx))}{20c^2e}
 \end{aligned}$$

Mathematica [A] time = 1.45193, size = 365, normalized size = 1.11

$$\frac{\sqrt{d+ex^2} \left(8ac^4(2d^2-dex^2-3e^2x^4) + 8bc^4\operatorname{sech}^{-1}(cx)(2d^2-dex^2-3e^2x^4) + be\sqrt{\frac{1-cx}{cx+1}}(cx+1)(c^2(7d+6ex^2)+9e) \right)}{120c^4e^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]), x]

[Out] -(Sqrt[d + e*x^2]*(8*a*c^4*(2*d^2 - d*e*x^2 - 3*e^2*x^4) + b*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(9*e + c^2*(7*d + 6*e*x^2)) + 8*b*c^4*(2*d^2 - d*e*x^2 - 3*e^2*x^4)*ArcSech[c*x]))/(120*c^4*e^2) - (b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*(Sqrt[-c^2]*Sqrt[-(c^2*d) - e]*Sqrt[e]*(15*c^4*d^2 - 10

```
*c^2*d*e - 9*e^2)*Sqrt[(c^2*(d + e*x^2))/(c^2*d + e)]*ArcSin[(c*Sqrt[e]*Sqrt[1 - c^2*x^2])/(Sqrt[-c^2]*Sqrt[-(c^2*d) - e])] + 16*c^7*d^(5/2)*Sqrt[-d - e*x^2]*ArcTan[(Sqrt[d]*Sqrt[1 - c^2*x^2])/Sqrt[-d - e*x^2]])/(120*c^7*e^2*(-1 + c*x)*Sqrt[d + e*x^2])
```

Maple [F] time = 1.626, size = 0, normalized size = 0.

$$\int x^3 (a + b \operatorname{arcsech}(cx)) \sqrt{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^3*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 7.63659, size = 3636, normalized size = 11.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/480*(16*b*c^5*d^(5/2)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) + (15*b*c^4*d^2 - 10*b*c^2*d*e - 9*b*e^2)*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e^2) + 32*(3*b*c^5*e^2*x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(24*a*c^5*e^2*x^4 + 8*a*c^5*d*e*x^2 - 16*a*c^5*d^2 - (6*b*c^4*e^2*x^3 + (7*b*c^4*d*e + 9*b*c^2*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^5*e^2), 1/240*(8*b*c^5*d^(5/2)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) + (15*b*c^4*d^2 - 10*b*c^2*d*e - 9*b*e^2)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d - e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e) + 16*(3*b*c^5*e^2*x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(24*a*c^5*e^2*x^4 + 8*a*c^5*d*e*x^2 - 16*a*c^5*d^2 - (6*b*c^4*e^2*x^3 + (7*b*c^4*d*e + 9*b*c^2*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d)]/
```



```
(c^5*e^2), 1/480*(32*b*c^5*sqrt(-d)*d^2*arctan(-1/2*((c^3*d - c*e)*x^3 - 2*
c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/(c^2*d*e*x^4
+ (c^2*d^2 - d*e)*x^2 - d^2)) + (15*b*c^4*d^2 - 10*b*c^2*d*e - 9*b*e^2)*sq
rt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2
- 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x
^2 - 1)/(c^2*x^2)) + e^2) + 32*(3*b*c^5*e^2*x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d
^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 4
*(24*a*c^5*e^2*x^4 + 8*a*c^5*d*e*x^2 - 16*a*c^5*d^2 - (6*b*c^4*e^2*x^3 + (7
*b*c^4*d*e + 9*b*c^2*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d
))/(c^5*e^2), 1/240*(16*b*c^5*sqrt(-d)*d^2*arctan(-1/2*((c^3*d - c*e)*x^3 -
2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/(c^2*d*e*
x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (15*b*c^4*d^2 - 10*b*c^2*d*e - 9*b*e^2)
*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d - e)*x)*sqrt(e*x^2 + d)*sqrt(e)*s
qrt(-(c^2*x^2 - 1)/(c^2*x^2)))/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e)) +
16*(3*b*c^5*e^2*x^4 + b*c^5*d*e*x^2 - 2*b*c^5*d^2)*sqrt(e*x^2 + d)*log((c*x
*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(24*a*c^5*e^2*x^4 + 8*a*c^5
*d*e*x^2 - 16*a*c^5*d^2 - (6*b*c^4*e^2*x^3 + (7*b*c^4*d*e + 9*b*c^2*e^2)*x)
*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^5*e^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(a+b*asech(c*x))*(e*x**2+d)**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex^2 + d}(b \operatorname{ar} \operatorname{sech}(cx) + a)x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)*x^3, x)

3.132 $\int x\sqrt{d+ex^2} \left(a + b\operatorname{sech}^{-1}(cx)\right) dx$

Optimal. Leaf size=221

$$\frac{(d+ex^2)^{3/2} (a+b\operatorname{sech}^{-1}(cx))}{3e} - \frac{bd^{3/2} \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3e} - \frac{b\sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \sqrt{1-c^2x^2} \sqrt{d+ex^2}}{6c^2} - \frac{b\sqrt{\frac{1}{cx+1}}}{b\sqrt{\frac{1}{cx+1}}}$$

[Out] $-(b*\operatorname{Sqrt}[(1+c*x)^{-1}]*\operatorname{Sqrt}[1+c*x]*\operatorname{Sqrt}[1-c^2*x^2]*\operatorname{Sqrt}[d+e*x^2])/ (6*c^2) + ((d+e*x^2)^{(3/2)}*(a+b*\operatorname{ArcSech}[c*x]))/(3*e) - (b*(3*c^2*d+e)* \operatorname{Sqrt}[(1+c*x)^{-1}]*\operatorname{Sqrt}[1+c*x]*\operatorname{ArcTan}[(\operatorname{Sqrt}[e]*\operatorname{Sqrt}[1-c^2*x^2])/(c*\operatorname{Sqrt}[d+e*x^2])])/(6*c^3*\operatorname{Sqrt}[e]) - (b*d^{(3/2)}*\operatorname{Sqrt}[(1+c*x)^{-1}]*\operatorname{Sqrt}[1+c*x]*\operatorname{ArcTanh}[\operatorname{Sqrt}[d+e*x^2]/(\operatorname{Sqrt}[d]*\operatorname{Sqrt}[1-c^2*x^2])])/(3*e)$

Rubi [A] time = 0.357428, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {6299, 517, 446, 102, 157, 63, 217, 203, 93, 207}

$$\frac{(d+ex^2)^{3/2} (a+b\operatorname{sech}^{-1}(cx))}{3e} - \frac{bd^{3/2} \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3e} - \frac{b\sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \sqrt{1-c^2x^2} \sqrt{d+ex^2}}{6c^2} - \frac{b\sqrt{\frac{1}{cx+1}}}{b\sqrt{\frac{1}{cx+1}}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x*\operatorname{Sqrt}[d+e*x^2]*(a+b*\operatorname{ArcSech}[c*x]),x]$

[Out] $-(b*\operatorname{Sqrt}[(1+c*x)^{-1}]*\operatorname{Sqrt}[1+c*x]*\operatorname{Sqrt}[1-c^2*x^2]*\operatorname{Sqrt}[d+e*x^2])/ (6*c^2) + ((d+e*x^2)^{(3/2)}*(a+b*\operatorname{ArcSech}[c*x]))/(3*e) - (b*(3*c^2*d+e)* \operatorname{Sqrt}[(1+c*x)^{-1}]*\operatorname{Sqrt}[1+c*x]*\operatorname{ArcTan}[(\operatorname{Sqrt}[e]*\operatorname{Sqrt}[1-c^2*x^2])/(c*\operatorname{Sqrt}[d+e*x^2])])/(6*c^3*\operatorname{Sqrt}[e]) - (b*d^{(3/2)}*\operatorname{Sqrt}[(1+c*x)^{-1}]*\operatorname{Sqrt}[1+c*x]*\operatorname{ArcTanh}[\operatorname{Sqrt}[d+e*x^2]/(\operatorname{Sqrt}[d]*\operatorname{Sqrt}[1-c^2*x^2])])/(3*e)$

Rule 6299

$\operatorname{Int}[(a_.) + \operatorname{ArcSech}[(c_.)*(x_.)]*(b_.)]*(x_.)*((d_.) + (e_.)*(x_.)^2)^{(p_.)}, x_Symbol] \rightarrow \operatorname{Simp}[(d+e*x^2)^{(p+1)}*(a+b*\operatorname{ArcSech}[c*x])]/(2*e*(p+1)), x] + \operatorname{Dist}[(b*\operatorname{Sqrt}[1+c*x]*\operatorname{Sqrt}[1/(1+c*x)])/(2*e*(p+1)), \operatorname{Int}[(d+e*x^2)^{(p+1)}/(x*\operatorname{Sqrt}[1-c*x]*\operatorname{Sqrt}[1+c*x]), x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, p\}, x] \&\& \operatorname{NeQ}[p, -1]$

Rule 517

$\operatorname{Int}[(u_.)*((c_.) + (d_.)*(x_.)^{(n_.)})^{(q_.)}*((a1_.) + (b1_.)*(x_.)^{(non2_.)})^{(p_.)}*((a2_.) + (b2_.)*(x_.)^{(non2_.)})^{(p_.)}, x_Symbol] \rightarrow \operatorname{Int}[u*(a1*a2 + b1*b2*x^n)^p*(c+d*x^n)^q, x] /; \operatorname{FreeQ}\{a1, b1, a2, b2, c, d, n, p, q\}, x] \&\& \operatorname{EqQ}[non2, n/2] \&\& \operatorname{EqQ}[a2*b1 + a1*b2, 0] \&\& (\operatorname{IntegerQ}[p] \|\ (\operatorname{GtQ}[a1, 0] \&\& \operatorname{GtQ}[a2, 0]))$

Rule 446

$\operatorname{Int}[(x_.)^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}*((c_.) + (d_.)*(x_.)^{(n_.)})^{(q_.)}, x_Symbol] \rightarrow \operatorname{Dist}[1/n, \operatorname{Subst}[\operatorname{Int}[x^{(\operatorname{Simplify}[(m+1)/n]-1)}*(a+b*x)^p*(c+d*x)^q, x], x, x^n], x] /; \operatorname{FreeQ}\{a, b, c, d, m, n, p, q\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{IntegerQ}[\operatorname{Simplify}[(m+1)/n]]$

Rule 102

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m + n) + c*f*(m + p)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int((((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/((a_.) + (b_.)*(x_)), x_Symbol] := Dist[h/b, Int[(c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 203

```
Int(((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 93

```
Int((((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 207

```
Int(((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int x\sqrt{d+ex^2}(a+b\operatorname{sech}^{-1}(cx))dx &= \frac{(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{3e} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{(d+ex^2)^{3/2}}{x\sqrt{1-cx}\sqrt{1+cx}}dx}{3e} \\
&= \frac{(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{3e} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{(d+ex^2)^{3/2}}{x\sqrt{1-c^2x^2}}dx}{3e} \\
&= \frac{(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{3e} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\frac{(d+ex)^{3/2}}{x\sqrt{1-c^2x}}dx,x,x^2\right)}{6e} \\
&= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2} + \frac{(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{3e} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{(d+ex)^{3/2}}{x\sqrt{1-c^2x}}dx}{6e} \\
&= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2} + \frac{(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{3e} + \frac{\left(bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{(d+ex)^{3/2}}{x\sqrt{1-c^2x}}dx}{6e} \\
&= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2} + \frac{(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{3e} + \frac{\left(bd^2\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{(d+ex)^{3/2}}{x\sqrt{1-c^2x}}dx}{6e} \\
&= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2} + \frac{(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{3e} - \frac{bd^{3/2}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}}{6e} \\
&= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{6c^2} + \frac{(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{3e} - \frac{b(3c^2d+bd^2)}{6e}
\end{aligned}$$

Mathematica [A] time = 1.28663, size = 307, normalized size = 1.39

$$\frac{\sqrt{d+ex^2}\left(2ac^2(d+ex^2)+2bc^2\operatorname{sech}^{-1}(cx)(d+ex^2)-be\sqrt{\frac{1-cx}{cx+1}}(cx+1)\right)}{6c^2e} + \frac{b\sqrt{\frac{1-cx}{cx+1}}\sqrt{1-c^2x^2}\left(2c^5d^{3/2}\sqrt{-d-ex^2}\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{-d-ex^2}}\right)\right)}{6e}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]),x]

[Out] (Sqrt[d + e*x^2]*(-(b*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)) + 2*a*c^2*(d + e*x^2) + 2*b*c^2*(d + e*x^2)*ArcSech[c*x]))/(6*c^2*e) + (b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*(Sqrt[-c^2]*Sqrt[-(c^2*d) - e]*Sqrt[e]*(3*c^2*d + e)*Sqrt[(c^2*(d + e*x^2))/(c^2*d + e)]*ArcSin[(c*Sqrt[e]*Sqrt[1 - c^2*x^2])/(Sqrt[-c^2]*Sqrt[-(c^2*d) - e]]) + 2*c^5*d^(3/2)*Sqrt[-d - e*x^2]*ArcTan[(Sqrt[d]*Sqrt[1 - c^2*x^2])/Sqrt[-d - e*x^2]]))/(6*c^5*e*(-1 + c*x)*Sqrt[d + e*x^2])

Maple [F] time = 1.395, size = 0, normalized size = 0.

$$\int x(a + b\operatorname{arcsech}(cx))\sqrt{ex^2 + d}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x)

[Out] $\int (x*(a+b*\operatorname{arcsech}(c*x))*(e*x^2+d)^{(1/2)}, x)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3} \left(\frac{(ex^2 + d)^{\frac{3}{2}} \log(\sqrt{cx+1}\sqrt{-cx+1} + 1)}{e} - 3 \int \frac{\sqrt{ex^2 + d} \left(6(c^2ex^2 - e)x \log(\sqrt{x}) + 3(c^2ex^2 \log(c) - e \log(c))x + \dots \right)}{3(c^2ex^2 + \dots)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")`

[Out] $\frac{1}{3}((e*x^2 + d)^{(3/2)}*\log(\sqrt{c*x + 1}*\sqrt{-c*x + 1} + 1)/e - 3*\int \dots)$

Fricas [B] time = 4.03671, size = 3011, normalized size = 13.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")`

[Out] $[1/24*(2*b*c^3*d^{(3/2)}*\log(\dots) - \dots)$

```
) * sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) / (c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e)
- 4*(b*c^3*e*x^2 + b*c^3*d) * sqrt(e*x^2 + d) * log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x))
- 2*(2*a*c^3*e*x^2 - b*c^2*e*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 2*a*c^3*d) * sqrt(e*x^2 + d) / (c^3*e)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int x(a + b \operatorname{asech}(cx)) \sqrt{d + ex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*asech(c*x))*(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x*(a + b*asech(c*x))*sqrt(d + e*x**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex^2 + d}(b \operatorname{arsech}(cx) + a)x dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)*x, x)
```

$$3.133 \quad \int \frac{\sqrt{d+ex^2} (a+b\operatorname{sech}^{-1}(cx))}{x} dx$$

Optimal. Leaf size=25

$$\text{Unintegrable} \left(\frac{\sqrt{d+ex^2} (a+b\operatorname{sech}^{-1}(cx))}{x}, x \right)$$

[Out] Unintegrable[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x, x]

Rubi [A] time = 0.098762, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sqrt{d+ex^2} (a+b\operatorname{sech}^{-1}(cx))}{x} dx$$

Verification is Not applicable to the result.

[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x,x]

[Out] Defer[Int] [(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x, x]

Rubi steps

$$\int \frac{\sqrt{d+ex^2} (a+b\operatorname{sech}^{-1}(cx))}{x} dx = \int \frac{\sqrt{d+ex^2} (a+b\operatorname{sech}^{-1}(cx))}{x} dx$$

Mathematica [A] time = 5.54482, size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex^2} (a+b\operatorname{sech}^{-1}(cx))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x,x]

[Out] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x, x]

Maple [A] time = 1.325, size = 0, normalized size = 0.

$$\int \frac{a + b\operatorname{arcsech}(cx)}{x} \sqrt{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x,x)

[Out] int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x,x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2 + d}(b \operatorname{arsech}(cx) + a)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="fricas")

[Out] integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/x, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{asech}(cx)) \sqrt{d + ex^2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asech(c*x))*(e*x**2+d)**(1/2)/x,x)

[Out] Integral((a + b*asech(c*x))*sqrt(d + e*x**2)/x, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2 + d}(b \operatorname{arsech}(cx) + a)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="giac")

[Out] integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/x, x)

$$3.134 \quad \int \frac{\sqrt{d+ex^2} (a+b\operatorname{sech}^{-1}(cx))}{x^3} dx$$

Optimal. Leaf size=25

$$\text{Unintegrable} \left(\frac{\sqrt{d+ex^2} (a+b\operatorname{sech}^{-1}(cx))}{x^3}, x \right)$$

[Out] Unintegrable[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^3, x]

Rubi [A] time = 0.104384, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sqrt{d+ex^2} (a+b\operatorname{sech}^{-1}(cx))}{x^3} dx$$

Verification is Not applicable to the result.

[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^3, x]

[Out] Defer[Int] [(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^3, x]

Rubi steps

$$\int \frac{\sqrt{d+ex^2} (a+b\operatorname{sech}^{-1}(cx))}{x^3} dx = \int \frac{\sqrt{d+ex^2} (a+b\operatorname{sech}^{-1}(cx))}{x^3} dx$$

Mathematica [A] time = 6.15173, size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex^2} (a+b\operatorname{sech}^{-1}(cx))}{x^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^3, x]

[Out] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^3, x]

Maple [A] time = 1.473, size = 0, normalized size = 0.

$$\int \frac{a + b\operatorname{arcsech}(cx)}{x^3} \sqrt{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^3, x)

[Out] int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^3, x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2 + d}(b \operatorname{arsech}(cx) + a)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="fricas")

[Out] integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/x^3, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{asech}(cx)) \sqrt{d + ex^2}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asech(c*x))*(e*x**2+d)**(1/2)/x**3,x)

[Out] Integral((a + b*asech(c*x))*sqrt(d + e*x**2)/x**3, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2 + d}(b \operatorname{arsech}(cx) + a)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="giac")

[Out] integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/x^3, x)

$$3.135 \quad \int x^2 \sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx)) dx$$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(x^2 \sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx)), x\right)$$

[Out] Unintegrable[x^2*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]), x]

Rubi [A] time = 0.0982496, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x^2 \sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Int[x^2*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]), x]

[Out] Defer[Int][x^2*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]), x]

Rubi steps

$$\int x^2 \sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx)) dx = \int x^2 \sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx)) dx$$

Mathematica [A] time = 12.8094, size = 0, normalized size = 0.

$$\int x^2 \sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Integrate[x^2*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]), x]

[Out] Integrate[x^2*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]), x]

Maple [A] time = 1.685, size = 0, normalized size = 0.

$$\int x^2 (a + b \operatorname{arcsech}(cx)) \sqrt{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2), x)

[Out] int(x^2*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2), x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(bx^2 \operatorname{arsech}(cx) + ax^2\right)\sqrt{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arcsech(c*x) + a*x^2)*sqrt(e*x^2 + d), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int x^2 (a + b \operatorname{asech}(cx)) \sqrt{d + ex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*asech(c*x))*(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*asech(c*x))*sqrt(d + e*x**2), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex^2 + d}(b \operatorname{arsech}(cx) + a)x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)*x^2, x)
```

$$3.136 \quad \int \sqrt{d + ex^2} \left(a + b \operatorname{sech}^{-1}(cx) \right) dx$$

Optimal. Leaf size=22

$$\text{Unintegrable} \left(\sqrt{d + ex^2} \left(a + b \operatorname{sech}^{-1}(cx) \right), x \right)$$

[Out] Unintegrable[Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]), x]

Rubi [A] time = 0.0363176, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \sqrt{d + ex^2} \left(a + b \operatorname{sech}^{-1}(cx) \right) dx$$

Verification is Not applicable to the result.

[In] Int[Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]), x]

[Out] Defer[Int][Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]), x]

Rubi steps

$$\int \sqrt{d + ex^2} \left(a + b \operatorname{sech}^{-1}(cx) \right) dx = \int \sqrt{d + ex^2} \left(a + b \operatorname{sech}^{-1}(cx) \right) dx$$

Mathematica [A] time = 2.9363, size = 0, normalized size = 0.

$$\int \sqrt{d + ex^2} \left(a + b \operatorname{sech}^{-1}(cx) \right) dx$$

Verification is Not applicable to the result.

[In] Integrate[Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]), x]

[Out] Integrate[Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]), x]

Maple [A] time = 1.279, size = 0, normalized size = 0.

$$\int (a + b \operatorname{arcsech}(cx)) \sqrt{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2), x)

[Out] int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2), x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{ex^2 + d}(b \operatorname{ar} \operatorname{sech}(cx) + a), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (a + b \operatorname{asech}(cx)) \sqrt{d + ex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asech(c*x))*(e*x**2+d)**(1/2),x)

[Out] Integral((a + b*asech(c*x))*sqrt(d + e*x**2), x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex^2 + d}(b \operatorname{ar} \operatorname{sech}(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a), x)

$$3.137 \quad \int \frac{\sqrt{d+ex^2} (a+b\operatorname{sech}^{-1}(cx))}{x^2} dx$$

Optimal. Leaf size=25

$$\text{Unintegrable} \left(\frac{\sqrt{d+ex^2} (a+b\operatorname{sech}^{-1}(cx))}{x^2}, x \right)$$

[Out] Unintegrable[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^2, x]

Rubi [A] time = 0.0874494, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{\sqrt{d+ex^2} (a+b\operatorname{sech}^{-1}(cx))}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^2, x]

[Out] Defer[Int] [(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^2, x]

Rubi steps

$$\int \frac{\sqrt{d+ex^2} (a+b\operatorname{sech}^{-1}(cx))}{x^2} dx = \int \frac{\sqrt{d+ex^2} (a+b\operatorname{sech}^{-1}(cx))}{x^2} dx$$

Mathematica [A] time = 1.84083, size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex^2} (a+b\operatorname{sech}^{-1}(cx))}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^2, x]

[Out] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^2, x]

Maple [A] time = 1.034, size = 0, normalized size = 0.

$$\int \frac{a + b\operatorname{arcsech}(cx)}{x^2} \sqrt{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^2, x)

[Out] int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^2, x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2 + d}(b \operatorname{arsech}(cx) + a)}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="fricas")

[Out] integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/x^2, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{asech}(cx)) \sqrt{d + ex^2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asech(c*x))*(e*x**2+d)**(1/2)/x**2,x)

[Out] Integral((a + b*asech(c*x))*sqrt(d + e*x**2)/x**2, x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2 + d}(b \operatorname{arsech}(cx) + a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="giac")

[Out] integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/x^2, x)

$$3.138 \quad \int \frac{\sqrt{d+ex^2} \left(a + b \operatorname{sech}^{-1}(cx) \right)}{x^4} dx$$

Optimal. Leaf size=312

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(c^2d+e)(2c^2d+3e)\sqrt{\frac{ex^2}{d}+1}\operatorname{EllipticF}\left(\sin^{-1}(cx),-\frac{e}{c^2d}\right)}{9cd\sqrt{d+ex^2}} - \frac{(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{3dx^3} + \frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9x^3}$$

```
[Out] (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(9
*x^3) + (2*b*(c^2*d + 2*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*
x^2]*Sqrt[d + e*x^2])/(9*d*x) - ((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/(3
*d*x^3) + (2*b*c*(c^2*d + 2*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[d +
e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(9*d*Sqrt[1 + (e*x^2)/d]) - (b
*(c^2*d + e)*(2*c^2*d + 3*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 + (e
*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(9*c*d*Sqrt[d + e*x^2])
```

Rubi [A] time = 0.379898, antiderivative size = 312, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {264, 6301, 12, 474, 583, 524, 426, 424, 421, 419}

$$\frac{(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{3dx^3} + \frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(c^2d+2e)\sqrt{d+ex^2}}{9dx} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^4,x]
```

```
[Out] (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(9
*x^3) + (2*b*(c^2*d + 2*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*
x^2]*Sqrt[d + e*x^2])/(9*d*x) - ((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/(3
*d*x^3) + (2*b*c*(c^2*d + 2*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[d +
e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(9*d*Sqrt[1 + (e*x^2)/d]) - (b
*(c^2*d + e)*(2*c^2*d + 3*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 + (e
*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(9*c*d*Sqrt[d + e*x^2])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)])*(b_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p +
3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 474

```
Int[((e_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))
^(q_), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(
q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
+ a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))
^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
+ 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0]
] && LtQ[m, -1]
```

Rule 524

```
Int[((e_) + (f_)*(x_)^(n_))/(Sqrt[(a_) + (b_)*(x_)^(n_)]*Sqrt[(c_) + (d_
)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] := Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0]
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[c]*Rt[-(d/c)
, 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{d+ex^2}(a+b\operatorname{sech}^{-1}(cx))}{x^4} dx &= -\frac{(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{3dx^3} + \left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int -\frac{(d+ex^2)^{3/2}}{3dx^4\sqrt{1-c^2x^2}} dx \\
&= -\frac{(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{3dx^3} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{(d+ex^2)^{3/2}}{x^4\sqrt{1-c^2x^2}} dx}{3d} \\
&= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9x^3} - \frac{(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{3dx^3} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{(d+ex^2)^{3/2}}{x^4\sqrt{1-c^2x^2}} dx}{3d} \\
&= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9x^3} + \frac{2b(c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9dx} \\
&= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9x^3} + \frac{2b(c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9dx} \\
&= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9x^3} + \frac{2b(c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9dx} \\
&= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9x^3} + \frac{2b(c^2d+2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{9dx}
\end{aligned}$$

Mathematica [C] time = 3.96936, size = 576, normalized size = 1.85

$$\frac{2ib\sqrt{\frac{1-cx}{cx+1}}(cx+1)(c\sqrt{d-i\sqrt{e}})^2 \sqrt{\frac{c(\sqrt{d-i\sqrt{e}})}{(cx+1)(c\sqrt{d-i\sqrt{e}})}} \sqrt{\frac{c(\sqrt{d+i\sqrt{e}})}{(cx+1)(c\sqrt{d+i\sqrt{e}})}} \left(\sqrt{e}(-3\sqrt{e}+2ic\sqrt{d})\operatorname{EllipticF}\left(i\sinh^{-1}\left(\sqrt{\frac{(1-cx)(c^2d+e)}{(cx+1)(c\sqrt{d+i\sqrt{e}})^2}}\right), \frac{(c\sqrt{d+i\sqrt{e}})^2}{(c\sqrt{d-i\sqrt{e}})^2}\right) + (c^2d+2e)E\left(i\sinh^{-1}\left(\sqrt{\frac{(1-cx)(c^2d+e)}{(cx+1)(c\sqrt{d+i\sqrt{e}})^2}}\right), \frac{(c\sqrt{d+i\sqrt{e}})^2}{(c\sqrt{d-i\sqrt{e}})^2}\right) \right)}{cd\sqrt{-\frac{(cx-1)(c\sqrt{d-i\sqrt{e}})}{(cx+1)(c\sqrt{d+i\sqrt{e}})}}}$$

$9\sqrt{d+ex^2}$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^4, x]

[Out] ((b*Sqrt[(1 - c*x)/(1 + c*x)]*(d + e*x^2))/x^3 + (b*c*Sqrt[(1 - c*x)/(1 + c*x)]*(d + e*x^2))/x^2 + (2*b*(c^2*d + 2*e)*Sqrt[(1 - c*x)/(1 + c*x)]*(d + e*x^2))/(d*x) - (3*a*(d + e*x^2)^2)/(d*x^3) - (3*b*(d + e*x^2)^2*ArcSech[c*x])/(d*x^3) - ((2*I)*b*(c*Sqrt[d] - I*Sqrt[e])^2*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[(c*(Sqrt[d] - I*Sqrt[e]*x))/((c*Sqrt[d] - I*Sqrt[e])*(1 + c*x))])*Sqrt[(c*(Sqrt[d] + I*Sqrt[e]*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x))]*(c^2*d + 2*e)*EllipticE[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2] + ((2*I)*c*Sqrt[d] - 3*Sqrt[e])*Sqrt[e]*EllipticF[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2]))/(c*d*Sqrt[-((c*Sqrt[d] - I*Sqrt[e])*(-1 + c*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x))]))/(9*Sqrt[d + e*x^2])

Maple [F] time = 1.318, size = 0, normalized size = 0.

$$\int \frac{a + b\operatorname{arcsech}(cx)}{x^4} \sqrt{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^4,x)`

[Out] `int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^4,x)`

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2 + d}(b \operatorname{arsech}(cx) + a)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="fricas")`

[Out] `integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/x^4, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + b \operatorname{asech}(cx)) \sqrt{d + ex^2}}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*asech(c*x))*(e*x**2+d)**(1/2)/x**4,x)`

[Out] `Integral((a + b*asech(c*x))*sqrt(d + e*x**2)/x**4, x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2 + d}(b \operatorname{arsech}(cx) + a)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="giac")`

[Out] `integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/x^4, x)`

$$3.139 \quad \int \frac{\sqrt{d+ex^2} (a+b\operatorname{sech}^{-1}(cx))}{x^6} dx$$

Optimal. Leaf size=446

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(c^2d+e)(24c^4d^2+7c^2de-30e^2)\sqrt{\frac{ex^2}{d}+1}\operatorname{EllipticF}\left(\sin^{-1}(cx),-\frac{e}{c^2d}\right)+\frac{2e(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{15d^2x^3}}{225cd^2\sqrt{d+ex^2}}$$

```
[Out] (b*(12*c^2*d - e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt
[d + e*x^2])/(225*d*x^3) + (b*(24*c^4*d^2 + 19*c^2*d*e - 31*e^2)*Sqrt[(1 +
c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(225*d^2*x) + (
b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*(d + e*x^2)^(3/2))/(
25*d*x^5) - ((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/(5*d*x^5) + (2*e*(d +
e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/(15*d^2*x^3) + (b*c*(24*c^4*d^2 + 19*c^2
*d*e - 31*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[d + e*x^2]*EllipticE
[ArcSin[c*x], -(e/(c^2*d))])/(225*d^2*Sqrt[1 + (e*x^2)/d]) - (b*(c^2*d + e)
*(24*c^4*d^2 + 7*c^2*d*e - 30*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[
1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(225*c*d^2*Sqrt[d + e*
x^2])
```

Rubi [A] time = 0.560598, antiderivative size = 446, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {271, 264, 6301, 12, 580, 583, 524, 426, 424, 421, 419}

$$\frac{2e(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{15d^2x^3} - \frac{(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{5dx^5} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(24c^4d^2+19c^2de-31e^2)\sqrt{\frac{ex^2}{d}+1}\operatorname{EllipticF}\left(\sin^{-1}(cx),-\frac{e}{c^2d}\right)}{225d^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^6, x]
```

```
[Out] (b*(12*c^2*d - e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt
[d + e*x^2])/(225*d*x^3) + (b*(24*c^4*d^2 + 19*c^2*d*e - 31*e^2)*Sqrt[(1 +
c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(225*d^2*x) + (
b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*(d + e*x^2)^(3/2))/(
25*d*x^5) - ((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/(5*d*x^5) + (2*e*(d +
e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/(15*d^2*x^3) + (b*c*(24*c^4*d^2 + 19*c^2
*d*e - 31*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[d + e*x^2]*EllipticE
[ArcSin[c*x], -(e/(c^2*d))])/(225*d^2*Sqrt[1 + (e*x^2)/d]) - (b*(c^2*d + e)
*(24*c^4*d^2 + 7*c^2*d*e - 30*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[
1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(225*c*d^2*Sqrt[d + e*
x^2])
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
```

p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

Rule 6301

Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))*((f_.)*(x_.))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 580

Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && GtQ[q, 0] && LtQ[m, -1] && !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])

Rule 583

Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) - e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]

Rule 524

Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || SimplerSqrtQ[-(b/a), -(d/c)]))))))

Rule 426

Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Dist[Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0]

Rule 424

Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rule 421

Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]

Rule 419

Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Simp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{d+ex^2}(a+b\operatorname{sech}^{-1}(cx))}{x^6} dx &= -\frac{(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{15d^2x^3} + \left(b\sqrt{\frac{1}{1+cx}}\right) \\ &= -\frac{(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{15d^2x^3} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\right)}{25dx^5} \\ &= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{25dx^5} - \frac{(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{5dx^5} + \frac{2e(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{15d^2x^3} \\ &= \frac{b(12c^2d-e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{225dx^3} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{25dx^5} \\ &= \frac{b(12c^2d-e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{225dx^3} + \frac{b(24c^4d^2+19c^2de-31e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{225dx^5} \\ &= \frac{b(12c^2d-e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{225dx^3} + \frac{b(24c^4d^2+19c^2de-31e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{225dx^5} \\ &= \frac{b(12c^2d-e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{225dx^3} + \frac{b(24c^4d^2+19c^2de-31e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{225dx^5} \\ &= \frac{b(12c^2d-e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{225dx^3} + \frac{b(24c^4d^2+19c^2de-31e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{225dx^5} \end{aligned}$$

Mathematica [C] time = 6.12054, size = 641, normalized size = 1.44

$$b\sqrt{\frac{1-cx}{cx+1}} \left(c^2(-24c^4d^2+19c^2de-31e^2)(d+ex^2) - \frac{i(cx+1)(c\sqrt{d-i\sqrt{e}})^2 \sqrt{\frac{c(\sqrt{d-i\sqrt{e}})}{(cx+1)(c\sqrt{d-i\sqrt{e}})}} \sqrt{\frac{c(\sqrt{d+i\sqrt{e}})}{(cx+1)(c\sqrt{d+i\sqrt{e}})}} \left(2\sqrt{e}(24ic^3d^{3/2}-36c^2d\sqrt{e}-29ic\sqrt{de}+30e^{3/2}) \operatorname{EllipticF}\left(i \sinh^{-1}\left(\sqrt{\frac{c}{cx+1}} \right) \right) \right)}{\sqrt{\frac{(cx-1)(c\sqrt{d-i\sqrt{e}})}{(cx+1)(c\sqrt{d+i\sqrt{e}})}}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/x^6, x]

[Out] ((15*a*(d + e*x^2)^2*(-3*d + 2*e*x^2))/x^5 + (b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(d + e*x^2)*(-31*e^2*x^4 + d*e*x^2*(8 + 19*c^2*x^2) + 3*d^2*(3 + 4

$$\frac{c^2 x^2 + 8c^4 x^4}{x^5} + (15b(d + ex^2)^2(-3d + 2ex^2) \operatorname{ArcSech}[cx])/x^5 + (b\sqrt{(1 - cx)/(1 + cx)}) \cdot (-c^2(24c^4 d^2 + 19c^2 d e - 31e^2)(d + ex^2) - (I(c\sqrt{d} - I\sqrt{e}))^2(1 + cx)\sqrt{(c(\sqrt{d} - I\sqrt{e}x))/((c\sqrt{d} - I\sqrt{e})(1 + cx))}) \cdot \sqrt{(c(\sqrt{d} + I\sqrt{e}x))/((c\sqrt{d} + I\sqrt{e})(1 + cx))}) \cdot ((24c^4 d^2 + 19c^2 d e - 31e^2) \operatorname{EllipticE}[I \operatorname{ArcSinh}[\sqrt{((c^2 d + e)(1 - cx))/((c\sqrt{d} + I\sqrt{e})^2(1 + cx))}]]], (c\sqrt{d} + I\sqrt{e})^2/(c\sqrt{d} - I\sqrt{e})^2] + 2\sqrt{e} \cdot ((24I)c^3 d^{3/2} - 36c^2 d \sqrt{e} - (29I)c\sqrt{d} e + 30e^{3/2}) \operatorname{EllipticF}[I \operatorname{ArcSinh}[\sqrt{((c^2 d + e)(1 - cx))/((c\sqrt{d} + I\sqrt{e})^2(1 + cx))}]]], (c\sqrt{d} + I\sqrt{e})^2/(c\sqrt{d} - I\sqrt{e})^2) \cdot \sqrt{-(((c\sqrt{d} - I\sqrt{e})(-1 + cx))/((c\sqrt{d} + I\sqrt{e})(1 + cx)))})/c)/(225d^2 \sqrt{d + ex^2})$$

Maple [F] time = 1.605, size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{arcsech}(cx)}{x^6} \sqrt{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^6,x)

[Out] int((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^6,x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{ex^2 + d}(b \operatorname{ar} \operatorname{sech}(cx) + a)}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="fricas")

[Out] integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/x^6, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asech(c*x))*(e*x**2+d)**(1/2)/x**6,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex^2 + d}(b \operatorname{ar} \operatorname{sech}(cx) + a)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="giac")

[Out] integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/x^6, x)

3.140 $\int x^3 (d + ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx)) dx$

Optimal. Leaf size=418

$$-\frac{d(d+ex^2)^{5/2}(a+b\operatorname{sech}^{-1}(cx))}{5e^2} + \frac{(d+ex^2)^{7/2}(a+b\operatorname{sech}^{-1}(cx))}{7e^2} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(3c^4d^2-38c^2de-25e^2)}{560c^6e}$$

```
[Out] (b*(3*c^4*d^2 - 38*c^2*d*e - 25*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(560*c^6*e) - (b*(13*c^2*d + 25*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*(d + e*x^2)^(3/2))/(840*c^4*e) - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*(d + e*x^2)^(5/2))/(42*c^2*e) - (d*(d + e*x^2)^(5/2)*(a + b*ArcSech[c*x]))/(5*e^2) + ((d + e*x^2)^(7/2)*(a + b*ArcSech[c*x]))/(7*e^2) + (b*(35*c^6*d^3 - 35*c^4*d^2*e - 63*c^2*d*e^2 - 25*e^3)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTan[(Sqrt[e]*Sqrt[1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(560*c^7*e^(3/2)) + (2*b*d^(7/2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/(35*e^2)
```

Rubi [A] time = 0.529505, antiderivative size = 418, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {266, 43, 6301, 12, 573, 154, 157, 63, 217, 203, 93, 207}

$$-\frac{d(d+ex^2)^{5/2}(a+b\operatorname{sech}^{-1}(cx))}{5e^2} + \frac{(d+ex^2)^{7/2}(a+b\operatorname{sech}^{-1}(cx))}{7e^2} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(3c^4d^2-38c^2de-25e^2)}{560c^6e}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]), x]
```

```
[Out] (b*(3*c^4*d^2 - 38*c^2*d*e - 25*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(560*c^6*e) - (b*(13*c^2*d + 25*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*(d + e*x^2)^(3/2))/(840*c^4*e) - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*(d + e*x^2)^(5/2))/(42*c^2*e) - (d*(d + e*x^2)^(5/2)*(a + b*ArcSech[c*x]))/(5*e^2) + ((d + e*x^2)^(7/2)*(a + b*ArcSech[c*x]))/(7*e^2) + (b*(35*c^6*d^3 - 35*c^4*d^2*e - 63*c^2*d*e^2 - 25*e^3)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTan[(Sqrt[e]*Sqrt[1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(560*c^7*e^(3/2)) + (2*b*d^(7/2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/(35*e^2)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))*((f_.)*(x_.))^(m_.)*((d_.) + (e_.)*(x_.)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 573

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((e_) + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.)*((e_.) + (f_.)*(x_.))^(p_.)*((g_.) + (h_.)*(x_.)), x_Symbol] := Simp[(h*(a + b*x)^m*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p + 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1)))]*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int((((c_.) + (d_.)*(x_.))^(n_.)*((e_.) + (f_.)*(x_.))^(p_.)*((g_.) + (h_.)*(x_.)))/((a_.) + (b_.)*(x_.)), x_Symbol] := Dist[h/b, Int[(c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
 \int x^3 (d + ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx)) dx &= -\frac{d(d + ex^2)^{5/2} (a + b \operatorname{sech}^{-1}(cx))}{5e^2} + \frac{(d + ex^2)^{7/2} (a + b \operatorname{sech}^{-1}(cx))}{7e^2} + \left(b \sqrt{\frac{1}{1+cx}} \right. \\
 &= -\frac{d(d + ex^2)^{5/2} (a + b \operatorname{sech}^{-1}(cx))}{5e^2} + \frac{(d + ex^2)^{7/2} (a + b \operatorname{sech}^{-1}(cx))}{7e^2} + \frac{\left(b \sqrt{\frac{1}{1+cx}} \right. \\
 &= -\frac{d(d + ex^2)^{5/2} (a + b \operatorname{sech}^{-1}(cx))}{5e^2} + \frac{(d + ex^2)^{7/2} (a + b \operatorname{sech}^{-1}(cx))}{7e^2} + \frac{\left(b \sqrt{\frac{1}{1+cx}} \right. \\
 &= -\frac{b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} (d + ex^2)^{5/2}}{42c^2e} - \frac{d(d + ex^2)^{5/2} (a + b \operatorname{sech}^{-1}(cx))}{5e^2} + \left(b \sqrt{\frac{1}{1+cx}} \right. \\
 &= -\frac{b(13c^2d + 25e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} (d + ex^2)^{3/2}}{840c^4e} - \frac{b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{42c^2e} \\
 &= \frac{b(3c^4d^2 - 38c^2de - 25e^2) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \sqrt{d + ex^2}}{560c^6e} - \frac{b(13c^2d + 25e)}{42c^2e} \\
 &= \frac{b(3c^4d^2 - 38c^2de - 25e^2) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \sqrt{d + ex^2}}{560c^6e} - \frac{b(13c^2d + 25e)}{42c^2e} \\
 &= \frac{b(3c^4d^2 - 38c^2de - 25e^2) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \sqrt{d + ex^2}}{560c^6e} - \frac{b(13c^2d + 25e)}{42c^2e} \\
 &= \frac{b(3c^4d^2 - 38c^2de - 25e^2) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \sqrt{d + ex^2}}{560c^6e} - \frac{b(13c^2d + 25e)}{42c^2e} \\
 &= \frac{b(3c^4d^2 - 38c^2de - 25e^2) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \sqrt{d + ex^2}}{560c^6e} - \frac{b(13c^2d + 25e)}{42c^2e} \\
 &= \frac{b(3c^4d^2 - 38c^2de - 25e^2) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \sqrt{d + ex^2}}{560c^6e} - \frac{b(13c^2d + 25e)}{42c^2e}
 \end{aligned}$$

Mathematica [A] time = 2.8559, size = 382, normalized size = 0.91

$$\frac{b \sqrt{\frac{1-cx}{cx+1}} \sqrt{c^2x^2 - 1} \left(\sqrt{c^2} \sqrt{e} \sqrt{c^2d + e} (-35c^4d^2e + 35c^6d^3 - 63c^2de^2 - 25e^3) \sqrt{\frac{c^2(d+ex^2)}{c^2d+e}} \sinh^{-1} \left(\frac{c \sqrt{e} \sqrt{c^2x^2 - 1}}{\sqrt{c^2} \sqrt{c^2d+e}} \right) + 32c^9d^{7/2} \sqrt{d + ex^2} \right)}{560c^9e^2(cx - 1) \sqrt{d + ex^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]),x]

[Out] $-(\sqrt{d + e x^2} (48 a c^6 (2 d - 5 e x^2) (d + e x^2)^2 + b e \sqrt{(1 - c x)/(1 + c x)} (1 + c x) (75 e^2 + 2 c^2 e (82 d + 25 e x^2) + c^4 (57 d^2 + 106 d e x^2 + 40 e^2 x^4)) + 48 b c^6 (2 d - 5 e x^2) (d + e x^2)^2 \operatorname{ArcSech}[c x])) / (1680 c^6 e^2) + (b \sqrt{(1 - c x)/(1 + c x)} \sqrt{-1 + c^2 x^2} (\sqrt{c^2} \sqrt{e} \sqrt{c^2 d + e} (35 c^6 d^3 - 35 c^4 d^2 e - 63 c^2 d e^2 - 25 e^3) \sqrt{(c^2 (d + e x^2))/(c^2 d + e)} \operatorname{ArcSinh}[(c \sqrt{e} \sqrt{-1 + c^2 x^2}) / (\sqrt{c^2} \sqrt{c^2 d + e})]) + 32 c^9 d^{7/2} \sqrt{d + e x^2} \operatorname{ArcTan}[(\sqrt{d} \sqrt{-1 + c^2 x^2}) / \sqrt{d + e x^2}])) / (560 c^9 e^2 (-1 + c x) \sqrt{d + e x^2})$

Maple [F] time = 1.135, size = 0, normalized size = 0.

$$\int x^3 (ex^2 + d)^{\frac{3}{2}} (a + b \operatorname{arcsech}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)

[Out] int(x^3*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 17.7434, size = 4365, normalized size = 10.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="fricas")

[Out] $[1/6720 * (96 * b * c^7 * d^{7/2} * \log(((c^4 * d^2 - 6 * c^2 * d * e + e^2) * x^4 - 8 * (c^2 * d^2 - d * e) * x^2 - 4 * ((c^3 * d - c * e) * x^3 - 2 * c * d * x) * \sqrt{e * x^2 + d} * \sqrt{d} * \sqrt{-(c^2 * x^2 - 1)/(c^2 * x^2)}) + 8 * d^2) / x^4) + 3 * (35 * b * c^6 * d^3 - 35 * b * c^4 * d^2 * e - 63 * b * c^2 * d * e^2 - 25 * b * e^3) * \sqrt{-e} * \log(8 * c^4 * e^2 * x^4 + c^4 * d^2 - 6 * c^2 * d * e + 8 * (c^4 * d * e - c^2 * e^2) * x^2 - 4 * (2 * c^4 * e * x^3 + (c^4 * d - c^2 * e) * x) * \sqrt{e * x^2 + d} * \sqrt{-e} * \sqrt{-(c^2 * x^2 - 1)/(c^2 * x^2)}) + e^2) + 192 * (5 * b * c^7 * e^3 * x^6 + 8 * b * c^7 * d * e^2 * x^4 + b * c^7 * d^2 * e * x^2 - 2 * b * c^7 * d^3) * \sqrt{e * x^2 + d} * \log((c * x * \sqrt{-(c^2 * x^2 - 1)/(c^2 * x^2)}) + 1) / (c * x)) + 4 * (240 * a * c^7 * e^3 * x^6 + 384 * a * c^7 * d * e^2 * x^4 + 48 * a * c^7 * d^2 * e * x^2 - 96 * a * c^7 * d^3 - (40 * b * c^6 * e^3 * x^5 + 2 * (53 * b * c^6 * d * e^2 + 25 * b * c^4 * e^3) * x^3 + (57 * b * c^6 * d^2 * e + 164 * b * c^4 * d * e^2 + 75 * b * c^2 * e^3) * x) * \sqrt{-(c^2 * x^2 - 1)/(c^2 * x^2)}) * \sqrt{e * x^2 + d}) / (c^7$

$*e^2)$, $1/3360*(48*b*c^7*d^{(7/2)}*\log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*((c^3*d - c*e)*x^3 - 2*c*d*x))*\sqrt{e*x^2 + d}*\sqrt{d}*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)} + 8*d^2)/x^4) + 3*(35*b*c^6*d^3 - 35*b*c^4*d^2*e - 63*b*c^2*d*e^2 - 25*b*e^3)*\sqrt{e}*\arctan(1/2*(2*c^2*e*x^3 + (c^2*d - e)*x))*\sqrt{e*x^2 + d}*\sqrt{e}*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)}/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e) + 96*(5*b*c^7*e^3*x^6 + 8*b*c^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*\sqrt{e*x^2 + d}*\log((c*x*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)} + 1)/(c*x)) + 2*(240*a*c^7*e^3*x^6 + 384*a*c^7*d*e^2*x^4 + 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 - (40*b*c^6*e^3*x^5 + 2*(53*b*c^6*d*e^2 + 25*b*c^4*e^3)*x^3 + (57*b*c^6*d^2*e + 164*b*c^4*d*e^2 + 75*b*c^2*e^3)*x))*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)}*\sqrt{e*x^2 + d}))/c^7*e^2)$, $1/6720*(192*b*c^7*\sqrt{-d}*d^3*\arctan(-1/2*((c^3*d - c*e)*x^3 - 2*c*d*x))*\sqrt{e*x^2 + d}*\sqrt{-d}*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)}/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + 3*(35*b*c^6*d^3 - 35*b*c^4*d^2*e - 63*b*c^2*d*e^2 - 25*b*e^3)*\sqrt{-e}*\log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x))*\sqrt{e*x^2 + d}*\sqrt{-e}*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)} + e^2) + 192*(5*b*c^7*e^3*x^6 + 8*b*c^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*\sqrt{e*x^2 + d}*\log((c*x*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)} + 1)/(c*x)) + 4*(240*a*c^7*e^3*x^6 + 384*a*c^7*d*e^2*x^4 + 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 - (40*b*c^6*e^3*x^5 + 2*(53*b*c^6*d*e^2 + 25*b*c^4*e^3)*x^3 + (57*b*c^6*d^2*e + 164*b*c^4*d*e^2 + 75*b*c^2*e^3)*x))*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)}*\sqrt{e*x^2 + d}))/c^7*e^2)$, $1/3360*(96*b*c^7*\sqrt{-d}*d^3*\arctan(-1/2*((c^3*d - c*e)*x^3 - 2*c*d*x))*\sqrt{e*x^2 + d}*\sqrt{-d}*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)}/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + 3*(35*b*c^6*d^3 - 35*b*c^4*d^2*e - 63*b*c^2*d*e^2 - 25*b*e^3)*\sqrt{e}*\arctan(1/2*(2*c^2*e*x^3 + (c^2*d - e)*x))*\sqrt{e*x^2 + d}*\sqrt{e}*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)}/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e) + 96*(5*b*c^7*e^3*x^6 + 8*b*c^7*d*e^2*x^4 + b*c^7*d^2*e*x^2 - 2*b*c^7*d^3)*\sqrt{e*x^2 + d}*\log((c*x*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)} + 1)/(c*x)) + 2*(240*a*c^7*e^3*x^6 + 384*a*c^7*d*e^2*x^4 + 48*a*c^7*d^2*e*x^2 - 96*a*c^7*d^3 - (40*b*c^6*e^3*x^5 + 2*(53*b*c^6*d*e^2 + 25*b*c^4*e^3)*x^3 + (57*b*c^6*d^2*e + 164*b*c^4*d*e^2 + 75*b*c^2*e^3)*x))*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)}*\sqrt{e*x^2 + d}))/c^7*e^2]$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(e*x**2+d)**(3/2)*(a+b*asech(c*x)), x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + d)^{\frac{3}{2}}(b \operatorname{ar} \operatorname{sech}(cx) + a)x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)), x, algorithm="giac")`

[Out] `integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)*x^3, x)`

3.141 $\int x (d + ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx)) dx$

Optimal. Leaf size=297

$$\frac{(d + ex^2)^{5/2} (a + b \operatorname{sech}^{-1}(cx))}{5e} - \frac{b \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} (15c^4 d^2 + 10c^2 de + 3e^2) \tan^{-1}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{40c^5 \sqrt{e}} - \frac{bd^{5/2} \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \tan^{-1}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{5e}$$

[Out] $-(b*(7*c^2*d + 3*e)*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[1 - c^2*x^2]*\operatorname{Sqrt}[d + e*x^2])/(40*c^4) - (b*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[1 - c^2*x^2]*(d + e*x^2)^{(3/2)})/(20*c^2) + ((d + e*x^2)^{(5/2)}*(a + b*\operatorname{ArcSech}[c*x]))/(5*e) - (b*(15*c^4*d^2 + 10*c^2*d*e + 3*e^2)*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{ArcTan}[(\operatorname{Sqrt}[e]*\operatorname{Sqrt}[1 - c^2*x^2])/(c*\operatorname{Sqrt}[d + e*x^2])])/(40*c^5*\operatorname{Sqrt}[e]) - (b*d^{(5/2)}*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{ArcTanh}[\operatorname{Sqrt}[d + e*x^2]/(\operatorname{Sqrt}[d]*\operatorname{Sqrt}[1 - c^2*x^2])])/(5*e)$

Rubi [A] time = 0.43235, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {6299, 517, 446, 102, 154, 157, 63, 217, 203, 93, 207}

$$\frac{(d + ex^2)^{5/2} (a + b \operatorname{sech}^{-1}(cx))}{5e} - \frac{b \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} (15c^4 d^2 + 10c^2 de + 3e^2) \tan^{-1}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{40c^5 \sqrt{e}} - \frac{bd^{5/2} \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \tan^{-1}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{5e}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[x*(d + e*x^2)^{(3/2)}*(a + b*\operatorname{ArcSech}[c*x]), x]$

[Out] $-(b*(7*c^2*d + 3*e)*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[1 - c^2*x^2]*\operatorname{Sqrt}[d + e*x^2])/(40*c^4) - (b*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[1 - c^2*x^2]*(d + e*x^2)^{(3/2)})/(20*c^2) + ((d + e*x^2)^{(5/2)}*(a + b*\operatorname{ArcSech}[c*x]))/(5*e) - (b*(15*c^4*d^2 + 10*c^2*d*e + 3*e^2)*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{ArcTan}[(\operatorname{Sqrt}[e]*\operatorname{Sqrt}[1 - c^2*x^2])/(c*\operatorname{Sqrt}[d + e*x^2])])/(40*c^5*\operatorname{Sqrt}[e]) - (b*d^{(5/2)}*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{ArcTanh}[\operatorname{Sqrt}[d + e*x^2]/(\operatorname{Sqrt}[d]*\operatorname{Sqrt}[1 - c^2*x^2])])/(5*e)$

Rule 6299

$\operatorname{Int}[(a + \operatorname{ArcSech}(c*x))*(b*x)*(d + e*x^2)^p, x_Symbol] \rightarrow \operatorname{Simp}[(d + e*x^2)^{p+1}*(a + b*\operatorname{ArcSech}(c*x))/(2*e*(p+1)), x] + \operatorname{Dist}[(b*\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[1/(1 + c*x)])/(2*e*(p+1)), \operatorname{Int}[(d + e*x^2)^{p+1}/(x*\operatorname{Sqrt}[1 - c*x]*\operatorname{Sqrt}[1 + c*x]), x, x] /; \operatorname{FreeQ}\{a, b, c, d, e, p\}, x] \&\& \operatorname{NeQ}[p, -1]$

Rule 517

$\operatorname{Int}[(u + (c + d*x^n)^q)*(a + b*x^{non2})^p, x_Symbol] \rightarrow \operatorname{Int}[u*(a*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x] /; \operatorname{FreeQ}\{a1, b1, a2, b2, c, d, n, p, q\}, x] \&\& \operatorname{EqQ}[non2, n/2] \&\& \operatorname{EqQ}[a2*b1 + a1*b2, 0] \&\& (\operatorname{IntegerQ}[p] \mid\mid (\operatorname{GtQ}[a1, 0] \&\& \operatorname{GtQ}[a2, 0]))$

Rule 446

$\operatorname{Int}(x^m*(a + b*x^n)^p*(c + d*x^n)^q, x_Symbol] \rightarrow \operatorname{Dist}[1/n, \operatorname{Subst}[\operatorname{Int}[x^{(m+1)/n} - 1]*(a + b*x)^p$

$*(c + d*x)^q, x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, m, n, p, q\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rule 102

$\text{Int}[(a + b*x)^m * (c + d*x)^n * (e + f*x)^p, x_Symbol] := \text{Simp}[(b*(a + b*x)^{m-1} * (c + d*x)^{n+1} * (e + f*x)^{p+1}) / (d*f*(m + n + p + 1)), x] + \text{Dist}[1 / (d*f*(m + n + p + 1)), \text{Int}[(a + b*x)^{m-2} * (c + d*x)^n * (e + f*x)^p * \text{Simp}[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m-1) + a*(d*e*(n+1) + c*f*(p+1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m+n) + c*f*(m+p))]*x, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n, p\}, x\} \&\& \text{GtQ}[m, 1] \&\& \text{NeQ}[m + n + p + 1, 0] \&\& \text{IntegersQ}[2*m, 2*n, 2*p]$

Rule 154

$\text{Int}[(a + b*x)^m * (c + d*x)^n * (e + f*x)^p * (g + h*x), x_Symbol] := \text{Simp}[(h*(a + b*x)^m * (c + d*x)^{n+1} * (e + f*x)^{p+1}) / (d*f*(m + n + p + 2)), x] + \text{Dist}[1 / (d*f*(m + n + p + 2)), \text{Int}[(a + b*x)^{m-1} * (c + d*x)^n * (e + f*x)^p * \text{Simp}[a*d*f*g*(m + n + p + 2) - h*(b*c*e*m + a*(d*e*(n+1) + c*f*(p+1))) + (b*d*f*g*(m + n + p + 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1)))]*x, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h, n, p\}, x\} \&\& \text{GtQ}[m, 0] \&\& \text{NeQ}[m + n + p + 2, 0] \&\& \text{IntegersQ}[2*m, 2*n, 2*p]$

Rule 157

$\text{Int}[(c + d*x)^n * (e + f*x)^p * (g + h*x), x_Symbol] := \text{Dist}[h/b, \text{Int}[(c + d*x)^n * (e + f*x)^p, x], x] + \text{Dist}[(b*g - a*h)/b, \text{Int}[(c + d*x)^n * (e + f*x)^p / (a + b*x), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h, n, p\}, x\}$

Rule 63

$\text{Int}[(a + b*x)^m * (c + d*x)^n, x_Symbol] := \text{With}\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{p*(m+1)-1} * (c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{1/p}], x] /; \text{FreeQ}\{a, b, c, d\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 217

$\text{Int}[1/\text{Sqrt}[a + b*x^2], x_Symbol] := \text{Subst}[\text{Int}[1/(1 - b*x^2), x], x, x/\text{Sqrt}[a + b*x^2]] /; \text{FreeQ}\{a, b\}, x\} \&\& !\text{GtQ}[a, 0]$

Rule 203

$\text{Int}[(a + b*x^2)^{-1}, x_Symbol] := \text{Simp}[(1 * \text{ArcTan}[(\text{Rt}[b, 2]*x)/\text{Rt}[a, 2]]) / (\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}\{a, b\}, x\} \&\& \text{PosQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{GtQ}[b, 0])$

Rule 93

$\text{Int}[(a + b*x)^m * (c + d*x)^n / (e + f*x), x_Symbol] := \text{With}\{q = \text{Denominator}[m]\}, \text{Dist}[q, \text{Subst}[\text{Int}[x^{q*(m+1)-1} / (b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^{1/q} / (c + d*x)^{1/q}], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\} \&\& \text{EqQ}[m + n + 1, 0] \&\& \text{RationalQ}[n] \&\& \text{LtQ}[-1, m, 0] \&\& \text{SimplerQ}[a + b*x, c + d*x]$

Rule 207


```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
 \int x(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))dx &= \frac{(d+ex^2)^{5/2}(a+b\operatorname{sech}^{-1}(cx))}{5e} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{(d+ex^2)^{5/2}}{x\sqrt{1-cx}\sqrt{1+cx}}dx}{5e} \\
 &= \frac{(d+ex^2)^{5/2}(a+b\operatorname{sech}^{-1}(cx))}{5e} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\int\frac{(d+ex^2)^{5/2}}{x\sqrt{1-c^2x^2}}dx}{5e} \\
 &= \frac{(d+ex^2)^{5/2}(a+b\operatorname{sech}^{-1}(cx))}{5e} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int\frac{(d+ex^2)^{5/2}}{x\sqrt{1-c^2x}}dx, x, x\right)}{10e} \\
 &= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{20c^2} + \frac{(d+ex^2)^{5/2}(a+b\operatorname{sech}^{-1}(cx))}{5e} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{20c^2} \\
 &= -\frac{b(7c^2d+3e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{40c^4} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{20c^2} \\
 &= -\frac{b(7c^2d+3e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{40c^4} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{20c^2} \\
 &= -\frac{b(7c^2d+3e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{40c^4} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{20c^2} \\
 &= -\frac{b(7c^2d+3e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{40c^4} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{20c^2} \\
 &= -\frac{b(7c^2d+3e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{40c^4} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{20c^2}
 \end{aligned}$$

Mathematica [A] time = 1.48017, size = 342, normalized size = 1.15

$$\frac{\sqrt{d+ex^2}\left(8ac^4(d+ex^2)^2 - be\sqrt{\frac{1-cx}{cx+1}}(cx+1)(c^2(9d+2ex^2)+3e) + 8bc^4\operatorname{sech}^{-1}(cx)(d+ex^2)^2\right)}{40c^4e} + \frac{b\sqrt{\frac{1-cx}{cx+1}}\sqrt{1-c^2x^2}}{20c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]), x]
```

```
[Out] (Sqrt[d + e*x^2]*(8*a*c^4*(d + e*x^2)^2 - b*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(3*e + c^2*(9*d + 2*e*x^2)) + 8*b*c^4*(d + e*x^2)^2*ArcSech[c*x]))/(40*c^4*e) + (b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*(Sqrt[-c^2]*Sqrt[-(c^2*d) - e]*Sqrt[e]*(15*c^4*d^2 + 10*c^2*d*e + 3*e^2)*Sqrt[(c^2*(d + e*x^2))/(c^2*d + e)]*ArcSin[(c*Sqrt[e]*Sqrt[1 - c^2*x^2])/(Sqrt[-c^2]*Sqrt[-(c^2*d) - e]]) + 8*c^7*d^(5/2)*Sqrt[-d - e*x^2]*ArcTan[(Sqrt[d]*Sqrt[1 - c^2*x^2])/Sqrt[-d - e*x^2]]))/(40*c^7*e*(-1 + c*x)*Sqrt[d + e*x^2])
```

Maple [F] time = 0.888, size = 0, normalized size = 0.

$$\int x (ex^2 + d)^{\frac{3}{2}} (a + b \operatorname{arcsech}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)`

[Out] `int(x*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(ex^2 + d)^{\frac{5}{2}} a}{5e} + \frac{1}{15} b \left(\frac{((3e^2x^4 + dex^2 - 2d^2)x^3 + 5(dex^4 + d^2x^2)x)\sqrt{ex^2 + d} \log(\sqrt{cx + 1}\sqrt{-cx + 1} + 1)}{ex^3} - 15 \int \frac{15(c^2e^2}{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="maxima")`

[Out] `1/5*(e*x^2 + d)^(5/2)*a/e + 1/15*b*(((3*e^2*x^4 + d*e*x^2 - 2*d^2)*x^3 + 5*(d*e*x^4 + d^2*x^2)*x)*sqrt(e*x^2 + d)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)/(e*x^3) - 15*integrate(1/15*(15*(c^2*e^2*x^4*log(c) - e^2*x^2*log(c))*x^3 + 15*(c^2*d*e*x^4*log(c) - d*e*x^2*log(c))*x + ((3*(5*e^2*log(c) + e^2)*c^2*x^4 - 2*c^2*d^2 + (c^2*d*e - 15*e^2*log(c))*x^2)*x^3 + 5*((3*d*e*log(c) + d*e)*c^2*x^4 + (c^2*d^2 - 3*d*e*log(c))*x^2)*x + 30*((c^2*e^2*x^4 - e^2*x^2)*x^3 + (c^2*d*e*x^4 - d*e*x^2)*x)*log(sqrt(x)))*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1)) + 30*((c^2*e^2*x^4 - e^2*x^2)*x^3 + (c^2*d*e*x^4 - d*e*x^2)*x)*log(sqrt(x))*sqrt(e*x^2 + d)/(c^2*e*x^4 - e*x^2 + (c^2*e*x^4 - e*x^2)*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1))), x)`

Fricas [B] time = 8.4908, size = 3606, normalized size = 12.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="fricas")`

[Out] `[1/160*(8*b*c^5*d^(5/2)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) - (15*b*c^4*d^2 + 10*b*c^2*d*e + 3*b*e^2)*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e^2) + 32*(b*c^5*e^2*x^4 + 2*b*c^5*d*e*x^2 + b*c^5*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(8*a*c^5*e^2*x^4 + 16*a*c^5*d*e*x^2 + 8*a*c^5*d^2 - (2*b*c^4*e^2*x^3 + 3*(3*b*c^4*d*e + b*c^2*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d)/(c^5*e), 1/80*(4*b*c^5*d^(5/2)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) - (15*b*c^4*d^2 + 10*b*c^2*d*e + 3*b*e^2)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d - e)*x)*sqrt(e`

$$\begin{aligned}
& x^2 + d) \sqrt{e} \sqrt{-(c^2 x^2 - 1)/(c^2 x^2)} / (c^2 e^2 x^4 + (c^2 d e - e^2) x^2 - d e) + 16 (b c^5 e^2 x^4 + 2 b c^5 d e x^2 + b c^5 d^2) \sqrt{e x^2 + d} \log((c x \sqrt{-(c^2 x^2 - 1)/(c^2 x^2)} + 1)/(c x)) + 2 (8 a c^5 e^2 x^4 + 16 a c^5 d e x^2 + 8 a c^5 d^2 - (2 b c^4 e^2 x^3 + 3 (3 b c^4 d e + b c^2 e^2) x) \sqrt{-(c^2 x^2 - 1)/(c^2 x^2)}) \sqrt{e x^2 + d} / (c^5 e), \\
& - 1/160 (16 b c^5 \sqrt{-d} d^2 \arctan(-1/2 ((c^3 d - c e) x^3 - 2 c d x) \sqrt{e x^2 + d} \sqrt{-d} \sqrt{-(c^2 x^2 - 1)/(c^2 x^2)}) / (c^2 d e x^4 + (c^2 d^2 - d e) x^2 - d^2) + (15 b c^4 d^2 + 10 b c^2 d e + 3 b e^2) \sqrt{-e} \log(8 c^4 e^2 x^4 + c^4 d^2 - 6 c^2 d e + 8 (c^4 d e - c^2 e^2) x^2 - 4 (2 c^4 e x^3 + (c^4 d - c^2 e) x) \sqrt{e x^2 + d} \sqrt{-e} \sqrt{-(c^2 x^2 - 1)/(c^2 x^2)} + e^2) - 32 (b c^5 e^2 x^4 + 2 b c^5 d e x^2 + b c^5 d^2) \sqrt{e x^2 + d} \log((c x \sqrt{-(c^2 x^2 - 1)/(c^2 x^2)} + 1)/(c x)) - 4 (8 a c^5 e^2 x^4 + 16 a c^5 d e x^2 + 8 a c^5 d^2 - (2 b c^4 e^2 x^3 + 3 (3 b c^4 d e + b c^2 e^2) x) \sqrt{-(c^2 x^2 - 1)/(c^2 x^2)}) \sqrt{e x^2 + d} / (c^5 e), \\
& - 1/80 (8 b c^5 \sqrt{-d} d^2 \arctan(-1/2 ((c^3 d - c e) x^3 - 2 c d x) \sqrt{e x^2 + d} \sqrt{-d} \sqrt{-(c^2 x^2 - 1)/(c^2 x^2)}) / (c^2 d e x^4 + (c^2 d^2 - d e) x^2 - d^2) + (15 b c^4 d^2 + 10 b c^2 d e + 3 b e^2) \sqrt{e} \arctan(1/2 (2 c^2 e x^3 + (c^2 d - e) x) \sqrt{e x^2 + d} \sqrt{e} \sqrt{-(c^2 x^2 - 1)/(c^2 x^2)}) / (c^2 e^2 x^4 + (c^2 d e - e^2) x^2 - d e) - 16 (b c^5 e^2 x^4 + 2 b c^5 d e x^2 + b c^5 d^2) \sqrt{e x^2 + d} \log((c x \sqrt{-(c^2 x^2 - 1)/(c^2 x^2)} + 1)/(c x)) - 2 (8 a c^5 e^2 x^4 + 16 a c^5 d e x^2 + 8 a c^5 d^2 - (2 b c^4 e^2 x^3 + 3 (3 b c^4 d e + b c^2 e^2) x) \sqrt{-(c^2 x^2 - 1)/(c^2 x^2)}) \sqrt{e x^2 + d} / (c^5 e)
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x**2+d)**(3/2)*(a+b*asech(c*x)),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + d)^{\frac{3}{2}} (b \operatorname{ar} \operatorname{sech}(cx) + a) x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="giac")

[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)*x, x)

$$3.142 \quad \int \frac{(d+ex^2)^{3/2} (a+b\operatorname{sech}^{-1}(cx))}{x} dx$$

Optimal. Leaf size=25

$$\operatorname{Unintegrable}\left(\frac{(d+ex^2)^{3/2} (a+b\operatorname{sech}^{-1}(cx))}{x}, x\right)$$

[Out] Unintegrable[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x, x]

Rubi [A] time = 0.119816, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(d+ex^2)^{3/2} (a+b\operatorname{sech}^{-1}(cx))}{x} dx$$

Verification is Not applicable to the result.

[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x, x]

[Out] Defer[Int] [((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x, x]

Rubi steps

$$\int \frac{(d+ex^2)^{3/2} (a+b\operatorname{sech}^{-1}(cx))}{x} dx = \int \frac{(d+ex^2)^{3/2} (a+b\operatorname{sech}^{-1}(cx))}{x} dx$$

Mathematica [A] time = 6.20457, size = 0, normalized size = 0.

$$\int \frac{(d+ex^2)^{3/2} (a+b\operatorname{sech}^{-1}(cx))}{x} dx$$

Verification is Not applicable to the result.

[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x, x]

[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x, x]

Maple [A] time = 0.828, size = 0, normalized size = 0.

$$\int \frac{a + b\operatorname{arcsech}(cx)}{x} (ex^2 + d)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x, x)

[Out] `int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x,x)`

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(aex^2 + ad + (bex^2 + bd) \operatorname{arsech}(cx))\sqrt{ex^2 + d}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x,x, algorithm="fricas")`

[Out] `integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsech(c*x))*sqrt(e*x^2 + d)/x, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)**(3/2)*(a+b*asech(c*x))/x,x)`

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^{\frac{3}{2}}(b \operatorname{arsech}(cx) + a)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x,x, algorithm="giac")`

[Out] `integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)/x, x)`

$$3.143 \quad \int \frac{(d+ex^2)^{3/2} (a+b\operatorname{sech}^{-1}(cx))}{x^3} dx$$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{(d+ex^2)^{3/2} (a+b\operatorname{sech}^{-1}(cx))}{x^3}, x\right)$$

[Out] Unintegrable[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^3, x]

Rubi [A] time = 0.124839, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(d+ex^2)^{3/2} (a+b\operatorname{sech}^{-1}(cx))}{x^3} dx$$

Verification is Not applicable to the result.

[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^3, x]

[Out] Defer[Int] [((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^3, x]

Rubi steps

$$\int \frac{(d+ex^2)^{3/2} (a+b\operatorname{sech}^{-1}(cx))}{x^3} dx = \int \frac{(d+ex^2)^{3/2} (a+b\operatorname{sech}^{-1}(cx))}{x^3} dx$$

Mathematica [A] time = 6.56331, size = 0, normalized size = 0.

$$\int \frac{(d+ex^2)^{3/2} (a+b\operatorname{sech}^{-1}(cx))}{x^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^3, x]

[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^3, x]

Maple [A] time = 0.937, size = 0, normalized size = 0.

$$\int \frac{a + b\operatorname{arcsech}(cx)}{x^3} (ex^2 + d)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^3, x)

[Out] `int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^3,x)`

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^3,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(aex^2 + ad + (bex^2 + bd) \operatorname{arsech}(cx))\sqrt{ex^2 + d}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^3,x, algorithm="fricas")`

[Out] `integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsech(c*x))*sqrt(e*x^2 + d)/x^3, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)**(3/2)*(a+b*asech(c*x))/x**3,x)`

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^{\frac{3}{2}}(b \operatorname{arsech}(cx) + a)}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^3,x, algorithm="giac")`

[Out] `integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)/x^3, x)`

$$3.144 \quad \int x^2 (d + ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx)) dx$$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(x^2 (d + ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx)), x\right)$$

[Out] Unintegrable[x^2*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]), x]

Rubi [A] time = 0.121631, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int x^2 (d + ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Int[x^2*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]), x]

[Out] Defer[Int][x^2*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]), x]

Rubi steps

$$\int x^2 (d + ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx)) dx = \int x^2 (d + ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx)) dx$$

Mathematica [A] time = 14.2335, size = 0, normalized size = 0.

$$\int x^2 (d + ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Integrate[x^2*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]), x]

[Out] Integrate[x^2*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]), x]

Maple [A] time = 1.113, size = 0, normalized size = 0.

$$\int x^2 (ex^2 + d)^{\frac{3}{2}} (a + b \operatorname{arcsech}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)), x)

[Out] int(x^2*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)), x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(aex^4 + adx^2 + (bex^4 + bdx^2)\text{arsech}(cx)\right)\sqrt{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="fricas")

[Out] integral((a*e*x^4 + a*d*x^2 + (b*e*x^4 + b*d*x^2)*arcsech(c*x))*sqrt(e*x^2 + d), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(e*x**2+d)**(3/2)*(a+b*asech(c*x)),x)

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + d)^{\frac{3}{2}}(b \text{arsech}(cx) + a)x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="giac")

[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)*x^2, x)

$$3.145 \quad \int (d + ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx)) dx$$

Optimal. Leaf size=22

$$\text{Unintegrable}\left((d + ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx)), x\right)$$

[Out] Unintegrable[(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]), x]

Rubi [A] time = 0.0444486, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (d + ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Int[(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]), x]

[Out] Defer[Int] [(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]), x]

Rubi steps

$$\int (d + ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx)) dx = \int (d + ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx)) dx$$

Mathematica [A] time = 4.29346, size = 0, normalized size = 0.

$$\int (d + ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Integrate[(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]), x]

[Out] Integrate[(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]), x]

Maple [A] time = 0.824, size = 0, normalized size = 0.

$$\int (ex^2 + d)^{\frac{3}{2}} (a + b \operatorname{arcsech}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x)), x)

[Out] int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x)), x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(aex^2 + ad + (bex^2 + bd) \operatorname{arsech}(cx)\right)\sqrt{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="fricas")

[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsech(c*x))*sqrt(e*x^2 + d), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**(3/2)*(a+b*asech(c*x)),x)

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + d)^{\frac{3}{2}}(b \operatorname{arsech}(cx) + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="giac")

[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a), x)

$$3.146 \quad \int \frac{(d+ex^2)^{3/2} (a+b\operatorname{sech}^{-1}(cx))}{x^2} dx$$

Optimal. Leaf size=25

$$\operatorname{Unintegrable}\left(\frac{(d+ex^2)^{3/2} (a+b\operatorname{sech}^{-1}(cx))}{x^2}, x\right)$$

[Out] Unintegrable[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^2, x]

Rubi [A] time = 0.104391, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(d+ex^2)^{3/2} (a+b\operatorname{sech}^{-1}(cx))}{x^2} dx$$

Verification is Not applicable to the result.

[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^2, x]

[Out] Defer[Int] [((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^2, x]

Rubi steps

$$\int \frac{(d+ex^2)^{3/2} (a+b\operatorname{sech}^{-1}(cx))}{x^2} dx = \int \frac{(d+ex^2)^{3/2} (a+b\operatorname{sech}^{-1}(cx))}{x^2} dx$$

Mathematica [A] time = 6.72403, size = 0, normalized size = 0.

$$\int \frac{(d+ex^2)^{3/2} (a+b\operatorname{sech}^{-1}(cx))}{x^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^2, x]

[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^2, x]

Maple [A] time = 0.806, size = 0, normalized size = 0.

$$\int \frac{a + b\operatorname{arcsech}(cx)}{x^2} (ex^2 + d)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^2, x)

[Out] `int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^2,x)`

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(aex^2 + ad + (bex^2 + bd) \operatorname{arsech}(cx))\sqrt{ex^2 + d}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^2,x, algorithm="fricas")`

[Out] `integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsech(c*x))*sqrt(e*x^2 + d)/x^2, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)**(3/2)*(a+b*asech(c*x))/x**2,x)`

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^{\frac{3}{2}}(b \operatorname{arsech}(cx) + a)}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^2,x, algorithm="giac")`

[Out] `integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)/x^2, x)`

$$3.147 \quad \int \frac{(d+ex^2)^{3/2} (a+b\operatorname{sech}^{-1}(cx))}{x^4} dx$$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{(d+ex^2)^{3/2} (a+b\operatorname{sech}^{-1}(cx))}{x^4}, x\right)$$

[Out] Unintegrable[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^4, x]

Rubi [A] time = 0.12844, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(d+ex^2)^{3/2} (a+b\operatorname{sech}^{-1}(cx))}{x^4} dx$$

Verification is Not applicable to the result.

[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^4, x]

[Out] Defer[Int] [((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^4, x]

Rubi steps

$$\int \frac{(d+ex^2)^{3/2} (a+b\operatorname{sech}^{-1}(cx))}{x^4} dx = \int \frac{(d+ex^2)^{3/2} (a+b\operatorname{sech}^{-1}(cx))}{x^4} dx$$

Mathematica [A] time = 16.0787, size = 0, normalized size = 0.

$$\int \frac{(d+ex^2)^{3/2} (a+b\operatorname{sech}^{-1}(cx))}{x^4} dx$$

Verification is Not applicable to the result.

[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^4, x]

[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^4, x]

Maple [A] time = 1.039, size = 0, normalized size = 0.

$$\int \frac{a + b\operatorname{arcsech}(cx)}{x^4} (ex^2 + d)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^4, x)

[Out] `int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^4,x)`

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^4,x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(aex^2 + ad + (bex^2 + bd) \operatorname{arsech}(cx))\sqrt{ex^2 + d}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^4,x, algorithm="fricas")`

[Out] `integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsech(c*x))*sqrt(e*x^2 + d)/x^4, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)**(3/2)*(a+b*asech(c*x))/x**4,x)`

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^{\frac{3}{2}}(b \operatorname{arsech}(cx) + a)}{x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^4,x, algorithm="giac")`

[Out] `integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)/x^4, x)`

$$3.148 \quad \int \frac{(d+ex^2)^{3/2} (a+b\operatorname{sech}^{-1}(cx))}{x^6} dx$$

Optimal. Leaf size=409

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(c^2d+e)(8c^4d^2+19c^2de+15e^2)\sqrt{\frac{ex^2}{d}+1}\operatorname{EllipticF}\left(\sin^{-1}(cx),-\frac{e}{c^2d}\right)}{75cd\sqrt{d+ex^2}} - \frac{(d+ex^2)^{5/2}(a+b\operatorname{sech}^{-1}(cx))}{5dx^5}$$

[Out] (4*b*(c^2*d + 2*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(75*x^3) + (b*(8*c^4*d^2 + 23*c^2*d*e + 23*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(75*d*x) + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*(d + e*x^2)^(3/2))/(25*x^5) - ((d + e*x^2)^(5/2)*(a + b*ArcSech[c*x]))/(5*d*x^5) + (b*c*(8*c^4*d^2 + 23*c^2*d*e + 23*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(75*d*Sqrt[1 + (e*x^2)/d]) - (b*(c^2*d + e)*(8*c^4*d^2 + 19*c^2*d*e + 15*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(75*c*d*Sqrt[d + e*x^2])

Rubi [A] time = 0.532908, antiderivative size = 409, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {264, 6301, 12, 474, 580, 583, 524, 426, 424, 421, 419}

$$-\frac{(d+ex^2)^{5/2}(a+b\operatorname{sech}^{-1}(cx))}{5dx^5} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(8c^4d^2+23c^2de+23e^2)\sqrt{d+ex^2}}{75dx} - \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(c^2d+e)}{5dx^5}$$

Antiderivative was successfully verified.

[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^6, x]

[Out] (4*b*(c^2*d + 2*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(75*x^3) + (b*(8*c^4*d^2 + 23*c^2*d*e + 23*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(75*d*x) + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*(d + e*x^2)^(3/2))/(25*x^5) - ((d + e*x^2)^(5/2)*(a + b*ArcSech[c*x]))/(5*d*x^5) + (b*c*(8*c^4*d^2 + 23*c^2*d*e + 23*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(75*d*Sqrt[1 + (e*x^2)/d]) - (b*(c^2*d + e)*(8*c^4*d^2 + 19*c^2*d*e + 15*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(75*c*d*Sqrt[d + e*x^2])

Rule 264

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

Rule 6301

Int[((a_.) + ArcSech[(c_.)*(x_)])*(b_.)*((f_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x]] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ

$[m + 2*p + 3, 0]) \vee (IGtQ[(m + 1)/2, 0] \wedge \neg(ILtQ[p, 0] \wedge GtQ[m + 2*p + 3, 0])) \vee (ILtQ[(m + 2*p + 1)/2, 0] \wedge \neg(ILtQ[(m - 1)/2, 0]))$

Rule 12

$\text{Int}[(a_*)*(u_*), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \wedge \neg \text{MatchQ}[u, (b_*)*(v_*)] /; \text{FreeQ}[b, x]$

Rule 474

$\text{Int}[(e_*)*(x_*)^{(m_*)}*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}*((c_*) + (d_*)*(x_*)^{(n_*)})^{(q_*)}, x_Symbol] \rightarrow \text{Simp}[(c*(e*x)^{(m+1)}*(a + b*x^n)^{(p+1)}*(c + d*x^n)^{(q-1)})/(a*e*(m+1)), x] - \text{Dist}[1/(a*e^n*(m+1)), \text{Int}[(e*x)^{(m+n)}*(a + b*x^n)^p*(c + d*x^n)^{(q-2)}*\text{Simp}[c*(c*b - a*d)*(m+1) + c*n*(b*c*(p+1) + a*d*(q-1)) + d*((c*b - a*d)*(m+1) + c*b*n*(p+q))*x^n, x], x] /; \text{FreeQ}\{a, b, c, d, e, p\}, x \wedge \neg \text{EqQ}[b*c - a*d, 0] \wedge IGtQ[n, 0] \wedge GtQ[q, 1] \wedge LtQ[m, -1] \wedge \text{IntBinomialQ}[a, b, c, d, e, m, n, p, q, x]$

Rule 580

$\text{Int}[(g_*)*(x_*)^{(m_*)}*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}*((c_*) + (d_*)*(x_*)^{(n_*)})^{(q_*)}*((e_*) + (f_*)*(x_*)^{(n_*)}), x_Symbol] \rightarrow \text{Simp}[(e*(g*x)^{(m+1)}*(a + b*x^n)^{(p+1)}*(c + d*x^n)^q)/(a*g*(m+1)), x] - \text{Dist}[1/(a*g^n*(m+1)), \text{Int}[(g*x)^{(m+n)}*(a + b*x^n)^p*(c + d*x^n)^{(q-1)}*\text{Simp}[c*(b*e - a*f)*(m+1) + e*n*(b*c*(p+1) + a*d*q) + d*((b*e - a*f)*(m+1) + b*e*n*(p+q+1))*x^n, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, p\}, x \wedge IGtQ[n, 0] \wedge GtQ[q, 0] \wedge LtQ[m, -1] \wedge \neg(\text{EqQ}[q, 1] \wedge \text{SimplerQ}[e + f*x^n, c + d*x^n])$

Rule 583

$\text{Int}[(g_*)*(x_*)^{(m_*)}*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}*((c_*) + (d_*)*(x_*)^{(n_*)})^{(q_*)}*((e_*) + (f_*)*(x_*)^{(n_*)}), x_Symbol] \rightarrow \text{Simp}[(e*(g*x)^{(m+1)}*(a + b*x^n)^{(p+1)}*(c + d*x^n)^{(q+1)})/(a*c*g*(m+1)), x] + \text{Dist}[1/(a*c*g^n*(m+1)), \text{Int}[(g*x)^{(m+n)}*(a + b*x^n)^p*(c + d*x^n)^q*\text{Simp}[a*f*c*(m+1) - e*(b*c + a*d)*(m+n+1) - e*n*(b*c*p + a*d*q) - b*e*d*(m+n*(p+q+2)+1)*x^n, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, p, q\}, x \wedge IGtQ[n, 0] \wedge LtQ[m, -1]$

Rule 524

$\text{Int}[(e_*) + (f_*)*(x_*)^{(n_*)}]/(\text{Sqrt}[(a_*) + (b_*)*(x_*)^{(n_*)}]*\text{Sqrt}[(c_*) + (d_*)*(x_*)^{(n_*)}]), x_Symbol] \rightarrow \text{Dist}[f/b, \text{Int}[\text{Sqrt}[a + b*x^n]/\text{Sqrt}[c + d*x^n], x], x] + \text{Dist}[(b*e - a*f)/b, \text{Int}[1/(\text{Sqrt}[a + b*x^n]*\text{Sqrt}[c + d*x^n]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x \wedge \neg(\text{EqQ}[n, 2] \wedge ((\text{PosQ}[b/a] \wedge \text{PosQ}[d/c]) \vee (\text{NegQ}[b/a] \wedge (\text{PosQ}[d/c] \vee (\text{GtQ}[a, 0] \wedge (\neg \text{GtQ}[c, 0] \vee \text{SimplerSqrtQ}[-(b/a), -(d/c)]))))))$

Rule 426

$\text{Int}[\text{Sqrt}[(a_*) + (b_*)*(x_*)^2]/\text{Sqrt}[(c_*) + (d_*)*(x_*)^2], x_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[a + b*x^2]/\text{Sqrt}[1 + (b*x^2)/a], \text{Int}[\text{Sqrt}[1 + (b*x^2)/a]/\text{Sqrt}[c + d*x^2], x], x] /; \text{FreeQ}\{a, b, c, d\}, x \wedge \text{NegQ}[d/c] \wedge GtQ[c, 0] \wedge \neg \text{GtQ}[a, 0]$

Rule 424

$\text{Int}[\text{Sqrt}[(a_*) + (b_*)*(x_*)^2]/\text{Sqrt}[(c_*) + (d_*)*(x_*)^2], x_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[a]*\text{EllipticE}[\text{ArcSin}[\text{Rt}[-(d/c), 2]*x], (b*c)/(a*d)])/(\text{Sqrt}[c]*\text{Rt}[-(d/c)$

), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rule 421

Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]

Rule 419

Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] :> Simp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])

Rubi steps

$$\int \frac{(d + ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{x^6} dx = -\frac{(d + ex^2)^{5/2} (a + b \operatorname{sech}^{-1}(cx))}{5dx^5} + \left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx} \right) \int -\frac{(d + ex^2)^{5/2}}{5dx^6\sqrt{1-c^2x^2}} dx$$

$$= -\frac{(d + ex^2)^{5/2} (a + b \operatorname{sech}^{-1}(cx))}{5dx^5} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx} \right) \int \frac{(d+ex^2)^{5/2}}{x^6\sqrt{1-c^2x^2}} dx}{5d}$$

$$= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2} (d + ex^2)^{3/2}}{25x^5} - \frac{(d + ex^2)^{5/2} (a + b \operatorname{sech}^{-1}(cx))}{5dx^5} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx} \right) \int \frac{(d+ex^2)^{5/2}}{x^6\sqrt{1-c^2x^2}} dx}{5d}$$

$$= \frac{4b(c^2d + 2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d + ex^2}}{75x^3} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2} (d + ex^2)^{3/2}}{25x^5}$$

$$= \frac{4b(c^2d + 2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d + ex^2}}{75x^3} + \frac{b(8c^4d^2 + 23c^2de + 23e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{75a}$$

$$= \frac{4b(c^2d + 2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d + ex^2}}{75x^3} + \frac{b(8c^4d^2 + 23c^2de + 23e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{75a}$$

$$= \frac{4b(c^2d + 2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d + ex^2}}{75x^3} + \frac{b(8c^4d^2 + 23c^2de + 23e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{75a}$$

$$= \frac{4b(c^2d + 2e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d + ex^2}}{75x^3} + \frac{b(8c^4d^2 + 23c^2de + 23e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{75a}$$

Mathematica [C] time = 6.27375, size = 620, normalized size = 1.52

$$b\sqrt{\frac{1-cx}{cx+1}} \left(c^2(-8c^4d^2+23c^2de+23e^2)(d+ex^2) - \frac{i^{(cx+1)}(c\sqrt{d-i\sqrt{e}})^2 \sqrt{\frac{c(\sqrt{d-i\sqrt{e}})}{(cx+1)(c\sqrt{d-i\sqrt{e}})}} \sqrt{\frac{c(\sqrt{d+i\sqrt{e}})}{(cx+1)(c\sqrt{d+i\sqrt{e}})}} \left(2\sqrt{e}(8ic^3d^3/2-12c^2d\sqrt{e}+7ic\sqrt{de}-15e^3/2) \operatorname{EllipticF}\left(i \sinh^{-1}\left(\sqrt{\frac{(1-cx)(c^2)}{(cx+1)(c\sqrt{d+i\sqrt{e}})}} \right)}{\sqrt{\frac{(cx-1)(c\sqrt{d-i\sqrt{e}})}{(cx+1)(c\sqrt{d+i\sqrt{e}})}}} \right)}{c} \right)$$

Antiderivative was successfully verified.

[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^6,x]

```
[Out] ((-15*a*(d + e*x^2)^3)/x^5 + (b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(d + e*x^2)*(23*e^2*x^4 + d*e*x^2*(11 + 23*c^2*x^2) + d^2*(3 + 4*c^2*x^2 + 8*c^4*x^4))/x^5 - (15*b*(d + e*x^2)^3*ArcSech[c*x])/x^5 + (b*Sqrt[(1 - c*x)/(1 + c*x)]*(-(c^2*(8*c^4*d^2 + 23*c^2*d*e + 23*e^2)*(d + e*x^2)) - (I*(c*Sqrt[d] - I*Sqrt[e])^2*(1 + c*x)*Sqrt[(c*(Sqrt[d] - I*Sqrt[e]*x))/((c*Sqrt[d] - I*Sqrt[e])*(1 + c*x))]*Sqrt[(c*(Sqrt[d] + I*Sqrt[e]*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x))])*((8*c^4*d^2 + 23*c^2*d*e + 23*e^2)*EllipticE[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2 + 2*Sqrt[e]*((8*I)*c^3*d^(3/2) - 12*c^2*d*Sqrt[e] + (7*I)*c*Sqrt[d]*e - 15*e^(3/2))*EllipticF[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2))/Sqrt[-((c*Sqrt[d] - I*Sqrt[e])*(-1 + c*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x))]))/c)/(75*d*Sqrt[d + e*x^2])
```

Maple [F] time = 1.224, size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{arcsech}(cx)}{x^6} (ex^2 + d)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^6,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^6,x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(aex^2 + ad + (bex^2 + bd) \operatorname{ar} \operatorname{sech}(cx)) \sqrt{ex^2 + d}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsech(c*x))*sqrt(e*x^2 + d)/x^6, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**(3/2)*(a+b*asech(c*x))/x**6,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^{\frac{3}{2}}(b \operatorname{ar} \operatorname{sech}(cx) + a)}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^6,x, algorithm="giac")

[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)/x^6, x)

$$3.149 \quad \int \frac{(d+ex^2)^{3/2} (a+b\operatorname{sech}^{-1}(cx))}{x^8} dx$$

Optimal. Leaf size=556

$$\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(c^2d+e)(204c^4d^2e+120c^6d^3+17c^2de^2-105e^3)\sqrt{\frac{ex^2}{d}+1}\operatorname{EllipticF}(\sin^{-1}(cx),-\frac{e}{c^2d})}{3675cd^2\sqrt{d+ex^2}} + \frac{2e(d+ex^2)^{3/2}}{7dx^7}$$

```
[Out] (b*(120*c^4*d^2 + 159*c^2*d*e - 37*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*
Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(3675*d*x^3) + (b*(240*c^6*d^3 + 528*c^4
*d^2*e + 193*c^2*d*e^2 - 247*e^3)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1
- c^2*x^2]*Sqrt[d + e*x^2])/(3675*d^2*x) + (b*(30*c^2*d + 11*e)*Sqrt[(1 +
c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*(d + e*x^2)^(3/2))/(1225*d*x^5)
+ (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*(d + e*x^2)^(5/2)
)/(49*d*x^7) - ((d + e*x^2)^(5/2)*(a + b*ArcSech[c*x]))/(7*d*x^7) + (2*e*(d
+ e*x^2)^(5/2)*(a + b*ArcSech[c*x]))/(35*d^2*x^5) + (b*c*(240*c^6*d^3 + 52
8*c^4*d^2*e + 193*c^2*d*e^2 - 247*e^3)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*S
qrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(3675*d^2*Sqrt[1 + (e*
x^2)/d]) - (2*b*(c^2*d + e)*(120*c^6*d^3 + 204*c^4*d^2*e + 17*c^2*d*e^2 - 1
05*e^3)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 + (e*x^2)/d]*EllipticF[Ar
cSin[c*x], -(e/(c^2*d))])/(3675*c*d^2*Sqrt[d + e*x^2])
```

Rubi [A] time = 0.757606, antiderivative size = 556, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {271, 264, 6301, 12, 580, 583, 524, 426, 424, 421, 419}

$$\frac{2e(d+ex^2)^{5/2}(a+b\operatorname{sech}^{-1}(cx))}{35d^2x^5} - \frac{(d+ex^2)^{5/2}(a+b\operatorname{sech}^{-1}(cx))}{7dx^7} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(528c^4d^2e+240c^6d^3-105e^3)}{3675d^2x}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^8, x]
```

```
[Out] (b*(120*c^4*d^2 + 159*c^2*d*e - 37*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*
Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(3675*d*x^3) + (b*(240*c^6*d^3 + 528*c^4
*d^2*e + 193*c^2*d*e^2 - 247*e^3)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1
- c^2*x^2]*Sqrt[d + e*x^2])/(3675*d^2*x) + (b*(30*c^2*d + 11*e)*Sqrt[(1 +
c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*(d + e*x^2)^(3/2))/(1225*d*x^5)
+ (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*(d + e*x^2)^(5/2)
)/(49*d*x^7) - ((d + e*x^2)^(5/2)*(a + b*ArcSech[c*x]))/(7*d*x^7) + (2*e*(d
+ e*x^2)^(5/2)*(a + b*ArcSech[c*x]))/(35*d^2*x^5) + (b*c*(240*c^6*d^3 + 52
8*c^4*d^2*e + 193*c^2*d*e^2 - 247*e^3)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*S
qrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(3675*d^2*Sqrt[1 + (e*
x^2)/d]) - (2*b*(c^2*d + e)*(120*c^6*d^3 + 204*c^4*d^2*e + 17*c^2*d*e^2 - 1
05*e^3)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 + (e*x^2)/d]*EllipticF[Ar
cSin[c*x], -(e/(c^2*d))])/(3675*c*d^2*Sqrt[d + e*x^2])
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p +
3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 580

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^((q_.)*((e_) + (f_.)*(x_)^(n_))), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1
))*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] && !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^((q_.)*((e_) + (f_.)*(x_)^(n_))), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplersqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex^2)^{3/2} (a+b\operatorname{sech}^{-1}(cx))}{x^8} dx &= -\frac{(d+ex^2)^{5/2} (a+b\operatorname{sech}^{-1}(cx))}{7dx^7} + \frac{2e(d+ex^2)^{5/2} (a+b\operatorname{sech}^{-1}(cx))}{35d^2x^5} + \left(b\sqrt{\frac{1}{1+cx}}\right) \\
&= -\frac{(d+ex^2)^{5/2} (a+b\operatorname{sech}^{-1}(cx))}{7dx^7} + \frac{2e(d+ex^2)^{5/2} (a+b\operatorname{sech}^{-1}(cx))}{35d^2x^5} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\right)}{1+cx} \\
&= \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{5/2}}{49dx^7} - \frac{(d+ex^2)^{5/2} (a+b\operatorname{sech}^{-1}(cx))}{7dx^7} + \frac{2e}{1+cx} \\
&= \frac{b(30c^2d+11e)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}(d+ex^2)^{3/2}}{1225dx^5} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{49dx^7} \\
&= \frac{b(120c^4d^2+159c^2de-37e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{3675dx^3} + \frac{b(30c^2d+11e)}{49dx^7} \\
&= \frac{b(120c^4d^2+159c^2de-37e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{3675dx^3} + \frac{b(240c^6d^3)}{3675dx^3} \\
&= \frac{b(120c^4d^2+159c^2de-37e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{3675dx^3} + \frac{b(240c^6d^3)}{3675dx^3} \\
&= \frac{b(120c^4d^2+159c^2de-37e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{3675dx^3} + \frac{b(240c^6d^3)}{3675dx^3} \\
&= \frac{b(120c^4d^2+159c^2de-37e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{3675dx^3} + \frac{b(240c^6d^3)}{3675dx^3}
\end{aligned}$$

Mathematica [C] time = 7.76306, size = 728, normalized size = 1.31

$$b\sqrt{\frac{1-cx}{cx+1}} \left(c^2(-528c^4d^2e+240c^6d^3+193c^2de^2-247e^3)(d+cx^2) - \frac{i^{(cx+1)}(c\sqrt{d-i\sqrt{e}})^2 \sqrt{\frac{c(\sqrt{d-i\sqrt{e}})}{(cx+1)(c\sqrt{d-i\sqrt{e}})}} \sqrt{\frac{c(\sqrt{d+i\sqrt{e}})}{(cx+1)(c\sqrt{d+i\sqrt{e}})}} \left(2\sqrt{e}(-360c^4d^2\sqrt{e}+48ic^3d^{3/2}e+240ic^5d^{5/2}-207c^2d^2) \right)}{3675dx^3} \right)$$

Antiderivative was successfully verified.

[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/x^8,x]

[Out] ((105*a*(d + e*x^2)^3*(-5*d + 2*e*x^2))/x^7 + (b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(d + e*x^2)*(-247*e^3*x^6 + d*e^2*x^4*(71 + 193*c^2*x^2) + 3*d^2*e*x^2*(61 + 83*c^2*x^2 + 176*c^4*x^4) + 15*d^3*(5 + 6*c^2*x^2 + 8*c^4*x^4 + 16*c^6*x^6)))/x^7 + (105*b*(d + e*x^2)^3*(-5*d + 2*e*x^2)*ArcSech[c*x])/x^7 + (b*Sqrt[(1 - c*x)/(1 + c*x)]*(-(c^2*(240*c^6*d^3 + 528*c^4*d^2*e + 193*c^2*d*e^2 - 247*e^3)*(d + e*x^2)) - (I*(c*Sqrt[d] - I*Sqrt[e])^2*(1 + c*x)*Sqrt[(c*(Sqrt[d] - I*Sqrt[e]*x))/((c*Sqrt[d] - I*Sqrt[e])*(1 + c*x))]*Sqrt[(c*(Sqrt[d] + I*Sqrt[e]*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x))]*((240*c^6*d^3 + 528*c^4*d^2*e + 193*c^2*d*e^2 - 247*e^3)*EllipticE[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2 + 2*Sqrt[e]*((240*I)*c^5*d^(5/2) - 360*c^4*d^2*Sqrt[e] + (48*I)*c^3*d^(3/2)*e - 207*c^2*d*e^(3/2) - (173*I)*c*Sqrt[d]*e^2 + 210*e^(5/2))*EllipticF[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2))/Sqrt[-((c*Sqrt[d] - I*Sqrt[e])*(-1 + c*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x))]))/c)/(3675*d^2*Sqrt[d + e*x^2])

Maple [F] time = 1.421, size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{arcsech}(cx)}{x^8} (ex^2 + d)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^8,x)

[Out] int((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^8,x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^8,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(aex^2 + ad + (bex^2 + bd) \operatorname{arsech}(cx))\sqrt{ex^2 + d}}{x^8}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^8,x, algorithm="fricas")

[Out] `integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsech(c*x))*sqrt(e*x^2 + d)/x^8, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x**2+d)**(3/2)*(a+b*asech(c*x))/x**8,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex^2 + d)^{\frac{3}{2}}(b \operatorname{arsech}(cx) + a)}{x^8} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x^2+d)^(3/2)*(a+b*arcsech(c*x))/x^8,x, algorithm="giac")`

[Out] `integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)/x^8, x)`

$$3.150 \quad \int \frac{x^5 (a + b \operatorname{sech}^{-1}(cx))}{\sqrt{d+ex^2}} dx$$

Optimal. Leaf size=356

$$\frac{d^2 \sqrt{d+ex^2} (a + b \operatorname{sech}^{-1}(cx))}{e^3} - \frac{2d (d+ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{3e^3} + \frac{(d+ex^2)^{5/2} (a + b \operatorname{sech}^{-1}(cx))}{5e^3} - \frac{b \sqrt{\frac{1}{cx+1}} \sqrt{cx+1}}{4}$$

```
[Out] (b*(19*c^2*d - 9*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(120*c^4*e^2) - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*(d + e*x^2)^(3/2))/(20*c^2*e^2) + (d^2*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/e^3 - (2*d*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/(3*e^3) + ((d + e*x^2)^(5/2)*(a + b*ArcSech[c*x]))/(5*e^3) - (b*(45*c^4*d^2 - 10*c^2*d*e + 9*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTan[(Sqrt[e]*Sqrt[1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(120*c^5*e^(5/2)) - (8*b*d^(5/2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/(15*e^3)
```

Rubi [A] time = 1.16028, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {266, 43, 6301, 12, 1615, 154, 157, 63, 217, 203, 93, 207}

$$\frac{d^2 \sqrt{d+ex^2} (a + b \operatorname{sech}^{-1}(cx))}{e^3} - \frac{2d (d+ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{3e^3} + \frac{(d+ex^2)^{5/2} (a + b \operatorname{sech}^{-1}(cx))}{5e^3} - \frac{b \sqrt{\frac{1}{cx+1}} \sqrt{cx+1}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2], x]
```

```
[Out] (b*(19*c^2*d - 9*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(120*c^4*e^2) - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*(d + e*x^2)^(3/2))/(20*c^2*e^2) + (d^2*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/e^3 - (2*d*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]))/(3*e^3) + ((d + e*x^2)^(5/2)*(a + b*ArcSech[c*x]))/(5*e^3) - (b*(45*c^4*d^2 - 10*c^2*d*e + 9*e^2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTan[(Sqrt[e]*Sqrt[1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(120*c^5*e^(5/2)) - (8*b*d^(5/2)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/(15*e^3)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))*((f_.)*(x_.))^(m_.)*((d_.) + (e_.)*(x_.)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.)*((e_.) + (f_.)*(x_.))^(p_.), x_Symbol] := With[{q = Expon[Px, x], k = Coeff[Px, x, Expon[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p + q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q - 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m + q + p)))*x], x], x] /; NeQ[m + n + p + q + 1, 0] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.)*((e_.) + (f_.)*(x_.))^(p_.)*((g_.) + (h_.)*(x_.)), x_Symbol] := Simp[(h*(a + b*x)^m*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p + 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1)))]*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int((((c_.) + (d_.)*(x_.))^(n_.)*((e_.) + (f_.)*(x_.))^(p_.)*((g_.) + (h_.)*(x_.)))/((a_.) + (b_.)*(x_.)), x_Symbol] := Dist[h/b, Int[(c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 93

Int((((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_)))/((e_) + (f_)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]

Rule 207

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{x^5 (a + b \operatorname{sech}^{-1}(cx))}{\sqrt{d + ex^2}} dx &= \frac{d^2 \sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{e^3} - \frac{2d (d + ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{3e^3} + \frac{(d + ex^2)^{5/2} (a + b \operatorname{sech}^{-1}(cx))}{5e^3} \\
 &= \frac{d^2 \sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{e^3} - \frac{2d (d + ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{3e^3} + \frac{(d + ex^2)^{5/2} (a + b \operatorname{sech}^{-1}(cx))}{5e^3} \\
 &= \frac{d^2 \sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{e^3} - \frac{2d (d + ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{3e^3} + \frac{(d + ex^2)^{5/2} (a + b \operatorname{sech}^{-1}(cx))}{5e^3} \\
 &= -\frac{b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} (d + ex^2)^{3/2}}{20c^2e^2} + \frac{d^2 \sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{e^3} - \frac{2d (d + ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{3e^3} \\
 &= \frac{b (19c^2d - 9e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \sqrt{d + ex^2}}{120c^4e^2} - \frac{b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} (d + ex^2)^{3/2}}{20c^2e^2} \\
 &= \frac{b (19c^2d - 9e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \sqrt{d + ex^2}}{120c^4e^2} - \frac{b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} (d + ex^2)^{3/2}}{20c^2e^2} \\
 &= \frac{b (19c^2d - 9e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \sqrt{d + ex^2}}{120c^4e^2} - \frac{b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} (d + ex^2)^{3/2}}{20c^2e^2} \\
 &= \frac{b (19c^2d - 9e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \sqrt{d + ex^2}}{120c^4e^2} - \frac{b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} (d + ex^2)^{3/2}}{20c^2e^2} \\
 &= \frac{b (19c^2d - 9e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \sqrt{d + ex^2}}{120c^4e^2} - \frac{b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} (d + ex^2)^{3/2}}{20c^2e^2}
 \end{aligned}$$

Mathematica [A] time = 1.56358, size = 366, normalized size = 1.03

$$\frac{\sqrt{d + ex^2} \left(8ac^4 (8d^2 - 4dex^2 + 3e^2x^4) + 8bc^4 \operatorname{sech}^{-1}(cx) (8d^2 - 4dex^2 + 3e^2x^4) - be \sqrt{\frac{1-cx}{cx+1}} (cx + 1) (c^2 (6ex^2 - 13d) + 9e) \right)}{120c^4e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (Sqrt[d + e*x^2]*(8*a*c^4*(8*d^2 - 4*d*e*x^2 + 3*e^2*x^4) - b*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(9*e + c^2*(-13*d + 6*e*x^2)) + 8*b*c^4*(8*d^2 - 4*d*e*x^2 + 3*e^2*x^4)*ArcSech[c*x]))/(120*c^4*e^3) + (b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*(Sqrt[-c^2]*Sqrt[-(c^2*d) - e]*Sqrt[e]*(45*c^4*d^2 - 10*c^2*d*e + 9*e^2)*Sqrt[(c^2*(d + e*x^2))/(c^2*d + e)]*ArcSin[(c*Sqrt[e]*Sqrt[1 - c^2*x^2])/(Sqrt[-c^2]*Sqrt[-(c^2*d) - e])]) + 64*c^7*d^(5/2)*Sqrt[-d - e*x^2]*ArcTan[(Sqrt[d]*Sqrt[1 - c^2*x^2])/Sqrt[-d - e*x^2]]))/(120*c^7*e^3*(-1 + c*x)*Sqrt[d + e*x^2])
```

Maple [F] time = 1.832, size = 0, normalized size = 0.

$$\int x^5 (a + b \operatorname{arcsech}(cx)) \frac{1}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 9.04671, size = 3663, normalized size = 10.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/480*(64*b*c^5*d^(5/2)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) - (45*b*c^4*d^2 - 10*b*c^2*d*e + 9*b*e^2)*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e^2) + 32*(3*b*c^5*e^2*x^4 - 4*b*c^5*d*e*x^2 + 8*b*c^5*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(24*a*c^5*e^2*x^4 - 32*a*c^5*d*e*x^2 + 64*a*c^5*d^2 - (6*b*c^4*e^2*x^3 - (13*b*c^4*d*e - 9*b*c^2*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d)]
```

```

rt(e*x^2 + d)/(c^5*e^3), 1/240*(32*b*c^5*d^(5/2)*log(((c^4*d^2 - 6*c^2*d*e
+ e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(
e*x^2 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) - (45*b*c^4
*d^2 - 10*b*c^2*d*e + 9*b*e^2)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d - e
)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/(c^2*e^2*x^4 +
(c^2*d*e - e^2)*x^2 - d*e)) + 16*(3*b*c^5*e^2*x^4 - 4*b*c^5*d*e*x^2 + 8*b*c
^5*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x))
+ 2*(24*a*c^5*e^2*x^4 - 32*a*c^5*d*e*x^2 + 64*a*c^5*d^2 - (6*b*c^4*e^2*x^3
- (13*b*c^4*d*e - 9*b*c^2*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x
^2 + d))/(c^5*e^3), -1/480*(128*b*c^5*sqrt(-d)*d^2*arctan(-1/2*((c^3*d - c*
e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/(
c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (45*b*c^4*d^2 - 10*b*c^2*d*e +
9*b*e^2)*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^
2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*s
qrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e^2) - 32*(3*b*c^5*e^2*x^4 - 4*b*c^5*d*e*x^
2 + 8*b*c^5*d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) +
1)/(c*x)) - 4*(24*a*c^5*e^2*x^4 - 32*a*c^5*d*e*x^2 + 64*a*c^5*d^2 - (6*b*c^
4*e^2*x^3 - (13*b*c^4*d*e - 9*b*c^2*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))
*sqrt(e*x^2 + d))/(c^5*e^3), -1/240*(64*b*c^5*sqrt(-d)*d^2*arctan(-1/2*((c^
3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2
*x^2)))/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (45*b*c^4*d^2 - 10*b*c^
2*d*e + 9*b*e^2)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d - e)*x)*sqrt(e*x^
2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/(c^2*e^2*x^4 + (c^2*d*e - e^2
)*x^2 - d*e)) - 16*(3*b*c^5*e^2*x^4 - 4*b*c^5*d*e*x^2 + 8*b*c^5*d^2)*sqrt(e
*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - 2*(24*a*c^5
*e^2*x^4 - 32*a*c^5*d*e*x^2 + 64*a*c^5*d^2 - (6*b*c^4*e^2*x^3 - (13*b*c^4*d
*e - 9*b*c^2*e^2)*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^5*
e^3)]

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*(a+b*asech(c*x))/(e*x**2+d)**(1/2), x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{ar} \operatorname{sech}(cx) + a)x^5}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2), x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)*x^5/sqrt(e*x^2 + d), x)

$$3.151 \quad \int \frac{x^3 (a + b \operatorname{sech}^{-1}(cx))}{\sqrt{d + ex^2}} dx$$

Optimal. Leaf size=251

$$\frac{d\sqrt{d+ex^2}(a+b\operatorname{sech}^{-1}(cx))}{e^2} + \frac{(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{3e^2} + \frac{2bd^{3/2}\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3e^2} + \frac{b\sqrt{\frac{1}{cx+1}}}{e^2}$$

[Out] $-(b*\operatorname{Sqrt}[(1+c*x)^{-1}]*\operatorname{Sqrt}[1+c*x]*\operatorname{Sqrt}[1-c^2*x^2]*\operatorname{Sqrt}[d+e*x^2])/((6*c^2*e)-(d*\operatorname{Sqrt}[d+e*x^2]*(a+b*\operatorname{ArcSech}[c*x]))/e^2+((d+e*x^2)^{(3/2)}*(a+b*\operatorname{ArcSech}[c*x]))/(3*e^2)+(b*(3*c^2*d-e)*\operatorname{Sqrt}[(1+c*x)^{-1}]*\operatorname{Sqrt}[1+c*x]*\operatorname{ArcTan}[(\operatorname{Sqrt}[e]*\operatorname{Sqrt}[1-c^2*x^2])/(c*\operatorname{Sqrt}[d+e*x^2])])/(6*c^3*e^{(3/2)})+(2*b*d^{(3/2)}*\operatorname{Sqrt}[(1+c*x)^{-1}]*\operatorname{Sqrt}[1+c*x]*\operatorname{ArcTanh}[\operatorname{Sqrt}[d+e*x^2]/(\operatorname{Sqrt}[d]*\operatorname{Sqrt}[1-c^2*x^2])])/(3*e^2))$

Rubi [A] time = 0.330097, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {266, 43, 6301, 12, 573, 154, 157, 63, 217, 203, 93, 207}

$$\frac{d\sqrt{d+ex^2}(a+b\operatorname{sech}^{-1}(cx))}{e^2} + \frac{(d+ex^2)^{3/2}(a+b\operatorname{sech}^{-1}(cx))}{3e^2} + \frac{2bd^{3/2}\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3e^2} + \frac{b\sqrt{\frac{1}{cx+1}}}{e^2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(x^3*(a+b*\operatorname{ArcSech}[c*x]))/\operatorname{Sqrt}[d+e*x^2],x]$

[Out] $-(b*\operatorname{Sqrt}[(1+c*x)^{-1}]*\operatorname{Sqrt}[1+c*x]*\operatorname{Sqrt}[1-c^2*x^2]*\operatorname{Sqrt}[d+e*x^2])/((6*c^2*e)-(d*\operatorname{Sqrt}[d+e*x^2]*(a+b*\operatorname{ArcSech}[c*x]))/e^2+((d+e*x^2)^{(3/2)}*(a+b*\operatorname{ArcSech}[c*x]))/(3*e^2)+(b*(3*c^2*d-e)*\operatorname{Sqrt}[(1+c*x)^{-1}]*\operatorname{Sqrt}[1+c*x]*\operatorname{ArcTan}[(\operatorname{Sqrt}[e]*\operatorname{Sqrt}[1-c^2*x^2])/(c*\operatorname{Sqrt}[d+e*x^2])])/(6*c^3*e^{(3/2)})+(2*b*d^{(3/2)}*\operatorname{Sqrt}[(1+c*x)^{-1}]*\operatorname{Sqrt}[1+c*x]*\operatorname{ArcTanh}[\operatorname{Sqrt}[d+e*x^2]/(\operatorname{Sqrt}[d]*\operatorname{Sqrt}[1-c^2*x^2])])/(3*e^2))$

Rule 266

$\operatorname{Int}[(x_)^{(m_*)}*((a_)+(b_)*(x_)^{(n_)})^{(p_)},x_Symbol] := \operatorname{Dist}[1/n, \operatorname{Subst}[\operatorname{Int}[x^{(\operatorname{Simplify}[(m+1)/n)-1}*(a+bx)^p, x], x, x^n], x] /; \operatorname{FreeQ}\{a, b, m, n, p\}, x] \&\& \operatorname{IntegerQ}[\operatorname{Simplify}[(m+1)/n]]$

Rule 43

$\operatorname{Int}[(a_)+(b_)*(x_)^{(m_*)}((c_)+(d_)*(x_)^{(n_)}),x_Symbol] := \operatorname{Int}[\operatorname{ExpandIntegrand}[(a+bx)^m*(c+dx)^n, x], x] /; \operatorname{FreeQ}\{a, b, c, d, n\}, x] \&\& \operatorname{NeQ}[b*c-a*d, 0] \&\& \operatorname{IGtQ}[m, 0] \&\& (!\operatorname{IntegerQ}[n] || (\operatorname{EqQ}[c, 0] \&\& \operatorname{LeQ}[7*m+4*n+4, 0]) || \operatorname{LtQ}[9*m+5*(n+1), 0] || \operatorname{GtQ}[m+n+2, 0])$

Rule 6301

$\operatorname{Int}[(a_)+\operatorname{ArcSech}[(c_)*(x_)]*(b_)*((f_)*(x_)^{(m_*)}((d_)+(e_)*(x_)^2)^{(p_)}),x_Symbol] := \operatorname{With}\{u = \operatorname{IntHide}[(f*x)^m*(d+e*x^2)^p, x]\}, \operatorname{Dist}[a+b*\operatorname{ArcSech}[c*x], u, x] + \operatorname{Dist}[b*\operatorname{Sqrt}[1+c*x]*\operatorname{Sqrt}[1/(1+c*x)], \operatorname{Int}[\operatorname{SimplifyIntegrand}[u/(x*\operatorname{Sqrt}[1-c*x]*\operatorname{Sqrt}[1+c*x]), x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, m, p\}, x] \&\& (!\operatorname{IGtQ}[p, 0] \&\& !(\operatorname{ILtQ}[(m-1)/2, 0] \&\& \operatorname{GtQ}[m+2*p+3, 0]) || (\operatorname{IGtQ}[(m+1)/2, 0] \&\& !(\operatorname{ILtQ}[p, 0] \&\& \operatorname{GtQ}[m+2*p+3, 0])$

3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 573

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_))^(r_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 154

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[(h*(a + b*x)^(m+1)*(e + f*x)^(p+1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p + 2)), Int[(a + b*x)^(m-1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1)))]*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0] && IntegersQ[2*m, 2*n, 2*p]

Rule 157

Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/((a_.) + (b_.)*(x_)), x_Symbol] := Dist[h/b, Int[(c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 93

Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]

&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{x^3 (a + b \operatorname{sech}^{-1}(cx))}{\sqrt{d + ex^2}} dx &= -\frac{d\sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{e^2} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{3e^2} + \left(b\sqrt{\frac{1}{1 + cx}} \sqrt{1 + cx} \right) \\
 &= -\frac{d\sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{e^2} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{3e^2} + \frac{\left(b\sqrt{\frac{1}{1 + cx}} \sqrt{1 + cx} \right) \int}{3e^2} \\
 &= -\frac{d\sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{e^2} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{3e^2} + \frac{\left(b\sqrt{\frac{1}{1 + cx}} \sqrt{1 + cx} \right) S}{3e^2} \\
 &= -\frac{b\sqrt{\frac{1}{1 + cx}} \sqrt{1 + cx} \sqrt{1 - c^2 x^2} \sqrt{d + ex^2}}{6c^2 e} - \frac{d\sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{e^2} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{3e^2} \\
 &= -\frac{b\sqrt{\frac{1}{1 + cx}} \sqrt{1 + cx} \sqrt{1 - c^2 x^2} \sqrt{d + ex^2}}{6c^2 e} - \frac{d\sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{e^2} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{3e^2} \\
 &= -\frac{b\sqrt{\frac{1}{1 + cx}} \sqrt{1 + cx} \sqrt{1 - c^2 x^2} \sqrt{d + ex^2}}{6c^2 e} - \frac{d\sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{e^2} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{3e^2} \\
 &= -\frac{b\sqrt{\frac{1}{1 + cx}} \sqrt{1 + cx} \sqrt{1 - c^2 x^2} \sqrt{d + ex^2}}{6c^2 e} - \frac{d\sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{e^2} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{3e^2} \\
 &= -\frac{b\sqrt{\frac{1}{1 + cx}} \sqrt{1 + cx} \sqrt{1 - c^2 x^2} \sqrt{d + ex^2}}{6c^2 e} - \frac{d\sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{e^2} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{3e^2}
 \end{aligned}$$

Mathematica [A] time = 1.26632, size = 406, normalized size = 1.62

$$\frac{\sqrt{d + ex^2} \left(2ac^2 (2d - ex^2) + 2bc^2 \operatorname{sech}^{-1}(cx) (2d - ex^2) + be\sqrt{\frac{1 - cx}{cx + 1}} (cx + 1) \right)}{6c^2 e^2} - \frac{b\sqrt{\frac{1 - cx}{cx + 1}} \sqrt{1 - c^2 x^2} \left(4c^5 d^{3/2} \sqrt{-d - ex^2} \right)}{6c^2 e^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2], x]

[Out] -(Sqrt[d + e*x^2]*(b*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x) + 2*a*c^2*(2*d - e*x^2) + 2*b*c^2*(2*d - e*x^2)*ArcSech[c*x]))/(6*c^2*e^2) - (b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*(-3*(-c^2)^(3/2)*d*Sqrt[-(c^2*d) - e]*Sqrt[e]*Sqrt[(c^2*(d + e*x^2))/(c^2*d + e)]*ArcSin[(c*Sqrt[e]*Sqrt[1 - c^2*x^2])/(Sqrt[-c^2]*Sqrt[-(c^2*d) - e]]) + Sqrt[-c^2]*Sqrt[-(c^2*d) - e]*e^(3/2)*Sqrt[(c^2*(d + e*x^2))/(c^2*d + e)]*ArcSin[(Sqrt[-c^2]*Sqrt[e]*Sqrt[1 - c^2*x^2])/(c*Sqrt[-(c^2*d) - e]]) + 4*c^5*d^(3/2)*Sqrt[-d - e*x^2]*ArcTan[(Sqrt[d]*Sqrt[1 - c^2*x^2])/Sqrt[-d - e*x^2]]))/(6*c^5*e^2*(-1 + c*x)*Sqrt[d +

$e^{x^2}]$)

Maple [F] time = 1.794, size = 0, normalized size = 0.

$$\int x^3 (a + b \operatorname{arcsech}(cx)) \frac{1}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)`

[Out] `int(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)`

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [B] time = 5.22589, size = 3032, normalized size = 12.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")`

[Out] `[1/24*(4*b*c^3*d^(3/2)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) + (3*b*c^2*d - b*e)*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2))) + e^2) + 8*(b*c^3*e*x^2 - 2*b*c^3*d)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(2*a*c^3*e*x^2 - b*c^2*e*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 4*a*c^3*d)*sqrt(e*x^2 + d))/(c^3*e^2), 1/12*(2*b*c^3*d^(3/2)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) + (3*b*c^2*d - b*e)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d - e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e) + 4*(b*c^3*e*x^2 - 2*b*c^3*d)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(2*a*c^3*e*x^2 - b*c^2*e*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 4*a*c^3*d)*sqrt(e*x^2 + d))/(c^3*e^2), 1/24*(8*b*c^3*sqrt(-d)*d*arctan(-1/2*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2) + (3*b*c^2*d - b*e)*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e^2) + 8*(b*c^3*e*x^2 - 2*b*c^3*d)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 -`

$$\begin{aligned} & 1/(c^2x^2) + 1/(cx) + 4*(2ac^3ex^2 - b^2c^2ex\sqrt{-(c^2x^2 - 1)/(c^2x^2)} - 4ac^3d)\sqrt{ex^2 + d}/(c^3e^2), \\ & 1/12*(4b^2c^3\sqrt{-d}\sqrt{ex^2 + d}\arctan(-1/2*((c^3d - ce)x^3 - 2cdx)\sqrt{ex^2 + d})\sqrt{-d})\sqrt{-d}\sqrt{-(c^2x^2 - 1)/(c^2x^2)}/(c^2de^2x^4 + (c^2d^2 - d^2e)x^2 - d^2) + (3 \\ & *b^2c^2d - b^2e)\sqrt{e}\arctan(1/2*(2c^2ex^3 + (c^2d - e)x)\sqrt{ex^2 + d})\sqrt{e}\sqrt{-(c^2x^2 - 1)/(c^2x^2)}/(c^2e^2x^4 + (c^2de - e^2)x^2 - d^2e) \\ & + 4*(b^2c^3ex^2 - 2b^2c^3d)\sqrt{ex^2 + d}\log((cx\sqrt{-(c^2x^2 - 1)/(c^2x^2)} + 1)/(cx)) + 2*(2ac^3ex^2 - b^2c^2ex\sqrt{-(c^2x^2 - 1)/(c^2x^2)} - 4ac^3d)\sqrt{ex^2 + d}/(c^3e^2) \end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 (a + b \operatorname{asech}(cx))}{\sqrt{d + ex^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(a+b*asech(c*x))/(e*x**2+d)**(1/2),x)

[Out] Integral(x**3*(a + b*asech(c*x))/sqrt(d + e*x**2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsech}(cx) + a)x^3}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)*x^3/sqrt(e*x^2 + d), x)

$$3.152 \quad \int \frac{x(a+b\operatorname{sech}^{-1}(cx))}{\sqrt{d+ex^2}} dx$$

Optimal. Leaf size=153

$$\frac{\sqrt{d+ex^2}(a+b\operatorname{sech}^{-1}(cx))}{e} - \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tan^{-1}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{c\sqrt{e}} - \frac{b\sqrt{d}\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{e}$$

[Out] (Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/e - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTan[(Sqrt[e]*Sqrt[1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(c*Sqrt[e]) - (b*Sqrt[d]*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/e

Rubi [A] time = 0.295115, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6299, 517, 446, 105, 63, 217, 203, 93, 207}

$$\frac{\sqrt{d+ex^2}(a+b\operatorname{sech}^{-1}(cx))}{e} - \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tan^{-1}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{c\sqrt{e}} - \frac{b\sqrt{d}\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{e}$$

Antiderivative was successfully verified.

[In] Int[(x*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2], x]

[Out] (Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/e - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTan[(Sqrt[e]*Sqrt[1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(c*Sqrt[e]) - (b*Sqrt[d]*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/e

Rule 6299

Int[((a_.) + ArcSech[(c_.)*(x_.)]*(b_.))*(x_.)*((d_.) + (e_.)*(x_.)^2)^(p_.), x_Symbol] := Simp[((d + e*x^2)^(p + 1)*(a + b*ArcSech[c*x]))/(2*e*(p + 1)), x] + Dist[(b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)])/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1]

Rule 517

Int[(u_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.)*((a1_.) + (b1_.)*(x_.)^(non2_.))^(p_.)*((a2_.) + (b2_.)*(x_.)^(non2_.))^(p_.), x_Symbol] := Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]))

Rule 446

Int[(x_.)^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] := Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dist[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m, 0] || (!RationalQ[m] && (SumSimplerQ[m, -1] || !SumSimplerQ[n, -1])))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 93

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)/((e_.) + (f_.)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x(a + b \operatorname{sech}^{-1}(cx))}{\sqrt{d + ex^2}} dx &= \frac{\sqrt{d + ex^2}(a + b \operatorname{sech}^{-1}(cx))}{e} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{\sqrt{d+ex^2}}{x\sqrt{1-cx}\sqrt{1+cx}} dx}{e} \\
&= \frac{\sqrt{d + ex^2}(a + b \operatorname{sech}^{-1}(cx))}{e} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{\sqrt{d+ex^2}}{x\sqrt{1-c^2x^2}} dx}{e} \\
&= \frac{\sqrt{d + ex^2}(a + b \operatorname{sech}^{-1}(cx))}{e} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \operatorname{Subst}\left(\int \frac{\sqrt{d+ex}}{x\sqrt{1-c^2x}} dx, x, x^2\right)}{2e} \\
&= \frac{\sqrt{d + ex^2}(a + b \operatorname{sech}^{-1}(cx))}{e} + \frac{1}{2} \left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1-c^2x}\sqrt{d+ex}} dx, x, x^2\right) \\
&= \frac{\sqrt{d + ex^2}(a + b \operatorname{sech}^{-1}(cx))}{e} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \operatorname{Subst}\left(\int \frac{1}{\sqrt{d+\frac{e}{c^2}-\frac{ex^2}{c^2}}} dx, x, \sqrt{1-c^2x^2}\right)}{c^2} + \dots \\
&= \frac{\sqrt{d + ex^2}(a + b \operatorname{sech}^{-1}(cx))}{e} - \frac{b\sqrt{d}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx} \tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{e} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) S}{e} \\
&= \frac{\sqrt{d + ex^2}(a + b \operatorname{sech}^{-1}(cx))}{e} - \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx} \tan^{-1}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{c\sqrt{e}} - \frac{b\sqrt{d}\sqrt{\frac{1}{1+cx}}\sqrt{1+cx} \tan}{e}
\end{aligned}$$

Mathematica [A] time = 0.536362, size = 239, normalized size = 1.56

$$\frac{\sqrt{d + ex^2}(a + b \operatorname{sech}^{-1}(cx))}{e} + \frac{b\sqrt{\frac{1-cx}{cx+1}}\sqrt{1-c^2x^2} \left(\sqrt{-c^2}\sqrt{e}\sqrt{c^2(-d)-e}\sqrt{\frac{c^2(d+ex^2)}{c^2d+e}} \sin^{-1}\left(\frac{c\sqrt{e}\sqrt{1-c^2x^2}}{\sqrt{-c^2}\sqrt{c^2(-d)-e}}\right) + c^3\sqrt{d}\sqrt{-d-ex^2} \right)}{c^3e(cx-1)\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2], x]

[Out] (Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/e + (b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*(Sqrt[-c^2]*Sqrt[-(c^2*d) - e]*Sqrt[e]*Sqrt[(c^2*(d + e*x^2))/(c^2*d + e)]*ArcSin[(c*Sqrt[e]*Sqrt[1 - c^2*x^2])/(Sqrt[-c^2]*Sqrt[-(c^2*d) - e])]) + c^3*Sqrt[d]*Sqrt[-d - e*x^2]*ArcTan[(Sqrt[d]*Sqrt[1 - c^2*x^2])/Sqrt[-d - e*x^2]])/(c^3*e*(-1 + c*x)*Sqrt[d + e*x^2])

Maple [F] time = 1.265, size = 0, normalized size = 0.

$$\int x(a + b \operatorname{arcsech}(cx)) \frac{1}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2), x)

[Out] int(x*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b \left(\frac{\sqrt{ex^2 + d} \log(\sqrt{cx + 1} \sqrt{-cx + 1} + 1)}{e} - \int \frac{2(c^2ex^2 - e)x \log(\sqrt{x}) + (c^2ex^2 \log(c) - e \log(c))x + (2(c^2ex^2 - e)x \log(\sqrt{x}) + (c^2ex^2 \log(c) - e \log(c))x)}{(c^2ex^2 + (c^2ex^2 - e)e^{\frac{1}{2} \log(cx + 1)})} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")

[Out] b*(sqrt(e*x^2 + d)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)/e - integrate((2*(c^2*e*x^2 - e)*x*log(sqrt(x)) + (c^2*e*x^2*log(c) - e*log(c))*x + (2*(c^2*e*x^2 - e)*x*log(sqrt(x)) + ((e*log(c) + e)*c^2*x^2 + c^2*d - e*log(c))*x)*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1)))/((c^2*e*x^2 + (c^2*e*x^2 - e)*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1)) - e)*sqrt(e*x^2 + d)), x) + sqrt(e*x^2 + d)*a/e

Fricas [B] time = 3.58199, size = 2419, normalized size = 15.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")

[Out] [1/4*(4*sqrt(e*x^2 + d)*b*c*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + b*c*sqrt(d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) + 4*sqrt(e*x^2 + d)*a*c - b*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e^2))/(c*e), 1/4*(4*sqrt(e*x^2 + d)*b*c*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + b*c*sqrt(d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) + 4*sqrt(e*x^2 + d)*a*c - 2*b*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d - e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e))/(c*e), -1/4*(2*b*c*sqrt(-d)*arctan(-1/2*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) - 4*sqrt(e*x^2 + d)*b*c*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - 4*sqrt(e*x^2 + d)*a*c + b*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e^2))/(c*e), -1/2*(b*c*sqrt(-d)*arctan(-1/2*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) - 2*sqrt(e*x^2 + d)*b*c*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - 2*sqrt(e*x^2 + d)*a*c + b*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d - e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e))/(c*e)]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x(a + b \operatorname{asech}(cx))}{\sqrt{d + ex^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*asech(c*x))/(e*x**2+d)**(1/2),x)

[Out] Integral(x*(a + b*asech(c*x))/sqrt(d + e*x**2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsech}(cx) + a)x}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)*x/sqrt(e*x^2 + d), x)

$$3.153 \quad \int \frac{a+b\operatorname{sech}^{-1}(cx)}{x\sqrt{d+ex^2}} dx$$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{a+b\operatorname{sech}^{-1}(cx)}{x\sqrt{d+ex^2}}, x\right)$$

[Out] Unintegrable[(a + b*ArcSech[c*x])/(x*Sqrt[d + e*x^2]), x]

Rubi [A] time = 0.0951646, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x\sqrt{d+ex^2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*ArcSech[c*x])/(x*Sqrt[d + e*x^2]), x]

[Out] Defer[Int][(a + b*ArcSech[c*x])/(x*Sqrt[d + e*x^2]), x]

Rubi steps

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x\sqrt{d+ex^2}} dx = \int \frac{a+b\operatorname{sech}^{-1}(cx)}{x\sqrt{d+ex^2}} dx$$

Mathematica [A] time = 3.10041, size = 0, normalized size = 0.

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x\sqrt{d+ex^2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*ArcSech[c*x])/(x*Sqrt[d + e*x^2]), x]

[Out] Integrate[(a + b*ArcSech[c*x])/(x*Sqrt[d + e*x^2]), x]

Maple [A] time = 1.22, size = 0, normalized size = 0.

$$\int \frac{a+b\operatorname{arcsech}(cx)}{x} \frac{1}{\sqrt{ex^2+d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))/x/(e*x^2+d)^(1/2), x)

[Out] int((a+b*arcsech(c*x))/x/(e*x^2+d)^(1/2), x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2 + d}(b \operatorname{arsech}(cx) + a)}{ex^3 + dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/(e*x^3 + d*x), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{asech}(cx)}{x\sqrt{d + ex^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asech(c*x))/x/(e*x**2+d)**(1/2),x)

[Out] Integral((a + b*asech(c*x))/(x*sqrt(d + e*x**2)), x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsech}(cx) + a}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)/(sqrt(e*x^2 + d)*x), x)

$$3.154 \quad \int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^3\sqrt{d+ex^2}} dx$$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{a+b\operatorname{sech}^{-1}(cx)}{x^3\sqrt{d+ex^2}}, x\right)$$

[Out] Unintegrable[(a + b*ArcSech[c*x])/(x^3*Sqrt[d + e*x^2]), x]

Rubi [A] time = 0.110568, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^3\sqrt{d+ex^2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*ArcSech[c*x])/(x^3*Sqrt[d + e*x^2]), x]

[Out] Defer[Int] [(a + b*ArcSech[c*x])/(x^3*Sqrt[d + e*x^2]), x]

Rubi steps

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^3\sqrt{d+ex^2}} dx = \int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^3\sqrt{d+ex^2}} dx$$

Mathematica [A] time = 22.1885, size = 0, normalized size = 0.

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^3\sqrt{d+ex^2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*ArcSech[c*x])/(x^3*Sqrt[d + e*x^2]), x]

[Out] Integrate[(a + b*ArcSech[c*x])/(x^3*Sqrt[d + e*x^2]), x]

Maple [A] time = 1.105, size = 0, normalized size = 0.

$$\int \frac{a+b\operatorname{arcsech}(cx)}{x^3} \frac{1}{\sqrt{ex^2+d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(1/2), x)

[Out] int((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(1/2), x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2 + d}(b \operatorname{ar} \operatorname{sech}(cx) + a)}{ex^5 + dx^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/(e*x^5 + d*x^3), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{asech}(cx)}{x^3 \sqrt{d + ex^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asech(c*x))/x**3/(e*x**2+d)**(1/2),x)

[Out] Integral((a + b*asech(c*x))/(x**3*sqrt(d + e*x**2)), x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{ar} \operatorname{sech}(cx) + a}{\sqrt{ex^2 + dx^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)/(sqrt(e*x^2 + d)*x^3), x)

$$3.155 \quad \int \frac{x^2 (a + b \operatorname{sech}^{-1}(cx))}{\sqrt{d + ex^2}} dx$$

Optimal. Leaf size=25

$$\text{Unintegrable} \left(\frac{x^2 (a + b \operatorname{sech}^{-1}(cx))}{\sqrt{d + ex^2}}, x \right)$$

[Out] Unintegrable[(x^2*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2], x]

Rubi [A] time = 0.0979238, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{x^2 (a + b \operatorname{sech}^{-1}(cx))}{\sqrt{d + ex^2}} dx$$

Verification is Not applicable to the result.

[In] Int[(x^2*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2], x]

[Out] Defer[Int] [(x^2*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2], x]

Rubi steps

$$\int \frac{x^2 (a + b \operatorname{sech}^{-1}(cx))}{\sqrt{d + ex^2}} dx = \int \frac{x^2 (a + b \operatorname{sech}^{-1}(cx))}{\sqrt{d + ex^2}} dx$$

Mathematica [A] time = 6.82845, size = 0, normalized size = 0.

$$\int \frac{x^2 (a + b \operatorname{sech}^{-1}(cx))}{\sqrt{d + ex^2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(x^2*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2], x]

[Out] Integrate[(x^2*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2], x]

Maple [A] time = 1.578, size = 0, normalized size = 0.

$$\int x^2 (a + b \operatorname{arcsech}(cx)) \frac{1}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2), x)

[Out] `int(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)`

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bx^2 \operatorname{arsech}(cx) + ax^2}{\sqrt{ex^2 + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")`

[Out] `integral((b*x^2*arcsech(c*x) + a*x^2)/sqrt(e*x^2 + d), x)`

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 (a + b \operatorname{asech}(cx))}{\sqrt{d + ex^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(a+b*asech(c*x))/(e*x**2+d)**(1/2),x)`

[Out] `Integral(x**2*(a + b*asech(c*x))/sqrt(d + e*x**2), x)`

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsech}(cx) + a)x^2}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")`

[Out] `integrate((b*arcsech(c*x) + a)*x^2/sqrt(e*x^2 + d), x)`

$$3.156 \quad \int \frac{a+b\operatorname{sech}^{-1}(cx)}{\sqrt{d+ex^2}} dx$$

Optimal. Leaf size=22

$$\text{Unintegrable}\left(\frac{a+b\operatorname{sech}^{-1}(cx)}{\sqrt{d+ex^2}}, x\right)$$

[Out] Unintegrable[(a + b*ArcSech[c*x])/Sqrt[d + e*x^2], x]

Rubi [A] time = 0.0354502, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{\sqrt{d+ex^2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*ArcSech[c*x])/Sqrt[d + e*x^2], x]

[Out] Defer[Int][(a + b*ArcSech[c*x])/Sqrt[d + e*x^2], x]

Rubi steps

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{\sqrt{d+ex^2}} dx = \int \frac{a+b\operatorname{sech}^{-1}(cx)}{\sqrt{d+ex^2}} dx$$

Mathematica [A] time = 1.01156, size = 0, normalized size = 0.

$$\int \frac{a+b\operatorname{sech}^{-1}(cx)}{\sqrt{d+ex^2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*ArcSech[c*x])/Sqrt[d + e*x^2], x]

[Out] Integrate[(a + b*ArcSech[c*x])/Sqrt[d + e*x^2], x]

Maple [A] time = 1.223, size = 0, normalized size = 0.

$$\int (a+b\operatorname{arcsech}(cx)) \frac{1}{\sqrt{ex^2+d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))/(e*x^2+d)^(1/2), x)

[Out] int((a+b*arcsech(c*x))/(e*x^2+d)^(1/2), x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b \operatorname{arsech}(cx) + a}{\sqrt{ex^2 + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")

[Out] integral((b*arcsech(c*x) + a)/sqrt(e*x^2 + d), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{asech}(cx)}{\sqrt{d + ex^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asech(c*x))/(e*x**2+d)**(1/2),x)

[Out] Integral((a + b*asech(c*x))/sqrt(d + e*x**2), x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsech}(cx) + a}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)/sqrt(e*x^2 + d), x)

$$3.157 \quad \int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^2\sqrt{d+ex^2}} dx$$

Optimal. Leaf size=221

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\operatorname{EllipticF}(\sin^{-1}(cx),-\frac{e}{c^2d})}{cd\sqrt{d+ex^2}} - \frac{\sqrt{d+ex^2}(a+b\operatorname{sech}^{-1}(cx))}{dx} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{dx}$$

```
[Out] (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(d
*x) - (Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/(d*x) + (b*c*Sqrt[(1 + c*x)^(-
1)]*Sqrt[1 + c*x]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(d*
Sqrt[1 + (e*x^2)/d]) - (b*(c^2*d + e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sq
rt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(c*d*Sqrt[d + e*x^2
])
```

Rubi [A] time = 0.257538, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {264, 6301, 12, 475, 21, 423, 426, 424, 421, 419}

$$\frac{\sqrt{d+ex^2}(a+b\operatorname{sech}^{-1}(cx))}{dx} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{dx} - \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(c^2d+e)\sqrt{\frac{ex^2}{d}+1}F(\sin^{-1}(cx))}{cd\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])/(x^2*Sqrt[d + e*x^2]),x]
```

```
[Out] (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(d
*x) - (Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/(d*x) + (b*c*Sqrt[(1 + c*x)^(-
1)]*Sqrt[1 + c*x]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(d*
Sqrt[1 + (e*x^2)/d]) - (b*(c^2*d + e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sq
rt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(c*d*Sqrt[d + e*x^2
])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)])*(b_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p +
3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1) + b*n*(p + q + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] := Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x, a + b*x])
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Dist[b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && NegQ[b/a]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Dist[Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Simp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\begin{aligned}
\int \frac{a + b \operatorname{sech}^{-1}(cx)}{x^2 \sqrt{d + ex^2}} dx &= -\frac{\sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{dx} - \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{\sqrt{d + ex^2}}{dx^2 \sqrt{1 - c^2 x^2}} dx \\
&= -\frac{\sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{dx} - \frac{\left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{\sqrt{d+ex^2}}{x^2 \sqrt{1-c^2x^2}} dx}{d} \\
&= \frac{b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \sqrt{d+ex^2}}{dx} - \frac{\sqrt{d+ex^2} (a + b \operatorname{sech}^{-1}(cx))}{dx} - \frac{\left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{\sqrt{d+ex^2}}{x^2 \sqrt{1-c^2x^2}} dx}{d} \\
&= \frac{b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \sqrt{d+ex^2}}{dx} - \frac{\sqrt{d+ex^2} (a + b \operatorname{sech}^{-1}(cx))}{dx} - \frac{\left(b e \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{\sqrt{d+ex^2}}{x^2 \sqrt{1-c^2x^2}} dx}{d} \\
&= \frac{b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \sqrt{d+ex^2}}{dx} - \frac{\sqrt{d+ex^2} (a + b \operatorname{sech}^{-1}(cx))}{dx} + \frac{\left(b c^2 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{\sqrt{d+ex^2}}{x^2 \sqrt{1-c^2x^2}} dx}{d} \\
&= \frac{b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \sqrt{d+ex^2}}{dx} - \frac{\sqrt{d+ex^2} (a + b \operatorname{sech}^{-1}(cx))}{dx} + \frac{\left(b c^2 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{d+ex^2} \right)}{d \sqrt{1-c^2x^2}} \\
&= \frac{b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \sqrt{d+ex^2}}{dx} - \frac{\sqrt{d+ex^2} (a + b \operatorname{sech}^{-1}(cx))}{dx} + \frac{b c \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{d+ex^2}}{d \sqrt{1-c^2x^2}}
\end{aligned}$$

Mathematica [C] time = 4.00984, size = 501, normalized size = 2.27

$$\frac{b \sqrt{\frac{1-cx}{cx+1}} (\sqrt{ex+i\sqrt{d}}) \sqrt{\frac{c(\sqrt{d+i\sqrt{ex}})}{(cx+1)(c\sqrt{d+i\sqrt{e}})}} \left(2i\sqrt{e} \operatorname{EllipticF} \left(i \sinh^{-1} \left(\sqrt{\frac{(1-cx)(c^2d+e)}{(cx+1)(c\sqrt{d+i\sqrt{e}})^2}}, \frac{(c\sqrt{d+i\sqrt{e}})^2}{(c\sqrt{d-i\sqrt{e}})^2} \right) + (c\sqrt{d-i\sqrt{e}}) E \left(i \sinh^{-1} \left(\sqrt{\frac{(dc^2+e)(1-cx)}{(\sqrt{dc+i\sqrt{e}})^2 (cx+1)}}, \frac{(\sqrt{dc+i\sqrt{e}})^2}{(c\sqrt{d-i\sqrt{e}})^2} \right) \right) \right)}{\sqrt{\frac{(cx-1)(c\sqrt{d-i\sqrt{e}})}{(cx+1)(c\sqrt{d+i\sqrt{e}})}} \sqrt{\frac{c(\sqrt{d-i\sqrt{ex}})}{(cx+1)(c\sqrt{d-i\sqrt{e}})}}}$$

$$d\sqrt{d+ex^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcSech[c*x])/(x^2*Sqrt[d + e*x^2]), x]

[Out] -((a*(d/x + e*x) + b*c*Sqrt[(1 - c*x)/(1 + c*x)]*(d + e*x^2) - (b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(d + e*x^2)))/x + (b*(d + e*x^2)*ArcSech[c*x])/x + (b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[(c*(Sqrt[d] + I*Sqrt[e]*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x))]*(I*Sqrt[d] + Sqrt[e]*x)*((c*Sqrt[d] - I*Sqrt[e])*EllipticE[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2] + (2*I)*Sqrt[e]*EllipticF[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2]))/(Sqrt[-((c*Sqrt[d] - I*Sqrt[e])*(-1 + c*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x))])*Sqrt[(c*(Sqrt[d] - I*Sqrt[e]*x))/((c*Sqrt[d] - I*Sqrt[e])*(1 + c*x))]))/(d*Sqrt[d + e*x^2]))

Maple [F] time = 1.015, size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{arcsech}(cx)}{x^2} \frac{1}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arcsech(c*x))/x^2/(e*x^2+d)^(1/2),x)`

[Out] `int((a+b*arcsech(c*x))/x^2/(e*x^2+d)^(1/2),x)`

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2 + d}(b \operatorname{arsech}(cx) + a)}{ex^4 + dx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/(e*x^4 + d*x^2), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{asech}(cx)}{x^2 \sqrt{d + ex^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*asech(c*x))/x**2/(e*x**2+d)**(1/2),x)`

[Out] `Integral((a + b*asech(c*x))/(x**2*sqrt(d + e*x**2)), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsech}(cx) + a}{\sqrt{ex^2 + dx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="giac")`

[Out] `integrate((b*arcsech(c*x) + a)/(sqrt(e*x^2 + d)*x^2), x)`

$$3.158 \quad \int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^4\sqrt{d+ex^2}} dx$$

Optimal. Leaf size=346

$$\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(c^2d-3e)(c^2d+e)\sqrt{\frac{ex^2}{d}+1}\operatorname{EllipticF}\left(\sin^{-1}(cx),-\frac{e}{c^2d}\right)}{9cd^2\sqrt{d+ex^2}} + \frac{2e\sqrt{d+ex^2}(a+b\operatorname{sech}^{-1}(cx))}{3d^2x} - \frac{\sqrt{d+ex^2}}{3d^2x}$$

```
[Out] (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(9
*d*x^3) + (b*(2*c^2*d - 5*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^
2*x^2]*Sqrt[d + e*x^2])/(9*d^2*x) - (Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/(
(3*d*x^3) + (2*e*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/(3*d^2*x) + (b*c*(2*
c^2*d - 5*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[d + e*x^2]*EllipticE[A
rcSin[c*x], -(e/(c^2*d))])/(9*d^2*Sqrt[1 + (e*x^2)/d]) - (2*b*(c^2*d - 3*e)
*(c^2*d + e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 + (e*x^2)/d]*Ellipti
cF[ArcSin[c*x], -(e/(c^2*d))])/(9*c*d^2*Sqrt[d + e*x^2])
```

Rubi [A] time = 0.484686, antiderivative size = 346, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {271, 264, 6301, 12, 580, 583, 524, 426, 424, 421, 419}

$$\frac{2e\sqrt{d+ex^2}(a+b\operatorname{sech}^{-1}(cx))}{3d^2x} - \frac{\sqrt{d+ex^2}(a+b\operatorname{sech}^{-1}(cx))}{3dx^3} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}(2c^2d-5e)\sqrt{d+ex^2}}{9d^2x} - \frac{2b\sqrt{d+ex^2}}{3d^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])/(x^4*Sqrt[d + e*x^2]),x]
```

```
[Out] (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(9
*d*x^3) + (b*(2*c^2*d - 5*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^
2*x^2]*Sqrt[d + e*x^2])/(9*d^2*x) - (Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/(
(3*d*x^3) + (2*e*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/(3*d^2*x) + (b*c*(2*
c^2*d - 5*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[d + e*x^2]*EllipticE[A
rcSin[c*x], -(e/(c^2*d))])/(9*d^2*Sqrt[1 + (e*x^2)/d]) - (2*b*(c^2*d - 3*e)
*(c^2*d + e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 + (e*x^2)/d]*Ellipti
cF[ArcSin[c*x], -(e/(c^2*d))])/(9*c*d^2*Sqrt[d + e*x^2])
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)])*(b_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
```

```
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p +
3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 580

```
Int[((g_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_
))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1
))*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] && !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 583

```
Int[((g_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_
))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2
) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 524

```
Int[((e_) + (f_)*(x_)^(n_))/(Sqrt[(a_) + (b_)*(x_)^(n_)]*Sqrt[(c_) + (d_
)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 426

```
Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] := Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0
]
```

Rule 424

```
Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] := Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rule 419

Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
 imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
 [-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
 [a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])

Rubi steps

$$\begin{aligned} \int \frac{a + b \operatorname{sech}^{-1}(cx)}{x^4 \sqrt{d + ex^2}} dx &= -\frac{\sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{3dx^3} + \frac{2e\sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{3d^2x} + \left(b\sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{\sqrt{d+ex^2}}{x^4 \sqrt{d+ex^2}} dx \\ &= -\frac{\sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{3dx^3} + \frac{2e\sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{3d^2x} + \frac{\left(b\sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{\sqrt{d+ex^2}}{x^4 \sqrt{d+ex^2}} dx}{3d^2} \\ &= \frac{b\sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \sqrt{d+ex^2}}{9dx^3} - \frac{\sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{3dx^3} + \frac{2e\sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{3d^2x} \\ &= \frac{b\sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \sqrt{d+ex^2}}{9dx^3} + \frac{b(2c^2d - 5e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \sqrt{d+ex^2}}{9d^2x} - \frac{\sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{3dx^3} \\ &= \frac{b\sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \sqrt{d+ex^2}}{9dx^3} + \frac{b(2c^2d - 5e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \sqrt{d+ex^2}}{9d^2x} - \frac{\sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{3dx^3} \\ &= \frac{b\sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \sqrt{d+ex^2}}{9dx^3} + \frac{b(2c^2d - 5e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \sqrt{d+ex^2}}{9d^2x} - \frac{\sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{3dx^3} \\ &= \frac{b\sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \sqrt{d+ex^2}}{9dx^3} + \frac{b(2c^2d - 5e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \sqrt{d+ex^2}}{9d^2x} - \frac{\sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{3dx^3} \end{aligned}$$

Mathematica [C] time = 4.64031, size = 612, normalized size = 1.77

$$\frac{b\sqrt{\frac{1-cx}{cx+1}}(\sqrt{ex+i\sqrt{d}})\sqrt{\frac{c(\sqrt{d+i\sqrt{ex}})}{(cx+1)(c\sqrt{d+i\sqrt{e}})}}\left(2\sqrt{e}(2ic^2d-c\sqrt{d}\sqrt{e}-6ie)\operatorname{EllipticF}\left(i\sinh^{-1}\left(\frac{(1-cx)(c^2d+e)}{\sqrt{(cx+1)(c\sqrt{d+i\sqrt{e}})^2}}\right),\frac{(c\sqrt{d+i\sqrt{e}})^2}{(c\sqrt{d-i\sqrt{e}})^2}\right)+(2c^3d^{3/2}-2ic^2d\sqrt{e}-5c\sqrt{de}+5ie^{3/2})E\left(i\sinh^{-1}\left(\frac{(1-cx)(c^2d+e)}{\sqrt{(cx+1)(c\sqrt{d+i\sqrt{e}})^2}}\right),\frac{(c\sqrt{d+i\sqrt{e}})^2}{(c\sqrt{d-i\sqrt{e}})^2}\right)\right)}{\sqrt{\frac{(cx-1)(c\sqrt{d-i\sqrt{e}})}{(cx+1)(c\sqrt{d+i\sqrt{e}})}}\sqrt{\frac{c(\sqrt{d-i\sqrt{ex}})}{(cx+1)(c\sqrt{d-i\sqrt{e}})}}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcSech[c*x])/(x^4*Sqrt[d + e*x^2]),x]

[Out] ((b*d*Sqrt[(1 - c*x)/(1 + c*x)]*(d + e*x^2))/x^3 + (b*c*d*Sqrt[(1 - c*x)/(1 + c*x)]*(d + e*x^2))/x^2 + (b*(2*c^2*d - 5*e)*Sqrt[(1 - c*x)/(1 + c*x)]*(d + e*x^2))/x - (3*a*(d - 2*e*x^2)*(d + e*x^2))/x^3 - (3*b*(d - 2*e*x^2)*(d + e*x^2)*ArcSech[c*x])/x^3 - (b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[(c*(Sqrt[d] + I*Sqrt[e]*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x))]*(I*Sqrt[d] + Sqrt[e]*x))*((2*c^3*d^(3/2) - (2*I)*c^2*d*Sqrt[e] - 5*c*Sqrt[d]*e + (5*I)*e^(3/2))*EllipticE[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2] + 2*((2*I)*c^2*d - c*Sqrt[d]*Sqrt[e] - (6*I)*e)*Sqrt[e]*EllipticF[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2)))/(Sqrt[-(((c*Sqrt[d] - I*Sqrt[e])*(-1 + c*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x)))]*Sqrt[(c*(Sqrt[d] - I*Sqrt[e])

$[e*x))/((c*\text{Sqrt}[d] - I*\text{Sqrt}[e])*(1 + c*x)))]/(9*d^2*\text{Sqrt}[d + e*x^2])$

Maple [F] time = 1.263, size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{arcsech}(cx)}{x^4} \frac{1}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arcsech(c*x))/x^4/(e*x^2+d)^(1/2),x)`

[Out] `int((a+b*arcsech(c*x))/x^4/(e*x^2+d)^(1/2),x)`

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsech(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{ex^2 + d}(b \operatorname{arsech}(cx) + a)}{ex^6 + dx^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsech(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/(e*x^6 + d*x^4), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*asech(c*x))/x**4/(e*x**2+d)**(1/2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsech}(cx) + a}{\sqrt{ex^2 + dx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arcsech(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)/(sqrt(e*x^2 + d)*x^4), x)
```

$$3.159 \quad \int \frac{x^5 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

Optimal. Leaf size=278

$$\frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} - \frac{2d \sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{e^3} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{3e^3} + \frac{8bd^{3/2} \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \tanh^{-1}}{3e^3}$$

[Out] $-(b \sqrt{(1 + cx)^{-1}} \sqrt{1 + cx} \sqrt{1 - c^2 x^2} \sqrt{d + ex^2}) / (6c^2 e^2 - (d^2 (a + b \operatorname{ArcSech}[cx])) / (e^3 \sqrt{d + ex^2}) - (2d \sqrt{d + ex^2} (a + b \operatorname{ArcSech}[cx])) / e^3 + ((d + ex^2)^{3/2} (a + b \operatorname{ArcSech}[cx])) / (3e^3) + (b(9c^2 d - e) \sqrt{(1 + cx)^{-1}} \sqrt{1 + cx} \operatorname{ArcTan}[\sqrt{e} \sqrt{1 - c^2 x^2}] / (c \sqrt{d + ex^2}])) / (6c^3 e^{5/2}) + (8bd^{3/2} \sqrt{(1 + cx)^{-1}} \sqrt{1 + cx} \operatorname{ArcTanh}[\sqrt{d + ex^2} / (\sqrt{d} \sqrt{1 - c^2 x^2}])) / (3e^3)$

Rubi [A] time = 1.11681, antiderivative size = 278, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {266, 43, 6301, 12, 1615, 157, 63, 217, 203, 93, 207}

$$\frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} - \frac{2d \sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{e^3} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{3e^3} + \frac{8bd^{3/2} \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \tanh^{-1}}{3e^3}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(x^5 (a + b \operatorname{ArcSech}[cx])) / (d + ex^2)^{3/2}, x]$

[Out] $-(b \sqrt{(1 + cx)^{-1}} \sqrt{1 + cx} \sqrt{1 - c^2 x^2} \sqrt{d + ex^2}) / (6c^2 e^2 - (d^2 (a + b \operatorname{ArcSech}[cx])) / (e^3 \sqrt{d + ex^2}) - (2d \sqrt{d + ex^2} (a + b \operatorname{ArcSech}[cx])) / e^3 + ((d + ex^2)^{3/2} (a + b \operatorname{ArcSech}[cx])) / (3e^3) + (b(9c^2 d - e) \sqrt{(1 + cx)^{-1}} \sqrt{1 + cx} \operatorname{ArcTan}[\sqrt{e} \sqrt{1 - c^2 x^2}] / (c \sqrt{d + ex^2}])) / (6c^3 e^{5/2}) + (8bd^{3/2} \sqrt{(1 + cx)^{-1}} \sqrt{1 + cx} \operatorname{ArcTanh}[\sqrt{d + ex^2} / (\sqrt{d} \sqrt{1 - c^2 x^2}])) / (3e^3)$

Rule 266

$\text{Int}[(x_)^{(m_.)} ((a_) + (b_.) (x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1) (a + bx)^p, x], x, x^n], x] /;$ FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 43

$\text{Int}(((a_) + (b_.) (x_))^{(m_.)} ((c_) + (d_.) (x_))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + bx)^m (c + dx)^n, x], x] /;$ FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 6301

$\text{Int}(((a_) + \operatorname{ArcSech}[(c_.) (x_)] (b_.) ((f_.) (x_))^{(m_.)} ((d_.) + (e_.) (x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{With}\{u = \text{IntHide}[(f*x)^m (d + ex^2)^p, x]\}, \text{Dist}[a + b \operatorname{ArcSech}[cx], u, x] + \text{Dist}[b \sqrt{1 + cx} \sqrt{1/(1 + cx)}, \text{Int}[\dots]]$

```
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x, x] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p +
3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 1615

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] := With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[(k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p +
1))/(d*f*b^(q - 1)*(m + n + p + q + 1)), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m +
n + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x], x], x] /; NeQ[m + n + p + q + 1, 0] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.)*((g_.) + (h_.)*(x
_)))/((a_.) + (b_.)*(x_)), x_Symbol] := Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.))/((e_.) + (f_.)*(x
_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
- 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x^5 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{3/2}} dx &= -\frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} - \frac{2d\sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{e^3} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{3e^3} \\ &= -\frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} - \frac{2d\sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{e^3} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{3e^3} \\ &= -\frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} - \frac{2d\sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{e^3} + \frac{(d + ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{3e^3} \\ &= -\frac{b\sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \sqrt{d+ex^2}}{6c^2e^2} - \frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} - \frac{2d\sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{e^3} \\ &= -\frac{b\sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \sqrt{d+ex^2}}{6c^2e^2} - \frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} - \frac{2d\sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{e^3} \\ &= -\frac{b\sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \sqrt{d+ex^2}}{6c^2e^2} - \frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} - \frac{2d\sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{e^3} \\ &= -\frac{b\sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \sqrt{d+ex^2}}{6c^2e^2} - \frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} - \frac{2d\sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{e^3} \\ &= -\frac{b\sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \sqrt{d+ex^2}}{6c^2e^2} - \frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} - \frac{2d\sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{e^3} \end{aligned}$$

Mathematica [A] time = 1.42311, size = 436, normalized size = 1.57

$$\frac{-2ac^2(8d^2 + 4dex^2 - e^2x^4) - 2bc^2 \operatorname{sech}^{-1}(cx)(8d^2 + 4dex^2 - e^2x^4) - be\sqrt{\frac{1-cx}{cx+1}}(cx+1)(d+ex^2) - b\sqrt{\frac{1-cx}{cx+1}}\sqrt{1-c^2x^2} \left(16c^2e^3\sqrt{d+ex^2}\right)}{6c^2e^3\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2), x]

[Out] $(-(b*e*\sqrt{(1-c*x)/(1+c*x)}*(1+c*x)*(d+e*x^2)) - 2*a*c^2*(8*d^2 + 4*d*e*x^2 - e^2*x^4) - 2*b*c^2*(8*d^2 + 4*d*e*x^2 - e^2*x^4)*\operatorname{ArcSech}[c*x]) / (6*c^2*e^3*\sqrt{d+e*x^2}) - (b*\sqrt{(1-c*x)/(1+c*x)}*\sqrt{1-c^2*x^2})*(-9*(-c^2)^(3/2)*d*\sqrt{-(c^2*d)-e}*\sqrt{e}*\sqrt{(c^2*(d+e*x^2))/(c^2*d+e)})*\operatorname{ArcSin}[(c*\sqrt{e}*\sqrt{1-c^2*x^2})/(\sqrt{-c^2}*\sqrt{-(c^2*d)-e})] + \sqrt{-c^2}*\sqrt{-(c^2*d)-e}*e^(3/2)*\sqrt{(c^2*(d+e*x^2))/(c^2*d+e)}*\operatorname{ArcSin}[(\sqrt{-c^2}*\sqrt{e}*\sqrt{1-c^2*x^2})/(c*\sqrt{-(c^2*d)-e})] + 16*c^5*d^(3/2)*\sqrt{-d-e*x^2}*\operatorname{ArcTan}[(\sqrt{d}*\sqrt{1-c^2*x^2})/\sqrt{-d-e*x^2}]) / (6*c^5*e^3*(-1+c*x)*\sqrt{d+e*x^2})$


```

d^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) +
4*(2*a*c^3*e^2*x^4 - 8*a*c^3*d*e*x^2 - 16*a*c^3*d^2 - (b*c^2*e^2*x^3 + b*c^
2*d*e*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^3*e^4*x^2 + c^
3*d*e^3), 1/12*(16*(b*c^3*d*e*x^2 + b*c^3*d^2)*sqrt(-d)*arctan(-1/2*((c^3*d
- c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^
2)))/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (9*b*c^2*d^2 - b*d*e + (9*
b*c^2*d*e - b*e^2)*x^2)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d - e)*x)*sq
rt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/(c^2*e^2*x^4 + (c^2*d*
e - e^2)*x^2 - d*e)) + 4*(b*c^3*e^2*x^4 - 4*b*c^3*d*e*x^2 - 8*b*c^3*d^2)*sq
rt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(2*a*
c^3*e^2*x^4 - 8*a*c^3*d*e*x^2 - 16*a*c^3*d^2 - (b*c^2*e^2*x^3 + b*c^2*d*e*x
)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d))/(c^3*e^4*x^2 + c^3*d*e^3
)]

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*asech(c*x))/(e*x**2+d)**(3/2), x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsech}(cx) + a)x^5}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2), x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x^5/(e*x^2 + d)^(3/2), x)
```

$$3.160 \quad \int \frac{x^3 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

Optimal. Leaf size=177

$$\frac{\sqrt{d+ex^2}(a+b\operatorname{sech}^{-1}(cx))}{e^2} + \frac{d(a+b\operatorname{sech}^{-1}(cx))}{e^2\sqrt{d+ex^2}} - \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tan^{-1}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{ce^{3/2}} - \frac{2b\sqrt{d}\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{e^2}$$

[Out] (d*(a + b*ArcSech[c*x]))/(e^2*Sqrt[d + e*x^2]) + (Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/e^2 - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTan[(Sqrt[e]*Sqrt[1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(c*e^(3/2)) - (2*b*Sqrt[d]*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/e^2

Rubi [A] time = 0.272205, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {266, 43, 6301, 12, 573, 157, 63, 217, 203, 93, 207}

$$\frac{\sqrt{d+ex^2}(a+b\operatorname{sech}^{-1}(cx))}{e^2} + \frac{d(a+b\operatorname{sech}^{-1}(cx))}{e^2\sqrt{d+ex^2}} - \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tan^{-1}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{ce^{3/2}} - \frac{2b\sqrt{d}\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{e^2}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2), x]

[Out] (d*(a + b*ArcSech[c*x]))/(e^2*Sqrt[d + e*x^2]) + (Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]))/e^2 - (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTan[(Sqrt[e]*Sqrt[1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(c*e^(3/2)) - (2*b*Sqrt[d]*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/e^2

Rule 266

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 43

Int[((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 6301

Int[((a_) + ArcSech[(c_)*(x_)])*(b_)*((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

Rule 12

$\text{Int}[(a_*)*(u_*), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_*)*(v_*)] /; \text{FreeQ}[b, x]$

Rule 573

$\text{Int}[(x_*)^{(m_*)}*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}*((c_*) + (d_*)*(x_*)^{(n_*)})^{(q_*)}*((e_*) + (f_*)*(x_*)^{(n_*)})^{(r_*)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r}, x], x, x^n], x] /; \text{FreeQ}[\{a, b, c, d, e, f, m, n, p, q, r\}, x] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m+1)/n]]$

Rule 157

$\text{Int}[(((c_*) + (d_*)*(x_*))^{(n_*)}*((e_*) + (f_*)*(x_*))^{(p_*)}*((g_*) + (h_*)*(x_*))) / ((a_*) + (b_*)*(x_*)), x_Symbol] \rightarrow \text{Dist}[h/b, \text{Int}[(c + d*x)^n*(e + f*x)^p, x], x] + \text{Dist}[(b*g - a*h)/b, \text{Int}[(c + d*x)^n*(e + f*x)^p / (a + b*x), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, h, n, p\}, x]$

Rule 63

$\text{Int}[(a_*) + (b_*)*(x_*)^{(m_*)}*((c_*) + (d_*)*(x_*)^{(n_*)}), x_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m+1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n}, x], x, (a + b*x)^{(1/p)}], x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 217

$\text{Int}[1/\text{Sqrt}[(a_*) + (b_*)*(x_*)^2], x_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(1 - b*x^2), x], x, x/\text{Sqrt}[a + b*x^2]] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ !\text{GtQ}[a, 0]$

Rule 203

$\text{Int}[(a_*) + (b_*)*(x_*)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTan}[(\text{Rt}[b, 2]*x)/\text{Rt}[a, 2]]) / (\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 93

$\text{Int}[(a_*) + (b_*)*(x_*)^{(m_*)}*((c_*) + (d_*)*(x_*)^{(n_*)}) / ((e_*) + (f_*)*(x_*)), x_Symbol] \rightarrow \text{With}[\{q = \text{Denominator}[m]\}, \text{Dist}[q, \text{Subst}[\text{Int}[x^{(q*(m+1) - 1)} / (b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^{(1/q)} / (c + d*x)^{(1/q)}], x]] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \ \&\& \ \text{EqQ}[m + n + 1, 0] \ \&\& \ \text{RationalQ}[n] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{SimplerQ}[a + b*x, c + d*x]$

Rule 207

$\text{Int}[(a_*) + (b_*)*(x_*)^2)^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTanh}[(\text{Rt}[b, 2]*x)/\text{Rt}[-a, 2]] / (\text{Rt}[-a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rubi steps

$$\begin{aligned}
\int \frac{x^3 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{3/2}} dx &= \frac{d(a + b \operatorname{sech}^{-1}(cx))}{e^2 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{e^2} + \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{2d}{e^2 x \sqrt{1-cx}} \\
&= \frac{d(a + b \operatorname{sech}^{-1}(cx))}{e^2 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{e^2} + \frac{\left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{2d+ex^2}{x \sqrt{1-c^2 x^2} \sqrt{d+ex^2}}}{e^2} \\
&= \frac{d(a + b \operatorname{sech}^{-1}(cx))}{e^2 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{e^2} + \frac{\left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \operatorname{Subst} \left(\int \frac{2}{x \sqrt{1-cx}} \right)}{2e^2} \\
&= \frac{d(a + b \operatorname{sech}^{-1}(cx))}{e^2 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{e^2} + \frac{\left(bd \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \operatorname{Subst} \left(\int \frac{2}{x \sqrt{1-cx}} \right)}{e^2} \\
&= \frac{d(a + b \operatorname{sech}^{-1}(cx))}{e^2 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{e^2} + \frac{\left(2bd \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \operatorname{Subst} \left(\int \frac{2}{-d} \right)}{e^2} \\
&= \frac{d(a + b \operatorname{sech}^{-1}(cx))}{e^2 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{e^2} - \frac{2b \sqrt{d} \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \tanh^{-1} \left(\frac{\sqrt{d}}{\sqrt{d+ex^2}} \right)}{e^2} \\
&= \frac{d(a + b \operatorname{sech}^{-1}(cx))}{e^2 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{e^2} - \frac{b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \tan^{-1} \left(\frac{\sqrt{e} \sqrt{1-c^2 x^2}}{c \sqrt{d+ex^2}} \right)}{ce^{3/2}}
\end{aligned}$$

Mathematica [A] time = 1.6662, size = 249, normalized size = 1.41

$$\frac{(2d + ex^2)(a + b \operatorname{sech}^{-1}(cx))}{e^2 \sqrt{d + ex^2}} + \frac{b \sqrt{\frac{1-cx}{cx+1}} \sqrt{1-c^2 x^2} \left(\sqrt{-c^2} \sqrt{e} \sqrt{c^2(-d)} - e \sqrt{\frac{c^2(d+ex^2)}{c^2 d + e}} \sin^{-1} \left(\frac{c \sqrt{e} \sqrt{1-c^2 x^2}}{\sqrt{-c^2} \sqrt{c^2(-d)-e}} \right) \right) + 2c^3 \sqrt{d} \sqrt{-d}}{c^3 e^2 (cx - 1) \sqrt{d + ex^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2), x]

[Out] ((2*d + e*x^2)*(a + b*ArcSech[c*x]))/(e^2*Sqrt[d + e*x^2]) + (b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*(Sqrt[-c^2]*Sqrt[-(c^2*d) - e]*Sqrt[e]*Sqrt[(c^2*(d + e*x^2))/(c^2*d + e)]*ArcSin[(c*Sqrt[e]*Sqrt[1 - c^2*x^2])/(Sqrt[-c^2]*Sqrt[-(c^2*d) - e])]) + 2*c^3*Sqrt[d]*Sqrt[-d - e*x^2]*ArcTan[(Sqrt[d]*Sqrt[1 - c^2*x^2])/Sqrt[-d - e*x^2]])/(c^3*e^2*(-1 + c*x)*Sqrt[d + e*x^2])

Maple [F] time = 1.487, size = 0, normalized size = 0.

$$\int x^3 (a + b \operatorname{arcsech}(cx)) (ex^2 + d)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2), x)

[Out] int(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2), x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 2.88903, size = 2867, normalized size = 16.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/4*((b*e*x^2 + b*d)*\sqrt{-e}*\log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8 \\ & *(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*\sqrt{e*x^2 + \\ & d}*\sqrt{-e}*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)} + e^2) - 4*(b*c*e*x^2 + 2*b*c*d \\ &)*\sqrt{e*x^2 + d}*\log((c*x*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)} + 1)/(c*x)) - 2*(\\ & b*c*e*x^2 + b*c*d)*\sqrt{d}*\log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 \\ & - d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*\sqrt{e*x^2 + d}*\sqrt{d}*\sqrt{ \\ & -(c^2*x^2 - 1)/(c^2*x^2)} + 8*d^2)/x^4) - 4*(a*c*e*x^2 + 2*a*c*d)*\sqrt{e*x \\ & ^2 + d}]/(c*e^3*x^2 + c*d*e^2), -1/2*((b*e*x^2 + b*d)*\sqrt{e}*\arctan(1/2*(2 \\ & *c^2*e*x^3 + (c^2*d - e)*x)*\sqrt{e*x^2 + d}*\sqrt{e}*\sqrt{-(c^2*x^2 - 1)/(c^ \\ & 2*x^2)})/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e)) - 2*(b*c*e*x^2 + 2*b*c*d \\ &)*\sqrt{e*x^2 + d}*\log((c*x*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)} + 1)/(c*x)) - (b \\ & c*e*x^2 + b*c*d)*\sqrt{d}*\log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 \\ & - d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*\sqrt{e*x^2 + d}*\sqrt{d}*\sqrt{ \\ & -(c^2*x^2 - 1)/(c^2*x^2)} + 8*d^2)/x^4) - 2*(a*c*e*x^2 + 2*a*c*d)*\sqrt{e*x^2 \\ & + d}]/(c*e^3*x^2 + c*d*e^2), -1/4*(4*(b*c*e*x^2 + b*c*d)*\sqrt{-d}*\arctan(- \\ & 1/2*((c^3*d - c*e)*x^3 - 2*c*d*x)*\sqrt{e*x^2 + d}*\sqrt{-d}*\sqrt{-(c^2*x^2 - \\ & 1)/(c^2*x^2)})/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (b*e*x^2 + b*d) \\ & *\sqrt{-e}*\log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x \\ & ^2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*\sqrt{e*x^2 + d}*\sqrt{-e}*\sqrt{-(c^ \\ & 2*x^2 - 1)/(c^2*x^2)} + e^2) - 4*(b*c*e*x^2 + 2*b*c*d)*\sqrt{e*x^2 + d}*\log(\\ & (c*x*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)} + 1)/(c*x)) - 4*(a*c*e*x^2 + 2*a*c*d)*\sqrt{ \\ & e*x^2 + d}]/(c*e^3*x^2 + c*d*e^2), -1/2*(2*(b*c*e*x^2 + b*c*d)*\sqrt{-d} \\ & *\arctan(-1/2*((c^3*d - c*e)*x^3 - 2*c*d*x)*\sqrt{e*x^2 + d}*\sqrt{-d}*\sqrt{-(\\ & c^2*x^2 - 1)/(c^2*x^2)})/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) + (b*e*x \\ & ^2 + b*d)*\sqrt{e}*\arctan(1/2*(2*c^2*e*x^3 + (c^2*d - e)*x)*\sqrt{e*x^2 + d} \\ & *\sqrt{e}*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)})/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - \\ & d*e)) - 2*(b*c*e*x^2 + 2*b*c*d)*\sqrt{e*x^2 + d}*\log((c*x*\sqrt{-(c^2*x^2 - \\ & 1)/(c^2*x^2)} + 1)/(c*x)) - 2*(a*c*e*x^2 + 2*a*c*d)*\sqrt{e*x^2 + d}]/(c*e^3 \\ & *x^2 + c*d*e^2)] \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(a+b*asech(c*x))/(e*x**2+d)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{ar} \operatorname{sech}(cx) + a)x^3}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)*x^3/(e*x^2 + d)^(3/2), x)

$$3.161 \quad \int \frac{x(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^{3/2}} dx$$

Optimal. Leaf size=87

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{\sqrt{de}} - \frac{a+b\operatorname{sech}^{-1}(cx)}{e\sqrt{d+ex^2}}$$

[Out] $-\left(\frac{a+b\operatorname{ArcSech}[c*x]}{e\sqrt{d+e*x^2}}\right) + \frac{b\sqrt{1+c*x}\operatorname{ArcTanh}\left[\frac{\sqrt{d+e*x^2}}{\sqrt{d}\sqrt{1-c^2*x^2}}\right]}{\sqrt{d}*e}$

Rubi [A] time = 0.245075, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {6299, 517, 446, 93, 207}

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{\sqrt{de}} - \frac{a+b\operatorname{sech}^{-1}(cx)}{e\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}\left[\frac{x(a+b\operatorname{ArcSech}[c*x])}{(d+e*x^2)^{3/2}}, x\right]$

[Out] $-\left(\frac{a+b\operatorname{ArcSech}[c*x]}{e\sqrt{d+e*x^2}}\right) + \frac{b\sqrt{1+c*x}\operatorname{ArcTanh}\left[\frac{\sqrt{d+e*x^2}}{\sqrt{d}\sqrt{1-c^2*x^2}}\right]}{\sqrt{d}*e}$

Rule 6299

$\operatorname{Int}\left[\left(\frac{a}{x} + \operatorname{ArcSech}\left[\frac{c}{x}\right]\right)\left(\frac{b}{x} + \frac{d}{e*x^2}\right)^p, x\right]$
 $\operatorname{Int}\left[\left(\frac{a}{x} + \operatorname{ArcSech}\left[\frac{c}{x}\right]\right)\left(\frac{b}{x} + \frac{d}{e*x^2}\right)^p, x\right] \rightarrow \operatorname{Simp}\left[\frac{(d+e*x^2)^{p+1}(a+b\operatorname{ArcSech}[c*x])}{2*e*(p+1)}, x\right] + \operatorname{Dist}\left[\frac{b\sqrt{1+c*x}\operatorname{ArcTanh}\left[\frac{\sqrt{d+e*x^2}}{\sqrt{d}\sqrt{1-c^2*x^2}}\right]}{2*e*(p+1)}, \operatorname{Int}\left[\frac{(d+e*x^2)^{p+1}}{x\sqrt{1-c*x}\sqrt{1+c*x}}, x\right], x\right]; \operatorname{FreeQ}\{a, b, c, d, e, p\}, x \&\& \operatorname{NeQ}[p, -1]$

Rule 517

$\operatorname{Int}\left[\left(\frac{a}{x} + \frac{d}{e*x^n}\right)^p \left(\frac{c}{x} + \frac{d}{e*x^n}\right)^q, x\right]$
 $\operatorname{Int}\left[\left(\frac{a}{x} + \frac{d}{e*x^n}\right)^p \left(\frac{c}{x} + \frac{d}{e*x^n}\right)^q, x\right] \rightarrow \operatorname{Int}\left[u(a_1*a_2 + b_1*b_2*x^n)^p (c + d*x^n)^q, x\right]; \operatorname{FreeQ}\{a_1, b_1, a_2, b_2, c, d, n, p, q\}, x \&\& \operatorname{EqQ}[non2, n/2] \&\& \operatorname{EqQ}[a_2*b_1 + a_1*b_2, 0] \&\& (\operatorname{IntegerQ}[p] \&\& (\operatorname{GtQ}[a_1, 0] \&\& \operatorname{GtQ}[a_2, 0]))$

Rule 446

$\operatorname{Int}\left[\left(\frac{a}{x} + \frac{d}{e*x^n}\right)^p \left(\frac{c}{x} + \frac{d}{e*x^n}\right)^q, x\right]$
 $\operatorname{Int}\left[\left(\frac{a}{x} + \frac{d}{e*x^n}\right)^p \left(\frac{c}{x} + \frac{d}{e*x^n}\right)^q, x\right] \rightarrow \operatorname{Dist}\left[\frac{1}{n}, \operatorname{Subst}\left[\operatorname{Int}\left[x^{\operatorname{Simplify}\left[\frac{m+1}{n}\right]-1}(a+b*x)^p (c+d*x)^q, x\right], x, x^n\right], x\right]; \operatorname{FreeQ}\{a, b, c, d, m, n, p, q\}, x \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{IntegerQ}\left[\operatorname{Simplify}\left[\frac{m+1}{n}\right]\right]$

Rule 93

$\operatorname{Int}\left[\left(\frac{a}{x} + \frac{d}{e*x^n}\right)^p \left(\frac{c}{x} + \frac{d}{e*x^n}\right)^q, x\right]$
 $\operatorname{Int}\left[\left(\frac{a}{x} + \frac{d}{e*x^n}\right)^p \left(\frac{c}{x} + \frac{d}{e*x^n}\right)^q, x\right] \rightarrow \operatorname{With}\left[q = \operatorname{Denominator}[m], \operatorname{Dist}\left[q, \operatorname{Subst}\left[\operatorname{Int}\left[x^{q*(m+1)-1}(a+b*x)^p (c+d*x)^q, x\right], x, (a+b*x)^{1/q}/(c+d*x)^{1/q}\right], x\right], x, (a+b*x)^{1/q}/(c+d*x)^{1/q}\right]$

], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{x(a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{3/2}} dx &= \frac{a + b \operatorname{sech}^{-1}(cx)}{e\sqrt{d + ex^2}} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}\sqrt{d+ex^2}} dx}{e} \\
 &= \frac{a + b \operatorname{sech}^{-1}(cx)}{e\sqrt{d + ex^2}} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{1}{x\sqrt{1-c^2x^2}\sqrt{d+ex^2}} dx}{e} \\
 &= \frac{a + b \operatorname{sech}^{-1}(cx)}{e\sqrt{d + ex^2}} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-c^2x}\sqrt{d+ex}} dx, x, x^2\right)}{2e} \\
 &= \frac{a + b \operatorname{sech}^{-1}(cx)}{e\sqrt{d + ex^2}} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \operatorname{Subst}\left(\int \frac{1}{-d+x^2} dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{1-c^2x^2}}\right)}{e} \\
 &= \frac{a + b \operatorname{sech}^{-1}(cx)}{e\sqrt{d + ex^2}} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx} \tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{\sqrt{de}}
 \end{aligned}$$

Mathematica [A] time = 0.59144, size = 135, normalized size = 1.55

$$\frac{a + b \operatorname{sech}^{-1}(cx)}{e\sqrt{d + ex^2}} - \frac{b\sqrt{\frac{1-cx}{cx+1}}\sqrt{1-c^2x^2}\sqrt{-d-ex^2} \tan^{-1}\left(\frac{\sqrt{d}\sqrt{1-c^2x^2}}{\sqrt{-d-ex^2}}\right)}{\sqrt{de}(cx-1)\sqrt{d + ex^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2), x]

[Out] -((a + b*ArcSech[c*x])/(e*Sqrt[d + e*x^2])) - (b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*Sqrt[-d - e*x^2]*ArcTan[(Sqrt[d]*Sqrt[1 - c^2*x^2])/Sqrt[-d - e*x^2]])/(Sqrt[d]*e*(-1 + c*x)*Sqrt[d + e*x^2])

Maple [F] time = 0.918, size = 0, normalized size = 0.

$$\int x(a + b \operatorname{arcsech}(cx))(ex^2 + d)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2), x)

[Out] int(x*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b \int \frac{x \log\left(\sqrt{\frac{1}{cx} + 1} \sqrt{\frac{1}{cx} - 1} + \frac{1}{cx}\right)}{(ex^2 + d)^{\frac{3}{2}}} dx - \frac{a}{\sqrt{ex^2 + de}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")

[Out] b*integrate(x*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e*x^2 + d)^(3/2), x) - a/(sqrt(e*x^2 + d)*e)

Fricas [B] time = 2.39633, size = 829, normalized size = 9.53

$$\frac{4\sqrt{ex^2 + d}bd \log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}+1}}{cx}\right) + 4\sqrt{ex^2 + d}ad - (bex^2 + bd)\sqrt{d} \log\left(\frac{(c^4d^2 - 6c^2de + e^2)x^4 - 8(c^2d^2 - de)x^2 - 4((c^3d - ce)x^3 - 2cdx)\sqrt{ex^2 + d}}{x^4}\right)}{4(d^2ex^2 + d^2e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")

[Out] [-1/4*(4*sqrt(e*x^2 + d)*b*d*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 4*sqrt(e*x^2 + d)*a*d - (b*e*x^2 + b*d)*sqrt(d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4))/(d*e^2*x^2 + d^2*e), -1/2*(2*sqrt(e*x^2 + d)*b*d*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) + 2*sqrt(e*x^2 + d)*a*d - (b*e*x^2 + b*d)*sqrt(-d)*arctan(-1/2*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2))/(d*e^2*x^2 + d^2*e)]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x(a + b \operatorname{asech}(cx))}{(d + ex^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*asech(c*x))/(e*x**2+d)**(3/2),x)

[Out] Integral(x*(a + b*asech(c*x))/(d + e*x**2)**(3/2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{ar} \operatorname{sech}(cx) + a)x}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x/(e*x^2 + d)^(3/2), x)
```

$$3.162 \quad \int \frac{a + b \operatorname{sech}^{-1}(cx)}{x(d + ex^2)^{3/2}} dx$$

Optimal. Leaf size=25

$$\text{Unintegrable} \left(\frac{a + b \operatorname{sech}^{-1}(cx)}{x(d + ex^2)^{3/2}}, x \right)$$

[Out] Unintegrable[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^(3/2)), x]

Rubi [A] time = 0.116189, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{a + b \operatorname{sech}^{-1}(cx)}{x(d + ex^2)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^(3/2)), x]

[Out] Defer[Int] [(a + b*ArcSech[c*x])/(x*(d + e*x^2)^(3/2)), x]

Rubi steps

$$\int \frac{a + b \operatorname{sech}^{-1}(cx)}{x(d + ex^2)^{3/2}} dx = \int \frac{a + b \operatorname{sech}^{-1}(cx)}{x(d + ex^2)^{3/2}} dx$$

Mathematica [A] time = 30.2695, size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{sech}^{-1}(cx)}{x(d + ex^2)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^(3/2)), x]

[Out] Integrate[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^(3/2)), x]

Maple [A] time = 0.841, size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{arcsech}(cx)}{x} (ex^2 + d)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))/x/(e*x^2+d)^(3/2), x)

[Out] `int((a+b*arcsech(c*x))/x/(e*x^2+d)^(3/2),x)`

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2+d}(b \operatorname{arsech}(cx) + a)}{e^2x^5 + 2dex^3 + d^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="fricas")`

[Out] `integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*asech(c*x))/x/(e*x**2+d)**(3/2),x)`

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsech}(cx) + a}{(ex^2 + d)^{\frac{3}{2}}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="giac")`

[Out] `integrate((b*arcsech(c*x) + a)/((e*x^2 + d)^(3/2)*x), x)`

$$3.163 \quad \int \frac{a + b \operatorname{sech}^{-1}(cx)}{x^3 (d + ex^2)^{3/2}} dx$$

Optimal. Leaf size=25

$$\text{Unintegrable} \left(\frac{a + b \operatorname{sech}^{-1}(cx)}{x^3 (d + ex^2)^{3/2}}, x \right)$$

[Out] Unintegrable[(a + b*ArcSech[c*x])/(x^3*(d + e*x^2)^(3/2)), x]

Rubi [A] time = 0.133081, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0$., Rules used = {}

$$\int \frac{a + b \operatorname{sech}^{-1}(cx)}{x^3 (d + ex^2)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*ArcSech[c*x])/(x^3*(d + e*x^2)^(3/2)), x]

[Out] Defer[Int] [(a + b*ArcSech[c*x])/(x^3*(d + e*x^2)^(3/2)), x]

Rubi steps

$$\int \frac{a + b \operatorname{sech}^{-1}(cx)}{x^3 (d + ex^2)^{3/2}} dx = \int \frac{a + b \operatorname{sech}^{-1}(cx)}{x^3 (d + ex^2)^{3/2}} dx$$

Mathematica [A] time = 36.6451, size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{sech}^{-1}(cx)}{x^3 (d + ex^2)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*ArcSech[c*x])/(x^3*(d + e*x^2)^(3/2)), x]

[Out] Integrate[(a + b*ArcSech[c*x])/(x^3*(d + e*x^2)^(3/2)), x]

Maple [A] time = 0.939, size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{arcsech}(cx)}{x^3} (ex^2 + d)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(3/2), x)

[Out] `int((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(3/2),x)`

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2+d}(b \operatorname{arsech}(cx) + a)}{e^2x^7 + 2dex^5 + d^2x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="fricas")`

[Out] `integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/(e^2*x^7 + 2*d*e*x^5 + d^2*x^3), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*asech(c*x))/x**3/(e*x**2+d)**(3/2),x)`

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsech}(cx) + a}{(ex^2 + d)^{\frac{3}{2}}x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="giac")`

[Out] `integrate((b*arcsech(c*x) + a)/((e*x^2 + d)^(3/2)*x^3), x)`

$$3.164 \quad \int \frac{x^4 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

Optimal. Leaf size=25

$$\text{Unintegrable} \left(\frac{x^4 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{3/2}}, x \right)$$

[Out] Unintegrable[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2), x]

Rubi [A] time = 0.11143, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{x^4 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2), x]

[Out] Defer[Int] [(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2), x]

Rubi steps

$$\int \frac{x^4 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{3/2}} dx = \int \frac{x^4 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

Mathematica [A] time = 9.46212, size = 0, normalized size = 0.

$$\int \frac{x^4 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2), x]

[Out] Integrate[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2), x]

Maple [A] time = 1.534, size = 0, normalized size = 0.

$$\int x^4 (a + b \operatorname{arcsech}(cx)) (ex^2 + d)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2), x)

[Out] `int(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)`

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^4 \operatorname{ar} \operatorname{sech}(cx) + ax^4)\sqrt{ex^2 + d}}{e^2x^4 + 2dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")`

[Out] `integral((b*x^4*arcsech(c*x) + a*x^4)*sqrt(e*x^2 + d)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*(a+b*asech(c*x))/(e*x**2+d)**(3/2),x)`

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{ar} \operatorname{sech}(cx) + a)x^4}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")`

[Out] `integrate((b*arcsech(c*x) + a)*x^4/(e*x^2 + d)^(3/2), x)`

$$3.165 \quad \int \frac{x^2 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

Optimal. Leaf size=25

$$\text{Unintegrable} \left(\frac{x^2 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{3/2}}, x \right)$$

[Out] Unintegrable[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2), x]

Rubi [A] time = 0.106315, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{x^2 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2), x]

[Out] Defer[Int] [(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2), x]

Rubi steps

$$\int \frac{x^2 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{3/2}} dx = \int \frac{x^2 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

Mathematica [A] time = 4.67383, size = 0, normalized size = 0.

$$\int \frac{x^2 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2), x]

[Out] Integrate[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2), x]

Maple [A] time = 1.47, size = 0, normalized size = 0.

$$\int x^2 (a + b \operatorname{arcsech}(cx)) (ex^2 + d)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2), x)

[Out] $\text{int}(x^2*(a+b*\text{arcsech}(c*x))/(e*x^2+d)^{(3/2)}, x)$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^2*(a+b*\text{arcsech}(c*x))/(e*x^2+d)^{(3/2)}, x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 \text{arsech}(cx) + ax^2)\sqrt{ex^2 + d}}{e^2x^4 + 2dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^2*(a+b*\text{arcsech}(c*x))/(e*x^2+d)^{(3/2)}, x, \text{algorithm}="fricas")$

[Out] $\text{integral}((b*x^2*\text{arcsech}(c*x) + a*x^2)*\text{sqrt}(e*x^2 + d)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)$

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 (a + b \text{asech}(cx))}{(d + ex^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x**2*(a+b*\text{asech}(c*x))/(e*x**2+d)**(3/2), x)$

[Out] $\text{Integral}(x**2*(a + b*\text{asech}(c*x))/(d + e*x**2)**(3/2), x)$

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \text{arsech}(cx) + a)x^2}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^2*(a+b*\text{arcsech}(c*x))/(e*x^2+d)^{(3/2)}, x, \text{algorithm}="giac")$

[Out] $\text{integrate}((b*\text{arcsech}(c*x) + a)*x^2/(e*x^2 + d)^{(3/2)}, x)$

$$3.166 \quad \int \frac{a + b \operatorname{sech}^{-1}(cx)}{(d + ex^2)^{3/2}} dx$$

Optimal. Leaf size=92

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{ex^2}{d}+1}\operatorname{EllipticF}\left(\sin^{-1}(cx), -\frac{e}{c^2d}\right)}{cd\sqrt{d+ex^2}} + \frac{x(a + b\operatorname{sech}^{-1}(cx))}{d\sqrt{d+ex^2}}$$

[Out] (x*(a + b*ArcSech[c*x]))/(d*Sqrt[d + e*x^2]) + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(c*d*Sqrt[d + e*x^2])

Rubi [A] time = 0.0786416, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {191, 6291, 12, 421, 419}

$$\frac{x(a + b\operatorname{sech}^{-1}(cx))}{d\sqrt{d+ex^2}} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{ex^2}{d}+1}F\left(\sin^{-1}(cx) \middle| -\frac{e}{c^2d}\right)}{cd\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcSech[c*x])/(d + e*x^2)^(3/2), x]

[Out] (x*(a + b*ArcSech[c*x]))/(d*Sqrt[d + e*x^2]) + (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(c*d*Sqrt[d + e*x^2])

Rule 191

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rule 6291

Int[((a_.) + ArcSech[(c_.)*(x_)])*(b_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p + 1/2, 0])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 421

Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]

Rule 419

Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Simp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt

$[-(d/c), 2]), x] /; \text{FreeQ}\{a, b, c, d, x\} \ \&\& \ \text{NegQ}[d/c] \ \&\& \ \text{GtQ}[c, 0] \ \&\& \ \text{GtQ}[a, 0] \ \&\& \ !(\text{NegQ}[b/a] \ \&\& \ \text{SimplerSqrtQ}[-(b/a), -(d/c)])$

Rubi steps

$$\begin{aligned} \int \frac{a + b \operatorname{sech}^{-1}(cx)}{(d + ex^2)^{3/2}} dx &= \frac{x(a + b \operatorname{sech}^{-1}(cx))}{d\sqrt{d + ex^2}} + \left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx} \right) \int \frac{1}{d\sqrt{1-c^2x^2}\sqrt{d+ex^2}} dx \\ &= \frac{x(a + b \operatorname{sech}^{-1}(cx))}{d\sqrt{d + ex^2}} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx} \right) \int \frac{1}{\sqrt{1-c^2x^2}\sqrt{d+ex^2}} dx}{d} \\ &= \frac{x(a + b \operatorname{sech}^{-1}(cx))}{d\sqrt{d + ex^2}} + \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}} \right) \int \frac{1}{\sqrt{1-c^2x^2}\sqrt{1+\frac{ex^2}{d}}} dx}{d\sqrt{d + ex^2}} \\ &= \frac{x(a + b \operatorname{sech}^{-1}(cx))}{d\sqrt{d + ex^2}} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1+\frac{ex^2}{d}} F\left(\sin^{-1}(cx) \middle| -\frac{e}{c^2d}\right)}{cd\sqrt{d + ex^2}} \end{aligned}$$

Mathematica [C] time = 1.32424, size = 334, normalized size = 3.63

$$\frac{x(a + b \operatorname{sech}^{-1}(cx))}{d\sqrt{d + ex^2}} + \frac{2ib\sqrt{\frac{1-cx}{cx+1}}(\sqrt{ex} - i\sqrt{d})\sqrt{\frac{(cx+1)(c\sqrt{d}+i\sqrt{e})}{(cx-1)(c\sqrt{d}-i\sqrt{e})}}\sqrt{\frac{c\left(x+\frac{i\sqrt{d}}{\sqrt{e}}\right)+\frac{i\sqrt{ex}}{\sqrt{d}}-1}{1-cx}} \operatorname{EllipticF}\left(\sin^{-1}\left(\sqrt{\frac{\frac{ic\sqrt{d}}{\sqrt{e}}+c(-x)+\frac{i\sqrt{ex}}{\sqrt{d}}+1}{2-2cx}}\right)\right)}{d(c\sqrt{d} + i\sqrt{e})\sqrt{d + ex^2}\sqrt{\frac{\frac{ic\sqrt{d}}{\sqrt{e}}+c(-x)+\frac{i\sqrt{ex}}{\sqrt{d}}+1}{1-cx}}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcSech[c*x])/(d + e*x^2)^(3/2), x]

[Out] (x*(a + b*ArcSech[c*x]))/(d*Sqrt[d + e*x^2]) + ((2*I)*b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x))/((c*Sqrt[d] - I*Sqrt[e])*(-1 + c*x))]*((-I)*Sqrt[d] + Sqrt[e]*x)*Sqrt[-((-1 + (I*Sqrt[e]*x)/Sqrt[d] + c*((I*Sqrt[d])/Sqrt[e] + x))/(1 - c*x))]*EllipticF[ArcSin[Sqrt[(1 + (I*c*Sqrt[d])/Sqrt[e] - c*x + (I*Sqrt[e]*x)/Sqrt[d])/(2 - 2*c*x)]], ((-4*I)*c*Sqrt[d]*Sqrt[e])/(c*Sqrt[d] - I*Sqrt[e])^2)/(d*(c*Sqrt[d] + I*Sqrt[e])*Sqrt[(1 + (I*c*Sqrt[d])/Sqrt[e] - c*x + (I*Sqrt[e]*x)/Sqrt[d])/(1 - c*x)]*Sqrt[d + e*x^2])

Maple [F] time = 1.067, size = 0, normalized size = 0.

$$\int (a + b \operatorname{arcsech}(cx))(ex^2 + d)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))/(e*x^2+d)^(3/2), x)

[Out] int((a+b*arcsech(c*x))/(e*x^2+d)^(3/2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b \int \frac{\log\left(\sqrt{\frac{1}{cx} + 1} \sqrt{\frac{1}{cx} - 1} + \frac{1}{cx}\right)}{(ex^2 + d)^{\frac{3}{2}}} dx + \frac{ax}{\sqrt{ex^2 + dd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")

[Out] b*integrate(log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e*x^2 + d)^(3/2), x) + a*x/(sqrt(e*x^2 + d)*d)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2 + d}(b \operatorname{arsech}(cx) + a)}{e^2x^4 + 2dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{asech}(cx)}{(d + ex^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asech(c*x))/(e*x**2+d)**(3/2),x)

[Out] Integral((a + b*asech(c*x))/(d + e*x**2)**(3/2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsech}(cx) + a}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)/(e*x^2 + d)^(3/2), x)

$$3.167 \quad \int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^2(d+ex^2)^{3/2}} dx$$

Optimal. Leaf size=249

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(c^2d+2e)\sqrt{\frac{ex^2}{d}+1}\operatorname{EllipticF}(\sin^{-1}(cx),-\frac{e}{c^2d})}{cd^2\sqrt{d+ex^2}} - \frac{2ex(a+b\operatorname{sech}^{-1}(cx))}{d^2\sqrt{d+ex^2}} - \frac{a+b\operatorname{sech}^{-1}(cx)}{dx\sqrt{d+ex^2}} + \frac{b\sqrt{\frac{1}{cx}}}{cx}$$

```
[Out] (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(d
^2*x) - (a + b*ArcSech[c*x])/(d*x*Sqrt[d + e*x^2]) - (2*e*x*(a + b*ArcSech[
c*x]))/(d^2*Sqrt[d + e*x^2]) + (b*c*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt
[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(d^2*Sqrt[1 + (e*x^2)/d])
- (b*(c^2*d + 2*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 + (e*x^2)/d]*
EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(c*d^2*Sqrt[d + e*x^2])
```

Rubi [A] time = 0.286656, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {271, 191, 6301, 12, 583, 524, 426, 424, 421, 419}

$$\frac{2ex(a+b\operatorname{sech}^{-1}(cx))}{d^2\sqrt{d+ex^2}} - \frac{a+b\operatorname{sech}^{-1}(cx)}{dx\sqrt{d+ex^2}} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}\sqrt{d+ex^2}}{d^2x} - \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(c^2d+2e)\sqrt{\frac{ex^2}{d}}}{cd^2\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])/(x^2*(d + e*x^2)^(3/2)), x]
```

```
[Out] (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*Sqrt[d + e*x^2])/(d
^2*x) - (a + b*ArcSech[c*x])/(d*x*Sqrt[d + e*x^2]) - (2*e*x*(a + b*ArcSech[
c*x]))/(d^2*Sqrt[d + e*x^2]) + (b*c*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt
[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(d^2*Sqrt[1 + (e*x^2)/d])
- (b*(c^2*d + 2*e)*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 + (e*x^2)/d]*
EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(c*d^2*Sqrt[d + e*x^2])
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)])*(b_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p +
```

3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 583

Int[((g_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) - e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]

Rule 524

Int[((e_) + (f_)*(x_)^(n_))/(Sqrt[(a_) + (b_)*(x_)^(n_)]*Sqrt[(c_) + (d_)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || SimplerSqrtQ[-(b/a), -(d/c)]))))))

Rule 426

Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] := Dist[Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0]

Rule 424

Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] := Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rule 421

Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]

Rule 419

Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := Simp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])

Rubi steps

$$\begin{aligned}
\int \frac{a + b \operatorname{sech}^{-1}(cx)}{x^2 (d + ex^2)^{3/2}} dx &= \frac{a + b \operatorname{sech}^{-1}(cx)}{dx \sqrt{d + ex^2}} - \frac{2ex (a + b \operatorname{sech}^{-1}(cx))}{d^2 \sqrt{d + ex^2}} + \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{-d - 2ex^2}{d^2 x^2 \sqrt{1 - c^2 x^2} \sqrt{d + ex^2}} dx \\
&= \frac{a + b \operatorname{sech}^{-1}(cx)}{dx \sqrt{d + ex^2}} - \frac{2ex (a + b \operatorname{sech}^{-1}(cx))}{d^2 \sqrt{d + ex^2}} + \frac{\left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{-d - 2ex^2}{x^2 \sqrt{1 - c^2 x^2} \sqrt{d + ex^2}} dx}{d^2} \\
&= \frac{b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1 - c^2 x^2} \sqrt{d + ex^2}}{d^2 x} - \frac{a + b \operatorname{sech}^{-1}(cx)}{dx \sqrt{d + ex^2}} - \frac{2ex (a + b \operatorname{sech}^{-1}(cx))}{d^2 \sqrt{d + ex^2}} - \frac{\left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{-d - 2ex^2}{x^2 \sqrt{1 - c^2 x^2} \sqrt{d + ex^2}} dx}{d^2} \\
&= \frac{b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1 - c^2 x^2} \sqrt{d + ex^2}}{d^2 x} - \frac{a + b \operatorname{sech}^{-1}(cx)}{dx \sqrt{d + ex^2}} - \frac{2ex (a + b \operatorname{sech}^{-1}(cx))}{d^2 \sqrt{d + ex^2}} + \frac{\left(bc^2 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{-d - 2ex^2}{x^2 \sqrt{1 - c^2 x^2} \sqrt{d + ex^2}} dx}{d^2} \\
&= \frac{b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1 - c^2 x^2} \sqrt{d + ex^2}}{d^2 x} - \frac{a + b \operatorname{sech}^{-1}(cx)}{dx \sqrt{d + ex^2}} - \frac{2ex (a + b \operatorname{sech}^{-1}(cx))}{d^2 \sqrt{d + ex^2}} + \frac{\left(bc^2 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{-d - 2ex^2}{x^2 \sqrt{1 - c^2 x^2} \sqrt{d + ex^2}} dx}{d^2} \\
&= \frac{b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1 - c^2 x^2} \sqrt{d + ex^2}}{d^2 x} - \frac{a + b \operatorname{sech}^{-1}(cx)}{dx \sqrt{d + ex^2}} - \frac{2ex (a + b \operatorname{sech}^{-1}(cx))}{d^2 \sqrt{d + ex^2}} + \frac{bc \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \int \frac{-d - 2ex^2}{x^2 \sqrt{1 - c^2 x^2} \sqrt{d + ex^2}} dx}{d^2}
\end{aligned}$$

Mathematica [C] time = 4.46121, size = 501, normalized size = 2.01

$$\frac{b \sqrt{\frac{1-cx}{cx+1}} \left(-c^2(d+ex^2) + \frac{(cx+1) \sqrt{\frac{c(\sqrt{d}-i\sqrt{e})}{(cx+1)(c\sqrt{d}-i\sqrt{e})}} \sqrt{\frac{c(\sqrt{d}+i\sqrt{e})}{(cx+1)(c\sqrt{d}+i\sqrt{e})}} \left(2\sqrt{e}(c\sqrt{d}-2i\sqrt{e}) \operatorname{EllipticF} \left(i \sinh^{-1} \left(\frac{(1-cx)(c^2 d+e)}{\sqrt{(cx+1)(c\sqrt{d}+i\sqrt{e})^2}} \right), \frac{(c\sqrt{d}+i\sqrt{e})^2}{(c\sqrt{d}-i\sqrt{e})^2} \right) - i(c\sqrt{d}-i\sqrt{e})^2 E \left(i \sinh^{-1} \left(\frac{(d+ex^2)}{\sqrt{(d+ex^2)(cx+1)}} \right), \frac{(d+ex^2)(cx+1)}{\sqrt{(d+ex^2)(cx+1)}} \right) \right)}{\sqrt{\frac{(cx-1)(c\sqrt{d}-i\sqrt{e})}{(cx+1)(c\sqrt{d}+i\sqrt{e})}} \right)}{c}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcSech[c*x])/(x^2*(d + e*x^2)^(3/2)), x]

[Out] ((b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(d + e*x^2))/x - (a*(d + 2*e*x^2))/x - (b*(d + 2*e*x^2)*ArcSech[c*x])/x + (b*Sqrt[(1 - c*x)/(1 + c*x)]*(-(c^2*(d + e*x^2)) + ((1 + c*x)*Sqrt[(c*(Sqrt[d] - I*Sqrt[e]*x))/((c*Sqrt[d] - I*Sqrt[e])*(1 + c*x))]*Sqrt[(c*(Sqrt[d] + I*Sqrt[e]*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x))]*((-I)*(c*Sqrt[d] - I*Sqrt[e])^2*EllipticE[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2 + 2*(c*Sqrt[d] - (2*I)*Sqrt[e])*Sqrt[e]*EllipticF[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2)))/Sqrt[-(((c*Sqrt[d] - I*Sqrt[e])*(-1 + c*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x)))))/c)/(d^2*Sqrt[d + e*x^2])

Maple [F] time = 0.809, size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{arcsech}(cx)}{x^2} (ex^2 + d)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))/x^2/(e*x^2+d)^(3/2), x)

[Out] `int((a+b*arcsech(c*x))/x^2/(e*x^2+d)^(3/2),x)`

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2 + d}(b \operatorname{arsech}(cx) + a)}{e^2x^6 + 2dex^4 + d^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="fricas")`

[Out] `integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/(e^2*x^6 + 2*d*e*x^4 + d^2*x^2), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*asech(c*x))/x**2/(e*x**2+d)**(3/2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{arsech}(cx) + a}{(ex^2 + d)^{\frac{3}{2}}x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsech(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="giac")`

[Out] `integrate((b*arcsech(c*x) + a)/((e*x^2 + d)^(3/2)*x^2), x)`

$$3.168 \quad \int \frac{x^5 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

Optimal. Leaf size=272

$$\frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{3e^3 (d + ex^2)^{3/2}} + \frac{2d (a + b \operatorname{sech}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{e^3} - \frac{bd \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \sqrt{1-c^2x^2}}{3e^2 (c^2d + e) \sqrt{d + ex^2}} - \frac{b \sqrt{\frac{1}{cx+1}}}{e^3}$$

[Out] $-(b*d*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[1 - c^2*x^2])/(3*e^2*(c^2*d + e)*\operatorname{Sqrt}[d + e*x^2]) - (d^2*(a + b*\operatorname{ArcSech}[c*x]))/(3*e^3*(d + e*x^2)^{(3/2)}) + (2*d*(a + b*\operatorname{ArcSech}[c*x]))/(e^3*\operatorname{Sqrt}[d + e*x^2]) + (\operatorname{Sqrt}[d + e*x^2]*(a + b*\operatorname{ArcSech}[c*x]))/e^3 - (b*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{ArcTan}[(\operatorname{Sqrt}[e]*\operatorname{Sqrt}[1 - c^2*x^2])/(c*\operatorname{Sqrt}[d + e*x^2])])/(c*e^{(5/2)}) - (8*b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{ArcTanh}[\operatorname{Sqrt}[d + e*x^2]/(\operatorname{Sqrt}[d]*\operatorname{Sqrt}[1 - c^2*x^2])])/(3*e^3)$

Rubi [A] time = 1.28104, antiderivative size = 272, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {266, 43, 6301, 12, 1614, 157, 63, 217, 203, 93, 207}

$$\frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{3e^3 (d + ex^2)^{3/2}} + \frac{2d (a + b \operatorname{sech}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{e^3} - \frac{bd \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \sqrt{1-c^2x^2}}{3e^2 (c^2d + e) \sqrt{d + ex^2}} - \frac{b \sqrt{\frac{1}{cx+1}}}{e^3}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(x^5*(a + b*\operatorname{ArcSech}[c*x]))/(d + e*x^2)^{(5/2)}, x]$

[Out] $-(b*d*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[1 - c^2*x^2])/(3*e^2*(c^2*d + e)*\operatorname{Sqrt}[d + e*x^2]) - (d^2*(a + b*\operatorname{ArcSech}[c*x]))/(3*e^3*(d + e*x^2)^{(3/2)}) + (2*d*(a + b*\operatorname{ArcSech}[c*x]))/(e^3*\operatorname{Sqrt}[d + e*x^2]) + (\operatorname{Sqrt}[d + e*x^2]*(a + b*\operatorname{ArcSech}[c*x]))/e^3 - (b*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{ArcTan}[(\operatorname{Sqrt}[e]*\operatorname{Sqrt}[1 - c^2*x^2])/(c*\operatorname{Sqrt}[d + e*x^2])])/(c*e^{(5/2)}) - (8*b*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{ArcTanh}[\operatorname{Sqrt}[d + e*x^2]/(\operatorname{Sqrt}[d]*\operatorname{Sqrt}[1 - c^2*x^2])])/(3*e^3)$

Rule 266

$\operatorname{Int}[(x_)^{(m_)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] := \operatorname{Dist}[1/n, \operatorname{Subst}[\operatorname{Int}[x^{(\operatorname{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p, x}], x, x^n], x] /;$ FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 43

$\operatorname{Int}[(a_ + (b_)*(x_))^{(m_)}*((c_ + (d_)*(x_))^{(n_)}), x_Symbol] := \operatorname{Int}[\operatorname{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$ FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 6301

$\operatorname{Int}[(a_ + \operatorname{ArcSech}[(c_)*(x_)]*(b_))*((f_)*(x_))^{(m_)}*((d_ + (e_)*(x_)^2)^{(p_)}), x_Symbol] := \operatorname{With}[\{u = \operatorname{IntHide}[(f*x)^m*(d + e*x^2)^p, x]\}, \operatorname{Di}$

```
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p +
3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 1614

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_
.)*(x_))^(p_), x_Symbol] := With[{Qx = PolynomialQuotient[Px, a + b*x, x],
R = PolynomialRemainder[Px, a + b*x, x]}, Simp[(b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
- b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1]
&& IntegersQ[2*m, 2*n, 2*p]
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] := Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
- 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{x^5 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{5/2}} dx &= -\frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{3e^3 (d + ex^2)^{3/2}} + \frac{2d (a + b \operatorname{sech}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{e^3} + \left(b \sqrt{\frac{1}{1+cx}} \right) \\
 &= -\frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{3e^3 (d + ex^2)^{3/2}} + \frac{2d (a + b \operatorname{sech}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{e^3} + \frac{\left(b \sqrt{\frac{1}{1+cx}} \right)}{\sqrt{d + ex^2}} \\
 &= -\frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{3e^3 (d + ex^2)^{3/2}} + \frac{2d (a + b \operatorname{sech}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx))}{e^3} + \frac{\left(b \sqrt{\frac{1}{1+cx}} \right)}{\sqrt{d + ex^2}} \\
 &= -\frac{bd \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{3e^2 (c^2d + e) \sqrt{d + ex^2}} - \frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{3e^3 (d + ex^2)^{3/2}} + \frac{2d (a + b \operatorname{sech}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2}}{\sqrt{d + ex^2}} \\
 &= -\frac{bd \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{3e^2 (c^2d + e) \sqrt{d + ex^2}} - \frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{3e^3 (d + ex^2)^{3/2}} + \frac{2d (a + b \operatorname{sech}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2}}{\sqrt{d + ex^2}} \\
 &= -\frac{bd \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{3e^2 (c^2d + e) \sqrt{d + ex^2}} - \frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{3e^3 (d + ex^2)^{3/2}} + \frac{2d (a + b \operatorname{sech}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2}}{\sqrt{d + ex^2}} \\
 &= -\frac{bd \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{3e^2 (c^2d + e) \sqrt{d + ex^2}} - \frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{3e^3 (d + ex^2)^{3/2}} + \frac{2d (a + b \operatorname{sech}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2}}{\sqrt{d + ex^2}} \\
 &= -\frac{bd \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{3e^2 (c^2d + e) \sqrt{d + ex^2}} - \frac{d^2 (a + b \operatorname{sech}^{-1}(cx))}{3e^3 (d + ex^2)^{3/2}} + \frac{2d (a + b \operatorname{sech}^{-1}(cx))}{e^3 \sqrt{d + ex^2}} + \frac{\sqrt{d + ex^2}}{\sqrt{d + ex^2}}
 \end{aligned}$$

Mathematica [A] time = 1.95521, size = 348, normalized size = 1.28

$$\frac{a(c^2d + e)(8d^2 + 12dex^2 + 3e^2x^4) + b(c^2d + e) \operatorname{sech}^{-1}(cx)(8d^2 + 12dex^2 + 3e^2x^4) - bde \sqrt{\frac{1-cx}{cx+1}}(cx+1)(d+ex^2)}{3e^3(c^2d + e)(d+ex^2)^{3/2}} + \dots$$

Antiderivative was successfully verified.

[In] Integrate[(x^5*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2), x]

[Out] $(-(b*d*e*\sqrt{(1-cx)/(1+cx)}*(1+cx)*(d+e*x^2)) + a*(c^2*d + e)*(8*d^2 + 12*d*e*x^2 + 3*e^2*x^4) + b*(c^2*d + e)*(8*d^2 + 12*d*e*x^2 + 3*e^2*x^4)*\operatorname{ArcSech}[c*x]) / (3*e^3*(c^2*d + e)*(d + e*x^2)^{(3/2)}) + (b*\sqrt{(1-cx)/(1+cx)}*\sqrt{1-c^2*x^2}*(3*\sqrt{-c^2}*\sqrt{-(c^2*d - e)}*\sqrt{e}*\sqrt{(c^2*(d + e*x^2))/(c^2*d + e)}*\operatorname{ArcSin}[(c*\sqrt{e}*\sqrt{1-c^2*x^2})]/(\sqrt{-c^2}*\sqrt{-(c^2*d - e)})) + 8*c^3*\sqrt{d}*\sqrt{-d - e*x^2}*\operatorname{ArcTan}[(\sqrt{d}*\sqrt{1-c^2*x^2})/\sqrt{-d - e*x^2}]) / (3*c^3*e^3*(-1 + cx)*\sqrt{d + e$

x^2)

Maple [F] time = 1.441, size = 0, normalized size = 0.

$$\int x^5 (a + b \operatorname{arcsech}(cx)) (ex^2 + d)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)`

[Out] `int(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)`

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [B] time = 4.4404, size = 5094, normalized size = 18.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")`

[Out] `[-1/12*(3*(b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(-e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^4*e*x^3 + (c^4*d - c^2*e)*x)*sqrt(e*x^2 + d)*sqrt(-e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + e^2) - 4*(8*b*c^3*d^3 + 8*b*c*d^2*e + 3*(b*c^3*d*e^2 + b*c*e^3)*x^4 + 12*(b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(e*x^2 + d)*log(((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - 8*(b*c^3*d^3 + b*c*d^2*e + (b*c^3*d*e^2 + b*c*e^3)*x^4 + 2*(b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) - 4*(8*a*c^3*d^3 + 8*a*c*d^2*e + 3*(a*c^3*d*e^2 + a*c*e^3)*x^4 + 12*(a*c^3*d^2*e + a*c*d*e^2)*x^2 - (b*c^2*d*e^2*x^3 + b*c^2*d^2*e*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d)/(c^3*d^3*e^3 + c*d^2*e^4 + (c^3*d*e^5 + c*e^6)*x^4 + 2*(c^3*d^2*e^4 + c*d*e^5)*x^2), -1/6*(3*(b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(e)*arctan(1/2*(2*c^2*e*x^3 + (c^2*d - e)*x)*sqrt(e*x^2 + d)*sqrt(e)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/(c^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e) - 2*(8*b*c^3*d^3 + 8*b*c*d^2*e + 3*(b*c^3*d*e^2 + b*c*e^3)*x^4 + 12*(b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(e*x^2 + d)*log(((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - 4*(b*c^3*d^3 + b*c*d^2*e + (b*c^3*d*e^2 + b*c*e^3)*x^4 + 2*(b*c^3*d^2*e + b*c*d*e^2)*x^2)*sqrt(d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 + 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e`

$$\begin{aligned}
& x^2 + d) \sqrt{d} \sqrt{-(c^2 x^2 - 1)/(c^2 x^2)} + 8d^2/x^4 - 2(8ac^3d^3 + 8ac^2d^2e + 3(a^3c^3d^2e^2 + ac^3e^3)x^4 + 12(a^3c^3d^2e + ac^3d^2e^2)x^2 - (bc^2d^2e^2x^3 + bc^2d^2e^2x) \sqrt{-(c^2x^2 - 1)/(c^2x^2)}) \sqrt{ex^2 + d}) / (c^3d^3e^3 + c^2d^2e^4 + (c^3d^2e^5 + ce^6)x^4 + 2(c^3d^2e^4 + cd^2e^5)x^2), \\
& -1/12(16(bc^3d^3 + bc^2d^2e + (bc^3d^2e^2 + bc^2e^3)x^4 + 2(bc^3d^2e + bc^2d^2e^2)x^2) \sqrt{-d} \arctan(-1/2((c^3d - ce)x^3 - 2cdx) \sqrt{ex^2 + d} \sqrt{-d} \sqrt{-(c^2x^2 - 1)/(c^2x^2)}) / (c^2d^2e^2x^4 + (c^2d^2 - d^2e)x^2 - d^2)) + 3(bc^2d^3 + (bc^2d^2e^2 + be^3)x^4 + bd^2e + 2(bc^2d^2e + bd^2e^2)x^2) \sqrt{-e} \log(8c^4e^2x^4 + c^4d^2 - 6c^2d^2e + 8(c^4d^2e - c^2e^2)x^2 - 4(2c^4e^2x^3 + (c^4d - c^2e)x) \sqrt{ex^2 + d} \sqrt{-e} \sqrt{-(c^2x^2 - 1)/(c^2x^2)}) + e^2) - 4(8bc^3d^3 + 8bc^2d^2e + 3(bc^3d^2e^2 + bc^2e^3)x^4 + 12(bc^3d^2e + bc^2d^2e^2)x^2) \sqrt{ex^2 + d} \log((cx \sqrt{-(c^2x^2 - 1)/(c^2x^2)}) + 1)/(cx)) - 4(8ac^3d^3 + 8ac^2d^2e + 3(a^3c^3d^2e^2 + ac^3e^3)x^4 + 12(a^3c^3d^2e + ac^3d^2e^2)x^2 - (bc^2d^2e^2x^3 + bc^2d^2e^2x) \sqrt{-(c^2x^2 - 1)/(c^2x^2)}) \sqrt{ex^2 + d}) / (c^3d^3e^3 + c^2d^2e^4 + (c^3d^2e^5 + ce^6)x^4 + 2(c^3d^2e^4 + cd^2e^5)x^2), \\
& -1/6(8(bc^3d^3 + bc^2d^2e + (bc^3d^2e^2 + bc^2e^3)x^4 + 2(bc^3d^2e + bc^2d^2e^2)x^2) \sqrt{-d} \arctan(-1/2((c^3d - ce)x^3 - 2cdx) \sqrt{ex^2 + d} \sqrt{-d} \sqrt{-(c^2x^2 - 1)/(c^2x^2)}) / (c^2d^2e^2x^4 + (c^2d^2 - d^2e)x^2 - d^2)) + 3(bc^2d^3 + (bc^2d^2e^2 + be^3)x^4 + bd^2e + 2(bc^2d^2e + bd^2e^2)x^2) \sqrt{e} \arctan(1/2(2c^2e^2x^3 + (c^2d - e)x) \sqrt{ex^2 + d} \sqrt{e} \sqrt{-(c^2x^2 - 1)/(c^2x^2)}) / (c^2e^2x^4 + (c^2d^2e - e^2)x^2 - d^2e)) - 2(8bc^3d^3 + 8bc^2d^2e + 3(bc^3d^2e^2 + bc^2e^3)x^4 + 12(bc^3d^2e + bc^2d^2e^2)x^2) \sqrt{ex^2 + d} \log((cx \sqrt{-(c^2x^2 - 1)/(c^2x^2)}) + 1)/(cx)) - 2(8ac^3d^3 + 8ac^2d^2e + 3(a^3c^3d^2e^2 + ac^3e^3)x^4 + 12(a^3c^3d^2e + ac^3d^2e^2)x^2 - (bc^2d^2e^2x^3 + bc^2d^2e^2x) \sqrt{-(c^2x^2 - 1)/(c^2x^2)}) \sqrt{ex^2 + d}) / (c^3d^3e^3 + c^2d^2e^4 + (c^3d^2e^5 + ce^6)x^4 + 2(c^3d^2e^4 + cd^2e^5)x^2)]
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*(a+b*asech(c*x))/(e*x**2+d)**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsech}(cx) + a)x^5}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)*x^5/(e*x^2 + d)^(5/2), x)

$$3.169 \quad \int \frac{x^3 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

Optimal. Leaf size=179

$$-\frac{a + b \operatorname{sech}^{-1}(cx)}{e^2 \sqrt{d + ex^2}} + \frac{d(a + b \operatorname{sech}^{-1}(cx))}{3e^2 (d + ex^2)^{3/2}} + \frac{2b \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3\sqrt{de}^2} + \frac{b \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \sqrt{1-c^2x^2}}{3e(c^2d + e) \sqrt{d + ex^2}}$$

[Out] (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(3*e*(c^2*d + e)*Sqrt[d + e*x^2]) + (d*(a + b*ArcSech[c*x]))/(3*e^2*(d + e*x^2)^(3/2)) - (a + b*ArcSech[c*x])/(e^2*Sqrt[d + e*x^2]) + (2*b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/(3*Sqrt[d]*e^2)

Rubi [A] time = 0.263708, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {266, 43, 6301, 12, 573, 152, 93, 207}

$$-\frac{a + b \operatorname{sech}^{-1}(cx)}{e^2 \sqrt{d + ex^2}} + \frac{d(a + b \operatorname{sech}^{-1}(cx))}{3e^2 (d + ex^2)^{3/2}} + \frac{2b \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3\sqrt{de}^2} + \frac{b \sqrt{\frac{1}{cx+1}} \sqrt{cx+1} \sqrt{1-c^2x^2}}{3e(c^2d + e) \sqrt{d + ex^2}}$$

Antiderivative was successfully verified.

[In] Int[(x^3*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2), x]

[Out] (b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(3*e*(c^2*d + e)*Sqrt[d + e*x^2]) + (d*(a + b*ArcSech[c*x]))/(3*e^2*(d + e*x^2)^(3/2)) - (a + b*ArcSech[c*x])/(e^2*Sqrt[d + e*x^2]) + (2*b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*ArcTanh[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[1 - c^2*x^2])])/(3*Sqrt[d]*e^2)

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 6301

Int[((a_.) + ArcSech[(c_.)*(x_)])*(b_.))*((f_.)*(x_)^(m_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 573

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p_.*((c_) + (d_.)*(x_)^(n_.))^q_.
*(e_) + (f_.)*(x_)^(n_.))^r_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^p_.*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
- a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.))/((e_.) + (f_.)*(x
_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
- 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{5/2}} dx &= \frac{d(a + b \operatorname{sech}^{-1}(cx))}{3e^2 (d + ex^2)^{3/2}} - \frac{a + b \operatorname{sech}^{-1}(cx)}{e^2 \sqrt{d + ex^2}} + \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{-2d - 3ex^2}{3e^2 x \sqrt{1-c^2x^2} (d + ex^2)^{3/2}} dx \\
&= \frac{d(a + b \operatorname{sech}^{-1}(cx))}{3e^2 (d + ex^2)^{3/2}} - \frac{a + b \operatorname{sech}^{-1}(cx)}{e^2 \sqrt{d + ex^2}} + \frac{\left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{-2d - 3ex^2}{x \sqrt{1-c^2x^2} (d + ex^2)^{3/2}} dx}{3e^2} \\
&= \frac{d(a + b \operatorname{sech}^{-1}(cx))}{3e^2 (d + ex^2)^{3/2}} - \frac{a + b \operatorname{sech}^{-1}(cx)}{e^2 \sqrt{d + ex^2}} + \frac{\left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \operatorname{Subst} \left(\int \frac{-2d - 3ex}{x \sqrt{1-c^2x} (d + ex)^{3/2}} dx, x, \sqrt{1-c^2x^2} \right)}{6e^2} \\
&= \frac{b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{3e (c^2d + e) \sqrt{d + ex^2}} + \frac{d(a + b \operatorname{sech}^{-1}(cx))}{3e^2 (d + ex^2)^{3/2}} - \frac{a + b \operatorname{sech}^{-1}(cx)}{e^2 \sqrt{d + ex^2}} - \frac{\left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) S}{3} \\
&= \frac{b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{3e (c^2d + e) \sqrt{d + ex^2}} + \frac{d(a + b \operatorname{sech}^{-1}(cx))}{3e^2 (d + ex^2)^{3/2}} - \frac{a + b \operatorname{sech}^{-1}(cx)}{e^2 \sqrt{d + ex^2}} - \frac{\left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) S}{3} \\
&= \frac{b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{3e (c^2d + e) \sqrt{d + ex^2}} + \frac{d(a + b \operatorname{sech}^{-1}(cx))}{3e^2 (d + ex^2)^{3/2}} - \frac{a + b \operatorname{sech}^{-1}(cx)}{e^2 \sqrt{d + ex^2}} - \frac{\left(2b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right)}{3} \\
&= \frac{b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{3e (c^2d + e) \sqrt{d + ex^2}} + \frac{d(a + b \operatorname{sech}^{-1}(cx))}{3e^2 (d + ex^2)^{3/2}} - \frac{a + b \operatorname{sech}^{-1}(cx)}{e^2 \sqrt{d + ex^2}} + \frac{2b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \operatorname{arctan} \left(\frac{\sqrt{1-c^2x^2}}{\sqrt{1+cx}} \right)}{3\sqrt{d + ex^2}}
\end{aligned}$$

Mathematica [A] time = 0.369211, size = 218, normalized size = 1.22

$$\frac{-a(c^2d + e)(2d + 3ex^2) - b(c^2d + e) \operatorname{sech}^{-1}(cx)(2d + 3ex^2) + be \sqrt{\frac{1-cx}{cx+1}}(cx + 1)(d + ex^2)}{3e^2 (c^2d + e) (d + ex^2)^{3/2}} - \frac{2b \sqrt{\frac{1-cx}{cx+1}} \sqrt{1-c^2x^2} \sqrt{-d - ex^2}}{3\sqrt{de^2}(cx - 1)\sqrt{d + ex^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2), x]

[Out] (b*e*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*(d + e*x^2) - a*(c^2*d + e)*(2*d + 3*e*x^2) - b*(c^2*d + e)*(2*d + 3*e*x^2)*ArcSech[c*x])/(3*e^2*(c^2*d + e)*(d + e*x^2)^(3/2)) - (2*b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^2*x^2]*Sqrt[-d - e*x^2]*ArcTan[(Sqrt[d]*Sqrt[1 - c^2*x^2])/Sqrt[-d - e*x^2]])/(3*Sqrt[d]*e^2*(-1 + c*x)*Sqrt[d + e*x^2])

Maple [F] time = 1.405, size = 0, normalized size = 0.

$$\int x^3 (a + b \operatorname{arcsech}(cx)) (ex^2 + d)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2), x)

[Out] int(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{3} a \left(\frac{3x^2}{(ex^2+d)^{\frac{3}{2}}e} + \frac{2d}{(ex^2+d)^{\frac{3}{2}}e^2} \right) + b \int \frac{x^3 \log \left(\sqrt{\frac{1}{cx}+1} \sqrt{\frac{1}{cx}-1} + \frac{1}{cx} \right)}{(ex^2+d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")

[Out] -1/3*a*(3*x^2/((e*x^2 + d)^(3/2)*e) + 2*d/((e*x^2 + d)^(3/2)*e^2)) + b*integrate(x^3*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e*x^2 + d)^(5/2), x)

Fricas [B] time = 3.21954, size = 1635, normalized size = 9.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")

[Out] [-1/6*(2*(2*b*c^2*d^3 + 2*b*d^2*e + 3*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(d)*log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 8*d^2)/x^4) + 2*(2*a*c^2*d^3 + 2*a*d^2*e + 3*(a*c^2*d^2*e + a*d*e^2)*x^2 - (b*c*d*e^2*x^3 + b*c*d^2*e*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d)/(c^2*d^4*e^2 + d^3*e^3 + (c^2*d^2*e^4 + d*e^5)*x^4 + 2*(c^2*d^3*e^3 + d^2*e^4)*x^2), 1/3*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(-d)*arctan(-1/2*((c^3*d - c*e)*x^3 - 2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/(c^2*d*e*x^4 + (c^2*d^2 - d*e)*x^2 - d^2) - (2*b*c^2*d^3 + 2*b*d^2*e + 3*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(e*x^2 + d)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (2*a*c^2*d^3 + 2*a*d^2*e + 3*(a*c^2*d^2*e + a*d*e^2)*x^2 - (b*c*d*e^2*x^3 + b*c*d^2*e*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x^2 + d)/(c^2*d^4*e^2 + d^3*e^3 + (c^2*d^2*e^4 + d*e^5)*x^4 + 2*(c^2*d^3*e^3 + d^2*e^4)*x^2)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(a+b*asech(c*x))/(e*x**2+d)**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{ar} \operatorname{sech}(cx) + a)x^3}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arcsech(c*x) + a)*x^3/(e*x^2 + d)^(5/2), x)
```


$$3.170 \quad \int \frac{x(a+b\operatorname{sech}^{-1}(cx))}{(d+ex^2)^{5/2}} dx$$

Optimal. Leaf size=154

$$-\frac{a+b\operatorname{sech}^{-1}(cx)}{3e(d+ex^2)^{3/2}} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3d^{3/2}e} - \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{3d(c^2d+e)\sqrt{d+ex^2}}$$

[Out] $-(b*\operatorname{Sqrt}[(1+c*x)^{-1}]*\operatorname{Sqrt}[1+c*x]*\operatorname{Sqrt}[1-c^2*x^2])/(3*d*(c^2*d+e)*\operatorname{Sqrt}[d+e*x^2]) - (a+b*\operatorname{ArcSech}[c*x])/(3*e*(d+e*x^2)^{(3/2)}) + (b*\operatorname{Sqrt}[(1+c*x)^{-1}]*\operatorname{Sqrt}[1+c*x]*\operatorname{ArcTanh}[\operatorname{Sqrt}[d+e*x^2]/(\operatorname{Sqrt}[d]*\operatorname{Sqrt}[1-c^2*x^2])])/(3*d^{(3/2)}*e)$

Rubi [A] time = 0.290899, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6299, 517, 446, 96, 93, 207}

$$-\frac{a+b\operatorname{sech}^{-1}(cx)}{3e(d+ex^2)^{3/2}} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right)}{3d^{3/2}e} - \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{3d(c^2d+e)\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(x*(a+b*\operatorname{ArcSech}[c*x]))/(d+e*x^2)^{(5/2)},x]$

[Out] $-(b*\operatorname{Sqrt}[(1+c*x)^{-1}]*\operatorname{Sqrt}[1+c*x]*\operatorname{Sqrt}[1-c^2*x^2])/(3*d*(c^2*d+e)*\operatorname{Sqrt}[d+e*x^2]) - (a+b*\operatorname{ArcSech}[c*x])/(3*e*(d+e*x^2)^{(3/2)}) + (b*\operatorname{Sqrt}[(1+c*x)^{-1}]*\operatorname{Sqrt}[1+c*x]*\operatorname{ArcTanh}[\operatorname{Sqrt}[d+e*x^2]/(\operatorname{Sqrt}[d]*\operatorname{Sqrt}[1-c^2*x^2])])/(3*d^{(3/2)}*e)$

Rule 6299

$\operatorname{Int}[(a_.) + \operatorname{ArcSech}[(c_.)*(x_.)]*(b_.)]*(x_.)*((d_.) + (e_.)*(x_.)^2)^{(p_.)}, x_Symbol] :> \operatorname{Simp}[(d+e*x^2)^{(p+1)}*(a+b*\operatorname{ArcSech}[c*x])]/(2*e*(p+1)), x] + \operatorname{Dist}[(b*\operatorname{Sqrt}[1+c*x]*\operatorname{Sqrt}[1/(1+c*x)])/(2*e*(p+1)), \operatorname{Int}[(d+e*x^2)^{(p+1)}/(x*\operatorname{Sqrt}[1-c*x]*\operatorname{Sqrt}[1+c*x]), x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, p\}, x] \&\& \operatorname{NeQ}[p, -1]$

Rule 517

$\operatorname{Int}[(u_.)*((c_.) + (d_.)*(x_.)^{(n_.)})^{(q_.)}*((a1_.) + (b1_.)*(x_.)^{(non2_.)})^{(p_.)}*((a2_.) + (b2_.)*(x_.)^{(non2_.)})^{(p_.)}, x_Symbol] :> \operatorname{Int}[u*(a1*a2 + b1*b2*x^n)^p*(c+d*x^n)^q, x] /; \operatorname{FreeQ}\{a1, b1, a2, b2, c, d, n, p, q\}, x] \&\& \operatorname{EqQ}[non2, n/2] \&\& \operatorname{EqQ}[a2*b1 + a1*b2, 0] \&\& (\operatorname{IntegerQ}[p] \|\| (\operatorname{GtQ}[a1, 0] \&\& \operatorname{GtQ}[a2, 0]))$

Rule 446

$\operatorname{Int}[(x_.)^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}*((c_.) + (d_.)*(x_.)^{(n_.)})^{(q_.)}, x_Symbol] :> \operatorname{Dist}[1/n, \operatorname{Subst}[\operatorname{Int}[x^{(\operatorname{Simplify}[(m+1)/n]-1)}*(a+b*x)^p*(c+d*x)^q, x], x, x^n], x] /; \operatorname{FreeQ}\{a, b, c, d, m, n, p, q\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{IntegerQ}[\operatorname{Simplify}[(m+1)/n]]$

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 207

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{x(a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{5/2}} dx &= -\frac{a + b \operatorname{sech}^{-1}(cx)}{3e(d + ex^2)^{3/2}} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{1}{x\sqrt{1-cx}\sqrt{1+cx}(d+ex^2)^{3/2}} dx}{3e} \\ &= -\frac{a + b \operatorname{sech}^{-1}(cx)}{3e(d + ex^2)^{3/2}} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \int \frac{1}{x\sqrt{1-c^2x^2}(d+ex^2)^{3/2}} dx}{3e} \\ &= -\frac{a + b \operatorname{sech}^{-1}(cx)}{3e(d + ex^2)^{3/2}} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-c^2x}(d+ex)^{3/2}} dx, x, x^2\right)}{6e} \\ &= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d(c^2d + e)\sqrt{d + ex^2}} - \frac{a + b \operatorname{sech}^{-1}(cx)}{3e(d + ex^2)^{3/2}} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-c^2x}\sqrt{d+ex}} dx, x, x^2\right)}{6de} \\ &= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d(c^2d + e)\sqrt{d + ex^2}} - \frac{a + b \operatorname{sech}^{-1}(cx)}{3e(d + ex^2)^{3/2}} - \frac{\left(b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\right) \operatorname{Subst}\left(\int \frac{1}{-d+x^2} dx, x, \frac{\sqrt{d+ex^2}}{\sqrt{d-ex^2}}\right)}{3de} \\ &= -\frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{3d(c^2d + e)\sqrt{d + ex^2}} - \frac{a + b \operatorname{sech}^{-1}(cx)}{3e(d + ex^2)^{3/2}} + \frac{b\sqrt{\frac{1}{1+cx}}\sqrt{1+cx} \tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d-ex^2}}\right)}{3d^{3/2}e} \end{aligned}$$

Mathematica [A] time = 0.30991, size = 204, normalized size = 1.32

$$\frac{-ad(c^2d + e) - bd(c^2d + e) \operatorname{sech}^{-1}(cx) - be\sqrt{\frac{1-cx}{cx+1}}(cx + 1)(d + ex^2)}{3de(c^2d + e)(d + ex^2)^{3/2}} - \frac{b\sqrt{\frac{1-cx}{cx+1}}\sqrt{1-c^2x^2}\sqrt{-d - ex^2} \tan^{-1}\left(\frac{\sqrt{d}\sqrt{1-c^2x^2}}{\sqrt{-d-ex^2}}\right)}{3d^{3/2}e(cx - 1)\sqrt{d + ex^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2), x]

[Out] $(-(a*d*(c^2*d + e)) - b*e*\sqrt{(1 - c*x)/(1 + c*x)}*(1 + c*x)*(d + e*x^2) - b*d*(c^2*d + e)*\text{ArcSech}[c*x]) / (3*d*e*(c^2*d + e)*(d + e*x^2)^{(3/2)}) - (b*\sqrt{(1 - c*x)/(1 + c*x)}*\sqrt{1 - c^2*x^2}*\sqrt{-d - e*x^2}*\text{ArcTan}[(\sqrt{d}*\sqrt{1 - c^2*x^2})/\sqrt{-d - e*x^2}]) / (3*d^{(3/2)}*e*(-1 + c*x)*\sqrt{d + e*x^2})$

Maple [F] time = 0.935, size = 0, normalized size = 0.

$$\int x(a + b\text{arcsech}(cx))(ex^2 + d)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2), x)

[Out] int(x*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$b \int \frac{x \log\left(\sqrt{\frac{1}{cx} + 1} \sqrt{\frac{1}{cx} - 1} + \frac{1}{cx}\right)}{(ex^2 + d)^{\frac{5}{2}}} dx - \frac{a}{3(ex^2 + d)^{\frac{3}{2}}e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2), x, algorithm="maxima")

[Out] $b*\text{integrate}(x*\log(\sqrt{1/(c*x) + 1}*\sqrt{1/(c*x) - 1} + 1/(c*x)))/(e*x^2 + d)^{(5/2)}, x) - 1/3*a/((e*x^2 + d)^{(3/2)}*e)$

Fricas [B] time = 3.17545, size = 1447, normalized size = 9.4

$$\frac{4(bc^2d^3 + bd^2e)\sqrt{ex^2 + d} \log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}+1}}{cx}\right) - (bc^2d^3 + (bc^2de^2 + be^3)x^4 + bd^2e + 2(bc^2d^2e + bde^2)x^2)\sqrt{d} \log\left(\frac{(c^4d^2 + d^2e^2 + c^2d^2e^2)\sqrt{d}}{c^4d^2 + d^2e^2 + c^2d^2e^2}\right)}{12(c^2d^5e + d^4e^2 + (c^2d^3e^3))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2), x, algorithm="fricas")

[Out] $[-1/12*(4*(b*c^2*d^3 + b*d^2*e)*\sqrt{e*x^2 + d}*\log((c*x*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)} + 1)/(c*x)) - (b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*\sqrt{d}*\log(((c^4*d^2 - 6*c^2*d*e + e^2)*x^4 - 8*(c^2*d^2 - d*e)*x^2 - 4*((c^3*d - c*e)*x^3 - 2*c*d*x)*\sqrt{e*x^2 + d}*\sqrt{d}*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)} + 8*d^2)/x^4) + 4*(a*c^2*d^3 + a*d^2*e + (b*c*d*e^2*x^3 + b*c*d^2*e*x)*\sqrt{-(c^2*x^2 - 1)/(c^2*x^2)})*\sqrt{d} + (b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*\sqrt{d}*\log\left(\frac{(c^4*d^2 + d^2e^2 + c^2d^2e^2)\sqrt{d}}{c^4d^2 + d^2e^2 + c^2d^2e^2}\right)]$

```
e*x^2 + d))/(c^2*d^5*e + d^4*e^2 + (c^2*d^3*e^3 + d^2*e^4)*x^4 + 2*(c^2*d^4
*e^2 + d^3*e^3)*x^2), 1/6*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e
+ 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(-d)*arctan(-1/2*((c^3*d - c*e)*x^3 -
2*c*d*x)*sqrt(e*x^2 + d)*sqrt(-d)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))/(c^2*d*e
x^4 + (c^2*d^2 - d*e)*x^2 - d^2)) - 2*(b*c^2*d^3 + b*d^2*e)*sqrt(e*x^2 + d)
*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - 2*(a*c^2*d^3 + a*d^2
*e + (b*c*d*e^2*x^3 + b*c*d^2*e*x)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)))*sqrt(e*x
^2 + d))/(c^2*d^5*e + d^4*e^2 + (c^2*d^3*e^3 + d^2*e^4)*x^4 + 2*(c^2*d^4*e
2 + d^3*e^3)*x^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*asech(c*x))/(e*x**2+d)**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{ar} \operatorname{sech}(cx) + a)x}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)*x/(e*x^2 + d)^(5/2), x)

$$3.171 \quad \int \frac{a + b \operatorname{sech}^{-1}(cx)}{x(d + ex^2)^{5/2}} dx$$

Optimal. Leaf size=25

$$\text{Unintegrable} \left(\frac{a + b \operatorname{sech}^{-1}(cx)}{x(d + ex^2)^{5/2}}, x \right)$$

[Out] Unintegrable[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^(5/2)), x]

Rubi [A] time = 0.122709, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{a + b \operatorname{sech}^{-1}(cx)}{x(d + ex^2)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^(5/2)), x]

[Out] Defer[Int] [(a + b*ArcSech[c*x])/(x*(d + e*x^2)^(5/2)), x]

Rubi steps

$$\int \frac{a + b \operatorname{sech}^{-1}(cx)}{x(d + ex^2)^{5/2}} dx = \int \frac{a + b \operatorname{sech}^{-1}(cx)}{x(d + ex^2)^{5/2}} dx$$

Mathematica [A] time = 42.6582, size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{sech}^{-1}(cx)}{x(d + ex^2)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^(5/2)), x]

[Out] Integrate[(a + b*ArcSech[c*x])/(x*(d + e*x^2)^(5/2)), x]

Maple [A] time = 0.829, size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{arcsech}(cx)}{x} (ex^2 + d)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))/x/(e*x^2+d)^(5/2), x)

[Out] `int((a+b*arcsech(c*x))/x/(e*x^2+d)^(5/2),x)`

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2+d}(b \operatorname{ar} \operatorname{sech}(cx) + a)}{e^3x^7 + 3de^2x^5 + 3d^2ex^3 + d^3x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="fricas")`

[Out] `integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*asech(c*x))/x/(e*x**2+d)**(5/2),x)`

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{ar} \operatorname{sech}(cx) + a}{(ex^2 + d)^{\frac{5}{2}}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsech(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="giac")`

[Out] `integrate((b*arcsech(c*x) + a)/((e*x^2 + d)^(5/2)*x), x)`

$$3.172 \quad \int \frac{a+b\operatorname{sech}^{-1}(cx)}{x^3(d+ex^2)^{5/2}} dx$$

Optimal. Leaf size=25

$$\text{Unintegrable} \left(\frac{a + b\operatorname{sech}^{-1}(cx)}{x^3 (d + ex^2)^{5/2}}, x \right)$$

[Out] Unintegrable[(a + b*ArcSech[c*x])/(x^3*(d + e*x^2)^(5/2)), x]

Rubi [A] time = 0.142136, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{a + b\operatorname{sech}^{-1}(cx)}{x^3 (d + ex^2)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*ArcSech[c*x])/(x^3*(d + e*x^2)^(5/2)), x]

[Out] Defer[Int][(a + b*ArcSech[c*x])/(x^3*(d + e*x^2)^(5/2)), x]

Rubi steps

$$\int \frac{a + b\operatorname{sech}^{-1}(cx)}{x^3 (d + ex^2)^{5/2}} dx = \int \frac{a + b\operatorname{sech}^{-1}(cx)}{x^3 (d + ex^2)^{5/2}} dx$$

Mathematica [A] time = 52.642, size = 0, normalized size = 0.

$$\int \frac{a + b\operatorname{sech}^{-1}(cx)}{x^3 (d + ex^2)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*ArcSech[c*x])/(x^3*(d + e*x^2)^(5/2)), x]

[Out] Integrate[(a + b*ArcSech[c*x])/(x^3*(d + e*x^2)^(5/2)), x]

Maple [A] time = 0.951, size = 0, normalized size = 0.

$$\int \frac{a + b\operatorname{arcsech}(cx)}{x^3} (ex^2 + d)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(5/2), x)

[Out] `int((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(5/2),x)`

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2+d}(b \operatorname{ar} \operatorname{sech}(cx) + a)}{e^3x^9 + 3de^2x^7 + 3d^2ex^5 + d^3x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="fricas")`

[Out] `integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/(e^3*x^9 + 3*d*e^2*x^7 + 3*d^2*e*x^5 + d^3*x^3), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*asech(c*x))/x**3/(e*x**2+d)**(5/2),x)`

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{ar} \operatorname{sech}(cx) + a}{(ex^2 + d)^{\frac{5}{2}} x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsech(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="giac")`

[Out] `integrate((b*arcsech(c*x) + a)/((e*x^2 + d)^(5/2)*x^3), x)`

$$3.173 \quad \int \frac{x^6 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

Optimal. Leaf size=25

$$\text{Unintegrable} \left(\frac{x^6 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{5/2}}, x \right)$$

[Out] Unintegrable[(x^6*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2), x]

Rubi [A] time = 0.121566, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{x^6 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(x^6*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2), x]

[Out] Defer[Int] [(x^6*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2), x]

Rubi steps

$$\int \frac{x^6 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{5/2}} dx = \int \frac{x^6 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

Mathematica [A] time = 13.6808, size = 0, normalized size = 0.

$$\int \frac{x^6 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(x^6*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2), x]

[Out] Integrate[(x^6*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2), x]

Maple [A] time = 1.511, size = 0, normalized size = 0.

$$\int x^6 (a + b \operatorname{arcsech}(cx)) (ex^2 + d)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2), x)

[Out] `int(x^6*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)`

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^6 \operatorname{ar} \operatorname{sech}(cx) + ax^6)\sqrt{ex^2 + d}}{e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")`

[Out] `integral((b*x^6*arcsech(c*x) + a*x^6)*sqrt(e*x^2 + d)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**6*(a+b*asech(c*x))/(e*x**2+d)**(5/2),x)`

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{ar} \operatorname{sech}(cx) + a)x^6}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^6*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")`

[Out] `integrate((b*arcsech(c*x) + a)*x^6/(e*x^2 + d)^(5/2), x)`

$$3.174 \quad \int \frac{x^4 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

Optimal. Leaf size=25

$$\text{Unintegrable} \left(\frac{x^4 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{5/2}}, x \right)$$

[Out] Unintegrable[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2), x]

Rubi [A] time = 0.111415, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{x^4 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Int[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2), x]

[Out] Defer[Int] [(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2), x]

Rubi steps

$$\int \frac{x^4 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{5/2}} dx = \int \frac{x^4 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

Mathematica [A] time = 13.229, size = 0, normalized size = 0.

$$\int \frac{x^4 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2), x]

[Out] Integrate[(x^4*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2), x]

Maple [A] time = 1.481, size = 0, normalized size = 0.

$$\int x^4 (a + b \operatorname{arcsech}(cx)) (ex^2 + d)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2), x)

[Out] `int(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)`

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^4 \operatorname{ar} \operatorname{sech}(cx) + ax^4)\sqrt{ex^2 + d}}{e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")`

[Out] `integral((b*x^4*arcsech(c*x) + a*x^4)*sqrt(e*x^2 + d)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*(a+b*asech(c*x))/(e*x**2+d)**(5/2),x)`

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{ar} \operatorname{sech}(cx) + a)x^4}{(ex^2 + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")`

[Out] `integrate((b*arcsech(c*x) + a)*x^4/(e*x^2 + d)^(5/2), x)`

$$3.175 \quad \int \frac{x^2 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{5/2}} dx$$

Optimal. Leaf size=246

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{ex^2}{d}+1}\operatorname{EllipticF}\left(\sin^{-1}(cx), -\frac{e}{c^2d}\right)}{3cde\sqrt{d+ex^2}} + \frac{x^3(a+b\operatorname{sech}^{-1}(cx))}{3d(d+ex^2)^{3/2}} - \frac{bx\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{3d(c^2d+e)\sqrt{d+ex^2}} - \frac{bc\sqrt{\frac{1}{cx+1}}}{3de}$$

```
[Out] -(b*x*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(3*d*(c^2*d + e)
)*Sqrt[d + e*x^2]) + (x^3*(a + b*ArcSech[c*x]))/(3*d*(d + e*x^2)^(3/2)) - (
b*c*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x
], -(e/(c^2*d))])/(3*d*e*(c^2*d + e)*Sqrt[1 + (e*x^2)/d]) + (b*Sqrt[(1 + c*
x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*
d))])/(3*c*d*e*Sqrt[d + e*x^2])
```

Rubi [A] time = 0.246232, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {264, 6301, 12, 471, 423, 426, 424, 421, 419}

$$\frac{x^3(a+b\operatorname{sech}^{-1}(cx))}{3d(d+ex^2)^{3/2}} - \frac{bx\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{3d(c^2d+e)\sqrt{d+ex^2}} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{ex^2}{d}+1}F\left(\sin^{-1}(cx)\middle|-\frac{e}{c^2d}\right)}{3cde\sqrt{d+ex^2}} - \frac{bc\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}}{3de}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2), x]
```

```
[Out] -(b*x*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(3*d*(c^2*d + e)
)*Sqrt[d + e*x^2]) + (x^3*(a + b*ArcSech[c*x]))/(3*d*(d + e*x^2)^(3/2)) - (
b*c*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x
], -(e/(c^2*d))])/(3*d*e*(c^2*d + e)*Sqrt[1 + (e*x^2)/d]) + (b*Sqrt[(1 + c*
x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*
d))])/(3*c*d*e*Sqrt[d + e*x^2])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)])*(b_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[
SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x] /; FreeQ[{
a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ
[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p +
3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m - n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n + 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Dist[b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PoSQ[d/c] && NegQ[b/a]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Dist[Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Simp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2 (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{5/2}} dx &= \frac{x^3 (a + b \operatorname{sech}^{-1}(cx))}{3d (d + ex^2)^{3/2}} + \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{x^2}{3d \sqrt{1-c^2x^2} (d + ex^2)^{3/2}} dx \\
&= \frac{x^3 (a + b \operatorname{sech}^{-1}(cx))}{3d (d + ex^2)^{3/2}} + \frac{\left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{x^2}{\sqrt{1-c^2x^2} (d+ex^2)^{3/2}} dx}{3d} \\
&= -\frac{bx \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{3d (c^2d + e) \sqrt{d + ex^2}} + \frac{x^3 (a + b \operatorname{sech}^{-1}(cx))}{3d (d + ex^2)^{3/2}} + \frac{\left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{\sqrt{1-c^2x^2}}{\sqrt{d+ex^2}} dx}{3d (c^2d + e)} \\
&= -\frac{bx \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{3d (c^2d + e) \sqrt{d + ex^2}} + \frac{x^3 (a + b \operatorname{sech}^{-1}(cx))}{3d (d + ex^2)^{3/2}} + \frac{\left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{1}{\sqrt{1-c^2x^2} \sqrt{d+ex^2}} dx}{3de} \\
&= -\frac{bx \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{3d (c^2d + e) \sqrt{d + ex^2}} + \frac{x^3 (a + b \operatorname{sech}^{-1}(cx))}{3d (d + ex^2)^{3/2}} - \frac{\left(bc^2 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{d + ex^2} \right) \int \frac{1}{\sqrt{1-c^2x^2} \sqrt{d+ex^2}} dx}{3de (c^2d + e) \sqrt{1 + \frac{ex^2}{d}}} \\
&= -\frac{bx \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{3d (c^2d + e) \sqrt{d + ex^2}} + \frac{x^3 (a + b \operatorname{sech}^{-1}(cx))}{3d (d + ex^2)^{3/2}} - \frac{bc \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{d + ex^2} E(\sin^{-1}(\frac{\sqrt{1-c^2x^2}}{\sqrt{d+ex^2}}))}{3de (c^2d + e) \sqrt{1 + \frac{ex^2}{d}}}
\end{aligned}$$

Mathematica [C] time = 2.47446, size = 488, normalized size = 1.98

$$\frac{b \sqrt{\frac{1-cx}{cx+1}} (cx+1) (d+ex^2) \sqrt{\frac{c(\sqrt{d+i\sqrt{ex}})}{(cx+1)(c\sqrt{d+i\sqrt{e}})}} \sqrt{\frac{c(\sqrt{ex+i\sqrt{d}})}{(cx+1)(\sqrt{e+ic\sqrt{d}})}} \left((\sqrt{e+ic\sqrt{d}}) E \left(i \sinh^{-1} \left(\sqrt{\frac{(dc^2+e)(1-cx)}{(\sqrt{dc+i\sqrt{e}})^2 (cx+1)}} \right) \right) \frac{(\sqrt{dc+i\sqrt{e}})^2}{(c\sqrt{d-i\sqrt{e}})^2} \right) - 2\sqrt{e} \operatorname{EllipticF} \left(i \sinh^{-1} \left(\sqrt{\frac{(1-cx)(c^2d+e)}{(cx+1)(c\sqrt{d+i\sqrt{e}})}} \right) \right)}{ce(c\sqrt{d+i\sqrt{e}}) \sqrt{\frac{(cx-1)(\sqrt{e+ic\sqrt{d}})}{(cx+1)(\sqrt{e-ic\sqrt{d}})}}} \frac{1}{3d (d + ex^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(a + b*ArcSech[c*x]))/(d + e*x^2)^(5/2), x]

[Out] (a*x^3 - (b*Sqrt[(1 - c*x)/(1 + c*x)]*(-(c*d) + e*x)*(d + e*x^2))/(e*(c^2*d + e)) + b*x^3*ArcSech[c*x] + (b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[(c*(Sqrt[d] + I*Sqrt[e]*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x))]*Sqrt[(c*(I*Sqrt[d] + Sqrt[e]*x))/((I*c*Sqrt[d] + Sqrt[e])*(1 + c*x))]*(d + e*x^2)*((I*c*Sqrt[d] + Sqrt[e])*EllipticE[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2 - 2*Sqrt[e]*EllipticF[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2))/(c*(c*Sqrt[d] + I*Sqrt[e])*e*Sqrt[(I*c*Sqrt[d] + Sqrt[e])*(-1 + c*x)]/((-I)*c*Sqrt[d] + Sqrt[e])*(1 + c*x)))/(3*d*(d + e*x^2)^(3/2))

Maple [F] time = 1.457, size = 0, normalized size = 0.

$$\int x^2 (a + b \operatorname{arcsech}(cx)) (ex^2 + d)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)`

[Out] `int(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{3}a\left(\frac{x}{(ex^2+d)^{\frac{3}{2}}e} - \frac{x}{\sqrt{ex^2+d}de}\right) + b\int\frac{x^2\log\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{(ex^2+d)^{\frac{5}{2}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")`

[Out] `-1/3*a*(x/((e*x^2 + d)^(3/2)*e) - x/(sqrt(e*x^2 + d)*d*e)) + b*integrate(x^2*log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e*x^2 + d)^(5/2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 \operatorname{arsech}(cx) + ax^2)\sqrt{ex^2 + d}}{e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")`

[Out] `integral((b*x^2*arcsech(c*x) + a*x^2)*sqrt(e*x^2 + d)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(a+b*asech(c*x))/(e*x**2+d)**(5/2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(b \operatorname{arsech}(cx) + a)x^2}{(ex^2 + d)^{\frac{5}{2}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")`

[Out] `integrate((b*arcsech(c*x) + a)*x^2/(e*x^2 + d)^(5/2), x)`

$$3.176 \quad \int \frac{a+b\operatorname{sech}^{-1}(cx)}{(d+ex^2)^{5/2}} dx$$

Optimal. Leaf size=266

$$\frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{ex^2}{d}+1}\operatorname{EllipticF}\left(\sin^{-1}(cx),-\frac{e}{c^2d}\right)}{3cd^2\sqrt{d+ex^2}} + \frac{2x(a+b\operatorname{sech}^{-1}(cx))}{3d^2\sqrt{d+ex^2}} + \frac{x(a+b\operatorname{sech}^{-1}(cx))}{3d(d+ex^2)^{3/2}} + \frac{bex\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}}{3d^2(c^2d+e)}$$

```
[Out] (b*e*x*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(3*d^2*(c^2*d
+ e)*Sqrt[d + e*x^2]) + (x*(a + b*ArcSech[c*x]))/(3*d*(d + e*x^2)^(3/2)) +
(2*x*(a + b*ArcSech[c*x]))/(3*d^2*Sqrt[d + e*x^2]) + (b*c*Sqrt[(1 + c*x)^(-
1)]*Sqrt[1 + c*x]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(3*
d^2*(c^2*d + e)*Sqrt[1 + (e*x^2)/d]) + (2*b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c
*x]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(3*c*d^2*Sqrt
[d + e*x^2])
```

Rubi [A] time = 0.20608, antiderivative size = 266, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {192, 191, 6291, 12, 527, 524, 426, 424, 421, 419}

$$\frac{2x(a+b\operatorname{sech}^{-1}(cx))}{3d^2\sqrt{d+ex^2}} + \frac{x(a+b\operatorname{sech}^{-1}(cx))}{3d(d+ex^2)^{3/2}} + \frac{bex\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{1-c^2x^2}}{3d^2(c^2d+e)\sqrt{d+ex^2}} + \frac{2b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}\sqrt{\frac{ex^2}{d}+1}F(\sin^{-1}(cx))}{3cd^2\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcSech[c*x])/(d + e*x^2)^(5/2), x]
```

```
[Out] (b*e*x*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(3*d^2*(c^2*d
+ e)*Sqrt[d + e*x^2]) + (x*(a + b*ArcSech[c*x]))/(3*d*(d + e*x^2)^(3/2)) +
(2*x*(a + b*ArcSech[c*x]))/(3*d^2*Sqrt[d + e*x^2]) + (b*c*Sqrt[(1 + c*x)^(-
1)]*Sqrt[1 + c*x]*Sqrt[d + e*x^2]*EllipticE[ArcSin[c*x], -(e/(c^2*d))])/(3*
d^2*(c^2*d + e)*Sqrt[1 + (e*x^2)/d]) + (2*b*Sqrt[(1 + c*x)^(-1)]*Sqrt[1 + c
*x]*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[c*x], -(e/(c^2*d))])/(3*c*d^2*Sqrt
[d + e*x^2])
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1
))/ (a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(
p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
&& NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^(p + 1
))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 6291

```
Int[((a_.) + ArcSech[(c_.)*(x_)])*(b_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Sym
bol] := With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcSech[c*x], u, x
] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[SimplifyIntegrand[u/(x*Sqrt
[1 - c*x]*Sqrt[1 + c*x]), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ
```

[p, 0] || ILtQ[p + 1/2, 0])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 527

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]

Rule 524

Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && (!GtQ[c, 0] || SimplerSqrtQ[-(b/a), -(d/c)]))))))

Rule 426

Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Dist[Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0]

Rule 424

Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Simp[(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]

Rule 421

Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Dist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]

Rule 419

Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Simp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])

Rubi steps

$$\begin{aligned}
\int \frac{a + b \operatorname{sech}^{-1}(cx)}{(d + ex^2)^{5/2}} dx &= \frac{x(a + b \operatorname{sech}^{-1}(cx))}{3d(d + ex^2)^{3/2}} + \frac{2x(a + b \operatorname{sech}^{-1}(cx))}{3d^2 \sqrt{d + ex^2}} + \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{3d + 2ex^2}{3d^2 \sqrt{1 - c^2 x^2} (d + ex^2)} dx \\
&= \frac{x(a + b \operatorname{sech}^{-1}(cx))}{3d(d + ex^2)^{3/2}} + \frac{2x(a + b \operatorname{sech}^{-1}(cx))}{3d^2 \sqrt{d + ex^2}} + \frac{\left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{3d + 2ex^2}{\sqrt{1 - c^2 x^2} (d + ex^2)^{3/2}} dx}{3d^2} \\
&= \frac{bex \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1 - c^2 x^2}}{3d^2 (c^2 d + e) \sqrt{d + ex^2}} + \frac{x(a + b \operatorname{sech}^{-1}(cx))}{3d(d + ex^2)^{3/2}} + \frac{2x(a + b \operatorname{sech}^{-1}(cx))}{3d^2 \sqrt{d + ex^2}} - \frac{\left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{3d + 2ex^2}{\sqrt{1 - c^2 x^2} (d + ex^2)^{3/2}} dx}{3d^2} \\
&= \frac{bex \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1 - c^2 x^2}}{3d^2 (c^2 d + e) \sqrt{d + ex^2}} + \frac{x(a + b \operatorname{sech}^{-1}(cx))}{3d(d + ex^2)^{3/2}} + \frac{2x(a + b \operatorname{sech}^{-1}(cx))}{3d^2 \sqrt{d + ex^2}} + \frac{\left(2b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{3d + 2ex^2}{\sqrt{1 - c^2 x^2} (d + ex^2)^{3/2}} dx}{3d^2} \\
&= \frac{bex \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1 - c^2 x^2}}{3d^2 (c^2 d + e) \sqrt{d + ex^2}} + \frac{x(a + b \operatorname{sech}^{-1}(cx))}{3d(d + ex^2)^{3/2}} + \frac{2x(a + b \operatorname{sech}^{-1}(cx))}{3d^2 \sqrt{d + ex^2}} + \frac{\left(bc^2 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \int \frac{3d + 2ex^2}{\sqrt{1 - c^2 x^2} (d + ex^2)^{3/2}} dx}{3d^2} \\
&= \frac{bex \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1 - c^2 x^2}}{3d^2 (c^2 d + e) \sqrt{d + ex^2}} + \frac{x(a + b \operatorname{sech}^{-1}(cx))}{3d(d + ex^2)^{3/2}} + \frac{2x(a + b \operatorname{sech}^{-1}(cx))}{3d^2 \sqrt{d + ex^2}} + \frac{bc \sqrt{\frac{1}{1+cx}} \sqrt{1+cx}}{3d^2}
\end{aligned}$$

Mathematica [C] time = 5.08504, size = 517, normalized size = 1.94

$$\frac{ib \sqrt{\frac{1-cx}{cx+1}} (cx+1)(d+ex^2) \sqrt{\frac{c(\sqrt{d-i\sqrt{e}})}{(cx+1)(c\sqrt{d-i\sqrt{e}})}} \sqrt{\frac{c(\sqrt{d+i\sqrt{e}})}{(cx+1)(c\sqrt{d+i\sqrt{e}})}} \left(c\sqrt{d-i\sqrt{e}} E \left(i \sinh^{-1} \left(\frac{(dc^2+e)(1-cx)}{\sqrt{(\sqrt{d+i\sqrt{e}})^2 (cx+1)}} \right) \right) \frac{(\sqrt{d+i\sqrt{e}})^2}{(c\sqrt{d-i\sqrt{e}})^2} \right) - 2(3c\sqrt{d+2i\sqrt{e}}) \operatorname{EllipticF} \left(i \sinh^{-1} \left(\frac{(\sqrt{d+i\sqrt{e}})^2}{(c\sqrt{d-i\sqrt{e}})^2} \right) \right)}{c(c\sqrt{d+i\sqrt{e}}) \sqrt{\frac{(cx-1)(c\sqrt{d-i\sqrt{e}})}{(cx+1)(c\sqrt{d+i\sqrt{e}})}}}$$

$$3d^2 (d + ex^2)^{3/2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcSech[c*x])/(d + e*x^2)^(5/2), x]

[Out] ((b*Sqrt[(1 - c*x)/(1 + c*x)]*(-(c*d) + e*x)*(d + e*x^2))/(c^2*d + e) + a*x*(3*d + 2*e*x^2) + b*x*(3*d + 2*e*x^2)*ArcSech[c*x] - (I*b*Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[(c*(Sqrt[d] - I*Sqrt[e]*x))/((c*Sqrt[d] - I*Sqrt[e])*(1 + c*x))])*Sqrt[(c*(Sqrt[d] + I*Sqrt[e]*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x))]*(d + e*x^2)*((c*Sqrt[d] - I*Sqrt[e])*EllipticE[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2 - 2*(3*c*Sqrt[d] + (2*I)*Sqrt[e])*EllipticF[I*ArcSinh[Sqrt[((c^2*d + e)*(1 - c*x))/((c*Sqrt[d] + I*Sqrt[e])^2*(1 + c*x))]]], (c*Sqrt[d] + I*Sqrt[e])^2/(c*Sqrt[d] - I*Sqrt[e])^2)))/(c*(c*Sqrt[d] + I*Sqrt[e])*Sqrt[-(((c*Sqrt[d] - I*Sqrt[e])*(-1 + c*x))/((c*Sqrt[d] + I*Sqrt[e])*(1 + c*x)))])))/(3*d^2*(d + e*x^2)^(3/2))

Maple [F] time = 1.012, size = 0, normalized size = 0.

$$\int (a + b \operatorname{arcsech}(cx)) (ex^2 + d)^{-5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)`

[Out] `int((a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{3}a\left(\frac{2x}{\sqrt{ex^2+dd^2}} + \frac{x}{(ex^2+d)^{\frac{3}{2}}d}\right) + b\int \frac{\log\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1} + \frac{1}{cx}\right)}{(ex^2+d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")`

[Out] `1/3*a*(2*x/(sqrt(e*x^2 + d)*d^2) + x/((e*x^2 + d)^(3/2)*d)) + b*integrate(log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(e*x^2 + d)^(5/2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex^2+d}(b\text{arsech}(cx) + a)}{e^3x^6 + 3de^2x^4 + 3d^2ex^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")`

[Out] `integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*asech(c*x))/(e*x**2+d)**(5/2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{b\text{arsech}(cx) + a}{(ex^2+d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*arcsech(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")`

[Out] `integrate((b*arcsech(c*x) + a)/(e*x^2 + d)^(5/2), x)`

3.177 $\int (fx)^m (d + ex^2)^3 (a + b \operatorname{sech}^{-1}(cx)) dx$

Optimal. Leaf size=596

$$b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(fx)^{m+1}\left(\frac{e^{(m+1)}(3c^4d^2(m^4+22m^3+179m^2+638m+840)+3c^2de(m+3)^2(m^2+13m+42)+e^2(m^2+8m+15)^2)}{(m+3)(m+5)(m+7)} + \frac{c^6d^3(m+2)(m+4)(m+6)}{m+1}\right)$$

$$c^6f(m+1)(m+2)(m+4)(m+6)$$

[Out] $-\left((b^*e*(e^2*(15 + 8*m + m^2)^2 + 3*c^2*d*e*(3 + m)^2*(42 + 13*m + m^2) + 3*c^4*d^2*(840 + 638*m + 179*m^2 + 22*m^3 + m^4))*(f*x)^{(1 + m)*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[1 - c^2*x^2]} / (c^6*f*(2 + m)*(3 + m)*(4 + m)*(5 + m)*(6 + m)*(7 + m)) - (b^*e^2*(e*(5 + m)^2 + 3*c^2*d*(42 + 13*m + m^2))*(f*x)^{(3 + m)*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[1 - c^2*x^2]} / (c^4*f^3*(4 + m)*(5 + m)*(6 + m)*(7 + m)) - (b^*e^3*(f*x)^{(5 + m)*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[1 - c^2*x^2]} / (c^2*f^5*(6 + m)*(7 + m)) + (d^3*(f*x)^{(1 + m)*(a + b*\operatorname{ArcSech}[c*x])}) / (f*(1 + m)) + (3*d^2*e*(f*x)^{(3 + m)*(a + b*\operatorname{ArcSech}[c*x])}) / (f^3*(3 + m)) + (3*d*e^2*(f*x)^{(5 + m)*(a + b*\operatorname{ArcSech}[c*x])}) / (f^5*(5 + m)) + (e^3*(f*x)^{(7 + m)*(a + b*\operatorname{ArcSech}[c*x])}) / (f^7*(7 + m)) + (b*((c^6*d^3*(2 + m)*(4 + m)*(6 + m)) / (1 + m) + (e*(1 + m)*(e^2*(15 + 8*m + m^2)^2 + 3*c^2*d*e*(3 + m)^2*(42 + 13*m + m^2) + 3*c^4*d^2*(840 + 638*m + 179*m^2 + 22*m^3 + m^4))) / ((3 + m)*(5 + m)*(7 + m)))*(f*x)^{(1 + m)*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{Hypergeometric2F1}[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2]} / (c^6*f*(1 + m)*(2 + m)*(4 + m)*(6 + m))\right)$

Rubi [A] time = 2.54533, antiderivative size = 576, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {270, 6301, 1809, 1267, 459, 364}

$$\frac{3d^2e(fx)^{m+3}(a + b \operatorname{sech}^{-1}(cx))}{f^3(m+3)} + \frac{d^3(fx)^{m+1}(a + b \operatorname{sech}^{-1}(cx))}{f(m+1)} + \frac{3de^2(fx)^{m+5}(a + b \operatorname{sech}^{-1}(cx))}{f^5(m+5)} + \frac{e^3(fx)^{m+7}(a + b \operatorname{sech}^{-1}(cx))}{f^7(m+7)}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(f*x)^m*(d + e*x^2)^3*(a + b*\operatorname{ArcSech}[c*x]),x]$

[Out] $-\left((b^*e*(e^2*(15 + 8*m + m^2)^2 + 3*c^2*d*e*(3 + m)^2*(42 + 13*m + m^2) + 3*c^4*d^2*(840 + 638*m + 179*m^2 + 22*m^3 + m^4))*(f*x)^{(1 + m)*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[1 - c^2*x^2]} / (c^6*f*(2 + m)*(3 + m)*(4 + m)*(5 + m)*(6 + m)*(7 + m)) - (b^*e^2*(e*(5 + m)^2 + 3*c^2*d*(42 + 13*m + m^2))*(f*x)^{(3 + m)*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[1 - c^2*x^2]} / (c^4*f^3*(4 + m)*(5 + m)*(6 + m)*(7 + m)) - (b^*e^3*(f*x)^{(5 + m)*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{Sqrt}[1 - c^2*x^2]} / (c^2*f^5*(6 + m)*(7 + m)) + (d^3*(f*x)^{(1 + m)*(a + b*\operatorname{ArcSech}[c*x])}) / (f*(1 + m)) + (3*d^2*e*(f*x)^{(3 + m)*(a + b*\operatorname{ArcSech}[c*x])}) / (f^3*(3 + m)) + (3*d*e^2*(f*x)^{(5 + m)*(a + b*\operatorname{ArcSech}[c*x])}) / (f^5*(5 + m)) + (e^3*(f*x)^{(7 + m)*(a + b*\operatorname{ArcSech}[c*x])}) / (f^7*(7 + m)) + (b*(d^3/(1 + m)^2 + (e*(e^2*(15 + 8*m + m^2)^2 + 3*c^2*d*e*(3 + m)^2*(42 + 13*m + m^2) + 3*c^4*d^2*(840 + 638*m + 179*m^2 + 22*m^3 + m^4))) / (c^6*(2 + m)*(3 + m)*(4 + m)*(5 + m)*(6 + m)*(7 + m)))*(f*x)^{(1 + m)*\operatorname{Sqrt}[(1 + c*x)^{-1}]*\operatorname{Sqrt}[1 + c*x]*\operatorname{Hypergeometric2F1}[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2]} / f\right)$

Rule 270

$\operatorname{Int}[(c_*)*(x_*)^{(m_*)}*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(c*x)^m*(a + b*x^n)^p, x], x] /; \operatorname{FreeQ}\{a, b, c, m, n\}, x] \&\&$

IGtQ[p, 0]

Rule 6301

Int[((a_.) + ArcSech[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))

Rule 1809

Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q - 1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m + q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && (!IGtQ[m, 0] || IGtQ[p + 1/2, -1])

Rule 1267

Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]

Rule 459

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]

Rule 364

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b*x^n)/a])]/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rubi steps

$$\begin{aligned}
\int (fx)^m (d + ex^2)^3 (a + b \operatorname{sech}^{-1}(cx)) dx &= \frac{d^3 (fx)^{1+m} (a + b \operatorname{sech}^{-1}(cx))}{f(1+m)} + \frac{3d^2 e (fx)^{3+m} (a + b \operatorname{sech}^{-1}(cx))}{f^3(3+m)} + \frac{3de^2 (fx)^{5+m} (a + b \operatorname{sech}^{-1}(cx))}{f^5(5+m)} \\
&= -\frac{be^3 (fx)^{5+m} \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{c^2 f^5 (6+m)(7+m)} + \frac{d^3 (fx)^{1+m} (a + b \operatorname{sech}^{-1}(cx))}{f(1+m)} + \frac{3de^2 (fx)^{5+m} (a + b \operatorname{sech}^{-1}(cx))}{f^5(5+m)} \\
&= -\frac{be^2 (e(5+m)^2 + 3c^2 d (42 + 13m + m^2)) (fx)^{3+m} \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{c^4 f^3 (4+m)(5+m)(6+m)(7+m)} \\
&= -\frac{be (e^2 (15 + 8m + m^2)^2 + 3c^2 de(3+m)^2 (42 + 13m + m^2) + 3c^4 d^2 (840 + 140m + 7m^2)) (fx)^{3+m} \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{c^6 f (2+m)(3+m)(4+m)(5+m)} \\
&= -\frac{be (e^2 (15 + 8m + m^2)^2 + 3c^2 de(3+m)^2 (42 + 13m + m^2) + 3c^4 d^2 (840 + 140m + 7m^2)) (fx)^{3+m} \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{c^6 f (2+m)(3+m)(4+m)(5+m)}
\end{aligned}$$

Mathematica [F] time = 0.241235, size = 0, normalized size = 0.

$$\int (fx)^m (d + ex^2)^3 (a + b \operatorname{sech}^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Integrate[(f*x)^m*(d + e*x^2)^3*(a + b*ArcSech[c*x]), x]

[Out] Integrate[(f*x)^m*(d + e*x^2)^3*(a + b*ArcSech[c*x]), x]

Maple [F] time = 2.759, size = 0, normalized size = 0.

$$\int (fx)^m (ex^2 + d)^3 (a + b \operatorname{arcsech}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x)^m*(e*x^2+d)^3*(a+b*arcsech(c*x)), x)

[Out] int((f*x)^m*(e*x^2+d)^3*(a+b*arcsech(c*x)), x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(e*x^2+d)^3*(a+b*arcsech(c*x)), x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

integral $\left(\left(ae^3x^6 + 3ade^2x^4 + 3ad^2ex^2 + ad^3 + \left(be^3x^6 + 3bde^2x^4 + 3bd^2ex^2 + bd^3\right)\operatorname{arsech}(cx)\right)(fx)^m, x\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(e*x^2+d)^3*(a+b*arcsech(c*x)),x, algorithm="fricas")

[Out] integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 + 3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arcsech(c*x))*(f*x)^m, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)**m*(e*x**2+d)**3*(a+b*asech(c*x)),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + d)^3 (b \operatorname{arsech}(cx) + a) (fx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(e*x^2+d)^3*(a+b*arcsech(c*x)),x, algorithm="giac")

[Out] integrate((e*x^2 + d)^3*(b*arcsech(c*x) + a)*(f*x)^m, x)

3.178 $\int (fx)^m (d + ex^2)^2 (a + b \operatorname{sech}^{-1}(cx)) dx$

Optimal. Leaf size=372

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(fx)^{m+1}\left(c^4d^2(m+2)(m+3)(m+4)(m+5) + e(m+1)^2\left(2c^2d(m^2+9m+20) + e(m+3)^2\right)\right)\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, c^2x^2\right]}{c^4f(m+1)^2(m+2)(m+3)(m+4)(m+5)}$$

```
[Out] -((b*e*(e*(3 + m)^2 + 2*c^2*d*(20 + 9*m + m^2))*(f*x)^(1 + m)*Sqrt[(1 + c*x)^(1 + m)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(c^4*f*(2 + m)*(3 + m)*(4 + m)*(5 + m))) - (b*e^2*(f*x)^(3 + m)*Sqrt[(1 + c*x)^(1 + m)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(c^2*f^3*(4 + m)*(5 + m)) + (d^2*(f*x)^(1 + m)*(a + b*ArcSech[c*x]))/(f*(1 + m)) + (2*d*e*(f*x)^(3 + m)*(a + b*ArcSech[c*x]))/(f^3*(3 + m)) + (e^2*(f*x)^(5 + m)*(a + b*ArcSech[c*x]))/(f^5*(5 + m)) + (b*(c^4*d^2*(2 + m)*(3 + m)*(4 + m)*(5 + m) + e*(1 + m)^2*(e*(3 + m)^2 + 2*c^2*d*(20 + 9*m + m^2)))*(f*x)^(1 + m)*Sqrt[(1 + c*x)^(1 + m)]*Sqrt[1 + c*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(c^4*f*(1 + m)^2*(2 + m)*(3 + m)*(4 + m)*(5 + m))
```

Rubi [A] time = 0.431709, antiderivative size = 352, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {270, 6301, 12, 1267, 459, 364}

$$\frac{d^2(fx)^{m+1}(a + b \operatorname{sech}^{-1}(cx))}{f(m+1)} + \frac{2de(fx)^{m+3}(a + b \operatorname{sech}^{-1}(cx))}{f^3(m+3)} + \frac{e^2(fx)^{m+5}(a + b \operatorname{sech}^{-1}(cx))}{f^5(m+5)} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(fx)^{m+1}\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, c^2x^2\right]}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)^2*(a + b*ArcSech[c*x]),x]
```

```
[Out] -((b*e*(e*(3 + m)^2 + 2*c^2*d*(20 + 9*m + m^2))*(f*x)^(1 + m)*Sqrt[(1 + c*x)^(1 + m)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(c^4*f*(2 + m)*(3 + m)*(4 + m)*(5 + m))) - (b*e^2*(f*x)^(3 + m)*Sqrt[(1 + c*x)^(1 + m)]*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(c^2*f^3*(4 + m)*(5 + m)) + (d^2*(f*x)^(1 + m)*(a + b*ArcSech[c*x]))/(f*(1 + m)) + (2*d*e*(f*x)^(3 + m)*(a + b*ArcSech[c*x]))/(f^3*(3 + m)) + (e^2*(f*x)^(5 + m)*(a + b*ArcSech[c*x]))/(f^5*(5 + m)) + (b*(d^2/(1 + m)^2 + (e*(e*(3 + m)^2 + 2*c^2*d*(20 + 9*m + m^2)))/(c^4*(2 + m)*(3 + m)*(4 + m)*(5 + m)))*(f*x)^(1 + m)*Sqrt[(1 + c*x)^(1 + m)]*Sqrt[1 + c*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/f
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]
```

Rule 6301

```
Int[((a_.) + ArcSech[(c_.)*(x_)])*(b_.)*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(p_.), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1 + c*x]*Sqrt[1/(1 + c*x)], Int[SimplifyIntegrand[u/(x*Sqrt[1 - c*x]*Sqrt[1 + c*x]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0])) || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 1267

```
Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (
c_)*(x_)^4)^(p_), x_Symbol] := Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^(
q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
+ 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x]
/; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0]
&& !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 459

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n
_)), x_Symbol] := Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
+ 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
+ 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 364

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b*x^n)/a
])/ (c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int (fx)^m (d + ex^2)^2 (a + b \operatorname{sech}^{-1}(cx)) dx &= \frac{d^2 (fx)^{1+m} (a + b \operatorname{sech}^{-1}(cx))}{f(1+m)} + \frac{2de (fx)^{3+m} (a + b \operatorname{sech}^{-1}(cx))}{f^3(3+m)} + \frac{e^2 (fx)^{5+m}}{f^5} \\ &= \frac{d^2 (fx)^{1+m} (a + b \operatorname{sech}^{-1}(cx))}{f(1+m)} + \frac{2de (fx)^{3+m} (a + b \operatorname{sech}^{-1}(cx))}{f^3(3+m)} + \frac{e^2 (fx)^{5+m}}{f^5} \\ &= -\frac{be^2 (fx)^{3+m} \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{c^2 f^3 (4+m)(5+m)} + \frac{d^2 (fx)^{1+m} (a + b \operatorname{sech}^{-1}(cx))}{f(1+m)} + \frac{2de}{f^5} \\ &= -\frac{be (e(3+m)^2 + 2c^2 d (20 + 9m + m^2)) (fx)^{1+m} \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{c^4 f(2+m)(4+m)(15 + 8m + m^2)} - \frac{2de}{f^5} \\ &= -\frac{be (e(3+m)^2 + 2c^2 d (20 + 9m + m^2)) (fx)^{1+m} \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{c^4 f(2+m)(4+m)(15 + 8m + m^2)} - \frac{2de}{f^5} \end{aligned}$$

Mathematica [F] time = 0.153183, size = 0, normalized size = 0.

$$\int (fx)^m (d + ex^2)^2 (a + b \operatorname{sech}^{-1}(cx)) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m*(d + e*x^2)^2*(a + b*ArcSech[c*x]), x]
```

[Out] Integrate[(f*x)^m*(d + e*x^2)^2*(a + b*ArcSech[c*x]), x]

Maple [F] time = 2.111, size = 0, normalized size = 0.

$$\int (fx)^m (ex^2 + d)^2 (a + b \operatorname{arcsech}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x)^m*(e*x^2+d)^2*(a+b*arcsech(c*x)), x)

[Out] int((f*x)^m*(e*x^2+d)^2*(a+b*arcsech(c*x)), x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*arcsech(c*x)), x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(ae^2x^4 + 2adex^2 + ad^2 + (be^2x^4 + 2bdex^2 + bd^2) \operatorname{arsech}(cx)\right) (fx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*arcsech(c*x)), x, algorithm="fricas")

[Out] integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^2)*arcsech(c*x))*(f*x)^m, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)**m*(e*x**2+d)**2*(a+b*asech(c*x)), x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + d)^2 (b \operatorname{arsech}(cx) + a) (fx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*arcsech(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arcsech(c*x) + a)*(f*x)^m, x)
```

3.179 $\int (fx)^m (d + ex^2) (a + b \operatorname{sech}^{-1}(cx)) dx$

Optimal. Leaf size=206

$$\frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(fx)^{m+1}(c^2d(m+2)(m+3)+e(m+1)^2)\operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{m+1}{2}, \frac{m+3}{2}, c^2x^2\right)}{c^2f(m+1)^2(m+2)(m+3)} + \frac{d(fx)^{m+1}(a + b \operatorname{sech}^{-1}(cx))}{f(m+1)}$$

```
[Out] -((b*e*(f*x)^(1+m)*Sqrt[(1+c*x)^(-1)]*Sqrt[1+c*x]*Sqrt[1-c^2*x^2])/(c^2*f*(2+m)*(3+m))) + (d*(f*x)^(1+m)*(a+b*ArcSech[c*x]))/(f*(1+m)) + (e*(f*x)^(3+m)*(a+b*ArcSech[c*x]))/(f^3*(3+m)) + (b*(e*(1+m)^2 + c^2*d*(2+m)*(3+m))*(f*x)^(1+m)*Sqrt[(1+c*x)^(-1)]*Sqrt[1+c*x]*Hypergeometric2F1[1/2, (1+m)/2, (3+m)/2, c^2*x^2])/(c^2*f*(1+m)^2*(2+m)*(3+m))
```

Rubi [A] time = 0.179143, antiderivative size = 192, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {14, 6301, 12, 459, 364}

$$\frac{d(fx)^{m+1}(a + b \operatorname{sech}^{-1}(cx))}{f(m+1)} + \frac{e(fx)^{m+3}(a + b \operatorname{sech}^{-1}(cx))}{f^3(m+3)} + \frac{b\sqrt{\frac{1}{cx+1}}\sqrt{cx+1}(fx)^{m+1}\left(\frac{e}{c^2(m+2)(m+3)} + \frac{d}{(m+1)^2}\right) {}_2F_1\left(\frac{1}{2}, \frac{m+1}{2}, \frac{m+3}{2}, c^2x^2\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*(a + b*ArcSech[c*x]),x]
```

```
[Out] -((b*e*(f*x)^(1+m)*Sqrt[(1+c*x)^(-1)]*Sqrt[1+c*x]*Sqrt[1-c^2*x^2])/(c^2*f*(2+m)*(3+m))) + (d*(f*x)^(1+m)*(a+b*ArcSech[c*x]))/(f*(1+m)) + (e*(f*x)^(3+m)*(a+b*ArcSech[c*x]))/(f^3*(3+m)) + (b*(d/(1+m)^2 + e/(c^2*(2+m)*(3+m)))*(f*x)^(1+m)*Sqrt[(1+c*x)^(-1)]*Sqrt[1+c*x]*Hypergeometric2F1[1/2, (1+m)/2, (3+m)/2, c^2*x^2])/f
```

Rule 14

```
Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 6301

```
Int[((a_) + ArcSech[(c_)*(x_)])*(b_)*((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(p_), x_Symbol] := With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist[a + b*ArcSech[c*x], u, x] + Dist[b*Sqrt[1+c*x]*Sqrt[1/(1+c*x)], Int[SimplifyIntegrand[u/(x*Sqrt[1-c*x]*Sqrt[1+c*x]), x], x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m-1)/2, 0] && GtQ[m+2*p+3, 0])) || (IGtQ[(m+1)/2, 0] && !(ILtQ[p, 0] && GtQ[m+2*p+3, 0])) || (ILtQ[(m+2*p+1)/2, 0] && !ILtQ[(m-1)/2, 0]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 459

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Simp[(d*(e*x)^(m+1)*(a + b*x^n)^(p+1))/(b*e*(m+n*(p
```

+ 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]

Rule 364

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -(b*x^n)/a])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rubi steps

$$\begin{aligned} \int (fx)^m (d + ex^2) (a + b \operatorname{sech}^{-1}(cx)) dx &= \frac{d(fx)^{1+m} (a + b \operatorname{sech}^{-1}(cx))}{f(1+m)} + \frac{e(fx)^{3+m} (a + b \operatorname{sech}^{-1}(cx))}{f^3(3+m)} + \left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right) \\ &= \frac{d(fx)^{1+m} (a + b \operatorname{sech}^{-1}(cx))}{f(1+m)} + \frac{e(fx)^{3+m} (a + b \operatorname{sech}^{-1}(cx))}{f^3(3+m)} + \frac{\left(b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right)}{f^3(3+m)} \\ &= -\frac{be(fx)^{1+m} \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{c^2 f(2+m)(3+m)} + \frac{d(fx)^{1+m} (a + b \operatorname{sech}^{-1}(cx))}{f(1+m)} + \frac{e(fx)^{3+m} (a + b \operatorname{sech}^{-1}(cx))}{f^3(3+m)} \\ &= -\frac{be(fx)^{1+m} \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{c^2 f(2+m)(3+m)} + \frac{d(fx)^{1+m} (a + b \operatorname{sech}^{-1}(cx))}{f(1+m)} + \frac{e(fx)^{3+m} (a + b \operatorname{sech}^{-1}(cx))}{f^3(3+m)} \end{aligned}$$

Mathematica [F] time = 0.116859, size = 0, normalized size = 0.

$$\int (fx)^m (d + ex^2) (a + b \operatorname{sech}^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Integrate[(f*x)^m*(d + e*x^2)*(a + b*ArcSech[c*x]), x]

[Out] Integrate[(f*x)^m*(d + e*x^2)*(a + b*ArcSech[c*x]), x]

Maple [F] time = 1.873, size = 0, normalized size = 0.

$$\int (fx)^m (ex^2 + d) (a + b \operatorname{arcsech}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x)^m*(e*x^2+d)*(a+b*arcsech(c*x)), x)

[Out] int((f*x)^m*(e*x^2+d)*(a+b*arcsech(c*x)), x)

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(aex^2 + ad + (bex^2 + bd)\text{arsech}(cx)\right)(fx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="fricas")

[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsech(c*x))*(f*x)^m, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (fx)^m (a + b\text{asech}(cx))(d + ex^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)**m*(e*x**2+d)*(a+b*asech(c*x)),x)

[Out] Integral((f*x)**m*(a + b*asech(c*x))*(d + e*x**2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + d)(b\text{arsech}(cx) + a)(fx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(e*x^2+d)*(a+b*arcsech(c*x)),x, algorithm="giac")

[Out] integrate((e*x^2 + d)*(b*arcsech(c*x) + a)*(f*x)^m, x)

$$3.180 \quad \int \frac{(fx)^m (a + b \operatorname{sech}^{-1}(cx))}{d + ex^2} dx$$

Optimal. Leaf size=25

$$\text{Unintegrable} \left(\frac{(fx)^m (a + b \operatorname{sech}^{-1}(cx))}{d + ex^2}, x \right)$$

[Out] Unintegrable[((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2), x]

Rubi [A] time = 0.0755577, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(fx)^m (a + b \operatorname{sech}^{-1}(cx))}{d + ex^2} dx$$

Verification is Not applicable to the result.

[In] Int[((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2), x]

[Out] Defer[Int][((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2), x]

Rubi steps

$$\int \frac{(fx)^m (a + b \operatorname{sech}^{-1}(cx))}{d + ex^2} dx = \int \frac{(fx)^m (a + b \operatorname{sech}^{-1}(cx))}{d + ex^2} dx$$

Mathematica [A] time = 2.35738, size = 0, normalized size = 0.

$$\int \frac{(fx)^m (a + b \operatorname{sech}^{-1}(cx))}{d + ex^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2), x]

[Out] Integrate[((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2), x]

Maple [A] time = 1.319, size = 0, normalized size = 0.

$$\int \frac{(fx)^m (a + b \operatorname{arcsech}(cx))}{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d), x)

[Out] int((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d), x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{ar} \operatorname{sech}(cx) + a)(fx)^m}{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="maxima")

[Out] integrate((b*arcsech(c*x) + a)*(f*x)^m/(e*x^2 + d), x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(b \operatorname{ar} \operatorname{sech}(cx) + a)(fx)^m}{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="fricas")

[Out] integral((b*arcsech(c*x) + a)*(f*x)^m/(e*x^2 + d), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx)^m (a + b \operatorname{asech}(cx))}{d + ex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)**m*(a+b*asech(c*x))/(e*x**2+d),x)

[Out] Integral((f*x)**m*(a + b*asech(c*x))/(d + e*x**2), x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{ar} \operatorname{sech}(cx) + a)(fx)^m}{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d),x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)*(f*x)^m/(e*x^2 + d), x)

$$3.181 \quad \int \frac{(fx)^m (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^2} dx$$

Optimal. Leaf size=25

$$\text{Unintegrable} \left(\frac{(fx)^m (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^2}, x \right)$$

[Out] Unintegrable[((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2)^2, x]

Rubi [A] time = 0.0749564, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(fx)^m (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^2} dx$$

Verification is Not applicable to the result.

[In] Int[((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]

[Out] Defer[Int][[(f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2)^2, x]

Rubi steps

$$\int \frac{(fx)^m (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^2} dx = \int \frac{(fx)^m (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^2} dx$$

Mathematica [A] time = 2.63164, size = 0, normalized size = 0.

$$\int \frac{(fx)^m (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2)^2,x]

[Out] Integrate[((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2)^2, x]

Maple [A] time = 1.441, size = 0, normalized size = 0.

$$\int \frac{(fx)^m (a + b \operatorname{arcsech}(cx))}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^2,x)`

[Out] `int((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^2,x)`

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{ar} \operatorname{sech}(cx) + a)(fx)^m}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="maxima")`

[Out] `integrate((b*arcsech(c*x) + a)*(f*x)^m/(e*x^2 + d)^2, x)`

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(b \operatorname{ar} \operatorname{sech}(cx) + a)(fx)^m}{e^2x^4 + 2dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="fricas")`

[Out] `integral((b*arcsech(c*x) + a)*(f*x)^m/(e^2*x^4 + 2*d*e*x^2 + d^2), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x)**m*(a+b*asech(c*x))/(e*x**2+d)**2,x)`

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{ar} \operatorname{sech}(cx) + a)(fx)^m}{(ex^2 + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^2,x, algorithm="giac")`

[Out] `integrate((b*arcsech(c*x) + a)*(f*x)^m/(e*x^2 + d)^2, x)`

$$3.182 \quad \int (fx)^m (d + ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx)) dx$$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\left(d + ex^2\right)^{3/2} (fx)^m (a + b \operatorname{sech}^{-1}(cx)), x\right)$$

[Out] Unintegrable[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]), x]

Rubi [A] time = 0.109657, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (fx)^m (d + ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Int[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]), x]

[Out] Defer[Int] [(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]), x]

Rubi steps

$$\int (fx)^m (d + ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx)) dx = \int (fx)^m (d + ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx)) dx$$

Mathematica [A] time = 1.05053, size = 0, normalized size = 0.

$$\int (fx)^m (d + ex^2)^{3/2} (a + b \operatorname{sech}^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Integrate[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]), x]

[Out] Integrate[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcSech[c*x]), x]

Maple [A] time = 0.862, size = 0, normalized size = 0.

$$\int (fx)^m (ex^2 + d)^{\frac{3}{2}} (a + b \operatorname{arcsech}(cx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)), x)

[Out] int((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)), x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + d)^{\frac{3}{2}} (b \operatorname{arsech}(cx) + a) (fx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="maxima")

[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)*(f*x)^m, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(aex^2 + ad + (bex^2 + bd) \operatorname{arsech}(cx)\right) \sqrt{ex^2 + d} (fx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="fricas")

[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arcsech(c*x))*sqrt(e*x^2 + d)*(f*x)^m, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)**m*(e*x**2+d)**(3/2)*(a+b*asech(c*x)),x)

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (ex^2 + d)^{\frac{3}{2}} (b \operatorname{arsech}(cx) + a) (fx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arcsech(c*x)),x, algorithm="giac")

[Out] integrate((e*x^2 + d)^(3/2)*(b*arcsech(c*x) + a)*(f*x)^m, x)

$$3.183 \quad \int (fx)^m \sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx)) dx$$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\sqrt{d + ex^2}(fx)^m (a + b \operatorname{sech}^{-1}(cx)), x\right)$$

[Out] Unintegrable[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]), x]

Rubi [A] time = 0.0986537, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int (fx)^m \sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Int[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]), x]

[Out] Defer[Int] [(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]), x]

Rubi steps

$$\int (fx)^m \sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx)) dx = \int (fx)^m \sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx)) dx$$

Mathematica [A] time = 0.117779, size = 0, normalized size = 0.

$$\int (fx)^m \sqrt{d + ex^2} (a + b \operatorname{sech}^{-1}(cx)) dx$$

Verification is Not applicable to the result.

[In] Integrate[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]), x]

[Out] Integrate[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcSech[c*x]), x]

Maple [A] time = 1.079, size = 0, normalized size = 0.

$$\int (fx)^m (a + b \operatorname{arcsech}(cx)) \sqrt{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x)^m*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2), x)

[Out] int((f*x)^m*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2), x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex^2 + d}(b \operatorname{ar} \operatorname{sech}(cx) + a)(fx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)*(f*x)^m, x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\sqrt{ex^2 + d}(b \operatorname{ar} \operatorname{sech}(cx) + a)(fx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)*(f*x)^m, x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int (fx)^m (a + b \operatorname{asech}(cx)) \sqrt{d + ex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)**m*(a+b*asech(c*x))*(e*x**2+d)**(1/2),x)

[Out] Integral((f*x)**m*(a + b*asech(c*x))*sqrt(d + e*x**2), x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{ex^2 + d}(b \operatorname{ar} \operatorname{sech}(cx) + a)(fx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(a+b*arcsech(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)*(f*x)^m, x)

$$3.184 \quad \int \frac{(fx)^m (a + b \operatorname{sech}^{-1}(cx))}{\sqrt{d+ex^2}} dx$$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{(fx)^m (a + b \operatorname{sech}^{-1}(cx))}{\sqrt{d+ex^2}}, x\right)$$

[Out] Unintegrable[((f*x)^m*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2], x]

Rubi [A] time = 0.0999564, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(fx)^m (a + b \operatorname{sech}^{-1}(cx))}{\sqrt{d+ex^2}} dx$$

Verification is Not applicable to the result.

[In] Int[((f*x)^m*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2], x]

[Out] Defer[Int](((f*x)^m*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2], x]

Rubi steps

$$\int \frac{(fx)^m (a + b \operatorname{sech}^{-1}(cx))}{\sqrt{d+ex^2}} dx = \int \frac{(fx)^m (a + b \operatorname{sech}^{-1}(cx))}{\sqrt{d+ex^2}} dx$$

Mathematica [A] time = 1.34371, size = 0, normalized size = 0.

$$\int \frac{(fx)^m (a + b \operatorname{sech}^{-1}(cx))}{\sqrt{d+ex^2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[((f*x)^m*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2], x]

[Out] Integrate[((f*x)^m*(a + b*ArcSech[c*x]))/Sqrt[d + e*x^2], x]

Maple [A] time = 1.279, size = 0, normalized size = 0.

$$\int (fx)^m (a + b \operatorname{arcsech}(cx)) \frac{1}{\sqrt{ex^2+d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2), x)

[Out] `int((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x)`

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{ar} \operatorname{sech}(cx) + a)(fx)^m}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")`

[Out] `integrate((b*arcsech(c*x) + a)*(f*x)^m/sqrt(e*x^2 + d), x)`

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(b \operatorname{ar} \operatorname{sech}(cx) + a)(fx)^m}{\sqrt{ex^2 + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")`

[Out] `integral((b*arcsech(c*x) + a)*(f*x)^m/sqrt(e*x^2 + d), x)`

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx)^m (a + b \operatorname{asech}(cx))}{\sqrt{d + ex^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x)**m*(a+b*asech(c*x))/(e*x**2+d)**(1/2),x)`

[Out] `Integral((f*x)**m*(a + b*asech(c*x))/sqrt(d + e*x**2), x)`

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{ar} \operatorname{sech}(cx) + a)(fx)^m}{\sqrt{ex^2 + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")`

[Out] `integrate((b*arcsech(c*x) + a)*(f*x)^m/sqrt(e*x^2 + d), x)`

$$3.185 \quad \int \frac{(fx)^m (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

Optimal. Leaf size=27

$$\text{Unintegrable} \left(\frac{(fx)^m (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{3/2}}, x \right)$$

[Out] Unintegrable[((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2), x]

Rubi [A] time = 0.110424, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{(fx)^m (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Int[((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2), x]

[Out] Defer[Int][[((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2), x]

Rubi steps

$$\int \frac{(fx)^m (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{3/2}} dx = \int \frac{(fx)^m (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

Mathematica [A] time = 1.7518, size = 0, normalized size = 0.

$$\int \frac{(fx)^m (a + b \operatorname{sech}^{-1}(cx))}{(d + ex^2)^{3/2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2), x]

[Out] Integrate[((f*x)^m*(a + b*ArcSech[c*x]))/(d + e*x^2)^(3/2), x]

Maple [A] time = 1.191, size = 0, normalized size = 0.

$$\int (fx)^m (a + b \operatorname{arcsech}(cx)) (ex^2 + d)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2), x)

[Out] `int((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x)`

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{ar} \operatorname{sech}(cx) + a)(fx)^m}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")`

[Out] `integrate((b*arcsech(c*x) + a)*(f*x)^m/(e*x^2 + d)^(3/2), x)`

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{ex^2 + d}(b \operatorname{ar} \operatorname{sech}(cx) + a)(fx)^m}{e^2x^4 + 2dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")`

[Out] `integral(sqrt(e*x^2 + d)*(b*arcsech(c*x) + a)*(f*x)^m/(e^2*x^4 + 2*d*e*x^2 + d^2), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x)**m*(a+b*asech(c*x))/(e*x**2+d)**(3/2),x)`

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{ar} \operatorname{sech}(cx) + a)(fx)^m}{(ex^2 + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x)^m*(a+b*arcsech(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")`

[Out] `integrate((b*arcsech(c*x) + a)*(f*x)^m/(e*x^2 + d)^(3/2), x)`

$$3.186 \quad \int \frac{x^{11} (a + b \operatorname{sech}^{-1}(cx))}{\sqrt{1 - c^4 x^4}} dx$$

Optimal. Leaf size=473

$$-\frac{(1 - c^4 x^4)^{5/2} (a + b \operatorname{sech}^{-1}(cx))}{10c^{12}} + \frac{(1 - c^4 x^4)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{3c^{12}} - \frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{sech}^{-1}(cx))}{2c^{12}} - \frac{b\sqrt{1 - c^2 x^2} (c^2 x^2 + 1)}{90c^{13} x \sqrt{\frac{1}{cx} - 1} \sqrt{\frac{1}{cx}}}$$

```
[Out] (-4*b*Sqrt[1 - c^2*x^2]*Sqrt[1 + c^2*x^2])/(15*c^13*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x) + (7*b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(3/2))/(90*c^13*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x) - (13*b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(5/2))/(150*c^13*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x) + (3*b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(7/2))/(70*c^13*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x) - (b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(9/2))/(90*c^13*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x) - (Sqrt[1 - c^4*x^4]*(a + b*ArcSech[c*x]))/(2*c^12) + ((1 - c^4*x^4)^(3/2)*(a + b*ArcSech[c*x]))/(3*c^12) - ((1 - c^4*x^4)^(5/2)*(a + b*ArcSech[c*x]))/(10*c^12) + (4*b*Sqrt[1 - c^2*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]])/(15*c^13*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x)
```

Rubi [A] time = 1.57949, antiderivative size = 473, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {266, 43, 6309, 12, 6742, 848, 50, 63, 208, 783}

$$-\frac{(1 - c^4 x^4)^{5/2} (a + b \operatorname{sech}^{-1}(cx))}{10c^{12}} + \frac{(1 - c^4 x^4)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{3c^{12}} - \frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{sech}^{-1}(cx))}{2c^{12}} - \frac{b\sqrt{1 - c^2 x^2} (c^2 x^2 + 1)}{90c^{13} x \sqrt{\frac{1}{cx} - 1} \sqrt{\frac{1}{cx}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(a + b*ArcSech[c*x]))/Sqrt[1 - c^4*x^4], x]
```

```
[Out] (-4*b*Sqrt[1 - c^2*x^2]*Sqrt[1 + c^2*x^2])/(15*c^13*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x) + (7*b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(3/2))/(90*c^13*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x) - (13*b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(5/2))/(150*c^13*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x) + (3*b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(7/2))/(70*c^13*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x) - (b*Sqrt[1 - c^2*x^2]*(1 + c^2*x^2)^(9/2))/(90*c^13*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x) - (Sqrt[1 - c^4*x^4]*(a + b*ArcSech[c*x]))/(2*c^12) + ((1 - c^4*x^4)^(3/2)*(a + b*ArcSech[c*x]))/(3*c^12) - ((1 - c^4*x^4)^(5/2)*(a + b*ArcSech[c*x]))/(10*c^12) + (4*b*Sqrt[1 - c^2*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]])/(15*c^13*Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]*x)
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 6309

Int[((a_.) + ArcSech[(c_.)*(x_)])*(b_.)*(u_), x_Symbol] := With[{v = IntHid
e[u, x]}, Dist[a + b*ArcSech[c*x], v, x] + Dist[(b*Sqrt[1 - c^2*x^2])/(c*x*
Sqrt[-1 + 1/(c*x)]*Sqrt[1 + 1/(c*x)]), Int[SimplifyIntegrand[v/(x*Sqrt[1 -
c^2*x^2]), x], x], x] /; InverseFunctionFreeQ[v, x] /; FreeQ[{a, b, c}, x]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]

Rule 848

Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_), x_Symbol] := Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 783

Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
.), x_Symbol] := Int[(d + e*x)^(m + p)*(f + g*x)*(a/d + (c*x)/e)^p, x] /; F
reeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))

Rubi steps

$$\begin{aligned}
\int \frac{x^{11} (a + b \operatorname{sech}^{-1}(cx))}{\sqrt{1 - c^4 x^4}} dx &= -\frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{sech}^{-1}(cx))}{2c^{12}} + \frac{(1 - c^4 x^4)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{3c^{12}} - \frac{(1 - c^4 x^4)^{5/2} (a + b \operatorname{sech}^{-1}(cx))}{10c^{12}} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{sech}^{-1}(cx))}{2c^{12}} + \frac{(1 - c^4 x^4)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{3c^{12}} - \frac{(1 - c^4 x^4)^{5/2} (a + b \operatorname{sech}^{-1}(cx))}{10c^{12}} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{sech}^{-1}(cx))}{2c^{12}} + \frac{(1 - c^4 x^4)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{3c^{12}} - \frac{(1 - c^4 x^4)^{5/2} (a + b \operatorname{sech}^{-1}(cx))}{10c^{12}} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{sech}^{-1}(cx))}{2c^{12}} + \frac{(1 - c^4 x^4)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{3c^{12}} - \frac{(1 - c^4 x^4)^{5/2} (a + b \operatorname{sech}^{-1}(cx))}{10c^{12}} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{sech}^{-1}(cx))}{2c^{12}} + \frac{(1 - c^4 x^4)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{3c^{12}} - \frac{(1 - c^4 x^4)^{5/2} (a + b \operatorname{sech}^{-1}(cx))}{10c^{12}} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{sech}^{-1}(cx))}{2c^{12}} + \frac{(1 - c^4 x^4)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{3c^{12}} - \frac{(1 - c^4 x^4)^{5/2} (a + b \operatorname{sech}^{-1}(cx))}{10c^{12}} \\
&= -\frac{4b\sqrt{1 - c^2 x^2} \sqrt{1 + c^2 x^2}}{15c^{13} \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}} x} - \frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{sech}^{-1}(cx))}{2c^{12}} + \frac{(1 - c^4 x^4)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{3c^{12}} \\
&= -\frac{4b\sqrt{1 - c^2 x^2} \sqrt{1 + c^2 x^2}}{15c^{13} \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}} x} + \frac{7b\sqrt{1 - c^2 x^2} (1 + c^2 x^2)^{3/2}}{90c^{13} \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}} x} - \frac{13b\sqrt{1 - c^2 x^2} (1 + c^2 x^2)^{5/2}}{150c^{13} \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}} x} + \frac{3b\sqrt{1 - c^2 x^2} (1 + c^2 x^2)^{7/2}}{315c^{13} \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}} x} \\
&= -\frac{4b\sqrt{1 - c^2 x^2} \sqrt{1 + c^2 x^2}}{15c^{13} \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}} x} + \frac{7b\sqrt{1 - c^2 x^2} (1 + c^2 x^2)^{3/2}}{90c^{13} \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}} x} - \frac{13b\sqrt{1 - c^2 x^2} (1 + c^2 x^2)^{5/2}}{150c^{13} \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}} x} + \frac{3b\sqrt{1 - c^2 x^2} (1 + c^2 x^2)^{7/2}}{315c^{13} \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}} x}
\end{aligned}$$

Mathematica [A] time = 0.418814, size = 213, normalized size = 0.45

$$\frac{-105a\sqrt{1 - c^4 x^4} (3c^8 x^8 + 4c^4 x^4 + 8) + \frac{b\sqrt{\frac{1-cx}{cx+1}} \sqrt{1 - c^4 x^4} (35c^8 x^8 + 5c^6 x^6 + 78c^4 x^4 + 36c^2 x^2 + 768)}{cx-1} - 840b \log\left(-\sqrt{\frac{1-cx}{cx+1}} \sqrt{1 - c^4 x^4} - cx + 1\right)}{3150c^{12}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^11*(a + b*ArcSech[c*x]))/Sqrt[1 - c^4*x^4], x]

[Out] (-105*a*Sqrt[1 - c^4*x^4]*(8 + 4*c^4*x^4 + 3*c^8*x^8) + (b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^4*x^4]*(768 + 36*c^2*x^2 + 78*c^4*x^4 + 5*c^6*x^6 + 35*c^8*x^8))/(-1 + c*x) - 105*b*Sqrt[1 - c^4*x^4]*(8 + 4*c^4*x^4 + 3*c^8*x^8)*ArcSech[c*x] + 840*b*Log[x*(1 - c*x)] - 840*b*Log[1 - c*x - Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^4*x^4]])/(3150*c^12)

Maple [F] time = 1.938, size = 0, normalized size = 0.

$$\int x^{11} (a + b \operatorname{arcsech}(cx)) \frac{1}{\sqrt{-c^4 x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^11*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x)`

[Out] `int(x^11*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{30} a \left(\frac{3(-c^4 x^4 + 1)^{\frac{5}{2}}}{c^{12}} - \frac{10(-c^4 x^4 + 1)^{\frac{3}{2}}}{c^{12}} + \frac{15 \sqrt{-c^4 x^4 + 1}}{c^{12}} \right) + \frac{1}{30} b \left(\frac{(3c^{12} x^{12} + c^8 x^8 + 4c^4 x^4 - 8) \log(\sqrt{cx+1} \sqrt{-cx-1})}{\sqrt{c^2 x^2 + 1} \sqrt{cx+1} \sqrt{-cx-1} c^{12}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^11*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")`

[Out] `-1/30*a*(3*(-c^4*x^4 + 1)^(5/2)/c^12 - 10*(-c^4*x^4 + 1)^(3/2)/c^12 + 15*sqrt(-c^4*x^4 + 1)/c^12) + 1/30*b*((3*c^12*x^12 + c^8*x^8 + 4*c^4*x^4 - 8)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)/(sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*c^12) - 30*integrate(1/30*(30*c^10*x^21*log(c) + 60*c^10*x^21*log(sqrt(x)) + (60*c^10*x^21*log(sqrt(x)) + (3*c^10*x^10*(10*log(c) + 1) + 3*c^8*x^8 + 4*c^6*x^6 + 4*c^4*x^4 + 8*c^2*x^2 + 8)*x^11)*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1)))/((c^10*x^10*e^(log(c*x + 1) + log(-c*x + 1)) + c^10*x^10*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1)))*sqrt(c^2*x^2 + 1)), x))`

Fricas [A] time = 2.17649, size = 852, normalized size = 1.8

$$105 (3bc^{10}x^{10} - 3bc^8x^8 + 4bc^6x^6 - 4bc^4x^4 + 8bc^2x^2 - 8b) \sqrt{-c^4x^4 + 1} \log \left(\frac{cx \sqrt{\frac{-c^2x^2 - 1}{c^2x^2 + 1}}}{cx} \right) - (35bc^9x^9 + 5bc^7x^7 + 78bc^5x^5 + 36bc^3x^3 + 768bcx) \sqrt{-c^4x^4 + 1} \sqrt{-(c^2x^2 - 1)/(c^2x^2 + 1)} + 420(b c^2 x^2 - b) \log((c^2 x^2 + \sqrt{-c^4 x^4 + 1}) c x \sqrt{-(c^2 x^2 - 1)/(c^2 x^2 + 1)}) - 420(b c^2 x^2 - b) \log(-(c^2 x^2 - \sqrt{-c^4 x^4 + 1}) c x \sqrt{-(c^2 x^2 - 1)/(c^2 x^2 + 1)}) + 105(3a c^{10} x^{10} - 3a c^8 x^8 + 4a c^6 x^6 - 4a c^4 x^4 + 8a c^2 x^2 - 8a) \sqrt{-c^4 x^4 + 1} / (c^{14} x^2 - c^{12})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^11*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")`

[Out] `-1/3150*(105*(3*b*c^10*x^10 - 3*b*c^8*x^8 + 4*b*c^6*x^6 - 4*b*c^4*x^4 + 8*b*c^2*x^2 - 8*b)*sqrt(-c^4*x^4 + 1)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2 + 1))/(c*x)) - (35*b*c^9*x^9 + 5*b*c^7*x^7 + 78*b*c^5*x^5 + 36*b*c^3*x^3 + 768*b*c*x)*sqrt(-c^4*x^4 + 1)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2 + 1)) + 420*(b*c^2*x^2 - b)*log((c^2*x^2 + sqrt(-c^4*x^4 + 1)*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2 + 1)) - 1)/(c^2*x^2 - 1)) - 420*(b*c^2*x^2 - b)*log(-(c^2*x^2 - sqrt(-c^4*x^4 + 1)*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2 + 1)) - 1)/(c^2*x^2 - 1)) + 105*(3*a*c^10*x^10 - 3*a*c^8*x^8 + 4*a*c^6*x^6 - 4*a*c^4*x^4 + 8*a*c^2*x^2 - 8*a)*sqrt(-c^4*x^4 + 1))/(c^14*x^2 - c^12)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**11*(a+b*asech(c*x))/(-c**4*x**4+1)**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{ar} \operatorname{sech}(cx) + a)x^{11}}{\sqrt{-c^4x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^11*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)*x^11/sqrt(-c^4*x^4 + 1), x)

$$3.187 \quad \int \frac{x^7 (a + b \operatorname{sech}^{-1}(cx))}{\sqrt{1 - c^4 x^4}} dx$$

Optimal. Leaf size=316

$$\frac{(1 - c^4 x^4)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{6c^8} - \frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{sech}^{-1}(cx))}{2c^8} - \frac{b\sqrt{1 - c^2 x^2} (c^2 x^2 + 1)^{5/2}}{30c^9 x \sqrt{\frac{1}{cx} - 1} \sqrt{\frac{1}{cx} + 1}} + \frac{b\sqrt{1 - c^2 x^2} (c^2 x^2 + 1)^{3/2}}{18c^9 x \sqrt{\frac{1}{cx} - 1} \sqrt{\frac{1}{cx} + 1}}$$

[Out] $-(b \operatorname{Sqrt}[1 - c^2 x^2] \operatorname{Sqrt}[1 + c^2 x^2]) / (3 c^9 \operatorname{Sqrt}[-1 + 1/(c x)] \operatorname{Sqrt}[1 + 1/(c x)] x) + (b \operatorname{Sqrt}[1 - c^2 x^2] (1 + c^2 x^2)^{(3/2)}) / (18 c^9 \operatorname{Sqrt}[-1 + 1/(c x)] \operatorname{Sqrt}[1 + 1/(c x)] x) - (b \operatorname{Sqrt}[1 - c^2 x^2] (1 + c^2 x^2)^{(5/2)}) / (30 c^9 \operatorname{Sqrt}[-1 + 1/(c x)] \operatorname{Sqrt}[1 + 1/(c x)] x) - (\operatorname{Sqrt}[1 - c^4 x^4] (a + b \operatorname{ArcSech}[c x])) / (2 c^8) + ((1 - c^4 x^4)^{(3/2)} (a + b \operatorname{ArcSech}[c x])) / (6 c^8) + (b \operatorname{Sqrt}[1 - c^2 x^2] \operatorname{ArcTanh}[\operatorname{Sqrt}[1 + c^2 x^2]]) / (3 c^9 \operatorname{Sqrt}[-1 + 1/(c x)] \operatorname{Sqrt}[1 + 1/(c x)] x)$

Rubi [A] time = 1.38294, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {266, 43, 6309, 12, 6742, 848, 50, 63, 208, 783}

$$\frac{(1 - c^4 x^4)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{6c^8} - \frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{sech}^{-1}(cx))}{2c^8} - \frac{b\sqrt{1 - c^2 x^2} (c^2 x^2 + 1)^{5/2}}{30c^9 x \sqrt{\frac{1}{cx} - 1} \sqrt{\frac{1}{cx} + 1}} + \frac{b\sqrt{1 - c^2 x^2} (c^2 x^2 + 1)^{3/2}}{18c^9 x \sqrt{\frac{1}{cx} - 1} \sqrt{\frac{1}{cx} + 1}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(x^7 (a + b \operatorname{ArcSech}[c x])) / \operatorname{Sqrt}[1 - c^4 x^4], x]$

[Out] $-(b \operatorname{Sqrt}[1 - c^2 x^2] \operatorname{Sqrt}[1 + c^2 x^2]) / (3 c^9 \operatorname{Sqrt}[-1 + 1/(c x)] \operatorname{Sqrt}[1 + 1/(c x)] x) + (b \operatorname{Sqrt}[1 - c^2 x^2] (1 + c^2 x^2)^{(3/2)}) / (18 c^9 \operatorname{Sqrt}[-1 + 1/(c x)] \operatorname{Sqrt}[1 + 1/(c x)] x) - (b \operatorname{Sqrt}[1 - c^2 x^2] (1 + c^2 x^2)^{(5/2)}) / (30 c^9 \operatorname{Sqrt}[-1 + 1/(c x)] \operatorname{Sqrt}[1 + 1/(c x)] x) - (\operatorname{Sqrt}[1 - c^4 x^4] (a + b \operatorname{ArcSech}[c x])) / (2 c^8) + ((1 - c^4 x^4)^{(3/2)} (a + b \operatorname{ArcSech}[c x])) / (6 c^8) + (b \operatorname{Sqrt}[1 - c^2 x^2] \operatorname{ArcTanh}[\operatorname{Sqrt}[1 + c^2 x^2]]) / (3 c^9 \operatorname{Sqrt}[-1 + 1/(c x)] \operatorname{Sqrt}[1 + 1/(c x)] x)$

Rule 266

$\operatorname{Int}[(x_)^{(m_.)} ((a_) + (b_.) (x_)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \operatorname{Dist}[1/n, \operatorname{Subst}[\operatorname{Int}[x^{(\operatorname{Simplify}[(m + 1)/n] - 1) (a + b x)^p}, x, x^n], x] /;$ $\operatorname{FreeQ}[\{a, b, m, n, p\}, x] \ \&\& \ \operatorname{IntegerQ}[\operatorname{Simplify}[(m + 1)/n]]$

Rule 43

$\operatorname{Int}[(a_. + (b_.) (x_.))^{(m_.)} ((c_.) + (d_.) (x_.))^{(n_.)}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(a + b x)^m (c + d x)^n, x], x] /;$ $\operatorname{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \operatorname{NeQ}[b c - a d, 0] \ \&\& \ \operatorname{IGtQ}[m, 0] \ \&\& \ (\! \operatorname{IntegerQ}[n] \ || \ (\operatorname{EqQ}[c, 0] \ \&\& \ \operatorname{LeQ}[7 m + 4 n + 4, 0]) \ || \ \operatorname{LtQ}[9 m + 5 (n + 1), 0] \ || \ \operatorname{GtQ}[m + n + 2, 0])$

Rule 6309

$\operatorname{Int}[(a_. + \operatorname{ArcSech}[(c_.) (x_.)]) (b_.) (u_.), x_Symbol] \rightarrow \operatorname{With}[\{v = \operatorname{IntHid}e[u, x]\}, \operatorname{Dist}[a + b \operatorname{ArcSech}[c x], v, x] + \operatorname{Dist}[(b \operatorname{Sqrt}[1 - c^2 x^2]) / (c x \operatorname{Sqrt}[-1 + 1/(c x)] \operatorname{Sqrt}[1 + 1/(c x)]), \operatorname{Int}[\operatorname{SimplifyIntegrand}[v / (x \operatorname{Sqrt}[1 -$

c^2x^2), x], x], x] /; InverseFunctionFreeQ[v, x]] /; FreeQ[{a, b, c}, x]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rule 848

Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))^(n_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))

Rule 50

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*(c + d*x)^n/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 783

Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[(d + e*x)^(m + p)*(f + g*x)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))

Rubi steps

$$\begin{aligned}
\int \frac{x^7 (a + b \operatorname{sech}^{-1}(cx))}{\sqrt{1 - c^4 x^4}} dx &= -\frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{sech}^{-1}(cx))}{2c^8} + \frac{(1 - c^4 x^4)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{6c^8} + \frac{(b\sqrt{1 - c^2 x^2}) \int \frac{(-2 - \frac{1}{cx})}{6c^9 \sqrt{-1 + \frac{1}{cx}}} dx}{c\sqrt{-1 + \frac{1}{cx}}} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{sech}^{-1}(cx))}{2c^8} + \frac{(1 - c^4 x^4)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{6c^8} + \frac{(b\sqrt{1 - c^2 x^2}) \int \frac{(-2 - \frac{1}{cx})}{6c^9 \sqrt{-1 + \frac{1}{cx}}} dx}{6c^9 \sqrt{-1 + \frac{1}{cx}}} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{sech}^{-1}(cx))}{2c^8} + \frac{(1 - c^4 x^4)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{6c^8} - \frac{(b\sqrt{1 - c^2 x^2}) \operatorname{Subst}(\int \frac{(-2 - \frac{1}{cx})}{6c^9 \sqrt{-1 + \frac{1}{cx}}} dx)}{12c^9 \sqrt{-1 + \frac{1}{cx}}} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{sech}^{-1}(cx))}{2c^8} + \frac{(1 - c^4 x^4)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{6c^8} - \frac{(b\sqrt{1 - c^2 x^2}) \operatorname{Subst}(\int \frac{(-2 - \frac{1}{cx})}{6c^9 \sqrt{-1 + \frac{1}{cx}}} dx)}{12c^9 \sqrt{-1 + \frac{1}{cx}}} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{sech}^{-1}(cx))}{2c^8} + \frac{(1 - c^4 x^4)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{6c^8} - \frac{(b\sqrt{1 - c^2 x^2}) \operatorname{Subst}(\int \frac{(-2 - \frac{1}{cx})}{6c^9 \sqrt{-1 + \frac{1}{cx}}} dx)}{6c^9 \sqrt{-1 + \frac{1}{cx}}} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{sech}^{-1}(cx))}{2c^8} + \frac{(1 - c^4 x^4)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{6c^8} - \frac{(b\sqrt{1 - c^2 x^2}) \operatorname{Subst}(\int \frac{(-2 - \frac{1}{cx})}{6c^9 \sqrt{-1 + \frac{1}{cx}}} dx)}{6c^9 \sqrt{-1 + \frac{1}{cx}}} \\
&= -\frac{b\sqrt{1 - c^2 x^2} \sqrt{1 + c^2 x^2}}{3c^9 \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}} - \frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{sech}^{-1}(cx))}{2c^8} + \frac{(1 - c^4 x^4)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{6c^8} \\
&= -\frac{b\sqrt{1 - c^2 x^2} \sqrt{1 + c^2 x^2}}{3c^9 \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}} + \frac{b\sqrt{1 - c^2 x^2} (1 + c^2 x^2)^{3/2}}{18c^9 \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}} - \frac{b\sqrt{1 - c^2 x^2} (1 + c^2 x^2)^{5/2}}{30c^9 \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}} - \frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{sech}^{-1}(cx))}{2c^8} + \frac{(1 - c^4 x^4)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{6c^8} \\
&= -\frac{b\sqrt{1 - c^2 x^2} \sqrt{1 + c^2 x^2}}{3c^9 \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}} + \frac{b\sqrt{1 - c^2 x^2} (1 + c^2 x^2)^{3/2}}{18c^9 \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}} - \frac{b\sqrt{1 - c^2 x^2} (1 + c^2 x^2)^{5/2}}{30c^9 \sqrt{-1 + \frac{1}{cx}} \sqrt{1 + \frac{1}{cx}}} - \frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{sech}^{-1}(cx))}{2c^8} + \frac{(1 - c^4 x^4)^{3/2} (a + b \operatorname{sech}^{-1}(cx))}{6c^8}
\end{aligned}$$

Mathematica [A] time = 0.359832, size = 178, normalized size = 0.56

$$\frac{-15a\sqrt{1 - c^4 x^4} (c^4 x^4 + 2) + \frac{b\sqrt{\frac{1-cx}{cx+1}} \sqrt{1 - c^4 x^4} (3c^4 x^4 + c^2 x^2 + 28)}{cx - 1} - 30b \log\left(-\sqrt{\frac{1-cx}{cx+1}} \sqrt{1 - c^4 x^4} - cx + 1\right) - 15b\sqrt{1 - c^4 x^4} (c^4 x^4 + 2)}{90c^8}$$

Antiderivative was successfully verified.

[In] Integrate[(x^7*(a + b*ArcSech[c*x]))/Sqrt[1 - c^4*x^4], x]

[Out] (-15*a*Sqrt[1 - c^4*x^4]*(2 + c^4*x^4) + (b*Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^4*x^4]*(28 + c^2*x^2 + 3*c^4*x^4))/(-1 + c*x) - 15*b*Sqrt[1 - c^4*x^4]*(2 + c^4*x^4)*ArcSech[c*x] + 30*b*Log[x*(1 - c*x)] - 30*b*Log[1 - c*x - Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^4*x^4]])/(90*c^8)

Maple [F] time = 2.082, size = 0, normalized size = 0.

$$\int x^7 (a + b \operatorname{arcsech}(cx)) \frac{1}{\sqrt{-c^4 x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^7*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x)`

[Out] `int(x^7*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{6} a \left(\frac{(-c^4 x^4 + 1)^{\frac{3}{2}}}{c^8} - \frac{3 \sqrt{-c^4 x^4 + 1}}{c^8} \right) + \frac{1}{6} b \left(\frac{(c^8 x^8 + c^4 x^4 - 2) \log(\sqrt{cx + 1} \sqrt{-cx + 1} + 1)}{\sqrt{c^2 x^2 + 1} \sqrt{cx + 1} \sqrt{-cx + 1} c^8} - 6 \int \frac{6 c^6 x^{13} \log(c) + 12 c^6 x^{13}}{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")`

[Out] `1/6*a*((-c^4*x^4 + 1)^(3/2)/c^8 - 3*sqrt(-c^4*x^4 + 1)/c^8) + 1/6*b*((c^8*x^8 + c^4*x^4 - 2)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)/(sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*c^8) - 6*integrate(1/6*(6*c^6*x^13*log(c) + 12*c^6*x^13*log(sqrt(x)) + (12*c^6*x^13*log(sqrt(x)) + (c^6*x^6*(6*log(c) + 1) + c^4*x^4 + 2*c^2*x^2 + 2)*x^7)*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1)))/((c^6*x^6*e^(log(c*x + 1) + log(-c*x + 1)) + c^6*x^6*e^(1/2*log(c*x + 1) + 1/2*log(-c*x + 1)))*sqrt(c^2*x^2 + 1)), x))`

Fricas [A] time = 1.9024, size = 705, normalized size = 2.23

$$15 (bc^6 x^6 - bc^4 x^4 + 2bc^2 x^2 - 2b) \sqrt{-c^4 x^4 + 1} \log \left(\frac{cx \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2} + 1}}{cx} \right) - (3bc^5 x^5 + bc^3 x^3 + 28bcx) \sqrt{-c^4 x^4 + 1} \sqrt{-\frac{c^2 x^2 - 1}{c^2 x^2}} + 15$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^7*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")`

[Out] `-1/90*(15*(b*c^6*x^6 - b*c^4*x^4 + 2*b*c^2*x^2 - 2*b)*sqrt(-c^4*x^4 + 1)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (3*b*c^5*x^5 + b*c^3*x^3 + 28*b*c*x)*sqrt(-c^4*x^4 + 1)*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 15*(b*c^2*x^2 - b)*log((c^2*x^2 + sqrt(-c^4*x^4 + 1)*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c^2*x^2 - 1)) - 15*(b*c^2*x^2 - b)*log(-(c^2*x^2 - sqrt(-c^4*x^4 + 1)*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c^2*x^2 - 1)) + 15*(a*c^6*x^6 - a*c^4*x^4 + 2*a*c^2*x^2 - 2*a)*sqrt(-c^4*x^4 + 1))/(c^10*x^2 - c^8)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**7*(a+b*asech(c*x))/(-c**4*x**4+1)**(1/2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{ar} \operatorname{sech}(cx) + a)x^7}{\sqrt{-c^4x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)*x^7/sqrt(-c^4*x^4 + 1), x)

$$3.188 \quad \int \frac{x^3 (a + b \operatorname{sech}^{-1}(cx))}{\sqrt{1 - c^4 x^4}} dx$$

Optimal. Leaf size=159

$$-\frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{sech}^{-1}(cx))}{2c^4} - \frac{b\sqrt{1 - c^2 x^2} \sqrt{c^2 x^2 + 1}}{2c^5 x \sqrt{\frac{1}{cx} - 1} \sqrt{\frac{1}{cx} + 1}} + \frac{b\sqrt{1 - c^2 x^2} \tanh^{-1}(\sqrt{c^2 x^2 + 1})}{2c^5 x \sqrt{\frac{1}{cx} - 1} \sqrt{\frac{1}{cx} + 1}}$$

[Out] $-(b \operatorname{Sqrt}[1 - c^2 x^2] \operatorname{Sqrt}[1 + c^2 x^2]) / (2 c^5 \operatorname{Sqrt}[-1 + 1/(c x)] \operatorname{Sqrt}[1 + 1/(c x)] x) - (\operatorname{Sqrt}[1 - c^4 x^4] (a + b \operatorname{ArcSech}[c x])) / (2 c^4) + (b \operatorname{Sqrt}[1 - c^2 x^2] \operatorname{ArcTanh}[\operatorname{Sqrt}[1 + c^2 x^2]]) / (2 c^5 \operatorname{Sqrt}[-1 + 1/(c x)] \operatorname{Sqrt}[1 + 1/(c x)] x)$

Rubi [A] time = 0.163726, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {261, 6309, 12, 1252, 848, 50, 63, 208}

$$-\frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{sech}^{-1}(cx))}{2c^4} - \frac{b\sqrt{1 - c^2 x^2} \sqrt{c^2 x^2 + 1}}{2c^5 x \sqrt{\frac{1}{cx} - 1} \sqrt{\frac{1}{cx} + 1}} + \frac{b\sqrt{1 - c^2 x^2} \tanh^{-1}(\sqrt{c^2 x^2 + 1})}{2c^5 x \sqrt{\frac{1}{cx} - 1} \sqrt{\frac{1}{cx} + 1}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(x^3 (a + b \operatorname{ArcSech}[c x])) / \operatorname{Sqrt}[1 - c^4 x^4], x]$

[Out] $-(b \operatorname{Sqrt}[1 - c^2 x^2] \operatorname{Sqrt}[1 + c^2 x^2]) / (2 c^5 \operatorname{Sqrt}[-1 + 1/(c x)] \operatorname{Sqrt}[1 + 1/(c x)] x) - (\operatorname{Sqrt}[1 - c^4 x^4] (a + b \operatorname{ArcSech}[c x])) / (2 c^4) + (b \operatorname{Sqrt}[1 - c^2 x^2] \operatorname{ArcTanh}[\operatorname{Sqrt}[1 + c^2 x^2]]) / (2 c^5 \operatorname{Sqrt}[-1 + 1/(c x)] \operatorname{Sqrt}[1 + 1/(c x)] x)$

Rule 261

$\operatorname{Int}[(x_)^{(m_)} ((a_) + (b_) (x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \operatorname{Simp}[(a + b x^n)^{(p+1)} / (b n (p+1)), x] /;$ FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rule 6309

$\operatorname{Int}[(a_) + \operatorname{ArcSech}[c_] (x_)] (b_) (u_), x_Symbol] \rightarrow \operatorname{With}[\{v = \operatorname{IntHide}[u, x]\}, \operatorname{Dist}[a + b \operatorname{ArcSech}[c x], v, x] + \operatorname{Dist}[(b \operatorname{Sqrt}[1 - c^2 x^2]) / (c x \operatorname{Sqrt}[-1 + 1/(c x)] \operatorname{Sqrt}[1 + 1/(c x)])], \operatorname{Int}[\operatorname{SimplifyIntegrand}[v / (x \operatorname{Sqrt}[1 - c^2 x^2])], x], x] /;$ InverseFunctionFreeQ[v, x] /; FreeQ[{a, b, c}, x]

Rule 12

$\operatorname{Int}[(a_) (u_), x_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /;$ FreeQ[a, x] && !MatchQ[u, (b_) (v_) /; FreeQ[b, x]]

Rule 1252

$\operatorname{Int}[(x_)^{(m_)} ((d_) + (e_) (x_)^2)^{(q_)} ((a_) + (c_) (x_)^4)^{(p_)}, x_Symbol] \rightarrow \operatorname{Dist}[1/2, \operatorname{Subst}[\operatorname{Int}[x^{(m-1)/2} (d + e x)^q (a + c x^2)^p, x], x, x^2], x] /;$ FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]

Rule 848

Int[((d_) + (e_)*(x_))^(m_)*((f_) + (g_)*(x_))^(n_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))

Rule 50

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
\int \frac{x^3 (a + b \operatorname{sech}^{-1}(cx))}{\sqrt{1 - c^4 x^4}} dx &= -\frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{sech}^{-1}(cx))}{2c^4} + \frac{(b\sqrt{1 - c^2 x^2}) \int -\frac{\sqrt{1 - c^4 x^4}}{2c^4 x \sqrt{1 - c^2 x^2}} dx}{c\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{sech}^{-1}(cx))}{2c^4} - \frac{(b\sqrt{1 - c^2 x^2}) \int \frac{\sqrt{1 - c^4 x^4}}{x\sqrt{1 - c^2 x^2}} dx}{2c^5\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{sech}^{-1}(cx))}{2c^4} - \frac{(b\sqrt{1 - c^2 x^2}) \operatorname{Subst}\left(\int \frac{\sqrt{1 - c^4 x^2}}{x\sqrt{1 - c^2 x}} dx, x, x^2\right)}{4c^5\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}} \\
&= -\frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{sech}^{-1}(cx))}{2c^4} - \frac{(b\sqrt{1 - c^2 x^2}) \operatorname{Subst}\left(\int \frac{\sqrt{1 + c^2 x}}{x} dx, x, x^2\right)}{4c^5\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}} \\
&= -\frac{b\sqrt{1 - c^2 x^2}\sqrt{1 + c^2 x^2}}{2c^5\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}} - \frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{sech}^{-1}(cx))}{2c^4} - \frac{(b\sqrt{1 - c^2 x^2}) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{1 + \frac{1}{cx}}} dx, x, x^2\right)}{4c^5\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}} \\
&= -\frac{b\sqrt{1 - c^2 x^2}\sqrt{1 + c^2 x^2}}{2c^5\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}} - \frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{sech}^{-1}(cx))}{2c^4} - \frac{(b\sqrt{1 - c^2 x^2}) \operatorname{Subst}\left(\int \frac{1}{-\frac{1}{c^2} + x} dx, x, x^2\right)}{2c^7\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}} \\
&= -\frac{b\sqrt{1 - c^2 x^2}\sqrt{1 + c^2 x^2}}{2c^5\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}} - \frac{\sqrt{1 - c^4 x^4} (a + b \operatorname{sech}^{-1}(cx))}{2c^4} + \frac{b\sqrt{1 - c^2 x^2} \tanh^{-1}\left(\sqrt{1 + c^2 x^2}\right)}{2c^5\sqrt{-1 + \frac{1}{cx}}\sqrt{1 + \frac{1}{cx}}}
\end{aligned}$$

Mathematica [A] time = 0.336357, size = 140, normalized size = 0.88

$$\frac{a\sqrt{1-c^4x^4} + \frac{b\sqrt{1-c^4x^4}}{\sqrt{\frac{1-cx}{cx+1}}(cx+1)} + b \log\left(-\sqrt{\frac{1-cx}{cx+1}}\sqrt{1-c^4x^4} - cx + 1\right) + b\sqrt{1-c^4x^4}\operatorname{sech}^{-1}(cx) - b \log(x(1-cx))}{2c^4}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3*(a + b*ArcSech[c*x]))/Sqrt[1 - c^4*x^4], x]

[Out] -(a*Sqrt[1 - c^4*x^4] + (b*Sqrt[1 - c^4*x^4])/(Sqrt[(1 - c*x)/(1 + c*x)]*(1 + c*x)) + b*Sqrt[1 - c^4*x^4]*ArcSech[c*x] - b*Log[x*(1 - c*x)] + b*Log[1 - c*x - Sqrt[(1 - c*x)/(1 + c*x)]*Sqrt[1 - c^4*x^4]])/(2*c^4)

Maple [F] time = 1.24, size = 0, normalized size = 0.

$$\int x^3 (a + b \operatorname{arcsech}(cx)) \frac{1}{\sqrt{-c^4x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2), x)

[Out] int(x^3*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{2} b \left(\frac{(c^4x^4 - 1) \log(\sqrt{cx+1}\sqrt{-cx+1} + 1)}{\sqrt{c^2x^2+1}\sqrt{cx+1}\sqrt{-cx+1}c^4} - 2 \int \frac{2c^2x^5 \log(c) + 4c^2x^5 \log(\sqrt{x}) + (4c^2x^5 \log(\sqrt{x}) + (c^2x^2(2 \log(c) + 1) * x^3) * e^{(1/2 \log(cx+1) + 1/2 \log(-cx+1))}) / ((c^2x^2 * e^{\log(cx+1) + \log(-cx+1)}) + c^2x^2 e^{(1/2 \log(cx+1) + 1/2 \log(-cx+1))})) * \sqrt{c^2x^2+1}}{2(c^2x^2 e^{(\log(cx+1)+\log(-cx+1))} + c^2x^2 e^{(\frac{1}{2} \log(cx+1) + \frac{1}{2} \log(-cx+1))})} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2), x, algorithm="maxima")

[Out] 1/2*b*((c^4*x^4 - 1)*log(sqrt(c*x + 1)*sqrt(-c*x + 1) + 1)/(sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*c^4) - 2*integrate(1/2*(2*c^2*x^5*log(c) + 4*c^2*x^5*log(sqrt(x)) + (4*c^2*x^5*log(sqrt(x)) + (c^2*x^2*(2*log(c) + 1) * x^3) * e^{(1/2*log(c*x + 1) + 1/2*log(-c*x + 1))}) / ((c^2*x^2 * e^{\log(c*x + 1) + \log(-c*x + 1)}) + c^2*x^2 * e^{(1/2*log(c*x + 1) + 1/2*log(-c*x + 1))})) * sqrt(c^2*x^2 + 1)), x) - 1/2*sqrt(-c^4*x^4 + 1)*a/c^4

Fricas [B] time = 2.11166, size = 575, normalized size = 3.62

$$\frac{2\sqrt{-c^4x^4+1}bcx\sqrt{-\frac{c^2x^2-1}{c^2x^2}} - 2\sqrt{-c^4x^4+1}(bc^2x^2-b)\log\left(\frac{cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}+1}{cx}\right) - (bc^2x^2-b)\log\left(\frac{c^2x^2+\sqrt{-c^4x^4+1}cx\sqrt{-\frac{c^2x^2-1}{c^2x^2}}-1}{c^2x^2-1}\right) + \dots}{4(c^6x^2-c^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")

[Out] 1/4*(2*sqrt(-c^4*x^4 + 1)*b*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 2*sqrt(-c^4*x^4 + 1)*(b*c^2*x^2 - b)*log((c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) + 1)/(c*x)) - (b*c^2*x^2 - b)*log((c^2*x^2 + sqrt(-c^4*x^4 + 1)*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c^2*x^2 - 1)) + (b*c^2*x^2 - b)*log(-(c^2*x^2 - sqrt(-c^4*x^4 + 1)*c*x*sqrt(-(c^2*x^2 - 1)/(c^2*x^2)) - 1)/(c^2*x^2 - 1)) - 2*sqrt(-c^4*x^4 + 1)*(a*c^2*x^2 - a))/(c^6*x^2 - c^4)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 (a + b \operatorname{asech}(cx))}{\sqrt{-(cx-1)(cx+1)(c^2x^2+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(a+b*asech(c*x))/(-c**4*x**4+1)**(1/2),x)

[Out] Integral(x**3*(a + b*asech(c*x))/sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b \operatorname{arsech}(cx) + a)x^3}{\sqrt{-c^4x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*arcsech(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)*x^3/sqrt(-c^4*x^4 + 1), x)

$$3.189 \quad \int \frac{a + b \operatorname{sech}^{-1}(cx)}{x \sqrt{1 - c^4 x^4}} dx$$

Optimal. Leaf size=28

$$\text{Unintegrable} \left(\frac{a + b \operatorname{sech}^{-1}(cx)}{x \sqrt{1 - c^4 x^4}}, x \right)$$

[Out] Unintegrable[(a + b*ArcSech[c*x])/(x*Sqrt[1 - c^4*x^4]), x]

Rubi [A] time = 0.0910904, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{a + b \operatorname{sech}^{-1}(cx)}{x \sqrt{1 - c^4 x^4}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*ArcSech[c*x])/(x*Sqrt[1 - c^4*x^4]), x]

[Out] Defer[Int] [(a + b*ArcSech[c*x])/(x*Sqrt[1 - c^4*x^4]), x]

Rubi steps

$$\int \frac{a + b \operatorname{sech}^{-1}(cx)}{x \sqrt{1 - c^4 x^4}} dx = \int \frac{a + b \operatorname{sech}^{-1}(cx)}{x \sqrt{1 - c^4 x^4}} dx$$

Mathematica [A] time = 0.391118, size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{sech}^{-1}(cx)}{x \sqrt{1 - c^4 x^4}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*ArcSech[c*x])/(x*Sqrt[1 - c^4*x^4]), x]

[Out] Integrate[(a + b*ArcSech[c*x])/(x*Sqrt[1 - c^4*x^4]), x]

Maple [A] time = 1.691, size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{arcsech}(cx)}{x} \frac{1}{\sqrt{-c^4 x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))/x/(-c^4*x^4+1)^(1/2), x)

[Out] int((a+b*arcsech(c*x))/x/(-c^4*x^4+1)^(1/2), x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{4}a\left(\log\left(\sqrt{-c^4x^4+1}+1\right)-\log\left(\sqrt{-c^4x^4+1}-1\right)\right)+b\int\frac{\log\left(\sqrt{\frac{1}{cx}+1}\sqrt{\frac{1}{cx}-1}+\frac{1}{cx}\right)}{\sqrt{-(c^2x^2+1)}(cx+1)(cx-1)x}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")

[Out] -1/4*a*(log(sqrt(-c^4*x^4 + 1) + 1) - log(sqrt(-c^4*x^4 + 1) - 1)) + b*integrate(log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(sqrt(-(c^2*x^2 + 1)*(c*x + 1)*(c*x - 1))*x), x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-c^4x^4+1}(b\text{arsech}(cx)+a)}{c^4x^5-x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(-c^4*x^4 + 1)*(b*arcsech(c*x) + a)/(c^4*x^5 - x), x)

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{a+b\text{asech}(cx)}{x\sqrt{-(cx-1)(cx+1)}(c^2x^2+1)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asech(c*x))/x/(-c**4*x**4+1)**(1/2),x)

[Out] Integral((a + b*asech(c*x))/(x*sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1))), x)

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int\frac{b\text{arsech}(cx)+a}{\sqrt{-c^4x^4+1}x}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)/(sqrt(-c^4*x^4 + 1)*x), x)

$$3.190 \quad \int \frac{a + b \operatorname{sech}^{-1}(cx)}{x^5 \sqrt{1 - c^4 x^4}} dx$$

Optimal. Leaf size=28

$$\text{Unintegrable} \left(\frac{a + b \operatorname{sech}^{-1}(cx)}{x^5 \sqrt{1 - c^4 x^4}}, x \right)$$

[Out] Unintegrable[(a + b*ArcSech[c*x])/(x^5*Sqrt[1 - c^4*x^4]), x]

Rubi [A] time = 0.101706, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$, Rules used = {}

$$\int \frac{a + b \operatorname{sech}^{-1}(cx)}{x^5 \sqrt{1 - c^4 x^4}} dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*ArcSech[c*x])/(x^5*Sqrt[1 - c^4*x^4]), x]

[Out] Defer[Int] [(a + b*ArcSech[c*x])/(x^5*Sqrt[1 - c^4*x^4]), x]

Rubi steps

$$\int \frac{a + b \operatorname{sech}^{-1}(cx)}{x^5 \sqrt{1 - c^4 x^4}} dx = \int \frac{a + b \operatorname{sech}^{-1}(cx)}{x^5 \sqrt{1 - c^4 x^4}} dx$$

Mathematica [A] time = 3.3729, size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{sech}^{-1}(cx)}{x^5 \sqrt{1 - c^4 x^4}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*ArcSech[c*x])/(x^5*Sqrt[1 - c^4*x^4]), x]

[Out] Integrate[(a + b*ArcSech[c*x])/(x^5*Sqrt[1 - c^4*x^4]), x]

Maple [A] time = 1.526, size = 0, normalized size = 0.

$$\int \frac{a + b \operatorname{arcsech}(cx)}{x^5} \frac{1}{\sqrt{-c^4 x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arcsech(c*x))/x^5/(-c^4*x^4+1)^(1/2), x)

[Out] int((a+b*arcsech(c*x))/x^5/(-c^4*x^4+1)^(1/2), x)

Maxima [A] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{8} \left(c^4 \log \left(\sqrt{-c^4 x^4 + 1} + 1 \right) - c^4 \log \left(\sqrt{-c^4 x^4 + 1} - 1 \right) + \frac{2 \sqrt{-c^4 x^4 + 1}}{x^4} \right) a + b \int \frac{\log \left(\sqrt{\frac{1}{cx} + 1} \sqrt{\frac{1}{cx} - 1} + \frac{1}{cx} \right)}{\sqrt{-(c^2 x^2 + 1)(cx + 1)(cx - 1)} x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")

[Out] -1/8*(c^4*log(sqrt(-c^4*x^4 + 1) + 1) - c^4*log(sqrt(-c^4*x^4 + 1) - 1) + 2*sqrt(-c^4*x^4 + 1)/x^4)*a + b*integrate(log(sqrt(1/(c*x) + 1)*sqrt(1/(c*x) - 1) + 1/(c*x))/(sqrt(-(c^2*x^2 + 1)*(c*x + 1)*(c*x - 1))*x^5), x)

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(-\frac{\sqrt{-c^4 x^4 + 1} (b \operatorname{ar} \operatorname{sech}(cx) + a)}{c^4 x^9 - x^5}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(-c^4*x^4 + 1)*(b*arcsech(c*x) + a)/(c^4*x^9 - x^5), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*asech(c*x))/x**5/(-c**4*x**4+1)**(1/2),x)

[Out] Timed out

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{b \operatorname{ar} \operatorname{sech}(cx) + a}{\sqrt{-c^4 x^4 + 1} x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arcsech(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="giac")

[Out] integrate((b*arcsech(c*x) + a)/(sqrt(-c^4*x^4 + 1)*x^5), x)

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
1 (* Original version thanks to Albert Rich emailed on 03/21/2017 *)
2 (* ::Package:: *)
3
4 (* ::Subsection:: *)
5 (*GradeAntiderivative[result,optimal]*)
6
7
8 (* ::Text:: *)
9 (*If result and optimal are mathematical expressions, *)
10 (*      GradeAntiderivative[result,optimal] returns*)
11 (* "F" if the result fails to integrate an expression that*)
12 (*   is integrable*)
13 (* "C" if result involves higher level functions than necessary*)
14 (* "B" if result is more than twice the size of the optimal*)
15 (*   antiderivative*)
16 (* "A" if result can be considered optimal*)
17
18
19 GradeAntiderivative[result_,optimal_] :=
20   If[ExpnType[result]<=ExpnType[optimal],
21     If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
22       If[LeafCount[result]<=2*LeafCount[optimal],
23         "A",
24         "B"],
25       "C"],
26     If[FreeQ[result,Integrate] && FreeQ[result,Int],
27       "C",
28       "F"]]
29
30
31 (* ::Text:: *)
32 (*The following summarizes the type number assigned an *)
33 (*expression based on the functions it involves*)
34 (*1 = rational function*)
35 (*2 = algebraic function*)
36 (*3 = elementary function*)
37 (*4 = special function*)
```



```

101 AppellFunctionQ[func_] :=
102   MemberQ[{AppellF1},func]

```

4.0.2 Maple grading function

```

1 # File: GradeAntiderivative.mpl
2 # Original version thanks to Albert Rich emailed on 03/21/2017
3
4 #Nasser 03/22/2017 Use Maple leaf count instead since buildin
5 #Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
6 #Nasser 03/24/2017 corrected the check for complex result
7 #Nasser 10/27/2017 check for leafsize and do not call ExpnType()
8 # if leaf size is "too large". Set at 500,000
9 #Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
10 # see problem 156, file Apostol_Problems
11
12 GradeAntiderivative := proc(result,optimal)
13 local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
14     debug:=false;
15
16     leaf_count_result:=leafcount(result);
17     #do NOT call ExpnType() if leaf size is too large. Recursion problem
18     if leaf_count_result > 500000 then
19         return "B";
20     fi;
21
22     leaf_count_optimal:=leafcount(optimal);
23
24     ExpnType_result:=ExpnType(result);
25     ExpnType_optimal:=ExpnType(optimal);
26
27     if debug then
28         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
29             ExpnType_optimal);
30     fi;
31
32 # If result and optimal are mathematical expressions,
33 # GradeAntiderivative[result,optimal] returns
34 # "F" if the result fails to integrate an expression that
35 # is integrable
36 # "C" if result involves higher level functions than necessary
37 # "B" if result is more than twice the size of the optimal
38 # antiderivative
39 # "A" if result can be considered optimal
40
41 #This check below actually is not needed, since I only
42 #call this grading only for passed integrals. i.e. I check
43 #for "F" before calling this. But no harm of keeping it here.
44 #just in case.
45
46 if not type(result,freeof('int')) then
47     return "F";
48 end if;
49
50 if ExpnType_result<=ExpnType_optimal then
51     if debug then
52         print("ExpnType_result<=ExpnType_optimal");
53     fi;
54     if is_contains_complex(result) then
55         if is_contains_complex(optimal) then
56             if debug then

```

```

57         print("both result and optimal complex");
58         fi;
59         #both result and optimal complex
60         if leaf_count_result<=2*leaf_count_optimal then
61             return "A";
62         else
63             return "B";
64         end if
65     else #result contains complex but optimal is not
66         if debug then
67             print("result contains complex but optimal is not");
68         fi;
69         return "C";
70     end if
71 else # result do not contain complex
72     # this assumes optimal do not as well
73     if debug then
74         print("result do not contain complex, this assumes optimal do
not as well");
75     fi;
76     if leaf_count_result<=2*leaf_count_optimal then
77         if debug then
78             print("leaf_count_result<=2*leaf_count_optimal");
79         fi;
80         return "A";
81     else
82         if debug then
83             print("leaf_count_result>2*leaf_count_optimal");
84         fi;
85         return "B";
86     end if
87 end if
88 else #ExpnType(result) > ExpnType(optimal)
89     if debug then
90         print("ExpnType(result) > ExpnType(optimal)");
91     fi;
92     return "C";
93 end if
94
95 end proc:
96
97 #
98 # is_contains_complex(result)
99 # takes expressions and returns true if it contains "I" else false
100 #
101 #Nasser 032417
102 is_contains_complex:= proc(expression)
103     return (has(expression,I));
104 end proc:
105
106 # The following summarizes the type number assigned an expression
107 # based on the functions it involves
108 # 1 = rational function
109 # 2 = algebraic function
110 # 3 = elementary function
111 # 4 = special function
112 # 5 = hyperpergeometric function
113 # 6 = appell function
114 # 7 = rootsum function
115 # 8 = integrate function
116 # 9 = unknown function
117
118 ExpnType := proc(expn)

```

```

119   if type(expn,'atomic') then
120     1
121   elif type(expn,'list') then
122     apply(max,map(ExpnType,expn))
123   elif type(expn,'sqrt') then
124     if type(op(1,expn),'rational') then
125       1
126     else
127       max(2,ExpnType(op(1,expn)))
128     end if
129   elif type(expn,'`^`') then
130     if type(op(2,expn),'integer') then
131       ExpnType(op(1,expn))
132     elif type(op(2,expn),'rational') then
133       if type(op(1,expn),'rational') then
134         1
135       else
136         max(2,ExpnType(op(1,expn)))
137       end if
138     else
139       max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
140     end if
141   elif type(expn,'`+`') or type(expn,'`*`') then
142     max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
143   elif ElementaryFunctionQ(op(0,expn)) then
144     max(3,ExpnType(op(1,expn)))
145   elif SpecialFunctionQ(op(0,expn)) then
146     max(4,apply(max,map(ExpnType,[op(expn)])))
147   elif HypergeometricFunctionQ(op(0,expn)) then
148     max(5,apply(max,map(ExpnType,[op(expn)])))
149   elif AppellFunctionQ(op(0,expn)) then
150     max(6,apply(max,map(ExpnType,[op(expn)])))
151   elif op(0,expn)='int' then
152     max(8,apply(max,map(ExpnType,[op(expn)]))) else
153     9
154   end if
155 end proc:
156
157
158 ElementaryFunctionQ := proc(func)
159   member(func,[
160     exp,log,ln,
161     sin,cos,tan,cot,sec,csc,
162     arcsin,arccos,arctan,arccot,arcsec,arccsc,
163     sinh,cosh,tanh,coth,sech,csch,
164     arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
165 end proc:
166
167 SpecialFunctionQ := proc(func)
168   member(func,[
169     erf,erfc,erfi,
170     FresnelS,FresnelC,
171     Ei,Ei,Li,Si,Ci,Shi,Chi,
172     GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
173     EllipticF,EllipticE,EllipticPi])
174 end proc:
175
176 HypergeometricFunctionQ := proc(func)
177   member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
178 end proc:
179
180 AppellFunctionQ := proc(func)
181   member(func,[AppellF1])

```

```

182 end proc:
183
184 # u is a sum or product. rest(u) returns all but the
185 # first term or factor of u.
186 rest := proc(u) local v;
187     if nops(u)=2 then
188         op(2,u)
189     else
190         apply(op(0,u),op(2..nops(u),u))
191     end if
192 end proc:
193
194 #leafcount(u) returns the number of nodes in u.
195 #Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
196 leafcount := proc(u)
197     MmaTranslator[Mma][LeafCount](u);
198 end proc:

```

4.0.3 Sympy grading function

```

1 #Dec 24, 2019. Nasser M. Abbasi:
2 #           Port of original Maple grading function by
3 #           Albert Rich to use with Sympy/Python
4 #Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
5 #           added 'exp_polar'
6 from sympy import *
7
8 def leaf_count(expr):
9     #sympy do not have leaf count function. This is approximation
10    return round(1.7*count_ops(expr))
11
12 def is_sqrt(expr):
13     if isinstance(expr,Pow):
14         if expr.args[1] == Rational(1,2):
15             return True
16         else:
17             return False
18     else:
19         return False
20
21 def is_elementary_function(func):
22     return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
23                    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
24                    asinh,acosh,atanh,acoth,asech,acsch
25                    ]
26
27 def is_special_function(func):
28     return func in [ erf,erfc,erfi,
29                    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
30                    gamma,loggamma,digamma,zeta,polylog,LambertW,
31                    elliptic_f,elliptic_e,elliptic_pi,exp_polar
32                    ]
33
34 def is_hypergeometric_function(func):
35     return func in [hyper]
36
37 def is_appell_function(func):
38     return func in [appellf1]
39
40 def is_atom(expn):
41     try:
42         if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
43             return True

```

```

44     else:
45         return False
46
47     except AttributeError as error:
48         return False
49
50 def expnType(expn):
51     debug=False
52     if debug:
53         print("expn=",expn,"type(expn)=",type(expn))
54
55     if is_atom(expn):
56         return 1
57     elif isinstance(expn,list):
58         return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
59     elif is_sqrt(expn):
60         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
61             return 1
62         else:
63             return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
64     elif isinstance(expn,Pow): #type(expn,``^`)
65         if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
66             return expnType(expn.args[0]) #ExpnType(op(1,expn))
67         elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
68             if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
69                 return 1
70             else:
71                 return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn
72 )))
73     else:
74         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
75 ExpnType(op(1,expn)),ExpnType(op(2,expn)))
76     elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,``+`) or
77 type(expn,``*`)
78     m1 = expnType(expn.args[0])
79     m2 = expnType(list(expn.args[1:]))
80     return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
81     elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
82     return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
83     elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
84     m1 = max(map(expnType, list(expn.args)))
85     return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
86     elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
87 expn))
88     m1 = max(map(expnType, list(expn.args)))
89     return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
90     elif is_appell_function(expn.func):
91     m1 = max(map(expnType, list(expn.args)))
92     return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
93     elif isinstance(expn,RootSum):
94     m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
95 ,Apply[List,expn]],7]],
96     return max(7,m1)
97     elif str(expn).find("Integral") != -1:
98     m1 = max(map(expnType, list(expn.args)))
99     return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
100     else:
101     return 9
102
103 #main function
104 def grade_antiderivative(result,optimal):
105
106     leaf_count_result = leaf_count(result)

```

```

102 leaf_count_optimal = leaf_count(optimal)
103
104 expnType_result = expnType(result)
105 expnType_optimal = expnType(optimal)
106
107 if str(result).find("Integral") != -1:
108     return "F"
109
110 if expnType_result <= expnType_optimal:
111     if result.has(I):
112         if optimal.has(I): #both result and optimal complex
113             if leaf_count_result <= 2*leaf_count_optimal:
114                 return "A"
115             else:
116                 return "B"
117         else: #result contains complex but optimal is not
118             return "C"
119     else: # result do not contain complex, this assumes optimal do not as
well
120         if leaf_count_result <= 2*leaf_count_optimal:
121             return "A"
122         else:
123             return "B"
124 else:
125     return "C"

```

4.0.4 SageMath grading function

```

1 #Dec 24, 2019. Nasser: Ported original Maple grading function by
2 #     Albert Rich to use with Sagemath. This is used to
3 #     grade Fracas, Giac and Maxima results.
4 #Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
5 #     'arctan2','floor','abs','log_integral'
6
7 from sage.all import *
8 from sage.symbolic.operators import add_vararg, mul_vararg
9
10 def tree(expr):
11     debug=False;
12     if debug:
13         print ("Enter tree(expr), expr=",expr)
14         print ("expr.operator()=",expr.operator())
15         print ("expr.operands()=",expr.operands())
16         print ("map(tree, expr.operands()=",map(tree, expr.operands()))
17
18     if expr.operator() is None:
19         return expr
20     else:
21         return [expr.operator()+list(map(tree, expr.operands()))
22
23 def leaf_count(anti):
24     debug=False;
25
26     if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))
27
28     if len(anti) == 0: #special check for optimal being 0 for some test cases.
29         if debug: print ("len(anti) == 0")
30         return 1
31     else:
32         if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
(flatten(tree(anti))))
33         return round(1.35*len(flatten(tree(anti)))) #fudge factor
34             #since this estimate of leaf count is bit lower than

```

```

35         #what it should be compared to Mathematica's
36
37 def is_sqrt(expr):
38     debug=False;
39     if expr.operator() == operator.pow: #isinstance(expr,Pow):
40         if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
41             if debug: print ("expr is sqrt")
42             return True
43         else:
44             return False
45     else:
46         return False
47
48 def is_elementary_function(func):
49     debug = False
50
51     m = func.name() in ['exp','log','ln',
52         'sin','cos','tan','cot','sec','csc',
53         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
54         'sinh','cosh','tanh','coth','sech','csch',
55         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
56         'arctan2','floor','abs'
57     ]
58     if debug:
59         if m:
60             print ("func ", func , " is elementary_function")
61         else:
62             print ("func ", func , " is NOT elementary_function")
63
64
65     return m
66
67 def is_special_function(func):
68     debug = False
69
70     if debug: print ("type(func)=", type(func))
71
72     m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
73         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
74     sinh_integral'
75         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
76         'polylog','lambert_w','elliptic_f','elliptic_e',
77         'elliptic_pi','exp_integral_e','log_integral']
78
79     if debug:
80         print ("m=",m)
81         if m:
82             print ("func ", func , " is special_function")
83         else:
84             print ("func ", func , " is NOT special_function")
85
86     return m
87
88
89 def is_hypergeometric_function(func):
90     return func.name() in ['hypergeometric','hypergeometric_M','
91     hypergeometric_U']
92
93 def is_appell_function(func):
94     return func.name() in ['hypergeometric'] #[appellf1] can't find this in
95     sagemath

```

```

95 def is_atom(expn):
96
97     #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
sagemath-equivalent-to-atomic-type-in-maple/
98     try:
99         if expn.parent() is SR:
100             return expn.operator() is None
101         if expn.parent() in (ZZ, QQ, AA, QQbar):
102             return expn in expn.parent() # Should always return True
103         if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
:
104             return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
105         return False
106
107     except AttributeError as error:
108         return False
109
110
111 def expnType(expn):
112     debug=False
113
114     if debug:
115         print(">>>>Enter expnType, expn=", expn)
116         print(">>>>is_atom(expn)=", is_atom(expn))
117
118     if is_atom(expn):
119         return 1
120     elif type(expn)==list: #isinstance(expn,list):
121         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
122     elif is_sqrt(expn):
123         if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
Rational):
124             return 1
125         else:
126             return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
127     elif expn.operator() == operator.pow: #isinstance(expn,Pow)
128         if type(expn.operands()[1])==Integer: #isinstance(expn.args[1],Integer
)
129             return expnType(expn.operands()[0]) #expnType(expn.args[0])
130         elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],
Rational)
131             if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
Rational)
132                 return 1
133             else:
134                 return max(2,expnType(expn.operands()[0])) #max(2,expnType(
expn.args[0]))
135         else:
136             return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
137     elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
138         m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
139         m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
140         return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn)))
141     elif is_elementary_function(expn.operator()): #is_elementary_function(expn
.func)
142         return max(3,expnType(expn.operands()[0]))
143     elif is_special_function(expn.operator()): #is_special_function(expn.func)
144         m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))

```



```

145     return max(4,m1)    #max(4,m1)
146     elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
147         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
148         return max(5,m1)    #max(5,m1)
149     elif is_appell_function(expn.operator()):
150         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
151         return max(6,m1)    #max(6,m1)
152     elif str(expn).find("Integral") != -1: #this will never happen, since it
153         #is checked before calling the grading function that is passed.
154         #but kept it here.
155         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
156         return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
157     else:
158         return 9
159
160 #main function
161 def grade_antiderivative(result,optimal):
162     debug = False;
163
164     if debug: print ("Enter grade_antiderivative for sagemath")
165
166     leaf_count_result = leaf_count(result)
167     leaf_count_optimal = leaf_count(optimal)
168
169     if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)
170
171
172     expnType_result = expnType(result)
173     expnType_optimal = expnType(optimal)
174
175     if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)
176
177     if expnType_result <= expnType_optimal:
178         if result.has(I):
179             if optimal.has(I): #both result and optimal complex
180                 if leaf_count_result <= 2*leaf_count_optimal:
181                     return "A"
182                 else:
183                     return "B"
184             else: #result contains complex but optimal is not
185                 return "C"
186         else: # result do not contain complex, this assumes optimal do not as
well
187             if leaf_count_result <= 2*leaf_count_optimal:
188                 return "A"
189             else:
190                 return "B"
191     else:
192         return "C"

```